Class X Session 2024-25
Subject - Mathematics (Standard)
Sample Question Paper - 4

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub- parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

Section A

1. $7 \times 11 \times 13 + 13$ is a/an: **[1]**

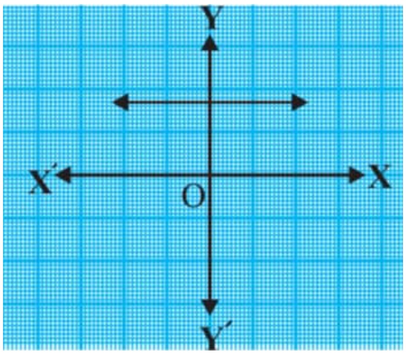
a) odd number but not composite

b) square number

c) prime number

d) composite number

2. The graph of y = p(x) in a figure given below, for some polynomial p(x). Find the number of zeroes of p(x). **[1]**



a) 4

b) 0

c) 1

d) 2

3. The pair of linear equations y = 0 and y = - 6 has: **[1]**

a) no solution

b) only solution (0, 0)

c) infinitely many solutions

d) a unique solution

4. The equation $x^2 - 8x + k = 0$ has real and distinct roots if **[1]**

a) k = 8

b) k > 16

c) k = 16

d) k < 16

5. If S,, denote the sum of n terms of an A.P. with first term a and common difference d such that $\frac{S_x}{S_{kx}}$ is independent of x, then **[1]**

a) a = 2d

b) d = a

c) d = - a

d) d = 2a

6. If three points (0,0), (3, $\sqrt{3}$) and (3, λ) form an equilateral triangle, then λ = **[1]**

a) -4

b) None of these

c) -3

d) 2

7. The coordinates of the mid-point of the line segment joining the points (-2, 3) and (4, -5) are **[1]**

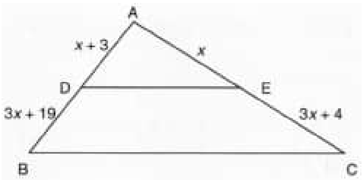
a) (0, 0)

b) (-1, 1)

c) (1, -1)

d) (-2, 4)

8. In the given figure value of x for which DE || BC is **[1]**



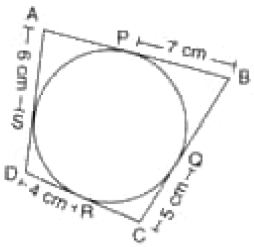
a) 3

b) 2

c) 4

d) 1

9. In the given figure, the perimeter of ABCD is **[1]**



a) 44 cm

b) 36 cm

c) 40 cm

d) 48 cm

10. Quadrilateral ABCD is circumscribed to a circle. If AB = 6 cm, BC = 7 cm and CD = 4 cm then the length of AD is **[1]**

a) 6 cm

b) 4 cm

c) 7 cm

d) 3 cm

11. If sinθ + cosθ =$\sqrt{2}$ cosθ, then the value of cosθ - sinθ is **[1]**

a) $\sqrt{2}\sin\theta$

b) 3sinθ

c) sinθ

d) 2sinθ

12. If $\tan\theta = \frac{a}{b}$, then $\frac{a\sin\theta+b\cos\theta}{a\sin\theta-b\cos\theta}$ is **[1]**

a) $\frac{a+b}{a-b}$

b) $\frac{a^2-b^2}{a^2+b^2}$

c) $\frac{a-b}{a+b}$

d) $\frac{a^2+b^2}{a^2-b^2}$

13. An observer 1.5 m tall is 28.5 m away from a tower and the angle of elevation of the top of the tower from the eye of the observer is 45°. The height of the tower is **[1]**

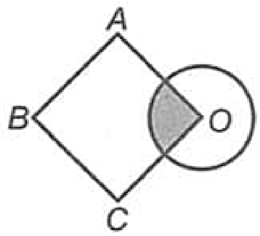
a) 30 m

b) 26.5 m

c) 28.5 m

d) 27 m

14. O is the centre of a circle of diameter 4 cm and OABC is a square, if the shaded area is $\frac{1}{3}$ area of the square, then the side of the square is _______. **[1]**



a) $\sqrt{3\pi}$ cm

b) $\pi\sqrt{3}$ cm

c) 3π cm

d) $3\sqrt{\pi}$ cm

15. Area of a segment of a circle of radius r and central angle 90^{o} is: **[1]**

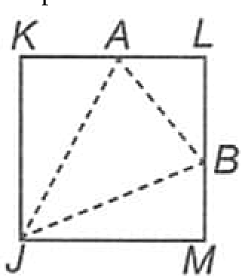
a) $\frac{2\pi r}{4} - \frac{1}{2}r^2$

b) $\frac{\pi r^2}{4} - \frac{1}{2}r^2$

c) $\frac{\pi r^2}{2} - \frac{1}{2}r^2$

d) $\frac{2\pi r}{4} - r^2 \sin 90^{o}$

16. In the given figure. JKLM is a square with sides of length 6 units. Points A and B are the mid-points of sides KL and LM respectively. If a point is selected at random from the interior of the square. What is the probability that the point will be chosen from the interior of $\triangle$JAB? **[1]**



a) $\frac{5}{8}$

b) $\frac{3}{8}$

c) $\frac{7}{8}$

d) $\frac{3}{4}$

17. A bag contains 5 red balls and n green balls. If the probability of drawing a green ball is three times that of a red ball, then the value of n is: **[1]**

a) 20

b) 18

c) 15

d) 10

18. Consider the frequency distribution of the heights of 60 students of a class: **[1]**

Height (in cm)	No. of Students	Cumulative Frequency
150-155	16	16
155-160	12	28
160-165	9	37
165-170	7	44
170-175	10	54
175-180	6	60

The sum of the lower limit of the modal class and the upper limit of the median class is

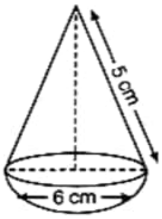
a) 320

b) 315

c) 330

d) 310

19. **Assertion (A):** The given figure represents a hemisphere surmounted by a conical block of wood. The diameter of their bases is 6 cm each and the slant height of the cone is 5 cm. The volume of the solid is 196 cm^3 **[1]**



Reason (R): The volume hemisphere is given by $\frac{2}{3}\pi r^3$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Sum of first 10 terms of the arithmetic progression -0.5, -1.0, -1.5, ... is 27.5 **[1]**

Reason (R): Sum of n terms of an A.P. is given as $S_n = \frac{n}{2}[2a + (n-1)d]$ where a = first term, d = common difference.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Find the least number which when divided by 12, 16 and 24 leaves remainder 7 in each case. **[2]**

22. In ΔABC, P and Q are points on sides AB and AC respectively such that $PQ||BC$. If AP = 4 cm, PB = 6 cm and PQ = 3 cm, determine BC. **[2]**

23. If a circle touches the side BC of a triangle ABC at P and extended sides AB and AC at Q and R, respectively, prove that **[2]**

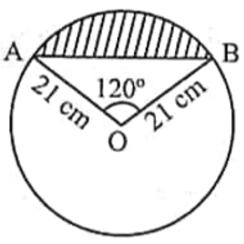
AQ = $\frac{1}{2}$(BC + CA + AB)

24. Prove that: $\sin^4 A - \cos^4 A = \sin^2 A - \cos^2 A = 2\sin^2 A - 1 = 1 - 2\cos^2 A$ **[2]**

OR

Prove that: $\frac{1-\cos A}{1+\cos A} = (\cot A - cosecA)^2$

25. Find the area of the segment shown in Fig., if radius of the circle is 21 cm and ∠AOB = 120^o (Use π = $\frac{22}{7}$) **[2]**



OR

A car has two wipers which do not overlap. Each wiper has a blade of length 25cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

Section C

26. Maya has two pieces of cloth. One piece is 36 inches wide and the other piece is 24 inches wide. She wants to cut both pieces into strips of equal width that are as wide as possible. How wide should she cut the strips? **[3]**

27. If α and β are zeroes of the quadratic polynomial $4x^2 + 4x + 1$, then form a quadratic polynomial whose zeroes are 2α and 2β. **[3]**

28. The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number. **[3]**

OR

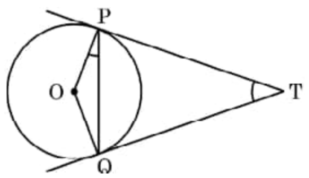
Graphically, solve the following pair of equations:

2x + y=6

2x - y + 2=0

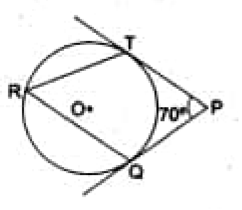
Find the ratio of the areas of the two triangles formed by the lines representing these equations with the x-axis and the lines with the y-axis.

29. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that ∠PTQ = 2 ∠OPQ. **[3]**



OR

In the given figure, O is the centre of a circle. PT and PQ are tangents to the circle from an external point P. If $\angle TPQ$ = 70°, find $\angle TRQ$.



30. If 1 + $\sin^2\theta$ = 3 sin θ cos θ, then prove that tan θ = 1, or $\frac{1}{2}$. **[3]**

31. Find median for the following data: **[3]**

Class Interval	**Frequency**
10 - 19	2
20 - 29	4

30 - 39	8
40 - 49	9
50 - 59	4
60 - 69	2
70 - 79	1

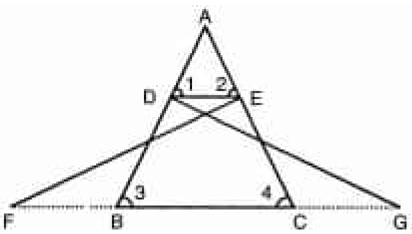
Section D

32. A train travels at a certain average speed for a distance of 360 km. It would have taken 48 minutes less to travel the same distance if its speed was 5 km/hour more. Find the original speed of the train. **[5]**

OR

If the roots of the quadratic equation (x - a) (x - b) + (x - b)(x - c) + (x - c)(x - a) = 0 are equal. Then show that a = b = c

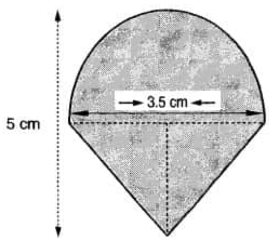
33. In the following figure, $\triangle FEC \cong \triangle GBD$ and ∠1 =∠2 Prove that $\triangle ADE \cong \triangle ABC$. **[5]**



34. A rocket is in the form of a right circular cylinder closed at the lower end and surmounted by a cone with the same radius as that of cylinder. The diameter and height of cylinder are 6 cm and 12 cm, respectively. If the slant height of the conical portion is 5 cm, then find the total surface area and volume of rocket. (Use π = 3.14) **[5]**

OR

Rasheed got a playing top (lattu) as his birthday present, which surprisingly had no colour on it. He wanted to colour it with his crayons. The top is shaped like a cone surmounted by a hemisphere. The entire top is 5 cm in height and the diameter of the top is 3.5 cm. Find the area he has to colour. (Take $\pi = \frac{22}{7}$).



35. Find the missing frequencies in the following distribution, if the sum of the frequencies is 120 and the mean is 50. **[5]**

Class	0-20	20-40	40-60	60-80	80-100
Frequency	17	f_1	32	f_2	19

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Deepa has to buy a scooty. She can buy scooty either making cashdown payment of ₹ 25,000 or by making 15 monthly instalments as below.

Ist month - ₹ 3425, IInd month - ₹ 3225, IIIrd month - ₹ 3025, IVth month - ₹ 2825 and so on

(a) Find the amount of 6th instalment.

(b) Total amount paid in 15 instalments.

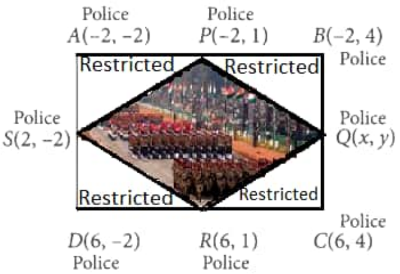
OR

If Deepa pays ₹2625 then find the number of instalment.

(c) Deepa paid 10th and 11th instalment together find the amount paid that month.

37. **Read the text carefully and answer the questions:** **[4]**

In order to facilitate smooth passage of the parade, movement of traffic on certain roads leading to the route of the Parade and Tableaux ah rays restricted. To avoid traffic on the road Delhi Police decided to construct a rectangular route plan, as shown in the figure.



(a) If Q is the mid point of BC, then what are the coordinates of Q?

(b) What is the length of the sides of quadrilateral PQRS?

OR

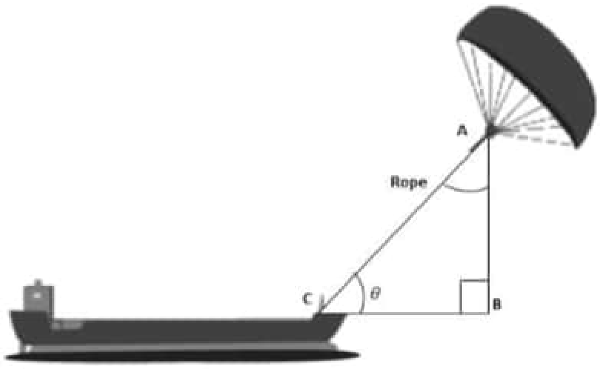
What is the length of route ABCD?

(c) What is the length of route PQRS?

38. **Read the text carefully and answer the questions:** **[4]**

Skysails is the genre of engineering science that uses extensive utilization of wind energy to move a vessel in the seawater. The 'Skysails' technology allows the towing kite to gain a height of anything between 100 metres - 300 metres. The sailing kite is made in such a way that it can be raised to its proper elevation and then brought back with the help of a 'telescopic mast' that enables the kite to be raised properly and effectively.

Based on the following figure related to sky sailing, answer the following questions:



(a) In the given figure, if $\sin\theta = \cos(\theta - 30^o)$, where θ and $\theta - 30^o$ are acute angles, then find the value of θ.

(b) What should be the length of the rope of the kite sail in order to pull the ship at the angle θ (calculated above) and be at a vertical height of 200m?

OR

What should be the length of the rope of the kite sail in order to pull the ship at the angle θ (calculated above) and be at a vertical height of 150m?

(c) In the given figure, if $\sin\theta = \cos(3\theta - 30^o)$, where θ and $3\theta - 30^o$ are acute angles, then find the value of θ.

Solution

Section A

1.

(d) composite number

Explanation: We have 7 × 11 × 13 + 13 = 13 (77 + 1) = 13 × 78. Since the given number has 2 more factors other than 1 and itself, therefore it is a composite number.

2.

(b) 0

Explanation: There is no zero as the graph does not intersect the x-axis at any point.

3. **(a)** no solution

Explanation: Since, we have y = 0 and y = -6 are two parallel lines.
therefore, no solution exists.

4.

(d) k < 16

Explanation: D > 0

$b^2 - 4ac > 0$

$(-8)^2 - 4(1)(k) > 0$

64 - 4k > 0

64 > 4k

$\left(\frac{64}{4}\right) > k$

16 > k

5.

(d) d = 2a

Explanation: Given AP in which

First term = a

Common difference = d

Number of terms = n

Given S_n denotes the sum of n terms

So

$S_{kx} = (kx/2)\{2a + (kx-1)d\}$

and $S_x = (x/2)\{2a + (x-1)d\}$

Now

$$S_x / S_{kx} = \frac{(\frac{x}{2})[2a+(x-1)d]}{(\frac{kx}{2})[2a+(kx-1)d]}$$

$$= \frac{[2a+(x-1)d]}{k[2a+(kx-1)d]}$$

$$= \frac{[2a+xd-d]}{k[2a+kxd-d]}$$

$$= \frac{[(2a-d)+xd]}{k[(2a-d)+kxd]]}$$

If d = 2a

then

$S_x / S_{kx} = [(2a - 2a) + x \times 2a]/[k \times \{(2a-2a) + k \times x \times 2a\}]$

$= (x \times 2a)/(k^2 \times x \times 2a)$

$= 1/k^2$

6.

(b) None of these

Explanation: Let the points (0, 0), $(3, \sqrt{3})$ and $(3, \lambda)$ from an equilateral triangle AB = BC = CA

$\Rightarrow AB^2 = BC^2 = CA^2$

Now, $AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$= (3-0)^2 + (\sqrt{3}-0)^2 = (3)^2 + (\sqrt{3})^2$

= 9 + 3 = 12

$\text{BC}^2 = (3-3)^2 + (\lambda - \sqrt{3})^2$

$= (0)^2 + (\lambda - \sqrt{3})^2 = (\lambda - 3)^2$

and $\text{CA}^2 = (0-3)^2 + (0-\lambda)^2 = (-3)^2 + (-\lambda)^2$

$= 9 + \lambda^2$

$AB^2 = CA^2 \Rightarrow 12 = 9 + \lambda^2$

$\Rightarrow \lambda^2 = 12 - 9 = 3$

$\therefore \lambda = \pm\sqrt{3}$

7.

(c) (1, -1)

Explanation: Let the coordinates of midpoint C(x, y) of the line segment joining the points A(-2, 3) and B(4, -5)

$\therefore$ x = $\frac{x_1+x_2}{2}$ = $\frac{-2+4}{2} = \frac{2}{2}$ = 1

And y = $\frac{y_1+y_2}{2}$ = $\frac{3-5}{2} = \frac{-2}{2}$ = -1

Therefore, the coordinates of mid-point C are (1, -1)

8.

(b) 2

Explanation: In $\triangle$ABC, DE || BC

$\therefore \frac{\text{AD}}{\text{DB}} = \frac{\text{AE}}{\text{EC}}$

$\Rightarrow \frac{x+3}{3x+19} = \frac{x}{3x+4}$

$\Rightarrow$ (x + 3) (3x + 4) = x (3x + 19)

$\Rightarrow 3x^2 + 4x + 9x + 12 = 3x^2 + 19x$

$\Rightarrow 3x^2 + 13x + 12 = 3x^2 + 19x$

$\Rightarrow 12 = 3x^2 + 19x - 3x^2 - 13x$

$\Rightarrow$ 12 = 6x $\Rightarrow$ x = $\frac{12}{6}$ = 2

$\therefore$ x = 2

9. **(a)** 44 cm

Explanation: Since tangents from an external point to a circle are equal in length.

$\therefore$ AS = AP = 6 cm and AB = 6 + 7 = 13 cm

PB = BQ = 7 cm and BC = 7 + 5 = 12 cm

CQ = CR = 5 cm and CD = 5 + 4 = 9 cm

RD = SD = 4 cm and AD = 4 + 6 = 10 cm

Therefore, perimeter of quadrilateral ABCD = 13 + 12 + 9 + 10 = 44 cm

10.

(d) 3 cm

Explanation: A quadrilateral ABCD is circumscribed to a circle with centre O.

AB = 6 cm, BC = 7 cm, CD = 4 cm, AD = 7 cm

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ABCD circumscribed to a circle.

AB + CD = BC + AD

$\Rightarrow$ 6 + 4 = 7 + AD

$\Rightarrow$ 10 = 7 + AD

AD = 10 - 7 = 3 cm

11. **(a)** $\sqrt{2}\sin\theta$

Explanation: Given: $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$

Squaring both sides, we get

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2\cos^2\theta$

$\Rightarrow \cos^2\theta - 2\sin\theta\cos\theta = \sin^2\theta$

$\Rightarrow \cos^2\theta$ - $2\sin\theta \cos\theta$ + $\sin^2\theta$ = $2\sin^2\theta$

$\Rightarrow (\cos\theta - \sin\theta)^2 = 2\sin^2\theta$

$\Rightarrow \cos\theta - \sin\theta = \sqrt{2}\sin\theta$

12.

(d) $\frac{a^2+b^2}{a^2-b^2}$

Explanation: $\tan\theta = \frac{a}{b}$

$\frac{a\sin\theta+b\cos\theta}{a\sin\theta-b\cos\theta} = \frac{a\frac{\sin\theta}{\cos\theta}+b\frac{\cos\theta}{\cos\theta}}{a\frac{\sin\theta}{\cos\theta}-b\frac{\cos\theta}{\cos\theta}}$ (Dividing by cos θ)

= $\frac{a\tan\theta+b}{a\tan\theta-b} = \frac{a\times\frac{a}{b}+b}{a\times\frac{a}{b}-b}$

= $\frac{\frac{a^2}{b}+b}{\frac{a^2}{b}-b} = \frac{\frac{a^2+b^2}{b}}{\frac{a^2-b^2}{b}}$

= $\frac{a^2+b^2}{b} \times \frac{b}{a^2-b^2}$

= $\frac{a^2+b^2}{a^2-b^2}$

13. **(a)** 30 m

Explanation: Let AB be the observer and CD =h metres be the tower.

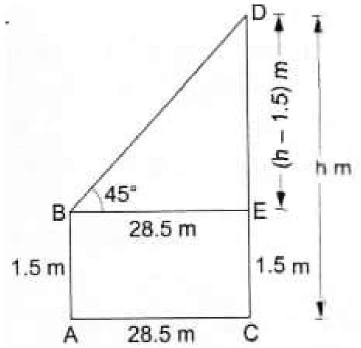
BE = AC = 28.5 m.

From right $\triangle$BED, we have

$\frac{DE}{BE}$ = tan 45^o $\Rightarrow$ $\frac{DE}{28.5\text{m}}$ = 1

$\Rightarrow$ De = 28.5 m

$\therefore$ h - 1.5 = 28.5 $\Rightarrow$ h = 30.



14. **(a)** $\sqrt{3\pi}$ cm

Explanation: Let the length of side of square be x cm

Then area of square = x^2 cm^2

Area of sector of circle

= $\frac{\theta}{360^\circ} \times \pi r^2$

= $\frac{90^\circ}{360^\circ} \times \pi r^2$ [$\because$ angle of square = θ = 90^o]

$\therefore$ Shaded area = $\frac{\pi\times4}{4}$ = π

According to question, Area of square = 3 $\times$ shaded area

$\Rightarrow 3\pi = x^2 \therefore x = \sqrt{3\pi}$ cm

15.

(b) $\frac{\pi r^2}{4} - \frac{1}{2}r^2$

Explanation: $\frac{\pi r^2}{4} - \frac{1}{2}r^2$

16.

(b) $\frac{3}{8}$

Explanation: Area of square JMLK = 6^2 = 36 sq. units

A and B are the mid-points of sides KL and LM.

$\therefore$ AL = KA = LB = BM = 3 units

Now, Area of $\triangle$ALB = $\frac{1}{2}$ $\times$ AL $\times$ LB = $\frac{1}{2}$ $\times$ 3 $\times$ 3 = $\frac{9}{2}$ sq. units

Area of $\triangle$JMB = $\frac{1}{2} \times$ BM $\times$ JM = $\frac{1}{2} \times 6 \times 3$ = 9 sq. units
Area of $\triangle$KAJ = $\frac{1}{2} \times$ KJ $\times$ KA = $\frac{1}{2} \times 6 \times 3$ = 9 sq. units
Total area of all the three triangles = ($\frac{9}{2}$ + 9 + 9)
= $\frac{45}{2}$ sq. units
$\therefore$ Area of $\triangle$JAB = (36 - $\frac{45}{2}$) = $\frac{27}{2}$ sq. units
$\therefore$ Required probability = $\frac{\frac{27}{2}}{36} = \frac{27}{2\times36} = \frac{3}{8}$

17.

(c) 15

Explanation: Given, Number of red ball = 5
Number of green ball = n
$\therefore$ Total ball = n + 5
Now P(red ball) = $\frac{5}{n+5}$
and P(green ball) = $\frac{n}{n+5}$
Now, according to the question
$\frac{n}{n+5} = \frac{3\times5}{n+5}$
n = 15
So, number of green ball = 15

18.

(b) 315

Explanation: Class having maximum frequency is the modal class.
hence, modal class : 150-155
$\therefore$ Lower limit of the modal class = 150
Also, $\mathrm{N} = 60 \Rightarrow \frac{\mathrm{N}}{2} = 30$
The cumulative frequency just greater than 30 is 37.
Hence, the median class is 160-165.
$\therefore$ Upper limit of the median class = 165
Required sum = 150 + 165 = 315

19.

(d) A is false but R is true.

Explanation: A is false but R is true.

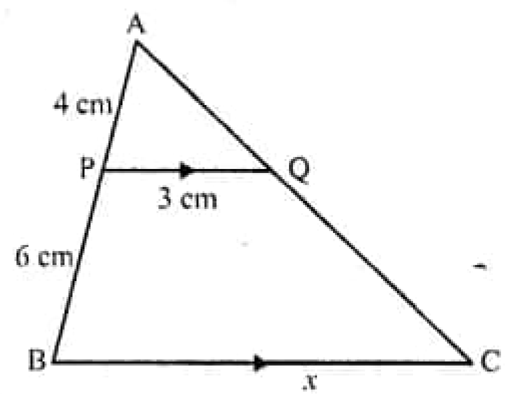
20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both are correct. Reason is the correct reasoning for Assertion.
Assertion, S_{10} = $\frac{10}{2}[2(-0.5) + (10-1)(-0.5)]$
= 5[-1 - 4.5]
= 5(-5.5) = 27.5

Section B

21. LCM of 12, 16, 24 = 48
Required number is 48 + 7 = 55.

22. Let BC = x cm



In Δ's APQ and ABC,we have,

$\angle A = \angle A$

$\angle APQ = \angle ABC$

Therefore,by AA criteria of similar Δ's ,we have,

$\because \mathrm{PQ} \| \mathrm{BC}$

$\therefore \Delta\mathrm{APQ} \sim \Delta\mathrm{ABC}$

$\therefore \frac{\mathrm{AP}}{\mathrm{AB}} = \frac{\mathrm{AQ}}{\mathrm{AC}} = \frac{\mathrm{PQ}}{\mathrm{BC}}$

$\Rightarrow \frac{\mathrm{AP}}{\mathrm{AP}+\mathrm{PB}} = \frac{\mathrm{PQ}}{\mathrm{BC}}$

$\Rightarrow \frac{4}{4+6} = \frac{3}{x} \Rightarrow \frac{4}{10} = \frac{3}{x}$

$\Rightarrow x = \frac{10\times 3}{4} = \frac{15}{2}$

$\therefore \mathrm{BC} = \frac{15}{2}\mathrm{cm} = 7.5$ cm

23. AQ = $\frac{1}{2}$(2AQ)

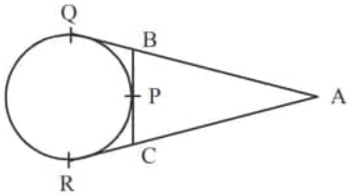
= $\frac{1}{2}$(AQ + AQ)

= $\frac{1}{2}$(AQ + AR)

= $\frac{1}{2}$(AB + BQ + AC + CR)

= $\frac{1}{2}$(AB + BC + CA)

$\because$ [BQ = BP, CR = CP]



24. We have,

LHS = $\sin^4$ A - $\cos^4$ A

$\Rightarrow$ LHS = $(\sin^2 A)^2 - (\cos^2 A)^2$

$\Rightarrow$ LHS = $(\sin^2 A + \cos^2 A)(\sin^2 A - \cos^2 A)$

$\Rightarrow$ LHS = $\sin^2 A - \cos^2 A$ ...[$\because \sin^2 A + \cos^2 A = 1$]

$\Rightarrow$ LHS = $\sin^2 A - (1 - \sin^2 A) = 2\sin^2 A - 1$

$\Rightarrow$ LHS = $2(1 - \cos^2 A) - 1 = 1 - 2\cos^2 A$ = RHS

OR

LHS $= \frac{1-\cos A}{1+\cos A}$

Multiplying numerator and denominator by 1 - cos A

$= \frac{(1-\cos A)(1-\cos A)}{(1+\cos A)(1-\cos A)}$

$= \frac{(1-\cos A)^2}{1-\cos^2 A} \left[\because (a+b)(a-b) = a^2 - b^2\right]$

$= \frac{(1-\cos A)^2}{\sin^2 A} \left[\because 1 - \cos^2 A = \sin^2 A\right]$

$= \left(\frac{1-\cos A}{\sin A}\right)^2$

$= \left(\frac{1}{\sin A} - \frac{\cos A}{\sin A}\right)^2$

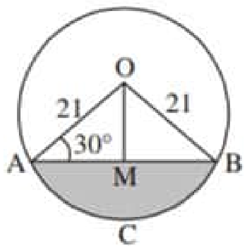
$= (\cos ecA - \cot A)^2 \left[\because \frac{1}{\sin A} = \cos ec\ A, \frac{\cos A}{\sin A} = \cot A\right]$

$= [-1(\cot A - \cos ecA)]^2$

= $(\cot A - \operatorname{cosec} A)^2$

Hence proved.

25. Draw OM $\perp$ AB



$\angle$OAB = $\angle$OBA = 30^o

sin 30^o = $\frac{1}{2} = \frac{\text{OM}}{21} \Rightarrow$ OM = $\frac{21}{2}$

cos 30^o = $\frac{\sqrt{3}}{2} = \frac{\text{AM}}{21} \Rightarrow$ AM = $\frac{21}{2}\sqrt{3}$

Area of $\triangle$OAB = $\frac{1}{2} \times \text{AB} \times \text{OM}$ = $\frac{1}{2} \times 21\sqrt{3} \times \frac{21}{2}$

= $\frac{441}{4}\sqrt{3}\,\text{cm}^2$

$\therefore$ Area of shaded region = Area (sector OACB) - Area ($\triangle$OAB)

= $\frac{22}{7} \times 21 \times 21 \times \frac{120}{360} - \frac{441}{4}\sqrt{3}$

= $\left(462 - 441\frac{\sqrt{3}}{4}\right)\text{cm}^2$ or 271.3 cm^2 (approx.)

OR

Radius of each wiper = 25cm, Angle = 115°

$\therefore$ θ = 115°

Total area cleaned at each sweep of the blades

$= 2\left[\frac{115}{360} \times \frac{22}{7} \times 25 \times 25\right]\left(\because \text{ Area } = \frac{\theta}{360}\pi r^2\right)$

$= \frac{230 \times 22 \times 5 \times 25}{72 \times 7}$

$= \frac{230 \times 11 \times 125}{36 \times 7}$

$= \frac{115 \times 11 \times 125}{18 \times 7}$

$= \frac{158125}{126}\text{cm}^2$

$= 1254.96\text{cm}^2$

Section C

26. This problem can be solved using H.C.F. because we are cutting or “dividing” the strips of cloth into smaller pieces of 36 and 24 and we are looking for the widest possible strips .

So,

H.C.F. of 36 and 24 is 12

So we can say that

Maya should cut each piece to be 12 inches wide.

27. Let the given polynomial is p(x) = $4x^2 + 4x + 1$

Since, α, β are zeroes of p(x),

$\therefore \alpha + \beta$= sum of zeroes = $\frac{-4}{4}$

Also, $\alpha.\beta$= Product of zeroes = $\alpha.\beta = \frac{1}{4}$

Now a quadratic polynomial whose zeroes are 2α and 2β

x^2 - (sum of zeroes)x + Product of zeroes

$= x^2 - (2\alpha + 2\beta)x + 2\alpha \times 2\beta$

$= x^2 - 2(\alpha + \beta)x + 4(\alpha\beta)$

$= x^2 - 2 \times (-1)x + 4 \times \frac{1}{4}$

= $x^2 + 2x + 1$

The quadratic polynomial whose zeroes are 2α and 2β is $x^2 + 2x + 1$

28. Let the digits at units and tens place of the given number be x and y respectively

Thus, the number is $10y + x$.

The sum of the two digits of the number is 9.

Thus, we have $x + y = 9$ .....(i)

After interchanging the digits, the number becomes $10x + y$.

Also, 9 times the number is equal to twice the number obtained by reversing the order of the digits.

Thus, we have

$9(10y + x) = 2(10x + y)$

$\Rightarrow 90y + 9x = 20x + 2y$

$\Rightarrow 20x + 2y - 90y - 9x = 0$

$\Rightarrow 11x - 88y = 0$

$\Rightarrow 11(x - 8y) = 0$

$\Rightarrow x - 8y = 0$ .....(ii)

So, we have the systems of equations

x + y = 9,
x - 8y = 0
Here x and y are unknowns.
Substituting $x = 8y$ from the second equation to the first equation, we get
$8y + y = 9$
$\Rightarrow 9y = 9$
$\Rightarrow y = \frac{9}{9}$
$\Rightarrow y = 1$
Substituting the value of y in the second equation, we have
$x - 8 \times 1 = 0$
$\Rightarrow x - 8 = 0$
$\Rightarrow x = 8$
$\therefore$ the number is $10 \times 1 + 8 = 18$

OR

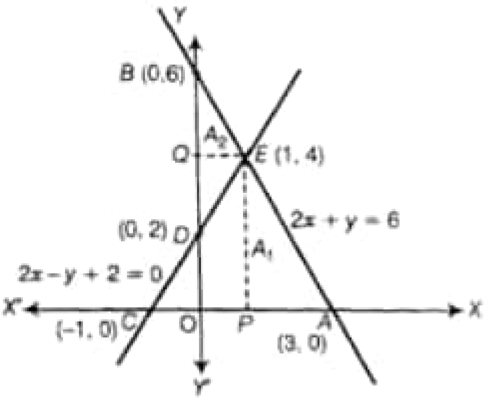
Given equation is $2x + y = 6$
$\Rightarrow$ y = 6 - 2x.......(i)
If, $x = 0, y = 6 - 2(0) = 6$
$x = 3, y = 6 - 2(3) = 0$

x	0	3
y	6	0
Points	B	A

Given equation is $2x - y + 2 = 0$
$\Rightarrow y = 2x + 2$ ...(ii)
If, $x = 0, y = 2(0) + 2 = 0 + 2 = 2$
$x = -1, y = 2(-1) + 2 = 0$

x	0	-1
y	2	0
Points	D	C

Plotting 2*x* + *y* = 6 and 2*x* - *y* + 2= 0, as shown below, we obtain two lines AB and CD respectively intersecting at point, *E* (1, 4).



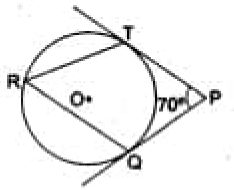
Now, A_1 = Area of ACE = $\frac{1}{2} \times AC \times PE$
$= \frac{1}{2} \times 4 \times 4 = 8$
And A_2 = Area of BDE = $\frac{1}{2} \times BD \times QE$
$= \frac{1}{2} \times 4 \times 1 = 2$
$\therefore A_1 : A_2$ = 8:2 = 4: 1
$\therefore$ Ratio of areas of two $\triangle s \quad = \frac{\text{Area } \Delta\text{ACE}}{\text{Area } \Delta\text{BDE}} = \frac{8}{2} = \frac{4}{1} = 4:1$

29. Given : A circle with centre O and an external point T and two tangents TP and TQ to the circle, where P, Q are the points of contact.
To Prove: ∠PTQ = 2∠OPQ

Proof: Let ∠PTQ = θ
Since TP, TQ are tangents drawn from point T to the circle.
TP = TQ
∴ TPQ is an isoscles triangle
∴ ∠TPQ = ∠TQP = $\frac{1}{2}$ (180^o - θ) = 90^o - $\frac{\theta}{2}$
Since, TP is a tangent to the circle at point of contact P
∴ ∠OPT = 90^o
∴ ∠OPQ = ∠OPT - ∠TPQ = 90^o - (90^o - $\frac{1}{2}$ θ) = $\frac{\theta}{2}$ = $\frac{1}{2}$∠PTQ
Thus, ∠PTQ = 2∠OPQ

OR

In the given figure, O is the centre of a circle. PT and PQ are tangents to the circle from an external point P. If $\angle TPQ$ = 70°, then,we have to find $\angle TRQ$.



We know that the radius and tangent are perpendicular at their point of contact.
$\angle OTP = \angle OQP = 90^\circ$
Now, In quadrilateral OQPT
$\angle QOT + \angle OTP + \angle OQP + \angle TPQ = 360^\circ$ [Angle sum property of a quadrilateral]
$\angle QOT + 90^\circ + 90^\circ + 70^\circ = 360^\circ$
$250^\circ + \angle QOT = 360^\circ$
$\angle QOT = 110^\circ$
We know that the angle subtended by an arc at the centre is double of the angle subtended by the arc at any point on the circumference of the circle.
$\angle TRQ = \frac{1}{2}\angle QOT \Rightarrow \angle TRQ = \frac{1}{2} \times 110^\circ = 55^\circ$

30. Given, 1 + $\sin^2\theta$ = 3 sin θ cos θ, then we have to prove that tan θ = 1, or $\frac{1}{2}$.

Now, 1 + $\sin^2\theta$ = 3 sinθ cosθ

[Dividing by $\sin^2\theta$ 0n both sides]
$\Rightarrow \frac{1}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{3\sin\theta\cos\theta}{\sin^2\theta}$
⇒ $\text{cosec}^2\theta$ + 1 = 3 cotθ
⇒ 1 + $\cot^2\theta$ + 1 – 3 cotθ = 0
⇒ $\cot^2\theta$ – 3 cot θ + 2 = 0
⇒ $\cot^2\theta$ – 2 cot θ –cot θ + 2 = 0
⇒ cot θ(cot θ – 2) –1(cot θ – 2) = 0
⇒ (cotθ – 2) (cot θ – 1) = 0
⇒ cotθ – 2 = 0 or (cot θ – 1) = 0
⇒ cotθ = 2 or cot θ = 1
⇒ tanθ = $\frac{1}{2}$ or tan θ = 1
Hence, either, tan θ = $\frac{1}{2}$, or 1

31. Calculation of Median:

Class Interval	**Frequency**	**c.f.**
9.5 - 19.5	2	2
19.5 - 29.5	4	6
29.5 - 39.5	8	14
39.5 - 49.5	9	23
49.5 - 59.5	4	27

59.5 - 69.5	2	29
69.5 - 79.5	1	30

n = 30, $\frac{n}{2}$ = 15, Median class = 39.5 - 49.5

$l = 39.5, c.f. = 14, f = 9, h = 10$

Median = 39.5 + $\left(\frac{15-14}{9}\right) \times 10$

= 39.5 + $\frac{1}{9} \times 10$

$= 39.5 + 1.11 = 40.61$

Section D

32. Given that a train travelling at a uniform speed for 360 km

Let the original speed of the train be x km/hr

Time taken $= \frac{\text{Distance}}{\text{Speed}} = \frac{360}{x}$

Time taken at increased speed $= \frac{360}{x+5}$ hours.

According to the question

$\frac{360}{x} - \frac{360}{x+5} = \frac{48}{60}$

$360\left[\frac{1}{x} - \frac{1}{x+5}\right] = \frac{4}{5}$

$or, \frac{360(x+5-x)}{x^2+5x} = \frac{4}{5}$

$or, \frac{1800}{x^2+5x} = \frac{4}{5}$

$\Rightarrow \quad x^2 + 5x - 2250 = 0$

$\Rightarrow \quad x^2 + (50-45)x - 2250 = 0$

$\Rightarrow \quad x^2 + 50x - 45x - 2250 = 0$

$\Rightarrow \quad (x+50)(x-45) = 0$

Either x = - 50 or x = 45

As speed cannot be negative

$\therefore$ Original speed of train = 45 km/hr.

OR

Given,

(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0

$\Rightarrow x^2 - ax - bx + ab + x^2 - bx - cx + bc + x^2 - cx - ax + ac = 0$

$\Rightarrow 3x^2 - 2ax - 2bx - 2cx + ab + bc + ca = 0$

For equal roots $B^2 - 4AC = 0$

or, $\{-2(a+b+c)\}^2 = 4 \times 3(ab+bc+ca)$

or, $4(a+b+c)^2 - 12(ab+bc+ca) = 0$

or, $a^2 + b^2 + c^2 + 2ab + 2bc + 2ac - 3ab - 3bc - 3ac = 0$

or, $\frac{1}{2}\left[2a^2 + 2b^2 + 2c^2 - 2ab - 2ac - 2bc\right] = 0$

or, $\frac{1}{2}\left[\left(a^2 + b^2 - 2ab\right) + \left(b^2 + c^2 - 2bc\right) + \left(c^2 + a^2 - 2ac\right)\right] = 0$

or, $\frac{1}{2}\left[\left(a^2 + b^2 - 2ab\right) + \left(b^2 + c^2 - 2bc\right) + \left(c^2 + a^2 - 2ac\right)\right] = 0$

or, $(a-b)^2 + (b-c)^2 + (c-a)^2 = 0$ if $a \neq b \neq c$

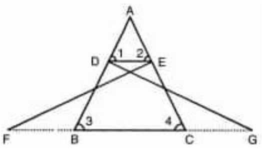
Since $(a-b)^2 > 0, (b-c)^2 > 0(c-a)^2 > 0$

Hence, $(a-b)^2 = 0 \Rightarrow a = b$

$(a-c)^2 = 0 \Rightarrow b = c$

$(c-a)^2 = 0 \Rightarrow c = a$

$\therefore a = b = c$ Hence Proved.

33. 

$\because \triangle FEC \cong \triangle GBD$

or, EC = BD ......(i)

It is given that $\angle 1 = \angle 2$

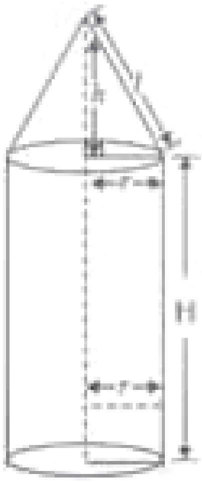
or, AE = AD ($\because$ Isosceles triangle property)...(ii)

From ,eqns. (i) and (ii),
$\frac{AE}{EC} = \frac{AD}{DB}$
or, DE || BC, ($\because$ converse of B.PT)
or, $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$ ($\because$ Corresponding angles)
Thus in $\triangle ADE$ and $\triangle ABC$,
$\angle$A $= \angle$A
$\angle 1 = \angle 3$
$\angle 2 = \angle 4$
$\triangle ADE \sim \triangle ABC$ ($\because$ AAA criterion of similarity)
$\triangle ADE \sim \triangle ABC$ Hence proved

34.

Cylinder	**Cone**
r = $\frac{6}{2}$ = 3 cm	r = 3 cm
H = 12 cm	l = 5 cm

For cone,
$\therefore$ $l^2 = r^2 + h^2$ + or $h^2 = l^2 - r^2$
$h^2 = 5^2 - 3^2 = 25 - 9 = 16$
$\Rightarrow$ h = $\sqrt{16}$ = 4 cm



Now, volume of rocket = Volume of cylinder + Volume of cone
$= \pi r^2 H + \frac{1}{3}\pi r^2 h = \pi r^2 \left[H + \frac{1}{3}h\right]$
= 3.14 × 3 × 3$\left[12 + \frac{1}{3} \times 4\right]$
= 3.14 × 9$\left[\frac{40}{3}\right]$ = 3.14 × 3 × 40 = 376.8 cm^3
$\therefore$ Volume of Rocket = 376.8 cm^3

Total surface area of rocket = Curved surface area of cylinder + Curved surface area of cone + Area of base of cylinder [As it is closed (Given)]
= 2πrH + πrl + πr^2 = πr [2H + l + r]
= 3.14 × 3[2 × 12 + 5 + 3]
= 3.14 × 3 × 32
= 301.44 cm^2
Hence, the surface area of rocket is 301.44 cm^2.

OR

Surface area to colour = surface area of hemisphere + curved surface area of cone
Diameter of hemisphere = 3.5 cm
So radius of hemispherical portion of the lattu = r = $\frac{3.5}{2}$cm = 1.75
r = Radius of the concial portion = $\frac{3.5}{2}$ = 1.75
Height of the conical portion = height of top - radius of hemisphere = 5 - 1.75 = 3.25 cm

Let I be the slant height of the conical part. Then,

$l^2 = h^2 + r^2$

$l^2 = (3.25)^2 + (1.75)^2$

$\Rightarrow l^2 = 10.5625 + 3.0625$

$\Rightarrow l^2 = 13.625$

$\Rightarrow l = \sqrt{13.625}$

$\Rightarrow l = 3.69$

Let S be the total surface area of the top. Then,

$S = 2\pi r^2 + \pi rl$

$\Rightarrow \quad S = \pi r(2r + l)$

$\Rightarrow S = \frac{22}{7} \times 1.75(2 \times 1.75 + 3.7)$

$= 5.5(3.5 + 3.7)$

$= 5.5(7.2)$

$= 39.6\ cm^2$

35.

Class Interval	**Frequency f_i**	**Mid-value x_i**	**f_i x_i**
0-20	17	10	170
20-40	f_1	30	$30f_1$
40-60	32	50	1600
60-80	f_2	70	$70f_2$
80-100	19	90	1710
	$\sum f_i = 68 + f_1 + f_2 = 120$		$\sum f_i x_i = 3480 + 30f_1 + 70f_2$

given mean = 50

Mean= $\frac{\sum f_i x_i}{\sum f_i}$

$\Rightarrow 50 = \frac{3480 + 30f_1 + 70f_2}{120}$

$\Rightarrow 6000 = 3480 + 30f_1 + 70f_2$

$\Rightarrow 30f_1 + 70f_2 = 252$ ...(i)

Also, $68 + f_1 + f_2 = 120$

$\Rightarrow f_1 = 52 - f_2$

Substituing in (i), we have

$3(52 - f_2) + 7f_2 = 252$

$\Rightarrow 4f_2 = 96$

$\Rightarrow f_2 = -24$

$\Rightarrow f_1 = 52 - 24 = 28$

Hence, $f_1 = 28$ and $f_2 = 24$

Section E

36. **Read the text carefully and answer the questions:**

Deepa has to buy a scooty. She can buy scooty either making cashdown payment of ₹ 25,000 or by making 15 monthly instalments as below.

Ist month - ₹ 3425, IInd month - ₹ 3225, IIIrd month - ₹ 3025, IVth month - ₹ 2825 and so on

(i) 1st installment = ₹3425
2nd installment = ₹3225

3rd installment = ₹3025

and so on

Now, 3425, 3225, 3025, ... are in AP, with

a = 3425, d = 3225 - 3425 = -200

Now 6th installment = a_n = a + 5d = 3425 + 5(-200) = ₹2425

(ii) Total amount paid = $\frac{15}{2}$(2a + 14d)

= $\frac{15}{2}$[2(3425) + 14(-200)] = $\frac{15}{2}$(6850 - 2800)

= $\frac{15}{2}$(4050) = ₹30375

OR

a_n = a + (n - 1)d given a_n = 2625

2625 = 3425 + (n - 1) × - 200

⇒ - 800 = (n - 1) × - 200

⇒ 4 = n - 1

⇒ n = 5

So, in 5th installment, she pays ₹2625.

(iii) a_n = a + (n - 1)d

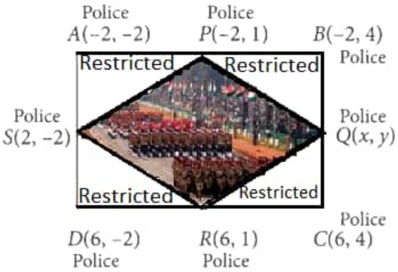
⇒ $a_{10} = 3425 + 9 \times (-200)$ = 1625

⇒ a_{11} = 3425 + 10 × (-200) = 1425

a_{10} + a_{11} = 1625 + 1425 = 3050

37. **Read the text carefully and answer the questions:**

In order to facilitate smooth passage of the parade, movement of traffic on certain roads leading to the route of the Parade and Tableaux ah rays restricted. To avoid traffic on the road Delhi Police decided to construct a rectangular route plan, as shown in the figure.



(i) Q(x, y) is mid-point of B(-2, 4) and C(6, 4)

∴ (x, y) = $\left(\frac{-2+6}{2}, \frac{4+4}{2}\right) = \left(\frac{4}{2}, \frac{8}{2}\right)$ = (2, 4)

(ii) Since PQRS is a rhombus, therefore, PQ = QR = RS = PS.

∴ PQ = $\sqrt{(-2-2)^2+(1-4)^2}$ = $\sqrt{16+9} = \sqrt{25}$ = 5 units

Thus, length of each side of PQRS is 5 units.

OR

Length of CD = 4 + 2 = 6 units and length of AD = 6 + 2 = 8 units

∴ Length of route ABCD - 2(6 + 8) = 28 units

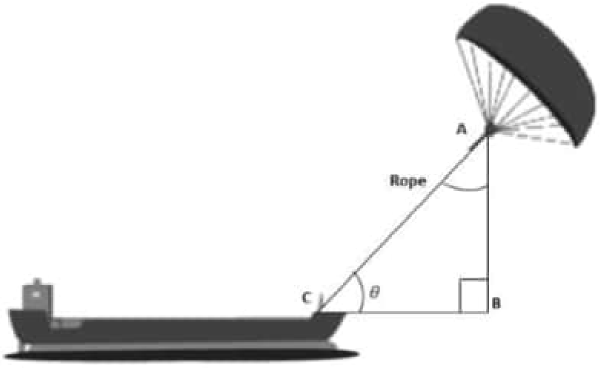
(iii) Length of route PQRS = 4 PQ

= 4 × 5 = 20 units

38. **Read the text carefully and answer the questions:**

Skysails is the genre of engineering science that uses extensive utilization of wind energy to move a vessel in the seawater. The 'Skysails' technology allows the towing kite to gain a height of anything between 100 metres - 300 metres. The sailing kite is made in such a way that it can be raised to its proper elevation and then brought back with the help of a 'telescopic mast' that enables the kite to be raised properly and effectively.

Based on the following figure related to sky sailing, answer the following questions:



(i) $\sin\theta = \cos(\theta - 30^o)$

$\cos(90^o - \theta) = \cos(\theta - 30^o)$

$\Rightarrow 90^\circ - \theta = \theta - 30^\circ$

$\Rightarrow \theta = 60^\circ$

(ii) $\frac{AB}{AC} = \sin 60^\circ$

$\therefore$ Length of rope, AC = $\frac{AB}{sin60^o}$ = $\frac{200}{\frac{\sqrt{3}}{2}}$ = $\frac{200\times2}{\sqrt{3}}$ = 230.94 m

OR

$\frac{AB}{AC} = \sin 30^\circ$

$\therefore$ Length of rope, AC = $\frac{AB}{sin30^o}$ = $\frac{150}{\frac{1}{2}}$ = 150×2 = 300 m

(iii) $\sin\theta = \cos(3\theta - 30^o)$

$\cos(90^o - \theta) = \cos(3\theta - 30^o)$

$\Rightarrow 90^\circ - \theta = 3\theta - 30^\circ \Rightarrow \theta = 30^\circ$