Class X Session 2024-25
Subject - Mathematics (Standard)
Sample Question Paper - 3

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub- parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

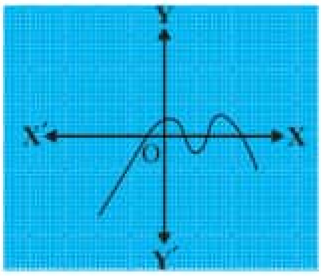
Section A

1. The sum of the exponents of the prime factors in the prime factorisation of 196, is **[1]**

a) 5 b) 3

c) 4 d) 2

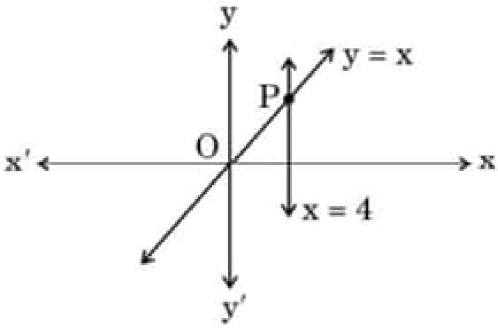
2. Find the number of zeroes of p(x) in the figure given below. **[1]**



a) 2 b) 4

c) 1 d) 3

3. The lines represented by the linear equations y = x and x = 4 intersect at P. The coordinates of the point P are: **[1]**



a) (4, 4)

b) (-4, 4)

c) (0, 4)

d) (4, 0)

4. If y = 1 is a common root of the equations $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$, then ab equals **[1]**

a) 3

b) -3

c) 6

d) -7/2

5. The common difference of the AP $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}, ...$is **[1]**

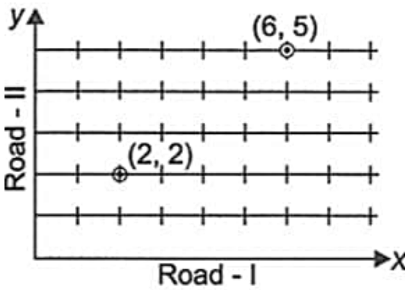
a) -1

b) $-\frac{1}{p}$

c) 1

d) $\frac{1}{p}$

6. A well-planned locality has two straight roads perpendicular to each other. There are 5 lanes parallel to Road - I. Each lane has 8 houses as seen in figure. Chaitanya lives in the 6^{th} house of the 5th lane and Hamida lives in the 2^{nd} house of the 2^{nd} lane. What will be the shortest distance between their houses? **[1]**



a) 12 units

b) 5 units

c) 6 units

d) 10 units

7. Three consecutive vertices of a parallelogram ABCD are A(1, 2), B(1, 0) and C(4, 0). The co – ordinates of the fourth vertex D are **[1]**

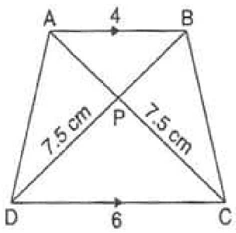
a) (– 4, 2)

b) (4, – 2)

c) (4, 2)

d) (– 4, – 2)

8. In the given figure, if AB||DC, then AP is equal to **[1]**



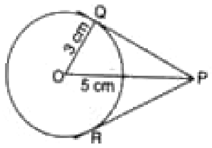
a) 5 cm.

b) 7 cm.

c) 6 cm.

d) 5.5 cm.

9. In the given figure, if OQ = 3 cm, OP = 5 m, then the length of PR is **[1]**



a) 4 cm
b) 3 cm
c) 5 cm
d) 6 cm

10. AB and CD are two parallel tangents to a circle of radius 5 cm. The distance between the tangents is **[1]**

a) 5 cm
b) $\sqrt{50}$ cm
c) $2\sqrt{5}$ cm
d) 10 cm

11. $1 + \frac{\cot^2\alpha}{1+\cos ec\alpha} =$ **[1]**

a) sinα
b) secα
c) cosecα
d) tanα

12. $\cos^2 30^\circ \cos^2 45^\circ + 4\sec^2 60^\circ + \frac{1}{2}\cos^2 90^\circ - 2\tan^2 60^\circ$ = ? **[1]**

a) $\frac{75}{8}$
b) $\frac{73}{8}$
c) $\frac{83}{8}$
d) $\frac{81}{8}$

13. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag staff of height 5 m. From a point on the plane the angles of elevation of the bottom and top of the flagstaff are respectively 30° and 60°. The height of the tower is **[1]**

a) 10 m
b) 5 m
c) 2.5 m
d) 2 m

14. The length of the minute hand of a clock is 21 cm. The area swept by the minute hand in 10 minutes is **[1]**

a) 252 cm^2
b) 126 cm^2
c) 231 cm^2
d) 210 cm^2

15. The length of an arc of a sector of angle $\theta°$ of a circle with radius R is **[1]**

a) $\frac{\pi R^2\theta}{180}$
b) $\frac{\pi R^2\theta}{360}$
c) $\frac{2\pi R\theta}{360}$
d) $\frac{2\pi R\theta}{180}$

16. If P(E) = 0.05, what will be the probability of 'not E'? **[1]**

a) 0.55
b) 0.59
c) 0.95
d) 0.095

17. An unbiased die is thrown once. The probability of getting a composite number is **[1]**

a) $\frac{2}{5}$
b) $\frac{1}{3}$
c) $\frac{2}{3}$
d) $\frac{1}{2}$

18. The arithmetic mean of 1, 2, 3, 4, ..., n is: **[1]**

a) $\frac{n-1}{2}$
b) $\frac{n(n+1)}{2}$
c) $\frac{n}{2}$
d) $\frac{n+1}{2}$

19. **Assrtion (A):** A toy is in the form of a cone mounted on a hemisphere with the same radius. The radius of the conical portion is 4 cm and its height is 3 cm. the surface area of the toy is 163.28 cm^2 . [Take π = 3.14] **[1]**

Reason (R): Volume of hemisphere is $\frac{2}{3}\pi r^2$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, 4\sqrt{3}$ this series forms an A.P. **[1]**

Reason (R): Since common difference is same and equal to $\sqrt{3}$ therefore given series is an AP.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

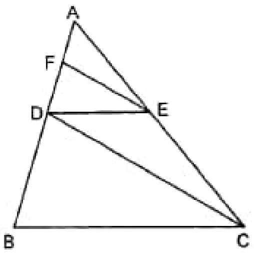
c) A is true but R is false.

d) A is false but R is true.

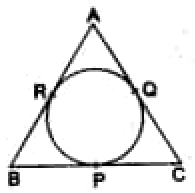
Section B

21. Find the LCM and HCF of the pairs of integers 336 and 54 and verify that LCM $\times$ HCF = product of the two numbers. **[2]**

22. In Fig. $DE||BC$ and $CD||EF$. Prove that AD^2 = AB $\times$ AF. **[2]**



23. A circle is inscribed in a $\triangle ABC$, touching BC, CA and AB at P, Q and R respectively, as shown in the given figure. If AB = 10 cm, AQ = 7 cm and CQ = 5 cm then find the length of BC. **[2]**



24. If A = B = 60°, verify that sin (A - B) = sin A cos B - cos A sinB **[2]**

OR

Prove the trigonometric identity:

$\tan^2 A \sec^2 B - \sec^2 A \tan^2 B = \tan^2 A - \tan^2 B$

25. Find the difference of the areas of a sector of angle 120° and its corresponding major sector of a circle of radius 21 cm. **[2]**

OR

A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding:

i. minor segment
ii. major sector.

Section C

26. For a morning walk, three persons steps off together. The measure of their steps is 80,85 and 90 cm respectively.What is the minimum distance each should walk so that all can cover the same distance in complete steps? **[3]**

Which value is preferred in this situation?

27. Find the zeroes of the quadratic polynomial $6x^2 - 3 - 7x$ and verify the relationship between the zeroes and the coefficients of the polynomial. **[3]**

28. Half of the difference between two numbers is 2. The sum of the greater number and twice the smaller number is 13. Find the numbers. **[3]**

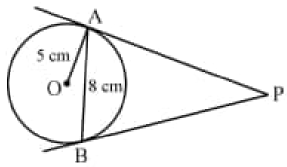
OR

Solve algebraically the following pair of linear equations for x and y

$31x + 29y = 33$

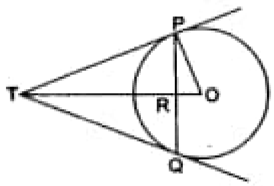
$29x + 31y = 27$

29. In a given figure, AB is a chord of length 8 cm of a circle of radius 5 cm. The tangents to the circle at A and B intersect at P. Find the length of AP. **[3]**



OR

PQ is a chord of length 4.8 cm of a circle of radius 3 cm. The tangents at P and Q intersect at a point T as shown in the figure. Find the length of TP.



30. Prove that: $\frac{1}{\operatorname{cosec} A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\operatorname{cosec} A + \cot A}$. **[3]**

31. Compute the median for the following cumulative frequency distribution: **[3]**

Less than 20	Less than 30	Less than 40	Less than 50	Less than 60	Less than 70	Less than 80	Less than 90	Less than 100
0	4	16	30	46	66	82	92	100

Section D

32. Solve for x **[5]**

$\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$ where $a + b + x \neq 0$ and $a, b\ x \neq 0$

OR

If x = - 4 is a root of the equation $x^2 + 2x + 4p = 0$, find the values of k for which the equation $x^2 + px(l + 3k) + 7(3 + 2k) = 0$ has equal roots.

33. In a trapezium ABCD, AB | | DC and DC = 2AB. EF | | AB, where E and F lie on BC and AD respectively such that $\frac{BE}{EC} = \frac{4}{3}$. Diagonal DB intersects EF at G. Prove that, 7EF = 11AB. **[5]**

34. A tent is in the form of a right circular cylinder surmounted by a cone. The diameter of the base of the cylinder or the cone is 24 m. The height of the cylinder is 11 m. If the vertex of the cone is 16 m above the ground, find the area of the canvas required for making the tent. (Use $\pi = \frac{22}{7}$) **[5]**

OR

From a cubical piece of wood of side 21 cm, a hemisphere is carved out in such a way that the diameter of the hemisphere is equal to the side of the cubical piece. Find the surface area and volume of the remaining piece.

35. Find the mean and the median of the following data: **[5]**

Marks	Number of Students
0 - 10	3
10 - 20	5
20 - 30	16
30 - 40	12
40 - 50	13
50 - 60	20
60 - 70	6
70 - 80	5

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Kamla and her husband were working in a factory in Seelampur, New Delhi. During the pandemic, they were asked to leave the job. As they have very limited resources to survive in a metro city, they decided to go back to their hometown in Himachal Pradesh. After a few months of struggle, they thought to grow roses in their fields and sell them to local vendors as roses have been always in demand. Their business started growing up and they hired many workers to manage their garden and do packaging of the flowers.

In their garden bed, there are 23 rose plants in the first row, 21 are in the 2nd, 19 in 3rd row and so on. There are 5 plants in the last row.

(a) How many rows are there of rose plants?

(b) Also, find the total number of rose plants in the garden.

OR

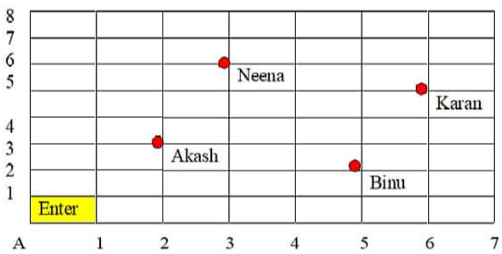
If total number of plants are 80 in the garden, then find number of rows?

(c) How many plants are there in 6th row.

37. **Read the text carefully and answer the questions:** **[4]**

Karan went to the Lab near to his home for COVID 19 test along with his family members.

The seats in the waiting area were as per the norms of distancing during this pandemic (as shown in the figure). His family member took their seats surrounded by red circular area.



(a) What is the distance between Neena and Karan?

(b) What are the coordinates of seat of Akash?

OR

Find distance between Binu and Karan.

(c) What will be the coordinates of a point exactly between Akash and Binu where a person can be?

38. **Read the text carefully and answer the questions:** **[4]**

An observer on the top of a 40m tall light house (including height of the observer) observes a ship at an angle of depression 30^{o} coming towards the base of the light house along straight line joining the ship and the base of the light house. The angle of depression of ship changes to 45^{o} after 6 seconds.

(a) Find the distance of ship from the base of the light house after 6 seconds from the initial position when angle of depression is 45^{o}.

(b) Find the distance between two positions of ship after 6 seconds?

OR

Find the distance of ship from the base of the light house when angle of depression is 30^{o}.

(c) Find the speed of the ship?

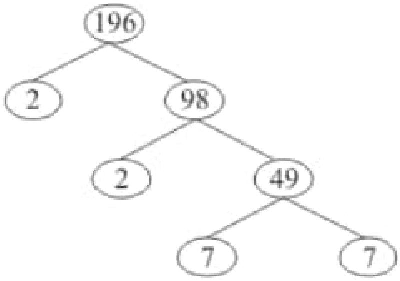
Solution

Section A

1.

(c) 4

Explanation: Using the factor tree for prime factorisation, we have:



Therefore,

$196 = 2 \times 2 \times 7 \times 7$

$196 = 2^2 \times 7^2$

The exponents of 2 and 7 are 2 and 2 respectively.

Thus the sum of the exponents is 4.

2.

(b) 4

Explanation: The number of zeroes is 4. as the graph intersect the x-axis at four point.

3. **(a)** (4, 4)

Explanation: (4, 4)

4. **(a)** 3

Explanation: Here it is given that y = 1 is a common root, so we have;

$ay^2 + ay + 3 = 0$

$\therefore$ a $\times$ $(1)^2$ + a (1) + 3 = 0

a + a + 3 = 0 $\Rightarrow$ 2a = -3

$\Rightarrow a = \frac{-3}{2}$

and $y^2 + y + b = 0$

$(1)^2$ + (1) + b = 0 $\Rightarrow$ 1 + 1 + b = 0

$\Rightarrow$ 2 + b = 0

$\therefore$ b = -2

ab = $\frac{-3}{2} \times$ (-2) = 3

5. **(a)** -1

Explanation: $d = \left\{\frac{1-p}{p} - \frac{1}{p}\right\} = \left(\frac{1-p-1}{p}\right) = \frac{-p}{p} = -1$.

6.

(b) 5 units

Explanation: According to question,

Coordinates of Chaitanya's house are (6, 5)

Coordinates of Hamida's house are (2, 2)

$\therefore$ Shortest distance between their houses

= $\sqrt{(6-2)^2 + (5-2)^2}$ = 5 units.

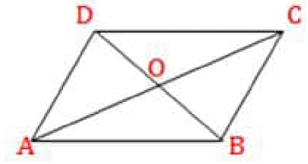
7.

(c) (4, 2)

Explanation: Coordinates are given for A(1, 2), B(1, 0) and C(4, 0)

Let coordinates of D be (x, y).

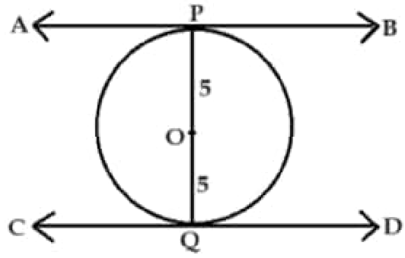
Since diagonals of a parallelogram bisect each other. at point O
Therefore O is the midpoint of diagonal AC
Therefore, coordinates of O will be $\left(\frac{1+4}{2}, \frac{2+0}{2}\right)$ = $\left(\frac{5}{2}, 1\right)$
O is also the midpoint of diagonal BD
$\therefore \frac{x+1}{2} = \frac{5}{2} \Rightarrow x = 4$
And $\frac{y+0}{2} = 1 \Rightarrow y = 2$ Therefore, the required coordinates are (4, 2).



8. **(a)** 5 cm.
Explanation: In triangles APB and CPD,
∠APB = ∠CPD [Vertically opposite angles] ∠BAP = ∠ACD [Alternate angles as $AB||CD$]
∴ΔAPB ~ ΔCPD [AA similarity]
$\therefore \frac{\text{AB}}{\text{CD}} = \frac{\text{CP}}{\text{AP}}$
$\Rightarrow \frac{4}{6} = \frac{\text{AP}}{7.5}$
⇒ AP = $\frac{7.5 \times 4}{6}$ = 5 cm

9. **(a)** 4 cm
Explanation: Here ∠Q = 90° [Angle between tangent and radius through the point of contact]
Now, in right angled triangle OPQ,
$OP^2 = OQ^2 + PQ^2$
$\Rightarrow (5)^2 = (3)^2 + PQ^2$
$\Rightarrow PQ^2 = 25 - 9 = 16$
⇒ PQ = 4 cm
But PQ = PR [Tangents from one point to a circle are equal]
Therefore, PR = 4 cm

10.
(d) 10 cm
Explanation: AB and CD are two parallel tangent to a circle
AB || CD
and OP ⊥ AB
OQ ⊥ CD
By fig clear that
distance between two tangent is OP + OQ
ie. 5 + 5 = 10 cm.



11.
(c) cosecα
Explanation: 1 + $\frac{\cot^2\alpha}{1+\cos ec\alpha}$
= 1 + $\frac{cosec^2\alpha - 1}{1+\cos ec\alpha}$
= 1 + $\frac{(cosec\,\alpha - 1)(cosec\,\alpha + 1)}{1+cosec\alpha}$
= 1 + cosecα - 1 = cosec α

12.

(c) $\frac{83}{8}$

Explanation: $\cos^2 30^\circ \cos^2 45^\circ + 4\sec^2 60^\circ + \frac{1}{2}\cos^2 90^\circ - 2\tan^2 60^\circ$

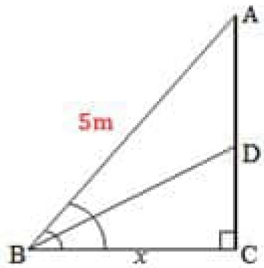
$= \left(\frac{\sqrt{3}}{2}\right)^2 \cdot \left(\frac{1}{\sqrt{2}}\right)^2 + \left(4 \times 2^2\right) + \left(\frac{1}{2} \times 0^2\right) - 2 \times (\sqrt{3})^2$

$= \left(\frac{3}{4} \times \frac{1}{2}\right) + 16 + 0 - 6 = \frac{3}{8} + 10 = \frac{83}{8}$

13.

(c) 2.5 m

Explanation:



Here Height of the tower = CD = h meters, height of the flagstaff = AD = 5 meters, angle of elevation of top of the tower = ∠DBC = 30° and angle of elevation of the top of the flagstaff from ground = ∠ABC = 60°

Now, in triangle DBC,

$\tan 30^\circ = \frac{h}{x}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x} \Rightarrow x = h\sqrt{3}$ .....(i)

And $\tan 60^\circ = \frac{h+5}{x}$

$\Rightarrow \sqrt{3} = \frac{h+5}{x}$

$\Rightarrow x = \frac{h+5}{\sqrt{3}}$ .....(ii)

From eq. (i) and (ii), we get,

$h\sqrt{3} = \frac{h+5}{\sqrt{3}}$

$\Rightarrow 3h = h + 5$

$\Rightarrow 2h = 5$

$\Rightarrow h = 2.5$ meters

14.

(c) 231 cm^2

Explanation: Area swept by minute hand in 60 minutes = πR^2

Area swept by it in 10 minutes

$= \left(\frac{\pi R^2}{60} \times 10\right) \text{cm}^2 = \left(\frac{22}{7} \times 21 \times 21 \times \frac{1}{6}\right) \text{cm}^2$

= 231 cm^2

15.

(c) $\frac{2\pi R\theta}{360}$

Explanation: $\frac{2\pi R\theta}{360}$

16.

(c) 0.95

Explanation: We know that

P(E) + P(not E) = 1

∴ P(not E) = 1 - P(E)

= 1 - 0.05

= 0.95

17.

(b) $\frac{1}{3}$

Explanation: Number of composite numbers on a dice = {4, 6} = 2

Number of possible outcomes = 2

Number of Total outcomes = 6

$\therefore$ Required Probability = $\frac{2}{6} = \frac{1}{3}$

18.

(d) $\frac{n+1}{2}$

Explanation: According to question,

Arithmetic Mean = $\frac{1+2+3+\ldots+n}{n}$

= $\frac{\frac{n(n+1)}{2}}{n}$

= $\frac{n+1}{2}$

19.

(c) A is true but R is false.

Explanation: A is true but R is false.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

Section B

21. 336 and 54

336 = 2 × 2 × 2 × 2 × 3 × 7 = 2^4 × 3 × 7

54 = 2 × 3 × 3 × 3 = 2 × 3^3

HCF = 2 × 3 = 6

LCM = 2^4 × 3^3 × 7 = 3024

Product of two numbers 336 and 54 = 336 × 54 = 18144

HCF × LCM = 6 × 3024 = 18144

Hence, product of two numbers = HCF × LCM`

22. In $\triangle$ABC, it is given that

$DE \| BC$

$\Rightarrow \quad \frac{AB}{AD} = \frac{AC}{AE}$ ....(i)

In $\triangle$ ADC, it is given that

$FE \| DC$

$\Rightarrow \quad \frac{AD}{AF} = \frac{AC}{AE}$ ....(ii)

From (i) and (ii), we get

$\frac{AB}{AD} = \frac{AD}{AF}$

$\Rightarrow$ AD^2 = AB $\times$ AF

23. Here it is given that, AB = 10 cm, AQ = 7 cm and CQ = 5 cm.

Now we know that the lengths of tangents drawn from an external point to a circle are equal.

$\therefore$ AR = AQ = 7 cm.

BR = (AB - AR) = (10 - 7) cm = 3 cm.

$\therefore$ BP = BR = 3 cm,

CP = CQ = 5 cm.

BC = (BP + CP) = (3 + 5) cm = 8 cm

Hence the length of BC = 8cm.

24. $\text{L.H.S.} = \sin(A - B) = \sin(60^\circ - 60^\circ)$

$= \sin 0^\circ = 0$

$\text{R. H. S.} = \sin \text{A} \cos \text{B} - \cos \text{A} \sin \text{B}$

$= \sin 60^\circ \cos 60^\circ - \cos 60^\circ \sin 60^\circ$

$= \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$

$\therefore$ $\text{L.H.S.} = \text{R. H. S}$

OR

LHS = $\tan^2$A $\sec^2$B - $\sec^2$A $\tan^2$B

= $\tan^2$A(1 + $\tan^2$B) - (1 + $\tan^2$A) $\tan^2$B $\left[\because 1 + \tan^2\theta = \sec^2\theta\right]$

$= \tan^2A + \tan^2A \tan^2B - \tan^2B - \tan^2A \tan^2B$

$= \tan^2A - \tan^2B$

= RHS

Hence proved.

25. Let A_1 and A_2 be the areas of the given sector and the corresponding major sector respectively.

Given, θ = 120° and it's radius is 21 cm. So, r = 21 cm.

$\therefore A_1 = \frac{\theta}{360} \times \pi r^2 = \frac{120}{360} \times \pi \times (21)^2 = 147\pi\text{cm}^2$

and, A_2 = Area of the circle - A_1

$\Rightarrow A_2 = \left\{\pi \times (21)^2 - 147\pi\right\} \text{cm}^2 = \pi(441 - 147)\text{cm}^2 = 294\pi\text{cm}^2$

Required differences $= A_2 - A_1$

$=(294\pi - 147\pi)\text{cm}^2 = 147\pi\text{cm}^2 = \left(147 \times \frac{22}{7}\right) \text{cm}^2$

= 462 cm^2

OR

i. r = 10 cm, $\theta = 90^\circ$

Area of minor sector = $\frac{\theta}{360} \times \pi r^2$

$= \frac{90}{360} \times 3.14 \times 10 \times 10 = 78.5\text{cm}^2$

Area of $\triangle OAB = \frac{OA \times OB}{2}$

$= \frac{10 \times 10}{2} = 50\text{cm}^2$

$\therefore$ Area of the minor segment

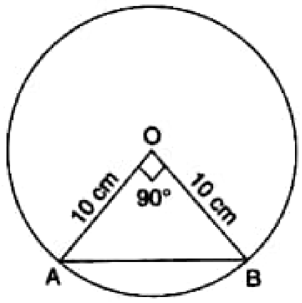
= Area of minor sector - Area of $\triangle OAB$

$= 78.5\text{cm}^2 - 50\text{cm}^2 = 28.5\text{cm}^2$

ii. Area of major sector = $\pi r^2 - area\ of\ minor\ sector$

$= 3.14 \times 10 \times 10 - 78.5$

$= 314 - 28.5 = 285.5\text{cm}^2$



Section C

26. Since, the three persons start walking together.

$\therefore$ The minimum distance each should walk so that all can cover the same distance in complete steps will be equal to LCM of 80,85 and 90.

Prime factors of 80,85 and are as following

$80 = 16 \times 5 = 2^4 \times 5$

$85 = 5 \times 17$

$90 = 2 \times 9 \times 5 = 2 \times 3^2 \times 5$

So LCM of 80,85 and 90 $= 2^4 \times 3^2 \times 5 \times 17$

=16×9×5×17=12240

$\therefore$ Each person should walk the minimum distance

=12240 cm =122 meter 40 cm

Value of morning walk :

“An early morning walk is a blessing for the whole day.”

27. The given polynomial is

$p(x) = 6x^2 - 7x - 3$

Factorize the above quadratic polynomial, we have

$6x^2 - 7x - 3 = 6x^2 - 9x + 2x - 3$

$= 3x(2x - 3) + 1(2x - 3)$

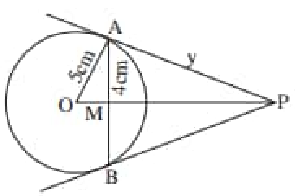
= (3x + 1)(2x - 3)
For p(x) = 0, either 3x + 1 = 0 or 2x - 3 = 0
$\Rightarrow \quad x = \frac{-1}{3}$ or $x = \frac{3}{2}$
Verification:we have a = 6, b = -7, c = -3
Sum of zeroes = $\frac{-1}{3} + \frac{3}{2} = \frac{7}{6}$
Also, $\frac{-b}{a} = \frac{-(-7)}{6} = \frac{7}{6}$
Now, product of zeroes= $\left(-\frac{1}{3}\right) \times \frac{3}{2} = \frac{-1}{2}$
Also, $\frac{c}{a} = \frac{-3}{6} = \frac{-1}{2}$

28. Let the greater numbers be 'x' and the smaller number 'y'.
It is said that half the difference between the two numbers is 2. So, we can write it as;
$\frac{1}{2} \times$ (x - y) = 2
$\Rightarrow$ (x - y) = 4 ...(1)
It is also said that the sum of the greater number and twice the smaller number is 13. So, we can write it as;
x + 2y = 13 ...(2)
Subtracting (2) from (1), we get;
(x - y) - (x + 2y) = 4 - 13
$\Rightarrow$ x - y - x - 2y = - 9
$\Rightarrow$ -3y = -9
$\Rightarrow$ y = $\frac{9}{3}$
$\Rightarrow$ y = 3
Putting y = 3 in (1), we get;
x - y = 4
$\Rightarrow$ x - 3 = 4
$\Rightarrow$ x = 4 + 3
$\Rightarrow$ x = 7
Hence, the greater number is 7. The smaller number is 3.

OR

$31x + 29y = 33$ ----------(1)
$29x + 31y = 27$ ------------ (2)
Multiply (1) by 29 and (2) by 31 (Since 29,31 are primes and Lcm is 29$\times$31)
(1) becomes 31x$\times$29 + 29$\times$29y = 33$\times$29 ------- (3)
(2) becomes 29x$\times$31+31$\times$31y=27$\times$31 ------- (4)
Subtracting (3) from (4),
$(312 - 292)y = 27 \times 31 - 33 \times 29 = -120$
$(31 - 29)(31 + 29)y = -120$
$120y = -120$
$y = -1$
Substituting in (1),
$31x - 29 = 33$
$31x = 62$
Hence,
$x = 2 \; and \; y = -1$

29. According to the question, radius of circle is = 5 cm.
Also, AB = 8 cm
Now, AM = $\frac{AB}{2}$ $=$4 cm

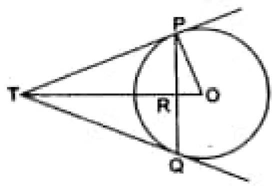


In $\triangle$OMA,
By using pythagoras theorem, we get

$OA^2 = OM^2 + AM^2$
$\therefore OM = \sqrt{5^2 - 4^2} = 3$ cm
Let AP = y cm, PM = x cm
In $\triangle$OAP,
By using pythagoras theorem, we get
$OP^2 = OA^2 = AP^2$
$(x + 3)^2 = y^2 + 25$
$\Rightarrow x^2 + 9 + 6x = y^2 + 25$ ...(i)
In $\triangle$AMP,
By using pythagoras theorem, we get
$x^2 + 4^2 = y^2$ ....(ii)
Substituting eq.(ii) in eq.(i), we get
$\Rightarrow x^2 + 6x + 9 = x^2 + 16 + 25$
$\Rightarrow 6x = 32$
$\Rightarrow x = \frac{32}{6} = \frac{16}{3}$ cm
Now, $y^2 = x^2 + 16 = \frac{256}{9} + 16 = \frac{400}{9}$
$\Rightarrow y = \frac{20}{3}$ cm.
Therefore, length of AP = y = $\frac{20}{3}$ cm.

OR

Given, Chord PQ = 4.8 cm , radius of circle = 3 cm
Also, The tangents at P and Q intersect at a point T as shown in the figure.We have to find the length of TP.



Let TR = y and TP= x
We know that the perpendicular drawn from the center to the chord bisects It.Hence, PR = RQ ....(1)
Since, PQ = 4.8
or, PR + RQ = 4.8
or, PR + PR = 4.8 [from (1)]
So, PR = 2.4

Now, in right triangle POR, Using Pythagoras theorem, we have
$PO^2 = OR^2 + PR^2$
$\Rightarrow 3^2 = OR^2 + (2.4)^2$
$\Rightarrow OR^2 = 3.24$
$\Rightarrow OR = 1.8$
Now, in right triangle TPR ,By Using Pythagoras theorem, we have
$TP^2 = TR^2 + PR^2$
$\Rightarrow x^2 = y^2 + (2.4)^2$
$\Rightarrow x^2 = y^2 + 5.76$ ...(2)
Again, In right triangle TPO By Using Pythagoras theorem, we have,
$TO^2 = TP^2 + PO^2$
$\Rightarrow (y + 1.8)^2 = x^2 + 3^2$
$\Rightarrow y^2 + 3.6y + 3.24 = x^2 + 9$
$\Rightarrow y^2 + 3.6y = x^2 + 5.76$ ......(3)
Solving (2) and (3), we get
x = 4cm and y = 3.2cm
$\therefore$ TP = 4cm.

30. We have,

$\Rightarrow \frac{1}{\operatorname{cosec} A-\cot A}-\frac{1}{\sin A}=\frac{1}{\sin A}-\frac{1}{\operatorname{cosec} A+\cot A}$

$\Rightarrow \frac{1}{\operatorname{cosec} A-\cot A}+\frac{1}{\operatorname{cosec} A+\cot A}=\frac{1}{\sin A}+\frac{1}{\sin A}$

$\Rightarrow \frac{1}{\operatorname{cosec} A-\cot A}+\frac{1}{\operatorname{cosec} A+\cot A}=\frac{2}{\sin A}$

LHS = $\frac{1}{\operatorname{cosec} A-\cot A}+\frac{1}{\operatorname{cosec} A+\cot A}$

$\Rightarrow \frac{\operatorname{cosec} A+\cot A+\operatorname{cosec} A-\cot A}{(\operatorname{cosec} A-\cot A)(\operatorname{cosec} A+\cot A)}$

$\Rightarrow \frac{2 \operatorname{cosec} A}{\operatorname{cosec}^2 A-\cot^2 A}$

$\Rightarrow \frac{\frac{2}{\sin A}}{1}=\frac{2}{\sin A}$ = RHS.

Hence Proved.

31. We are given the cumulative frequency distribution. So, we first construct a frequency table from the given cumulative frequency distribution and then we will make necessary computations to compute median

Class intervals	Frequency (f)	Cumulative frequency (c.f.)
20-30	4	4
30-40	12	16
40-50	14	30
50-60	16	46
60-70	20	66
70-80	16	82
80-90	10	92
90-100	8	100
		N = Σf_i = 100

Here, N = Σf_i = 100 $\therefore \frac{N}{2}$ = 50

We observe that the cumulative frequency just greater than $\frac{N}{2}$ = 50 is 66 and the corresponding class is 60-70.

So, 60-70 is the median class.

$\therefore$l = 60, f = 20, F = 46 and h = 10

Now, Median = l + $\frac{\frac{N}{2}-F}{f} \times$ h

$\Rightarrow$ Median = 60 + $\frac{50-46}{20} \times$ 10 = 62

Section D

32. $\frac{1}{a+b+x}=\frac{1}{a}+\frac{1}{b}+\frac{1}{x}$

$\Rightarrow \frac{1}{a+b+x}-\frac{1}{x}=\frac{1}{a}+\frac{1}{b}$

$\Rightarrow \quad \frac{-(a+b)}{x^2+(a+b) x}=\frac{b+a}{a b}$

$\Rightarrow \quad x^2+(a+b) x+a b=0$

$\Rightarrow \quad (x+a)(x+b)=0$

$\Rightarrow \quad x=-a, x=-b$

Hence ,x = - a, -b.

OR

x = - 4 is the root of the equation $x^2+2 x+4 p=0$

$(-4)^2+(2 \times-4)+4 p=0$

or, p = -2

Equation $x^2-2(1+3 k) x+7(3+2 k)=0$ has equal roots.

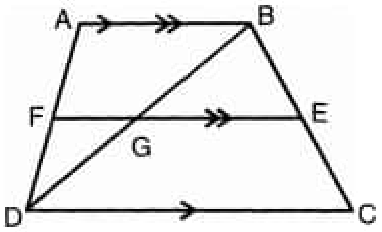
$\therefore 4(1+3 x)^2-28(3+2 k)=0$

or, $9 k^2-8 k-20=0$

or, (9k +10)(k-2) = 0

or, $k=\frac{-10}{9}, 2$

Hence, the value of k = $-\frac{10}{9}, 2$

33. 

In a trapezium ABCD, AB|| DC ,. EF || AB and CD=2AB

and also $\frac{BE}{EC}=\frac{4}{3}$ -------------(1)

AB || CD and AB || EF

$\therefore \frac{AF}{FD}=\frac{BE}{EC}=\frac{4}{3}$

In ΔBGE and ΔBDC

$\angle BEG=\angle BCD$ ($\because$ corresponding angles)

$\angle GBE=\angle DBC$ (Common)

$\therefore \Delta BGE\sim\Delta BDC$ [By AA similarity]

$\Rightarrow \frac{EG}{CD}=\frac{BE}{BC}$ ....(2)

Now, from (1) $\frac{BE}{EC}=\frac{4}{3}$

$\Rightarrow \frac{EC}{BE}=\frac{3}{4}$

$\Rightarrow \frac{EC}{BE}+1=\frac{3}{4}+1$

$\Rightarrow \frac{EC+BE}{BE}=\frac{7}{4}$

$\Rightarrow \frac{BC}{BE}=\frac{7}{4}$ or $\frac{BE}{BC}=\frac{4}{7}$

from equation (2), $\frac{EG}{CD}=\frac{4}{7}$

So $EG=\frac{4}{7}CD$ ......(3)

Similarly, $\Delta DGF\sim\Delta DBA$ (by AA similarity)

$\Rightarrow \frac{DF}{DA}=\frac{FG}{AB}$

$\Rightarrow \frac{FG}{AB}=\frac{3}{7}$

$\Rightarrow FG=\frac{3}{7}AB$ ...(4)

$$\left[\begin{array}{l}\because \frac{AF}{AD}=\frac{4}{7}=\frac{BE}{BC}\\ \Rightarrow \frac{EC}{BC}=\frac{3}{7}=\frac{DE}{DA}\end{array}\right]$$

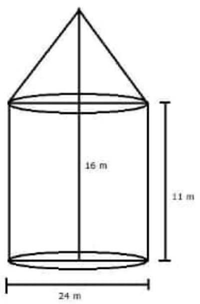
Adding equations (3) and (4),we get,

$EG+FG=\frac{4}{7}CD+\frac{3}{7}AB$

$\Rightarrow EF=\frac{4}{7}\times(2AB)+\frac{3}{7}AB$

$=\frac{8}{7}AB+\frac{3}{7}AB=\frac{11}{7}AB$

$\therefore \quad 7EF=11AB$

34. 

Diameter of cyliner = 24m

$\therefore$ radius of cylinder = radius of cone = 12m

Height of cylinder = 11m

Total height of tent = 16m

$\therefore$ Height of cone = 16 - 11 = 5m

Now, $l^2 = r^2 + h^2$

$\Rightarrow l^2 = 12^2 + 5^2$

$\Rightarrow l^2 = 144 + 25 = 169$

$\Rightarrow l = \sqrt{169} = 13m$

∴ Canvas required for tent = curved surface area of cone + curved surface area of cylinder

$= \pi rl + 2\pi rh$

$= \frac{22}{7} \times 12 \times 13 + 2 \times \frac{22}{7} \times 12 \times 11$

$= \frac{22}{7} \times 12 [13 + 2 \times 11]$

$= \frac{22}{7} \times 12 \times 35$

$= 22 \times 12 \times 5 = 1320 m^2$

OR

Given side of a cube = 21 cm

Diameter of the hemisphere is equal to the side of the cubical piece (d) = 21 cm

⇒ Radius of the hemisphere = 10.5 cm

Volume of cube = $Side^3$

$= (21)^3$

$= 9261\ cm^3$

Surface area of cubical piece of wood = $6a^2$

$= 6 \times 21 \times 21\ cm^2$

$= 2646\ cm^2$

Volume of the hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times 10.5 \times 10.5 \times 10.5$

$= 44 \times 0.5 \times 10.5 \times 10.5$

$= 2425.5\ cm^3$

Surface area of hemisphere = $2\pi r^2$

$= 2 \times \pi \times 10.5 \times 10.5$ cm

= 693 cm

Volume of remaining solid = Volume of cubical piece of wood – Volume of hemisphere

⇒ Volume of the remaining solid = 9261 - 2425.5

$= 6835.5\ cm^3$

Surface area remaining piece of solid = surface area of cubical piece of wood – Area of circular base of hemisphere + Curved Surface area of hemisphere

$= 6a^2 - \pi r^2 + 2\pi r^2$

$= (2646 - \pi \times 10.5^2 + 693)\ cm^2$

$= 2992.5\ cm^2$.

35.

Marks	**x**	**f**	**u = $\frac{x-35}{10}$**	**fu**	**cf**
0-10	5	3	-3	-9	3
10-20	15	5	-2	-10	8
20-30	25	16	-1	-16	24
30-40	35	12	0	0	36
40-50	45	13	1	13	49
50-60	55	20	2	40	69
60-70	65	6	3	18	75
70-80	75	5	4	20	80
		80		56	

Mean = $35 + \left(10 \times \frac{56}{80}\right) = 42$

Median class: 40 - 50

Median = $40 + \frac{10}{13}(40 - 36) = 43$

Section E

36. **Read the text carefully and answer the questions:**

Kamla and her husband were working in a factory in Seelampur, New Delhi. During the pandemic, they were asked to leave the job. As they have very limited resources to survive in a metro city, they decided to go back to their hometown in Himachal Pradesh. After a few months of struggle, they thought to grow roses in their fields and sell them to local vendors as roses have been always in demand. Their business started growing up and they hired many workers to manage their garden and do packaging of the flowers.

In their garden bed, there are 23 rose plants in the first row, 21 are in the 2nd, 19 in 3rd row and so on. There are 5 plants in the last row.

(i) The number of rose plants in the 1st, 2nd, ... are 23, 21, 19, ... 5

$a = 23$, $d = 21 - 23 = -2$, $a_n = 5$

$\therefore a_n = a + (n - 1)d$

or, $5 = 23 + (n - 1)(-2)$

or, $5 = 23 - 2n + 2$

or, $5 = 25 - 2n$

or, $2n = 20$

or, $n = 10$

(ii) Total number of rose plants in the flower bed,

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2(23) + (10-1)(-2)]$

$S_{10} = 5[46 - 20 + 2]$

$S_{10} = 5(46 - 18)$

$S_{10} = 5(28)$

$S_{10} = 140$

OR

$S_n = 80$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 80 = \frac{n}{2}[2 \times 23 + (n-1) \times -2]$

$\Rightarrow 80 = 23n - n^2 + n$

$\Rightarrow n^2 - 24n + 80 = 0$

$\Rightarrow (n - 4)(n - 20) = 0$

$\Rightarrow n = 4$ or $n = 20$

$n = 20$ not possible

$a_{20} = 23 + 19 \times (-2) = -15$

Number of plants cannot be negative.

$n = 4$

(iii) $a_n = a + (n - 1)d$

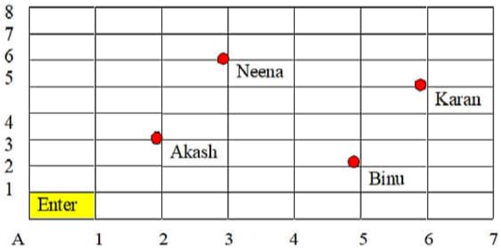
$\Rightarrow a_6 = 23 + 5 \times (-2)$

$\Rightarrow a_6 = 13$

37. **Read the text carefully and answer the questions:**

Karan went to the Lab near to his home for COVID 19 test along with his family members.

The seats in the waiting area were as per the norms of distancing during this pandemic (as shown in the figure). His family member took their seats surrounded by red circular area.



(i) Position of Neena = (3, 6)

Position of Karan = (6, 5)

Distance between Neena and Karan = $\sqrt{(6-3)^2+(5-6)^2}$

= $\sqrt{9+(-1)^2}$

= $\sqrt{10}$

(ii) Co-ordinate of seat of Akash = 2, 3

OR

Binu = (5, 5); Karan = (6, 5)

Distance = $\sqrt{(6-5)^2+(5-2)^2}$

$=\sqrt{1+9}$

$=\sqrt{10}$

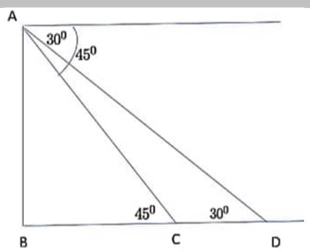
(iii) 

Co-ordinate of middle point = $\left(\frac{2+5}{2}, \frac{3+2}{2}\right)$

= 3.5, 2.5

38. **Read the text carefully and answer the questions:**

An observer on the top of a 40m tall light house (including height of the observer) observes a ship at an angle of depression 30^o coming towards the base of the light house along straight line joining the ship and the base of the light house. The angle of depression of ship changes to 45^o after 6 seconds.

(i) 

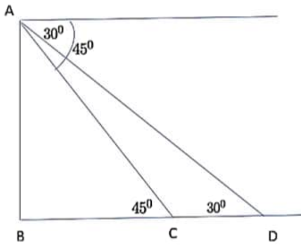
The distance of ship from the base of the light house after 6 seconds from the initial position when angle of depression is 45^o.

In $\triangle$ABC

tan 45^o = $\frac{AB}{BC}$

$\Rightarrow 1 = \frac{40}{BC}$

$\Rightarrow$ BC = 40 m

(ii) 

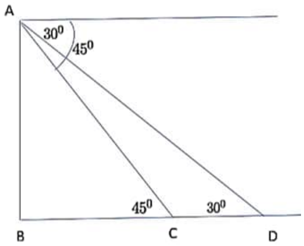
The distance between two positions of ship after 6 seconds

CD = BD - BC

$\Rightarrow$ CD = $40\sqrt{3}$ - 40 = $40(\sqrt{3} - 1)$

$\Rightarrow$ CD = 29.28 m

OR



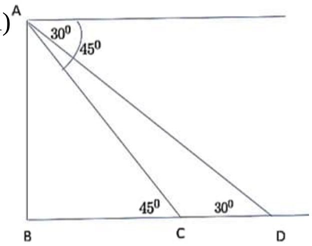
The distance of ship from the base of the light house when angle of depression is 30^o.

In $\triangle$ABD

$\tan 30^o = \frac{AB}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{40}{BD}$

$\Rightarrow$ BD = $40\sqrt{3}$ m

(iii) 

Speed of ship = $\frac{Distance}{Time} = \frac{29.28}{6}$ = 4.88 m/sec