Series : WXYZ/S

SET~1

प्रश्न-पत्र कोड
Q.P. Code **65/S/1**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित
MATHEMATICS

निर्धारित समय : ***3*** *घण्टे*
Time allowed : ***3*** *hours*

अधिकतम अंक : ***80***
Maximum Marks : ***80***

नोट

(I) कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं।

(II) प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।

(III) कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं।

(IV) **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में यथा स्थान पर प्रश्न का क्रमांक अवश्य लिखें।**

(V) इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।

NOTE

(I) Please check that this question paper contains **23** printed pages.

(II) Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(III) Please check that this question paper contains **38** questions.

(IV) **Please write down the Serial Number of the question in the answer-book at the given place before attempting it.**

(V) 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period. #

सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख़्ती से पालन कीजिए :

(i) *इस प्रश्न-पत्र में **38** प्रश्न हैं। **सभी** प्रश्न **अनिवार्य** हैं।*

(ii) *यह प्रश्न-पत्र **पाँच** खण्डों में विभाजित है – **क, ख, ग, घ** एवं **ङ**।*

(iii) ***खण्ड क** में प्रश्न संख्या **1** से **18** तक बहुविकल्पीय (MCQ) तथा प्रश्न संख्या **19** एवं **20** अभिकथन एवं तर्क आधारित **1** अंक के प्रश्न हैं।*

(iv) ***खण्ड ख** में प्रश्न संख्या **21** से **25** तक अति लघु-उत्तरीय (VSA) प्रकार के **2** अंकों के प्रश्न हैं।*

(v) ***खण्ड ग** में प्रश्न संख्या **26** से **31** तक लघु-उत्तरीय (SA) प्रकार के **3** अंकों के प्रश्न हैं।*

(vi) ***खण्ड घ** में प्रश्न संख्या **32** से **35** तक दीर्घ-उत्तरीय (LA) प्रकार के **5** अंकों के प्रश्न हैं।*

(vii) ***खण्ड ङ** में प्रश्न संख्या **36** से **38** तक प्रकरण अध्ययन आधारित **4** अंकों के प्रश्न हैं।*

(viii) *प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड ख के 2 प्रश्नों में, खण्ड ग के 3 प्रश्नों में, खण्ड घ के 2 प्रश्नों में तथा खण्ड ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है।*

(ix) *कैल्कुलेटर का उपयोग **वर्जित** है।*

खण्ड क

इस खण्ड में बहुविकल्पीय प्रश्न (MCQ) हैं, जिनमें प्रत्येक प्रश्न 1 अंक का है।

1. यदि आव्यूह $A = [a_{ij}]_{2\times 2}$ में $a_{ij} = \begin{cases} 1, \text{यदि } i \neq j \\ 0, \text{यदि } i = j \end{cases}$ है, तो $A + A^2$ बराबर है :

(A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A**, **B**, **C**, **D** and **E**.*

(iii) *In **Section A**, Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B**, Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C**, Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D**, Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E**, Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculator is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If the matrix $A = [a_{ij}]_{2\times 2}$ is such that $a_{ij} = \begin{cases} 1, & \text{if } i \neq j \\ 0, & \text{if } i = j \end{cases}$, then $A + A^2$ is equal to :

(A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

2. $f(x) = \cos^{-1}(2x)$ का प्रान्त है :

(A) $[-1, 1]$ (B) $\left[0, \frac{1}{2}\right]$

(C) $[-2, 2]$ (D) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

3. सारणिक $\begin{vmatrix} \cos 75^\circ & \sin 75^\circ \\ \sin 15^\circ & \cos 15^\circ \end{vmatrix}$ का मान है :

(A) 1 (B) शून्य

(C) $\frac{1}{2}$ (D) $\frac{\sqrt{3}}{2}$

4. एक व्युत्क्रमणीय आव्यूह X के लिए, यदि $X^2 = I$ है, तो X^{-1} बराबर है :

(A) X (B) X^2

(C) I (D) O

5. सारणिक $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix}$ में अवयव a_{32} का सहखंड है :

(A) ± 5 (B) -5

(C) 5 (D) 0

6. यदि A कोटि n का एक तत्समक आव्यूह है, तो A (Adj A) है एक :

(A) तत्समक आव्यूह

(B) पंक्ति आव्यूह

(C) शून्य आव्यूह

(D) विषम सममित आव्यूह

2. The domain of $f(x) = \cos^{-1}(2x)$ is :

(A) $[-1, 1]$ (B) $\left[0, \frac{1}{2}\right]$

(C) $[-2, 2]$ (D) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

3. The value of the determinant $\begin{vmatrix} \cos 75^o & \sin 75^o \\ \sin 15^o & \cos 15^o \end{vmatrix}$ is :

(A) 1 (B) zero

(C) $\frac{1}{2}$ (D) $\frac{\sqrt{3}}{2}$

4. For a non-singular matrix X, if $X^2 = I$, then X^{-1} is equal to :

(A) X (B) X^2

(C) I (D) O

5. The cofactor of the element a_{32} in the determinant $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix}$ is :

(A) ± 5 (B) -5

(C) 5 (D) 0

6. If A is an identity matrix of order n, then A (Adj A) is a/an :

(A) identity matrix

(B) row matrix

(C) zero matrix

(D) skew symmetric matrix

7. यदि $x = t^3$ तथा $y = t^2$ है, तो $t = 1$ पर $\frac{d^2y}{dx^2}$ का मान है :

(A) $\frac{3}{2}$ (B) $-\frac{2}{9}$

(C) $-\frac{3}{2}$ (D) $-\frac{2}{3}$

8. परवलय $x^2 = y$ तथा रेखा $y = 1$ द्वारा परिबद्ध क्षेत्र का क्षेत्रफल है :

(A) $\frac{2}{3}$ वर्ग इकाई (B) $\frac{1}{3}$ वर्ग इकाई

(C) $\frac{4}{3}$ वर्ग इकाई (D) 2 वर्ग इकाई

9. यदि एक गोले के आयतन के परिवर्तन की दर उसकी त्रिज्या के परिवर्तन की दर से दुगुनी है, तो गोले का पृष्ठीय क्षेत्रफल है :

(A) 1 वर्ग इकाई

(B) 2 वर्ग इकाई

(C) 3 वर्ग इकाई

(D) 4 वर्ग इकाई

10. $\int \frac{3\cos\sqrt{x}}{\sqrt{x}}\,dx$ बराबर है :

(A) $-6\sin\sqrt{x} + C$

(B) $-6\cos\sqrt{x} + C$

(C) $6\cos\sqrt{x} + C$

(D) $6\sin\sqrt{x} + C$

7. If $x = t^3$ and $y = t^2$, then $\frac{d^2y}{dx^2}$ at $t = 1$ is :

(A) $\frac{3}{2}$ (B) $-\frac{2}{9}$

(C) $-\frac{3}{2}$ (D) $-\frac{2}{3}$

8. The area bounded by the parabola $x^2 = y$ and the line $y = 1$ is :

(A) $\frac{2}{3}$ sq unit (B) $\frac{1}{3}$ sq unit

(C) $\frac{4}{3}$ sq units (D) 2 sq units

9. If the rate of change of volume of a sphere is twice the rate of change of its radius, then the surface area of the sphere is :

(A) 1 sq unit

(B) 2 sq units

(C) 3 sq units

(D) 4 sq units

10. $\int \frac{3\cos\sqrt{x}}{\sqrt{x}}\,dx$ is equal to :

(A) $-6\sin\sqrt{x} + C$

(B) $-6\cos\sqrt{x} + C$

(C) $6\cos\sqrt{x} + C$

(D) $6\sin\sqrt{x} + C$

11. यदि $\frac{d}{dx} f(x) = 3x^2 - \frac{3}{x^4}$ इस प्रकार है कि $f(1) = 0$ है, तो $f(x)$ है :

(A) $6x + \frac{12}{x^5}$

(B) $x^4 - \frac{1}{x^3} + 2$

(C) $x^3 + \frac{1}{x^3} - 2$

(D) $x^3 + \frac{1}{x^3} + 2$

12. एक LPP में, रैखिक निकाय व्यवरोधों द्वारा बने सुसंगत क्षेत्र के कोनीय बिंदु (1, 1), (3, 0) तथा (0, 3) हैं। यदि $Z = ax + by$, जहाँ $a, b > 0$ का न्यूनतमीकरण करना हो और Z का न्यूनतम मान (3, 0) तथा (1, 1) पर हो, तो a तथा b के बीच का संबंध होगा :

(A) $a = 2b$

(B) $a = \frac{b}{2}$

(C) $a = 3b$

(D) $a = b$

13. व्यवरोधों $x + y \leq 1,\ x,\ y \geq 0$ के अंतर्गत $Z = 3x + 4y$ का अधिकतम मान है :

(A) 3 (B) 4

(C) 7 (D) 0

14. अवकल समीकरण $\frac{dy}{dx} = 2x \cdot e^{x^2 + y}$ का व्यापक हल है :

(A) $e^{x^2 + y} = C$ (B) $e^{x^2} + e^{-y} = C$

(C) $e^{x^2} = e^y + C$ (D) $e^{x^2 - y} = C$

11. If $\frac{d}{dx} f(x) = 3x^2 - \frac{3}{x^4}$ such that $f(1) = 0$, then $f(x)$ is :

(A) $6x + \frac{12}{x^5}$

(B) $x^4 - \frac{1}{x^3} + 2$

(C) $x^3 + \frac{1}{x^3} - 2$

(D) $x^3 + \frac{1}{x^3} + 2$

12. In an LPP, corner points of the feasible region determined by the system of linear constraints are (1, 1), (3, 0) and (0, 3). If $Z = ax + by$, where $a, b > 0$ is to be minimized, the condition on a and b, so that the minimum of Z occurs at (3, 0) and (1, 1), will be :

(A) $a = 2b$

(B) $a = \frac{b}{2}$

(C) $a = 3b$

(D) $a = b$

13. The maximum value of $Z = 3x + 4y$ subject to the constraints $x + y \leq 1$, $x, y \geq 0$ is :

(A) 3 (B) 4

(C) 7 (D) 0

14. The general solution of the differential equation $\frac{dy}{dx} = 2x \cdot e^{x^2 + y}$ is :

(A) $e^{x^2 + y} = C$ (B) $e^{x^2} + e^{-y} = C$

(C) $e^{x^2} = e^y + C$ (D) $e^{x^2 - y} = C$

15. यदि 'm' तथा 'n' क्रमश: अवकल समीकरण $1 + \left(\frac{dy}{dx}\right)^3 = \frac{d^2y}{dx^2}$ की घात तथा कोटि हैं, तो $(m + n)$ का मान है :

(A) 4

(B) 3

(C) 2

(D) 5

16. यदि $|\vec{a}| = 1$, $|\vec{b}| = 2$ तथा $\vec{a} \cdot \vec{b} = 2$ है, तो $|\vec{a} + \vec{b}|$ का मान है :

(A) 9

(B) 3

(C) -3

(D) 2

17. दो सदिश $\vec{a}$ तथा $\vec{b}$ इस प्रकार हैं कि $|\vec{a} \times \vec{b}| = \vec{a} \cdot \vec{b}$ है। दोनों सदिशों के बीच का कोण है :

(A) 30° (B) 60°

(C) 45° (D) 90°

18. एक सिक्का 3 बार उछाला गया। कम-से-कम दो बार चित आने की प्रायिकता है :

(A) $\frac{1}{2}$

(B) $\frac{3}{8}$

(C) $\frac{1}{8}$

(D) $\frac{1}{4}$

15. If 'm' and 'n' are the degree and order respectively of the differential equation $1 + \left(\frac{dy}{dx}\right)^3 = \frac{d^2y}{dx^2}$, then the value of (m + n) is :

(A) 4

(B) 3

(C) 2

(D) 5

16. If $|\vec{a}| = 1$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 2$, then the value of $|\vec{a} + \vec{b}|$ is :

(A) 9

(B) 3

(C) – 3

(D) 2

17. Two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a} \times \vec{b}| = \vec{a} \cdot \vec{b}$. The angle between the two vectors is :

(A) 30° (B) 60°

(C) 45° (D) 90°

18. A coin is tossed three times. The probability of getting at least two heads is :

(A) $\frac{1}{2}$

(B) $\frac{3}{8}$

(C) $\frac{1}{8}$

(D) $\frac{1}{4}$

*प्रश्न संख्या **19** और **20** अभिकथन एवं तर्क आधारित प्रश्न हैं। दो कथन दिए गए हैं जिनमें एक को अभिकथन (A) तथा दूसरे को तर्क (R) द्वारा अंकित किया गया है। इन प्रश्नों के सही उत्तर नीचे दिए गए कोड (A), (B), (C) और (D) में से चुनकर दीजिए।*

(A) अभिकथन (A) और तर्क (R) दोनों सही हैं और तर्क (R), अभिकथन (A) की सही व्याख्या करता है।

(B) अभिकथन (A) और तर्क (R) दोनों सही हैं, परन्तु तर्क (R), अभिकथन (A) की सही व्याख्या ***नहीं*** करता है।

(C) अभिकथन (A) सही है, परन्तु तर्क (R) ग़लत है।

(D) अभिकथन (A) ग़लत है, परन्तु तर्क (R) सही है।

19. फलन $f : R \longrightarrow R$ पर विचार कीजिए, जिसे $f(x) = x^3$ के रूप में परिभाषित किया गया है।

अभिकथन (A) : $f(x)$ एक एकैकी फलन है।

तर्क (R) : यदि फलन का सहप्रान्त इसके परिसर के समान हो, तो $f(x)$ एकैकी फलन होता है।

20. *अभिकथन (A) :* महत्तम पूर्णांक फलन $f(x) = [x]$, $x \in \mathbb{R}$ में, $x = 2$ पर अवकलनीय नहीं है।

तर्क (R) : महत्तम पूर्णांक फलन किसी भी पूर्णांकीय मान पर संतत नहीं होता।

खण्ड ख

इस खण्ड में अति लघु-उत्तरीय (VSA) प्रकार के 5 प्रश्न हैं, जिनमें प्रत्येक के 2 अंक हैं।

21. (क) $\cos^{-1}\left(-\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$ का मुख्य मान ज्ञात कीजिए।

अथवा

(ख) सिद्ध कीजिए कि :

$$\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x \in [0, 1]$$

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. Consider the function $f : R \longrightarrow R$, defined as $f(x) = x^3$.

Assertion (A) : $f(x)$ is a one-one function.

Reason (R) : $f(x)$ is a one-one function, if co-domain = range.

20. *Assertion (A) :* $f(x) = [x]$, $x \in \mathbb{R}$, the greatest integer function is not differentiable at $x = 2$.

Reason (R) : The greatest integer function is not continuous at any integral value.

SECTION B

This section comprises 5 very short answer (VSA) type questions of 2 marks each.

21. (a) Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$.

OR

(b) Prove that :

$$\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x \in [0, 1]$$

22. यदि $e^y (x + 1) = 1$ है, तो सिद्ध कीजिए कि $\frac{dy}{dx} = -e^y$.

23. 13 m लंबी एक सीढ़ी दीवार के सहारे झुकी है। सीढ़ी का नीचे का सिरा, भूमि के अनुदिश दीवार से दूर 2 m/s की दर से खींचा जाता है। दीवार पर इसकी ऊँचाई किस दर से घट रही है, जबकि सीढ़ी के नीचे का सिरा दीवार से 12 m दूर है ?

24. (क) यदि बिंदु $(-1, -1, 2)$, $(2, 8, \lambda)$ तथा $(3, 11, 6)$ संरेख हैं, तो λ का मान ज्ञात कीजिए।

अथवा

(ख) $\vec{a}$ तथा $\vec{b}$ दो सह-प्रारंभिक सदिश (co-initial vectors) हैं जो एक समांतर चतुर्भुज की संलग्न भुजाएँ बनाते हैं और $|\vec{a}| = 10$, $|\vec{b}| = 2$ तथा $\vec{a} . \vec{b} = 12$ है। समांतर चतुर्भुज का क्षेत्रफल ज्ञात कीजिए ।

25. रेखाओं $\vec{r} = (3 + 2\lambda)\hat{i} - (2 - 2\lambda)\hat{j} + (6 + 2\lambda)\hat{k}$ तथा

$\vec{r} = (2\hat{j} - 5\hat{k}) + \mu(6\hat{i} + 3\hat{j} + 2\hat{k})$ के बीच का कोण ज्ञात कीजिए।

खण्ड ग

इस खण्ड में लघु-उत्तरीय (SA) प्रकार के 6 प्रश्न हैं, जिनमें प्रत्येक के 3 अंक हैं।

26. वक्र $y = -x^3 + 3x^2 + 9x - 30$ की प्रवणता (ढाल) का अधिकतम मान ज्ञात कीजिए।

27. (क) ज्ञात कीजिए :

$$\int \sqrt{4x^2 - 4x + 10}\ dx$$

अथवा

(ख) मान ज्ञात कीजिए :

$$\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x}\ dx$$

22. If $e^y (x + 1) = 1$, prove that $\frac{dy}{dx} = -e^y$.

23. A ladder 13 m long is leaning against the wall. The bottom of the ladder is pulled along the ground away from the wall at the rate of 2 m/s. How fast is the height on the wall decreasing when the foot of the ladder is 12 m away from the wall ?

24. (a) Find the value of λ, if the points $(-1, -1, 2)$, $(2, 8, \lambda)$ and $(3, 11, 6)$ are collinear.

OR

(b) $\vec{a}$ and $\vec{b}$ are two co-initial vectors forming the adjacent sides of a parallelogram such that $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$. Find the area of the parallelogram.

25. Find the angle between the lines

$\vec{r} = (3 + 2\lambda)\hat{i} - (2 - 2\lambda)\hat{j} + (6 + 2\lambda)\hat{k}$ and

$\vec{r} = (2\hat{j} - 5\hat{k}) + \mu(6\hat{i} + 3\hat{j} + 2\hat{k})$.

SECTION C

This section comprises 6 short answer (SA) type questions of 3 marks each.

26. Find the maximum slope of the curve $y = -x^3 + 3x^2 + 9x - 30$.

27. (a) Find :

$$\int \sqrt{4x^2 - 4x + 10}\ dx$$

OR

(b) Evaluate :

$$\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x}\ dx$$

28. निम्नलिखित रैखिक प्रोग्रामन समस्या को ग्राफ द्वारा हल कीजिए :

व्यवरोधों $x + 4y \leq 8$

$2x + 3y \leq 12$

$3x + y \leq 9$

$x \geq 0, y \geq 0$

के अंतर्गत $Z = 2x + 3y$ का अधिकतमीकरण कीजिए।

29. (क) अवकल समीकरण $(2x^2 + y)\, dx = x\, dy$ का व्यापक हल ज्ञात कीजिए।

अथवा

(ख) अवकल समीकरण $\frac{dy}{dx} - \frac{y}{x} + \text{cosec}\left(\frac{y}{x}\right) = 0$ का विशिष्ट हल ज्ञात कीजिए, यदि $x = 1$ के लिए $y = 0$ दिया गया है।

30. यदि $\hat{a}$, $\hat{b}$ तथा $\hat{c}$ मात्रक सदिश हैं तथा $\hat{a}.\hat{b} = \hat{a}.\hat{c} = 0$ है, $\hat{b}$ तथा $\hat{c}$ के बीच का कोण $\frac{\pi}{6}$ है, तो सिद्ध कीजिए कि $\hat{a} = \pm 2\,(\hat{b} \times \hat{c})$.

31. (क) कक्षा XII के चार विद्यार्थियों को एक समस्या स्वतंत्र रूप से हल करने के लिए दी गई है। उनकी समस्या को हल कर पाने की संभावनाएँ क्रमश: $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}$ तथा $\frac{1}{5}$ हैं। उनमें से अधिकतम एक द्वारा समस्या हल कर पाने की प्रायिकता ज्ञात कीजिए।

अथवा

(ख) एक यादृच्छिक चर X का प्रायिकता बंटन नीचे दिया गया है :

X	1	2	4	2k	3k	5k
P(X)	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{3}{25}$	$\frac{1}{10}$	$\frac{1}{25}$	$\frac{1}{25}$

यदि $E(X) = 2{\cdot}94$ है, तो k ज्ञात कीजिए और $P(X \leq 4)$ भी ज्ञात कीजिए।

28. Solve the following LPP graphically :

Maximize $Z = 2x + 3y$

subject to the constraints $x + 4y \leq 8$

$2x + 3y \leq 12$

$3x + y \leq 9$

$x \geq 0, y \geq 0.$

29. (a) Find the general solution of the differential equation

$(2x^2 + y)\, dx = x\, dy.$

OR

(b) For the differential equation $\frac{dy}{dx} - \frac{y}{x} + \text{cosec}\left(\frac{y}{x}\right) = 0$, find the particular solution, given that $y = 0$ when $x = 1$.

30. If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are unit vectors such that $\hat{a}.\hat{b} = \hat{a}.\hat{c} = 0$ and the angle between $\hat{b}$ and $\hat{c}$ is $\frac{\pi}{6}$, then prove that $\hat{a} = \pm 2(\hat{b} \times \hat{c})$.

31. (a) Four students of class XII are given a problem to solve independently. Their chances of solving the problem respectively are $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$ and $\frac{1}{5}$. Find the probability that at most one of them will solve the problem.

OR

(b) The probability distribution of a random variable X is given below :

X	1	2	4	2k	3k	5k
P(X)	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{3}{25}$	$\frac{1}{10}$	$\frac{1}{25}$	$\frac{1}{25}$

Find k, if $E(X) = 2{\cdot}94$ and also find $P(X \leq 4)$.

खण्ड घ

इस खण्ड में 4 दीर्घ-उत्तरीय (LA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 5 अंक हैं।

32. यदि $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$ है, तो A^{-1} ज्ञात कीजिए। A^{-1} के प्रयोग से दिए गए समीकरणों के निकाय $3x + 4y + 7z = 14;\ 2x - y + 3z = 4;\ x + 2y - 3z = 0$ को हल कीजिए।

33. (क) यदि $y = \cos x^2 + \cos^2 x + \cos^2 (x^2) + \cos (x^x)$ है, तो $\frac{dy}{dx}$ ज्ञात कीजिए।

अथवा

(ख) उन अंतरालों को ज्ञात कीजिए जिनमें दिया गया फलन :

$$f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$$

(i) निरंतर वर्धमान है।

(ii) निरंतर ह्रासमान है।

34. समाकलन के प्रयोग से क्षेत्र $\{(x, y): 0 \le y \le x^2,\ 0 \le y \le x,\ 0 \le x \le 3\}$ का क्षेत्रफल ज्ञात कीजिए।

35. (क) नीचे दी गई रेखाओं l_1 तथा l_2 में न्यूनतम दूरी ज्ञात कीजिए :

$$l_1 : \vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(4\hat{i} + 6\hat{j} + 12\hat{k})$$

और $l_2 : \vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(6\hat{i} + 9\hat{j} + 18\hat{k})$

अथवा

(ख) दर्शाइए कि रेखाएँ $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ तथा $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1}$ प्रतिच्छेदी रेखाएँ हैं। रेखाओं का प्रतिच्छेदन बिंदु भी ज्ञात कीजिए।

SECTION D

This section comprises 4 long answer (LA) type questions of 5 marks each.

32. If $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the given system of equations $3x + 4y + 7z = 14$; $2x - y + 3z = 4$; $x + 2y - 3z = 0$.

33. (a) If $y = \cos x^2 + \cos^2 x + \cos^2 (x^2) + \cos (x^x)$, find $\frac{dy}{dx}$.

OR

(b) Find the intervals in which the function given by

$f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ is :

(i) strictly increasing.

(ii) strictly decreasing.

34. Using integration, find the area of the region

$\{(x, y) : 0 \le y \le x^2, \ 0 \le y \le x, \ 0 \le x \le 3\}$.

35. (a) Find the shortest distance between the lines l_1 and l_2 given by :

$l_1 : \vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(4\hat{i} + 6\hat{j} + 12\hat{k})$

and $l_2 : \vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(6\hat{i} + 9\hat{j} + 18\hat{k})$

OR

(b) Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1}$ intersect. Also, find their point of intersection.

खण्ड ङ

इस खण्ड में 3 प्रकरण-अध्ययन आधारित प्रश्न हैं, जिनमें प्रत्येक के 4 अंक हैं।

प्रकरण अध्ययन – 1

36. एक खिड़की एक आयत के रूप में है जिसके ऊपर लंबाई पर एक समबाहु त्रिभुज अध्यारोपित है। माना आयताकार भाग की लंबाई x मीटर तथा चौड़ाई y मीटर है।

उपर्युक्त सूचना के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

(i) यदि खिड़की का परिमाप 12 m है, तो x तथा y के बीच संबंध ज्ञात कीजिए। *1*

(ii) (i) में प्राप्त व्यंजक के प्रयोग से, खिड़की के क्षेत्रफल का केवल x के फलन के रूप में व्यंजक लिखिए। *1*

(iii) (क) आयत की वह विमाएँ ज्ञात कीजिए जिनसे खिड़की से अधिक-से-अधिक प्रकाश आ सके। ((ii) में प्राप्त व्यंजक का प्रयोग कीजिए) *2*

अथवा

(iii) (ख) यदि यह दिया गया हो कि खिड़की का क्षेत्रफल 50 m^2 है, तो खिड़की के परिमाप का x के पदों में व्यंजक ज्ञात कीजिए। *2*

प्रकरण अध्ययन – 2

37. त्योहारों के मौसम में, एक सोसाइटी के आवासीय कल्याण संघ ने साथ वाले पार्क में एक मेले का आयोजन किया। मेले का मुख्य आकर्षण, पार्क के एक कोने में लगा झूला था जो झूलते समय फलन $x^2 = y$ का परवलय पथ पूरा करता था।

उपर्युक्त सूचना के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

(i) माना $f : N \to R$, $f(x) = x^2$ द्वारा परिभाषित है। इसका परिसर क्या होगा ? *1*

(ii) माना $f : N \to N$, $f(x) = x^2$ द्वारा परिभाषित है, तो जाँच कीजिए कि फलन एकैकी है या नहीं। *1*

(iii) (क) माना $f : \{1, 2, 3, 4 .....\} \longrightarrow \{1, 4, 9, 16 .....\}$ में $f(x) = x^2$ द्वारा परिभाषित है, तो सिद्ध कीजिए कि फलन एकैकी-आच्छादी है। *2*

अथवा

(iii) (ख) माना $f : \mathbb{R} \longrightarrow \mathbb{R}$ में $f(x) = x^2$ द्वारा परिभाषित है, तो दर्शाइए कि f न तो एकैकी है और न ही आच्छादी है। *2*

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. A window is in the form of a rectangle surmounted by an equilateral triangle on its length. Let the rectangular part have length and breadth x and y metres respectively.

Based on the given information, answer the following questions :

(i) If the perimeter of the window is 12 m, find the relation between x and y. *1*

(ii) Using the expression obtained in (i), write an expression for the area of the window as a function of x only. *1*

(iii) (a) Find the dimensions of the rectangle that will allow maximum light through the window. (use expression obtained in (ii)) *2*

OR

(iii) (b) If it is given that the area of the window is 50 m^2, find an expression for its perimeter in terms of x. *2*

Case Study – 2

37. During the festival season, there was a mela organized by the Resident Welfare Association at a park, near the society. The main attraction of the mela was a huge swing installed at one corner of the park. The swing is traced to follow the path of a parabola given by $x^2 = y$.

Based on the above information, answer the following questions :

(i) Let $f : N \longrightarrow R$ is defined by $f(x) = x^2$. What will be the range ? *1*

(ii) Let $f : N \longrightarrow N$ is defined by $f(x) = x^2$. Check if the function is injective or not. *1*

(iii) (a) Let $f : \{1, 2, 3, 4 .....\} \longrightarrow \{1, 4, 9, 16 .....\}$ be defined by $f(x) = x^2$. Prove that the function is bijective. *2*

OR

(iii) (b) Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ is defined by $f(x) = x^2$. Show that f is neither injective nor surjective. *2*

प्रकरण अध्ययन – 3

38. एक संगठन की प्रबंध समिति के पद के लिए दो व्यक्ति प्रतिस्पर्धा कर रहे हैं। पहले तथा दूसरे व्यक्ति के चयन की प्रायिकता क्रमश: 0·5 तथा 0·6 है। इसके अतिरिक्त, यदि पहले व्यक्ति का चयन होता है, तो इसके द्वारा अपशिष्ट उपचार संयंत्र शुरू करने की प्रायिकता 0·7 है, जबकि दूसरे व्यक्ति का चयन होता है, तो उसकी संबंधित प्रायिकता 0·4 है।

उपर्युक्त सूचना के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

(i) अपशिष्ट उपचार संयंत्र के शुरू होने की प्रायिकता क्या है ? *2*

(ii) चयन के बाद, यदि अपशिष्ट उपचार संयंत्र शुरू हो गया है तो इसकी क्या प्रायिकता है कि पहले व्यक्ति ने इसे शुरू किया है ? *2*

Case Study – 3

38. Two persons are competing for a position on the Managing Committee of an organisation. The probabilities that the first and the second person will be appointed are 0·5 and 0·6 respectively. Also, if the first person gets appointed, then the probability of introducing waste treatment plant is 0·7 and the corresponding probability is 0·4, if the second person gets appointed.

Based on the above information, answer the following questions :

(i) What is the probability that the waste treatment plant is introduced ? *2*

(ii) After the selection, if the waste treatment plant is introduced, what is the probability that the first person had introduced it ? *2*

Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Senior Secondary School Supplementary Examination, 2025
SUBJECT- MATHEMATICS (041) (Q.P. CODE – 65/S/1)

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its leakage to the public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in Newspaper/Website, etc. may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. The Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-XII, while evaluating the competency-based questions, please try to understand the given answer and even if reply is not from a marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.**
4	The Marking Scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS 'X' be marked. Evaluators will not put right (✓) while evaluating which gives the impression that the answer is correct, and no marks are awarded. **This is the most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	If a student has attempted an extra question, answer to the question deserving more marks should be retained and the other answer scored out with a note **"Extra Question"**.

10	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
11	A full scale of marks (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
12	Every examiner must necessarily do evaluation work for full working hours, i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
13	Ensure that you do not make the following common types of errors committed by the Examiner in the past: - ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of the answer marked correct and the rest as wrong, but no marks
14	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
15	Any unassessed portion, non-carrying over of marks to the title page, or total error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
16	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for Spot Evaluation**" before starting the actual evaluation.
17	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
18	The candidates are entitled to obtain a photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

Q. No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION - A Questions no. 1 to 18 are multiple choice questions (MCQs) of **1 mark each**.	
Q1.	If the matrix $A = [a_{ij}]_{2\times 2}$ is such that $a_{ij} = \begin{cases} 1, & \text{if } i \neq j \\ 0, & \text{if } i = j \end{cases}$, then $A + A^2$ is equal to : (A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	
Ans	(C) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$	1
Q2.	The domain of $f(x) = \cos^{-1}(2x)$ is : (A) $[-1, 1]$ (B) $\left[0, \frac{1}{2}\right]$ (C) $[-2, 2]$ (D) $\left[-\frac{1}{2}, \frac{1}{2}\right]$	
Ans	(D) $\left[-\frac{1}{2}, \frac{1}{2}\right]$	1
Q3.	The value of the determinant $\begin{vmatrix} \cos 75^\circ & \sin 75^\circ \\ \sin 15^\circ & \cos 15^\circ \end{vmatrix}$ is : (A) 1 (B) zero (C) $\frac{1}{2}$ (D) $\frac{\sqrt{3}}{2}$	
Ans	(B) zero	1

Q4.	For a non-singular matrix X, if $X^2 = I$, then X^{-1} is equal to : (A) X (B) X^2 (C) I (D) O	
Ans	(A) X	1
Q5.	The cofactor of the element a_{32} in the determinant $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix}$ is : (A) ± 5 (B) -5 (C) 5 (D) 0	
Ans	(C) 5	1
Q6.	If A is an identity matrix of order n, then A (Adj A) is a/an : (A) identity matrix (B) row matrix (C) zero matrix (D) skew symmetric matrix	
Ans	(A) identity matrix	1
Q7.	If $x = t^3$ and $y = t^2$, then $\frac{d^2y}{dx^2}$ at $t = 1$ is : (A) $\frac{3}{2}$ (B) $-\frac{2}{9}$ (C) $-\frac{3}{2}$ (D) $-\frac{2}{3}$	
Ans	(B) $-\frac{2}{9}$	1

Q8.	The area bounded by the parabola $x^2 = y$ and the line $y = 1$ is : (A) $\frac{2}{3}$ sq unit (B) $\frac{1}{3}$ sq unit (C) $\frac{4}{3}$ sq units (D) 2 sq units	
Ans	(C) $\frac{4}{3}$ sq units	1
Q9.	If the rate of change of volume of a sphere is twice the rate of change of its radius, then the surface area of the sphere is : (A) 1 sq unit (B) 2 sq units (C) 3 sq units (D) 4 sq units	
Ans	(B) 2 sq units	1
Q10.	$\int \frac{3\cos\sqrt{x}}{\sqrt{x}}\,dx$ is equal to : (A) $-6\sin\sqrt{x} + C$ (B) $-6\cos\sqrt{x} + C$ (C) $6\cos\sqrt{x} + C$ (D) $6\sin\sqrt{x} + C$	
Ans	(D) $6\sin\sqrt{x} + C$	1

Q11.	If $\frac{d}{dx}f(x) = 3x^2 - \frac{3}{x^4}$ such that $f(1) = 0$, then $f(x)$ is : (A) $6x + \frac{12}{x^5}$ (B) $x^4 - \frac{1}{x^3} + 2$ (C) $x^3 + \frac{1}{x^3} - 2$ (D) $x^3 + \frac{1}{x^3} + 2$	
Ans	(C) $x^3 + \frac{1}{x^3} - 2$	1
Q12.	In an LPP, corner points of the feasible region determined by the system of linear constraints are (1, 1), (3, 0) and (0, 3). If $Z = ax + by$, where $a, b > 0$ is to be minimized, the condition on a and b, so that the minimum of Z occurs at (3, 0) and (1, 1), will be : (A) $a = 2b$ (B) $a = \frac{b}{2}$ (C) $a = 3b$ (D) $a = b$	
Ans	(B) $a = \frac{b}{2}$	1
Q13.	The maximum value of $Z = 3x + 4y$ subject to the constraints $x + y \leq 1$, $x, y \geq 0$ is : (A) 3 (B) 4 (C) 7 (D) 0	
Ans	(B) 4	1

Q14.	The general solution of the differential equation $\frac{dy}{dx} = 2x \cdot e^{x^2+y}$ is : (A) $e^{x^2+y} = C$ (B) $e^{x^2} + e^{-y} = C$ (C) $e^{x^2} = e^{y} + C$ (D) $e^{x^2-y} = C$	
Ans	(B) $e^{x^2} + e^{-y} = C$	1
Q15.	If 'm' and 'n' are the degree and order respectively of the differential equation $1 + \left(\frac{dy}{dx}\right)^3 = \frac{d^2y}{dx^2}$, then the value of (m + n) is : (A) 4 (B) 3 (C) 2 (D) 5	
Ans	(B) 3	1
Q16.	If $\lvert \vec{a} \rvert = 1$, $\lvert \vec{b} \rvert = 2$ and $\vec{a} \cdot \vec{b} = 2$, then the value of $\lvert \vec{a} + \vec{b} \rvert$ is : (A) 9 (B) 3 (C) – 3 (D) 2	
Ans	(B) 3	1
Q17.	Two vectors $\vec{a}$ and $\vec{b}$ are such that $\lvert \vec{a} \times \vec{b} \rvert = \vec{a} \cdot \vec{b}$. The angle between the two vectors is : (A) 30° (B) 60° (C) 45° (D) 90°	
Ans	(C) 45°	1

Q18.	A coin is tossed three times. The probability of getting at least two heads is : (A) $\frac{1}{2}$ (B) $\frac{3}{8}$ (C) $\frac{1}{8}$ (D) $\frac{1}{4}$	
Ans	(A) $\frac{1}{2}$	1

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Q19.	Consider the function f : R $\longrightarrow$ R, defined as $f(x) = x^3$. *Assertion (A) :* f(x) is a one-one function. *Reason (R) :* f(x) is a one-one function, if co-domain = range.	
Ans	(C) Assertion (A) is true, but Reason (R) is false.	1
Q20.	*Assertion (A) :* f(x) = [x], $x \in \mathbb{R}$, the greatest integer function is not differentiable at x = 2. *Reason (R) :* The greatest integer function is not continuous at any integral value.	
Ans	(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).	1

SECTION B

This section comprises very short answer (VSA) type questions of **2 marks each**.

Q21.

(a) Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$.

OR

(b) Prove that :

$$\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x \in [0, 1]$$

Ans(a)

$\cos^{-1}\left(-\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$ — 1 + ½

$= \left(\pi - \frac{\pi}{3}\right) + 2\left(\frac{\pi}{6}\right)$

$= \pi$ — ½

OR

Ans(b)

Put $x = \tan^2\theta \Rightarrow \theta = \tan^{-1}\sqrt{x}$ — ½

RHS $= \frac{1}{2}\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$= \frac{1}{2}\cos^{-1}(\cos 2\theta)$ — 1

$= \frac{1}{2}(2\theta)$

$= \theta = \tan^{-1}\sqrt{x}$ = LHS — ½

Q22.

If $e^y(x + 1) = 1$, prove that $\frac{dy}{dx} = -e^y$.

Ans

$e^y(x+1) = 1 \Rightarrow e^y = \frac{1}{x+1}$

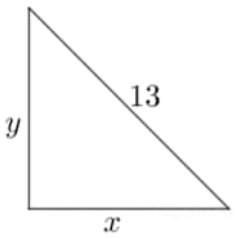
$\Rightarrow y = -\log(x+1)$ — ½

$\Rightarrow \frac{dy}{dx} = -\frac{1}{x+1}$ — 1

$= -e^y \quad \left[\because \frac{1}{x+1} = e^y\right]$ — ½

Q23.

A ladder 13 m long is leaning against the wall. The bottom of the ladder is pulled along the ground away from the wall at the rate of 2 m/s. How fast is the height on the wall decreasing when the foot of the ladder is 12 m away from the wall ?

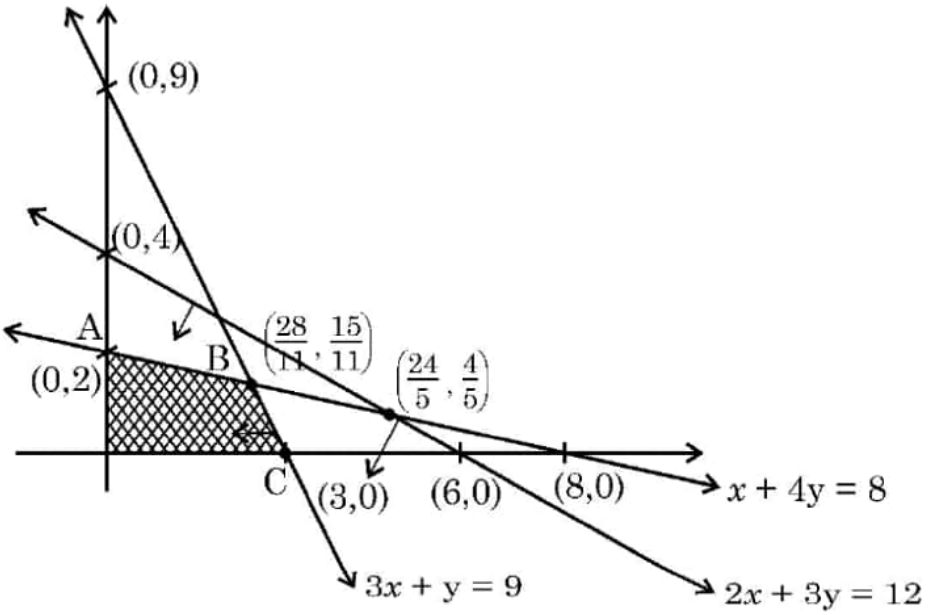
Ans	$x^2 + y^2 = 169$ Differentiate both sides w.r.t. t $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$ $\Rightarrow 12(2) + 5\left(\frac{dy}{dt}\right) = 0[\because \text{when } x = 12m, y = 5m]$ $\Rightarrow \frac{dy}{dt} = -\frac{24}{5}$ Hence, the height decreases at the rate of $\frac{24}{5}$m/s 	½ 1 ½
Q24.	(a) Find the value of λ, if the points (–1, –1, 2), (2, 8, λ) and (3, 11, 6) are collinear. **OR** (b) $\vec{a}$ and $\vec{b}$ are two co-initial vectors forming the adjacent sides of a parallelogram such that $\lvert\vec{a}\rvert = 10$, $\lvert\vec{b}\rvert = 2$ and $\vec{a}.\vec{b} = 12$. Find the area of the parallelogram.	
Ans(a)	$A(-1,-1,2), B(2,8,\lambda), C(3,11,6)$ $\overrightarrow{AB} = 3\hat{i} + 9\hat{j} + (\lambda - 2)\hat{k}$ and $\overrightarrow{BC} = \hat{i} + 3\hat{j} + (6 - \lambda)\hat{k}$ Since A, B and C are collinear, $\frac{3}{1} = \frac{9}{3} = \frac{\lambda - 2}{6 - \lambda}$ $\Rightarrow \lambda = 5$	1 ½ ½
	OR	
Ans(b)	Let θ is the angle between $\vec{a}$ and $\vec{b}$. $\vec{a}.\vec{b} = 12 \Rightarrow \lvert\vec{a}\rvert\lvert\vec{b}\rvert \cos\theta = 12$ $\Rightarrow (10)(2)\cos\theta = 12 \Rightarrow \cos\theta = \frac{3}{5}$ $\therefore \sin\theta = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \frac{4}{5}$ Now, area of parallelogram $= \lvert\vec{a} \times \vec{b}\rvert = \lvert\vec{a}\rvert\lvert\vec{b}\rvert \sin\theta$ $= (10)(2)\left(\frac{4}{5}\right) = 16$ $\therefore$ area of parallelogram $= 16$	½ ½ ½ ½
Q25.	Find the angle between the lines $\vec{r} = (3 + 2\lambda)\hat{i} - (2 - 2\lambda)\hat{j} + (6 + 2\lambda)\hat{k}$ and $\vec{r} = (2\hat{j} - 5\hat{k}) + \mu(6\hat{i} + 3\hat{j} + 2\hat{k})$.	

Ans	Given lines are: $\vec{r}=\left(3\hat{i}-2\hat{j}+6\hat{k}\right)+\lambda\left(2\hat{i}+2\hat{j}+2\hat{k}\right)$ and $\vec{r}=\left(2\hat{j}-5\hat{k}\right)+\mu\left(6\hat{i}+3\hat{j}+2\hat{k}\right)$ Let θ be the angle between these two lines. $\cos\theta=\frac{2(6)+2(3)+2(2)}{\sqrt{4+4+4}\sqrt{36+9+4}}=\frac{22}{2\sqrt{3}\times 7}$ $\Rightarrow\cos\theta=\frac{11}{21}\sqrt{3}\Rightarrow\theta=\cos^{-1}\left(\frac{11}{21}\sqrt{3}\right)$	½ 1 ½

SECTION C

This section comprises short answer (SA) type questions of **3 marks each**.

Q26.	Find the maximum slope of the curve $y=-x^3+3x^2+9x-30$.	
Ans	$y=-x^3+3x^2+9x-30$ Slope of the curve, $m=\frac{dy}{dx}=-3x^2+6x+9$ $\Rightarrow\frac{dm}{dx}=-6x+6$ For maximum/ minimum slope, put $\frac{dm}{dx}=0$ $\Rightarrow x=1$ As $\frac{d^2m}{dx^2}=-6<0\ \therefore m$ is maximum at $x=1$ Maximum slope $=-3(1)^2+6(1)+9=12$	 1 ½ ½ ½ ½
Q27.	(a) Find : $\int\sqrt{4x^2-4x+10}\ dx$ **OR** (b) Evaluate : $\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$	
Ans(a)	$I=\int\sqrt{4x^2-4x+10}\,dx$ $=\int\sqrt{(2x-1)^2+(3)^2}dx$ $=\frac{1}{2}\left[\left(\frac{2x-1}{2}\right)\sqrt{4x^2-4x+10}+\frac{9}{2}log\left\lvert(2x-1)+\sqrt{4x^2-4x+10}\right\rvert\right]+C$	 1 1+1
OR		

Ans(b)	$I=\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx \quad \ldots(i)$ $=\int_0^{\pi}\frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}dx$ $I=\int_0^{\pi}\frac{(\pi-x)\sin x}{1+\cos^2 x}dx \quad \ldots(ii)$ Adding (i) and (ii) $2I=\int_0^{\pi}\frac{\pi\sin x}{1+\cos^2 x}dx$ $\Rightarrow I=\frac{\pi}{2}\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}dx$ Put $\cos x=t\Rightarrow -\sin x\,dx=dt$ $I=-\frac{\pi}{2}\int_1^{-1}\frac{dt}{1+t^2}$ $=\pi\left[\tan^{-1}t\right]_0^1=\frac{\pi^2}{4}$	½ ½ ½ ½ 1
Q28.	Solve the following LPP graphically : Maximize Z = 2x + 3y subject to the constraints $x + 4y \le 8$ $2x + 3y \le 12$ $3x + y \le 9$ $x \ge 0, y \ge 0.$	
Ans		For correct graph and shading 2

Corner Point	Value of $Z=2x+3y$
$O(0,0)$	0
$A(0,2)$	6
$B\left(\frac{28}{11},\frac{15}{11}\right)$	$\frac{101}{11}$ Maximum
$C(3,0)$	6

For correct table 1

$Z_{\max}=\frac{101}{11}$ when $x=\frac{28}{11}, y=\frac{15}{11}$

Q29.

(a) Find the general solution of the differential equation

$(2x^2 + y)\,dx = x\,dy.$

OR

(b) For the differential equation $\frac{dy}{dx} - \frac{y}{x} + \text{cosec}\left(\frac{y}{x}\right) = 0$, find the particular solution, given that $y = 0$ when $x = 1$.

Ans(a)

$\left(2x^2+y\right)dx = x\,dy$

$\Rightarrow \frac{dy}{dx}-\frac{1}{x}.y=2x$ — 1

I.F. $=e^{-\int\frac{1}{x}dx}=\frac{1}{x}$ — 1

Solution is given by,

$y.\left(\frac{1}{x}\right)=\int 2x.\frac{1}{x}dx$ — ½

$\Rightarrow \frac{y}{x}=2x+C \text{ or } y=2x^2+Cx$ — ½

OR

Ans(b)

$\frac{dy}{dx}=\frac{y}{x}-\cos ec\left(\frac{y}{x}\right)$

Put $\frac{y}{x}=v$ i.e. $y=vx$

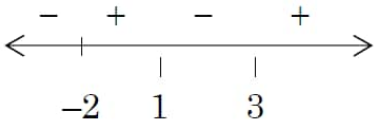
$\Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$ — ½

The differential equation reduces to

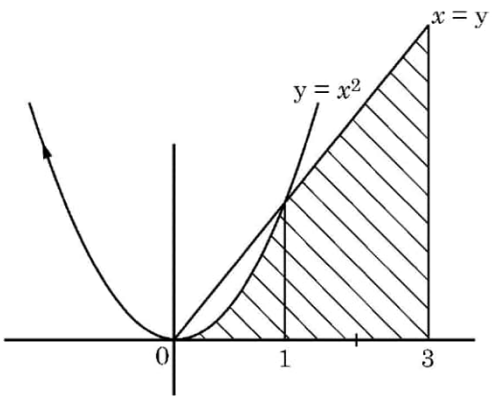
$v+x\frac{dv}{dx}=v-\cos ecv$ — ½

	$\sin v\,dv = -\frac{1}{x}dx \Rightarrow \int \sin v\,dv = -\int \frac{dx}{x}$ $\Rightarrow -\cos v = -\log\lvert x\rvert - C$ $\text{or}, \cos v = \log\lvert x\rvert + C'$ $\Rightarrow \cos\left(\frac{y}{x}\right) = \log\lvert x\rvert + C'$ $y = 0, x = 1 \text{ gives } C' = 1$ $\therefore$ Solution is given by $\cos\left(\frac{y}{x}\right) = \log\lvert x\rvert + 1$	1 ½ ½
Q30.	If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are unit vectors such that $\hat{a}.\hat{b} = \hat{a}.\hat{c} = 0$ and the angle between $\hat{b}$ and $\hat{c}$ is $\frac{\pi}{6}$, then prove that $\hat{a} = \pm 2(\hat{b} \times \hat{c})$.	
Ans)	$\hat{a}.\hat{b} = 0 \Rightarrow \hat{a} \perp \hat{b}, \hat{a}.\hat{c} = 0 \Rightarrow \hat{a} \perp \hat{c}$ $\Rightarrow \hat{a}$ is perpendicular to both $\hat{b}$ and $\hat{c} \Rightarrow \hat{a} \parallel \left(\hat{b} \times \hat{c}\right)$ Let $\hat{a} = \lambda\left(\hat{b} \times \hat{c}\right)$ $\Rightarrow \lvert\hat{a}\rvert = \lvert\lambda\rvert\left\lvert\left(\hat{b} \times \hat{c}\right)\right\rvert \Rightarrow \lvert\hat{a}\rvert = \lvert\lambda\rvert\lvert\hat{b}\rvert\lvert\hat{c}\rvert \sin\frac{\pi}{6}$ $\Rightarrow \lvert\lambda\rvert = 2 \Rightarrow \lambda = \pm 2$ $\therefore \hat{a} = \pm 2\left(\hat{b} \times \hat{c}\right)$	1 ½ 1 ½
Q31.	(a) Four students of class XII are given a problem to solve independently. Their chances of solving the problem respectively are $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}$ and $\frac{1}{5}$. Find the probability that at most one of them will solve the problem. **OR** (b) The probability distribution of a random variable X is given below : X: 1, 2, 4, 2k, 3k, 5k P(X): $\frac{1}{2}, \frac{1}{5}, \frac{3}{25}, \frac{1}{10}, \frac{1}{25}, \frac{1}{25}$ Find k, if E(X) = 2·94 and also find $P(X \le 4)$.	
Ans(a)	$P(\text{at most one of them will solve the problem})$ $= P(\text{none of them solves the problem}) + P(\text{only one of them solves the problem})$ $= \left(\frac{1}{2}\times\frac{2}{3}\times\frac{1}{3}\times\frac{4}{5}\right) + \left(\frac{1}{2}\times\frac{2}{3}\times\frac{1}{3}\times\frac{4}{5} + \frac{1}{2}\times\frac{1}{3}\times\frac{1}{3}\times\frac{4}{5} + \frac{1}{2}\times\frac{2}{3}\times\frac{2}{3}\times\frac{4}{5} + \frac{1}{2}\times\frac{2}{3}\times\frac{1}{3}\times\frac{1}{5}\right)$ $= \frac{19}{45}$	2½ ½

	OR	
Ans(b)	$E(X) = 2.94$ $\Rightarrow 1\left(\frac{1}{2}\right) + 2\left(\frac{1}{5}\right) + 4\left(\frac{3}{25}\right) + 2k\left(\frac{1}{10}\right) + 3k\left(\frac{1}{25}\right) + 5k\left(\frac{1}{25}\right) = 2.94$ $\Rightarrow k = \frac{1.56}{0.52} \Rightarrow k = 3$ Now, $P(X \leq 4) = P(X = 1) + P(X = 2) + P(X = 4)$ $= \frac{1}{2} + \frac{1}{5} + \frac{3}{25}$ $= \frac{41}{50}$	1 1 ½ ½
	SECTION D This section comprises long answer (LA) type questions of **5 marks each**.	
Q32.	If A $= \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the given system of equations $3x + 4y + 7z = 14$; $2x - y + 3z = 4$; $x + 2y - 3z = 0$.	
Ans	$\lvert A\rvert = 3(-3) - 2(-26) + 1(19) = 62 \neq 0 \Rightarrow A^{-1}$ exists. cofactor Matrix $= \begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix}$ $adjA = \begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix}$ $A^{-1} = \frac{1}{62}\begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix}$ Given system of equations can be written as $A'.X = B$ where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$ Now, $A'.X = B \Rightarrow X = (A')^{-1}.B$ $\Rightarrow X = \left(A^{-1}\right)'.B = \frac{1}{62}\begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix}\begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$ $= \frac{1}{62}\begin{bmatrix} 62 \\ 62 \\ 62 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\Rightarrow x = 1, y = 1, z = 1$	½ 2 ½ ½ ½ 1

Q33.	(a) If $y = \cos x^2 + \cos^2 x + \cos^2 (x^2) + \cos (x^x)$, find $\frac{dy}{dx}$. **OR** (b) Find the intervals in which the function given by $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ is : (i) strictly increasing. (ii) strictly decreasing.	
Ans(a)	$y = \cos x^2 + \cos^2 x + \cos^2(x^2) + \cos(x^x)$ $\frac{d}{dx}(\cos x^2) = -2x \sin x^2$ $\frac{d}{dx}(\cos^2 x) = 2\cos x(-\sin x) = -2\sin x\cos x$ $\frac{d}{dx}(\cos^2(x^2)) = 2\cos(x^2)(-\sin(x^2))(2x) = -4x \sin x^2 \cos x^2$ $\frac{d}{dx}(\cos(x^x)) = -\sin(x^x)[x^x(1 + \log x)]$ $\frac{dy}{dx} = -2x \sin x^2 - 2\sin x\cos x - 4x \sin x^2 \cos x^2 - \sin(x^x)[x^x(1 + \log x)]$	1 1 1 1½ ½
	OR	
Ans(b)	$f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ $\Rightarrow f'(x) = \frac{6}{5}x^3 - \frac{12}{5}x^2 - 6x + \frac{36}{5} = \frac{6}{5}(x^3 - 2x^2 - 5x + 6)$ $= \frac{6}{5}(x-1)(x+2)(x-3)$  For strictly inc/dec, put $f'(x) = 0$ $\Rightarrow x = 1, -2, 3$ (i) f is strictly increasing when $x \in (-2,1) \cup (3, \infty)$ (ii) f is strictly decreasing when $x \in (-\infty, -2) \cup (1,3)$ Note: Closed intervals are also acceptable.	2 1 1 1 1
Q34.	Using integration, find the area of the region $\{(x, y): 0 \le y \le x^2, \ 0 \le y \le x, \ 0 \le x \le 3\}$.	

Ans



1 mark for correct figure

Required Area

$$= \int_0^1 x^2\,dx + \int_1^3 x\,dx \qquad 1+1$$

$$= \frac{x^3}{3}\Big]_0^1 + \frac{x^2}{2}\Big]_1^3 \qquad 1$$

$$= \frac{1}{3} + 4 = \frac{13}{3} \qquad 1$$

Q35.

(a) Find the shortest distance between the lines l_1 and l_2 given by :

$$l_1 : \vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(4\hat{i} + 6\hat{j} + 12\hat{k})$$

and $l_2 : \vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(6\hat{i} + 9\hat{j} + 18\hat{k})$

OR

(b) Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1}$ intersect. Also, find their point of intersection.

Ans(a)

Given lines are : $\vec{r} = \left(\hat{i} + 2\hat{j} - 4\hat{k}\right) + 2\lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$

and $\vec{r} = \left(\hat{i} + 2\hat{j} - 4\hat{k}\right) + 3\mu\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$

Clearly, the given lines are parallel.

Here, $\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}$, $\vec{a}_2 = \hat{i} + 2\hat{j} - 4\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ — 1

$\vec{a}_2 - \vec{a}_1 = 2\hat{i} + \hat{j} - \hat{k}$ — ½

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix} = 9\hat{i} - 14\hat{j} + 4\hat{k}$$ — 1 ½

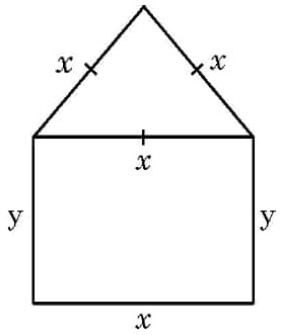
$\therefore \left|(\vec{a}_2 - \vec{a}_1) \times \vec{b}\right| = \sqrt{81 + 196 + 16} = \sqrt{293}$ — 1

Also, $\left|\vec{b}\right| = \sqrt{4 + 9 + 36} = 7$ — ½

$$\text{S.D.} = \frac{\left|(\vec{a}_2 - \vec{a}_1) \times \vec{b}\right|}{\left|\vec{b}\right|}$$

$$= \frac{\sqrt{293}}{7}$$ — ½

	OR	
Ans(b)	Let the given lines be $l_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$ and $l_2 : \frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1} = \mu$ Any point on the line l_1 is $(2\lambda+1, 3\lambda+2, 4\lambda+3)$ Any point on the line l_2 is $(5\mu+4, 2\mu+1, \mu)$ For the given lines to intersect, there must be a common point. $\therefore 2\lambda+1=5\mu+4 \Rightarrow 2\lambda-5\mu=3 \quad ...(i)$ $3\lambda+2=2\mu+1 \Rightarrow 3\lambda-2\mu=-1 \quad ...(ii)$ $4\lambda+3=\mu \Rightarrow 4\lambda-\mu=-3 \quad ...(iii)$ Solving (i) and (ii) gives, $\lambda=\mu=-1$ We notice that $\lambda=\mu=-1$ also satisfies equation (iii) $\therefore$ The given lines intersect. Point of intersection is $(2(-1)+1, 3(-1)+2, 4(-1)+3)$ i.e. $(-1,-1,-1)$	1 1 1 1 1
	SECTION E This section comprises 3 case study-based questions of **4 marks each**.	
Q36.	**Case Study – 1** A window is in the form of a rectangle surmounted by an equilateral triangle on its length. Let the rectangular part have length and breadth x and y metres respectively. Based on the given information, answer the following questions : (i) If the perimeter of the window is 12 m, find the relation between x and y. *1* (ii) Using the expression obtained in (i), write an expression for the area of the window as a function of x only. *1* (iii) (a) Find the dimensions of the rectangle that will allow maximum light through the window. (use expression obtained in (ii)) *2* **OR** (iii) (b) If it is given that the area of the window is 50 m^2, find an expression for its perimeter in terms of x. *2*	

Ans	(i) Perimeter $(P)=3x+2y=12$ 	1
	(ii) Area $(A)=xy+\frac{\sqrt{3}}{4}x^2$ $=x\left(\frac{12-3x}{2}\right)+\frac{\sqrt{3}}{4}x^2$ $=6x-\frac{3}{2}x^2+\frac{\sqrt{3}}{4}x^2$	1
	$(iii)(a)\frac{dA}{dx}=6-3x+\frac{\sqrt{3}}{2}x$	½
	For maximum light, $\frac{dA}{dx}=0$	
	$\Rightarrow 6-3x+\frac{\sqrt{3}}{2}x=0\Rightarrow x=\frac{12}{6-\sqrt{3}}m$	½
	Also, $\frac{d^2A}{dx^2}=-3+\frac{\sqrt{3}}{2}<0\therefore A$ is maximum when $x=\frac{12}{6-\sqrt{3}}m$	½
	Now, $y=\frac{12-3x}{2}=6-\frac{3}{2}\left(\frac{12}{6-\sqrt{3}}\right)=\frac{18-6\sqrt{3}}{6-\sqrt{3}}m$	½
	OR	
	$(iii)(b)\,xy+\frac{\sqrt{3}}{4}x^2=50$	½
	$\Rightarrow y=\frac{50}{x}-\frac{\sqrt{3}}{4}x$	1
	Now, $P=3x+2y$ $=3x+2\left(\frac{50}{x}-\frac{\sqrt{3}}{4}x\right)m$	½

Q37. **Case Study – 2**

During the festival season, there was a mela organized by the Resident Welfare Association at a park, near the society. The main attraction of the mela was a huge swing installed at one corner of the park. The swing is traced to follow the path of a parabola given by $x^2 = y$.

Based on the above information, answer the following questions :

(i) Let $f : N \longrightarrow R$ is defined by $f(x) = x^2$. What will be the range ? *1*

(ii) Let $f : N \longrightarrow N$ is defined by $f(x) = x^2$. Check if the function is injective or not. *1*

(iii) (a) Let $f : \{1, 2, 3, 4 .....\} \longrightarrow \{1, 4, 9, 16 .....\}$ be defined by $f(x) = x^2$. Prove that the function is bijective. *2*

OR

(iii) (b) Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ is defined by $f(x) = x^2$. Show that f is neither injective nor surjective. *2*

Ans

$(i)\ R_f = \{1,4,9,16,...\}$ i.e. set of perfect squares of natural numbers. 1

(ii) Let $x_1, x_2 \in N$ (domain)

Let $f(x_1) = f(x_2)$

$\Rightarrow x_1^2 = x_2^2$ 1

$\Rightarrow x_1 = \pm x_2$

$\Rightarrow x_1 = x_2$ as $x_1, x_2 \in N$

$\therefore f$ is injective.

$(iii)(a)\ f(x) = x^2$

Let $x_1, x_2 \in \{1,2,3,4,...\}$

Let $f(x_1) = f(x_2)$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = x_2$

$\therefore f$ is one-one. 1

As Co-domain = Range $= \{1,4,9,16,...\}$

$\therefore f$ is onto. 1

Since, f is one-one and onto, so f is bijective.

OR

$(iii)(b)\ f : R \to R, f(x) = x^2$

$-1, 1 \in R$ (domain)

As $f(-1) = f(1) = 1$ but $-1 \neq 1$ 1

$\therefore f$ is not injective.

Co-domain $= R$, but Range $= [0, \infty)$ 1

Since Co-domain $\neq$ Range, f is not surjective.

Q38.

Case Study – 3

Two persons are competing for a position on the Managing Committee of an organisation. The probabilities that the first and the second person will be appointed are 0·5 and 0·6 respectively. Also, if the first person gets appointed, then the probability of introducing waste treatment plant is 0·7 and the corresponding probability is 0·4, if the second person gets appointed.

Based on the above information, answer the following questions :

(i) What is the probability that the waste treatment plant is introduced ? 2

(ii) After the selection, if the waste treatment plant is introduced, what is the probability that the first person had introduced it ? 2

Ans

E_1 : Event that the first person is appointed.

E_2 : Event that the second person is appointed.

A: Event that the waste treatment plant is introducted.

Here, $P(E_1)=0.5, P(E_2)=0.6$

$P(A|E_1)=0.7, P(A|E_2)=0.4$

$(i)\ P(\text{waste treatment plant is introducted})$

$=P(E_1)P(A|E_1)+P(E_2)P(A|E_2)$

$=(0.5)(0.7)+(0.6)(0.4)$ 1

$=0.35+0.24=0.59$ 1

$(ii)\ P(E_1|A)=\dfrac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)}$

$=\dfrac{(0.5)(0.7)}{(0.5)(0.7)+(0.6)(0.4)}$ 1 ½

$=\dfrac{0.35}{0.59}=\dfrac{35}{59}$ ½

Note: Full marks to be awarded, in case a student writes "Sum of probabilities of selecting first person and second person should not be greater than 1".