Class X Session 2024-25
Subject - Mathematics (Standard)
Sample Question Paper - 6

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment carrying 04 marks each.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

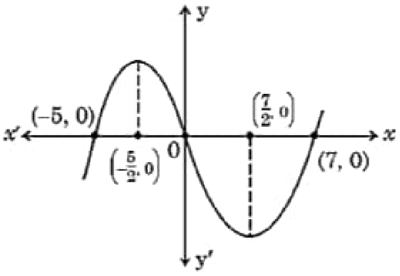
Section A

1. If the HCF of 65 and 117 is expressible in the form 65m – 117, then the value of ‘m’ is **[1]**

   a) 3 b) 1

   c) 2 d) 4

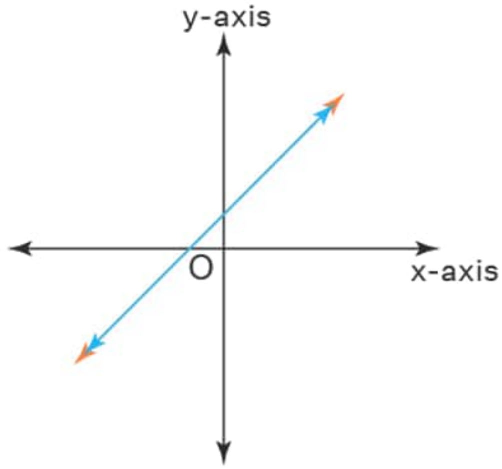
2. The graph of y = p(x) is given in the adjoining figure. Zeroes of the polynomial p(x) are **[1]**



   a) $-5, \frac{-5}{2}, \frac{7}{2}, 7$ b) -5, 7

   c) -5, 0, 7 d) $\frac{-5}{2}, \frac{-7}{2}$

3. The number of solutions of two linear equations representing coincident lines is/are **[1]**



a) infinite solution

b) 0

c) 1

d) 5

4. The values of k for which the quadratic equation $16x^2 + 4kx + 9 = 0$ has real and equal roots are **[1]**

a) 6, - 6

b) $\frac{3}{4}, -\frac{3}{4}$

c) 36,-36

d) $6, -\frac{1}{6}$

5. The next term of the A.P. $\sqrt{18}$, $\sqrt{32}$ and $\sqrt{50}$ is **[1]**

a) $\sqrt{72}$

b) $\sqrt{84}$

c) $\sqrt{64}$

d) $\sqrt{80}$

6. The distance of the point (5, 0) from the origin is **[1]**

a) 5^2

b) 5

c) 0

d) $\sqrt{5}$

7. If A(1, 3), B(-1, 2), C(2, 5) and D(x, 4) are the vertices of a ||gm ABCD then the value of x is **[1]**

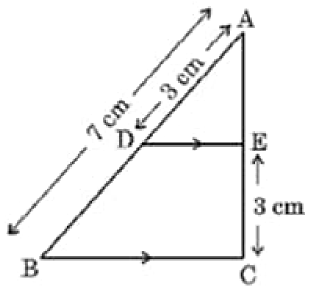
a) 0

b) 3

c) $\frac{3}{2}$

d) 4

8. In the given figure, DE||BC. If AD = 3 cm, AB = 7 cm and EC = 3 cm, then the length of AE is **[1]**



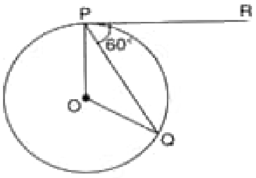
a) 4 cm

b) 2.25 cm

c) 2 cm

d) 3.5 cm

9. If O is the centre of a circle, PQ is a chord and tangent PR at P makes an angle of 60° with PQ, then $\angle POQ$ is equal to **[1]**



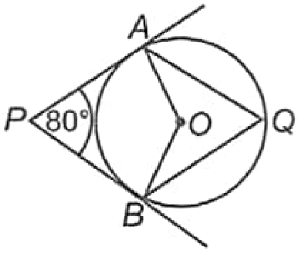
a) 110°

b) 120°

c) 100°

d) 90°

10. In the given figure, O is the centre of the circle. If PA and PB are tangents, then the value of $\angle$AQB is **[1]**



a) 80^o

b) 60^o

c) 50^o

d) 100^o

11. $\sqrt{\frac{1+\cos A}{1-\cos A}} = ?$ **[1]**

a) cosec A - cot A

b) -cosec A cot A

c) cosec A + cot A

d) cosec A cot A

12. If 2 cosθ = 1, then the value of θ is **[1]**

a) 60^o

b) 30^o

c) 45^o

d) 90^o

13. A pole 6 m high casts a shadow $2\sqrt{3}$ m long on the ground, then the sun's elevation is **[1]**

a) 30°

b) 60°

c) 45°

d) 90°

14. A piece of wire 20cm long is bent into the form of an arc of a circle subtending an angle of 60^o at its centre. The radius of the circle is **[1]**

a) $\frac{20}{6+\pi}$ cm

b) $\frac{30}{6+\pi}$ cm

c) $\frac{60}{\pi}$ cm

d) $\frac{15}{6+\pi}$ cm

15. If the area of a sector of a circle is $\frac{7}{20}$ of the area of the circle, then the angle at the centre is equal to **[1]**

a) 110°

b) 100°

c) 130°

d) 126°

16. A coin is tossed thrice. The probability of getting at least two tails is **[1]**

a) $\frac{4}{5}$

b) $\frac{2}{3}$

c) $\frac{1}{4}$

d) $\frac{1}{2}$

17. A card is drawn at random from a pack of 52 cards. The probability that the card is drawn is a black king. **[1]**

a) $\frac{11}{13}$

b) $\frac{3}{13}$

c) $\frac{1}{26}$ d) $\frac{1}{52}$

18. The mean and median of a statistical data are 21 and 23 respectively. The mode of the data is: **[1]**

a) 27 b) 23

c) 22 d) 17

19. **Assertion (A):** A sphere of radius 7 cm is mounted on the solid cone of radius 6 cm and height 8 cm. The volume of the combined solid is 1737.97 cm^3. **[1]**

Reason (R): Volume of sphere is $\frac{4}{3}\pi r^3$.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** Arithmetic mean between 8 and 12 is 10. **[1]**

Reason (R): Arithmetic mean between two numbers a and b is given as $\frac{a+b}{2}$.

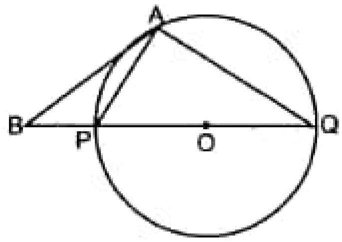
a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Find by prime factorisation the LCM of the numbers 18180 and 7575. Also, find the HCF of the two numbers. **[2]**

22. $\triangle$ABD is a right triangle right-angled at A and AC$\perp$ BD. Show that $\frac{AB^2}{AC^2} = \frac{BD}{DC}$ **[2]**

23. The tangent at a point A of a circle with centre O intersects the diameter PQ of the circle(when extended) at the point B. If $\angle$BAQ = 105°, find APQ. **[2]**



24. Prove the trigonometric identity: $\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta} = 2\sec^2\theta$ **[2]**

OR

Prove that: $\tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta$

25. Find the diameter of the circle whose area is equal to the sum of the areas of two circles having radii 4 cm and 3 cm. **[2]**

OR

From a circular piece of carboard of radius 3 cm two sectors of 90° have been cut off. Find the perimeter of the remaining portion nearest hundredth centimeters. (Take π = 22/7).

Section C

26. In a school there are two sections, namely A and B, of class X. There are 30 students in section A and 28 students in section B. Find the minimum number of books required for their class library so that they can be distributed equally among students of section A or section B. **[3]**

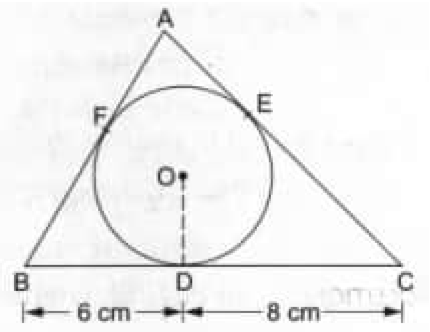
27. Find the zeroes of the quadratic polynomial $7y^2 - \frac{11}{3}y - \frac{2}{3}$ and verify the relationship between the zeroes and the coefficients. **[3]**

28. Solve the pair of linear equations 3x + 4y = 10 and 2x – 2y = 2 by elimination and substitution method. **[3]**

OR

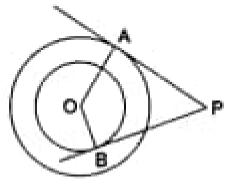
If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction? Solve the pair of the linear equation obtained by the elimination method.

29. In the given figure, a triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 6 cm and 8 cm respectively. Find the side AB, if the area of $\triangle$ABC is 63 cm^2 **[3]**



OR

Tangents PA and PB are drawn from an external point P to two concentric circles with centre O and radii 8 cm and 5 cm respectively, as shown in the figure. If AP = 15 cm then find the length of BP.



30. If $\sin\theta + \cos\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$, show that $q(p^2 - 1) = 2p$. **[3]**

31. Find the median of the following frequency distribution: **[3]**

Weekly wages (in ₹)	60-69	70-79	80-89	90-99	100-109	110-119
No. of days	5	15	20	30	20	8

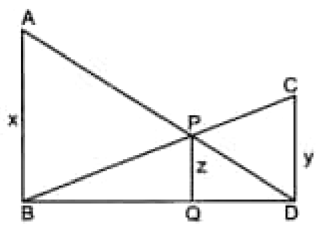
Section D

32. A shopkeeper buys a number of books for Rs.1200. If he had bought 10 more books for the same amount, each book would have cost him Rs.20 less. Find how many books did he buy? **[5]**

OR

A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/h more than the first speed. If it takes 3 hours to complete the total journey, what was its first average speed?

33. In figure AB || PQ || CD, AB = x units, CD = y units and PQ = z units, prove that $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$ **[5]**



34. A solid is in the shape of a right-circular cone surmounted on a hemisphere, the radius of each of them is being 3.5 cm and the total height of solid is 9.5 cm. Find the volume of the solid. **[5]**

OR

A toy is in the form of a cone mounted on a hemisphere of radius 3.5 cm. The total height of the toy is 15.5 cm; find the total surface area and volume of the toy.

35. The monthly income of 100 families are given as below: **[5]**

Income in (in ₹.)	**Number of families**
0-5000	8
5000-10000	26
10000-15000	41
15000-20000	16
20000-25000	3
25000-30000	3
30000-35000	2
35000-40000	1

Calculate the modal income.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

Your friend Varun wants to participate in a 200m race. He can currently run that distance in 51 seconds and with each day of practice it takes him 2 seconds less. He wants to do in 31 seconds.

i. Write first four terms are in AP for the given situations. (1)

ii. What is the minimum number of days he needs to practice till his goal is achieved? (1)

iii. How many second takes after 5^{th} days? (2)

OR

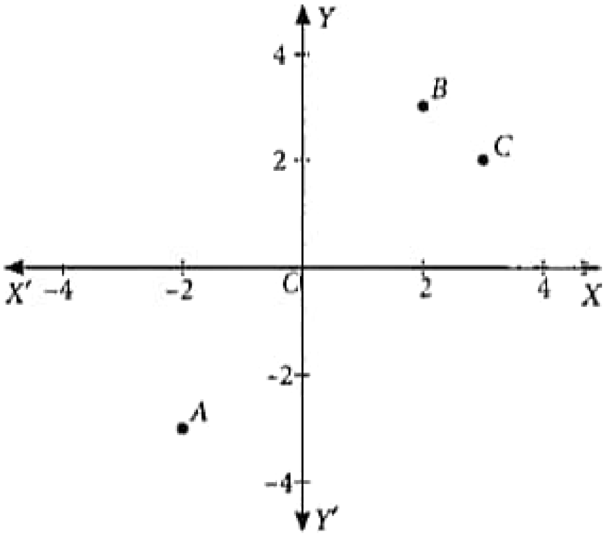
Out of 41, 30, 37 and 39 which term is not in the AP of the above given situation? (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

There are two routes to travel from source A to destination B by bus. First bus reaches at B via point C and second bus reaches from A to B directly. The position of A, B and C are represented in the following graph: Based on the above information, answer the following questions.

Scale: x-axis : 1 unit = 1 km

y-axis: 1 unit = 1 km



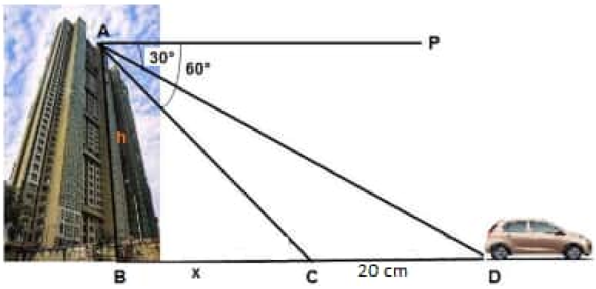
i. If the fare for the second bus is ₹15/km, then what will be the fare to reach to the destination by this bus? (1)
ii. What is the distance between A and B? (1)
iii. What is the distance between A and C? (2)

OR

If it is assumed that both buses have same speed, then by which bus do you want to travel from A to B? (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Vijay lives in a flat in a multi-story building. Initially, his driving was rough so his father keeps eye on his driving. Once he drives from his house to Faridabad. His father was standing on the top of the building at point A as shown in the figure. At point C, the angle of depression of a car from the building was 60^{o}. After accelerating 20 m from point C, Vijay stops at point D to buy ice cream and the angle of depression changed to 30^{o}.



i. Find the value of x. (1)
ii. Find the height of the building AB. (1)
iii. Find the distance between top of the building and a car at position D? (2)

OR

Find the distance between top of the building and a car at position C? (2)

Solution

Section A

1.

(c) 2

Explanation: First, find the HCF of 65 and 117

117 = 65 $\times$ 1 + 52

65 = 52 $\times$ 1 + 13

52 = 13 $\times$ 4 + 0 (zero remainder)

Therefore, HCF (117 , 65) is 13

Now,

$\therefore$ 65m – 117 = 13

$\Rightarrow$ 65m = 13 + 117

$\Rightarrow$ 65m = 130

$\Rightarrow$ m = 2

2.

(c) -5, 0, 7

Explanation: The graph intersect the x-axis at three distinct Points -5, 0, 7. So, there are three zeroes of P(x) which are -5, 0, 7.

3. **(a)** infinite solution

Explanation: The number of solutions of two linear equations representing coincident lines are ∞ because two linear equations representing coincident lines has infinitely many solutions.

4. **(a)** 6, - 6

Explanation: Given equation is; $16x^2 + 4kx + 9 = 0$

Here a =16, b = 4k, c = 9

Now $D = b^2 - 4ac = (4k)^2 - 4\times16\times9 = 16k^2 - 576$

Roots are real and equal if D = 0 i,e. $b^2 - 4ac = 0$

$\Rightarrow 16k^2 - 576 = 0$

$\Rightarrow k^2 - 36 = 0$

$\Rightarrow k^2 = 36 = (\pm6)^2$

$\Rightarrow k = \pm6$

5. **(a)** $\sqrt{72}$

Explanation: Given: $\sqrt{18}, \sqrt{32}, \sqrt{50}$

$\Rightarrow 3\sqrt{2}, 4\sqrt{2}, 5\sqrt{2}$

$\therefore$ d = $4\sqrt{2}$ - $3\sqrt{2}$ = $\sqrt{2}$

Therefore, next term is $5\sqrt{2} + \sqrt{2}$

= $6\sqrt{2} = \sqrt{72}$

6.

(b) 5

Explanation: Distance from origin = $\sqrt{(5-0)^2 - (0-0)^2}$

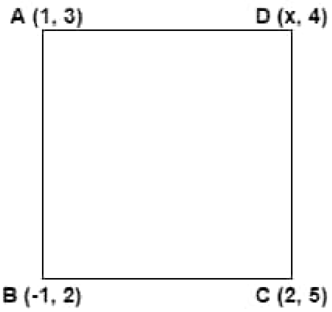
= $\sqrt{25}$

= 5 units

7.

(d) 4

Explanation:



Since ABCD is a ||gm, the diagonals bisect eachother. so
M is the mid- point of BD as well as AC.
$\frac{1+2}{2} = \frac{x-1}{2}$
1 + 2 = $x - 1$
$x = 4$

8.

(b) 2.25 cm

Explanation: Given, AD = 3 cm, AB = 7 cm, EC = 3 cm.

Let AE = x cm

∴ AC = AE + EC = x + 3 cm

As we know that

$\frac{AD}{AB} = \frac{AE}{AC}$

$\Rightarrow \frac{3}{7} = \frac{x}{x+3}$

⇒ 3(x + 3) = 7x

⇒ 3x + 9 = 7x

⇒ 7x - 3x = 9

⇒ 4x = 9

⇒ x = $\frac{9}{4}$ = 2.25 cm

∴ length of AE = 2.25 cm

9.

(b) 120°

Explanation: Here ∠RPO = 90°

∠RPQ = 60° (given)

∴∠OPQ = 90° - 60° = 30° ∠PQO = 30° Also [Opposite angles of equal radii] Now, In triangle OPQ,

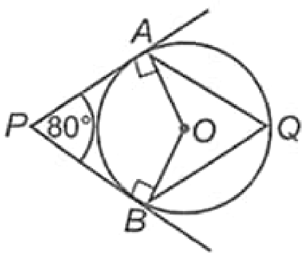
$\angle \text{OPQ} + \angle \text{PQO} + \angle \text{QOP} = 180^\circ$

⇒30° + 30° + ∠QOP = 180°

⇒ ∠QOP = 120°

10.

(c) 50^o

Explanation: 

Since, PA and PB are tangents.

Also, tangent is perpendicular to radius at the point of contact.

∴ ∠PAO = 90^o and ∠PBO = 90^o

In quadrilateral APBO;

∠APB + ∠PAO + ∠PBO + ∠AOB = 360^o

$80^o + 90^o + 90^o + \angle AOB = 360^o$

$\Rightarrow \angle AOB = 100^o \Rightarrow \angle AQB = \frac{1}{2}\angle AOB = 50^o$

11.

(c) cosec A + cot A

Explanation: $\sqrt{\frac{1+\cos A}{1-\cos A}} = \sqrt{\frac{(1+\cos A)}{(1-\cos A)} \times \frac{(1+\cos A)}{(1+\cos A)}} = \frac{(1+\cos A)}{\sqrt{1-\cos^2 A}} = \frac{(1+\cos A)}{\sqrt{\sin^2 A}}$

$= \frac{(1+\cos A)}{\sin A} = \left(\frac{1}{\sin A} + \frac{\cos A}{\sin A}\right) = (cosec\ A + \cot A)$

12. **(a)** 60^o

Explanation: 60^o

13.

(b) 60°

Explanation: Let height = 6m

length of shadow = $2\sqrt{3m}$

θ is angle of elevation

tanθ = (height) / (shadow length)

$= \frac{6}{2}\sqrt{3} = \sqrt{3}$

$\theta = \frac{\pi}{3}$

Angle of inclination is = 60^o

14.

(c) $\frac{60}{\pi}$ cm

Explanation: Given: Length of arc = 20 cm

$\Rightarrow \frac{\theta}{360^\circ} \times 2\pi r = 20$

$\Rightarrow \frac{60^\circ}{360^\circ} \times 2\pi r = 20$

$\Rightarrow \frac{\pi r}{3} = 20$

$\Rightarrow r\left(\frac{\pi}{3}\right) = 20$

$\Rightarrow r\left(\frac{\pi}{3}\right) = 20$

$\Rightarrow$ r = $\frac{60}{\pi}$ cm

15.

(d) 126°

Explanation: We have given that area of the sector is $\frac{7}{20}$ of the area of the circle.

Therefore, area of the sector $= \frac{7}{20} \times$ area of the circle

$\therefore \frac{\theta}{360} \times \pi r^2 = \frac{7}{20} \times \pi r^2$

Now we will simplify the equation as below,

$\frac{\theta}{360} = \frac{7}{20}$

Now we will multiply both sides of the equation by 360,

$\therefore \theta = \frac{7}{20} \times 360$

$\therefore \theta = 126$

Therefore, sector angle is 126°.

16.

(d) $\frac{1}{2}$

Explanation: Total outcomes = = {HHH, TTT, HHT, HTH, HTT, THH, THT, TTH} = 8

Number of possible outcomes (at least two tails) = 4

$\therefore$ Required Probability = $\frac{4}{8} = \frac{1}{2}$

17.

(c) $\frac{1}{26}$

Explanation: Total number of outcomes = 52

Favourable outcomes, in this case, = 2 {2 black kings}

$\therefore$ P (black king) = $\frac{Favourable\ outcomes}{Total\ outcomes} = \frac{2}{52} = \frac{1}{26}$

18. **(a)** 27

Explanation: We know that

3 Median= Mode + 2 Mean

3×23 = Mode + 2×21

69 - 42 = Mode

Mode = 27

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

Section B

21. 18180 = $2^2 \times 3^2 \times 5 \times 101$

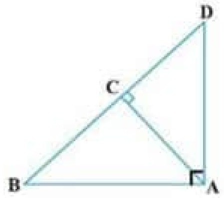
7575 = $3 \times 5^2 \times 101$

LCM = $2^2 \times 3^2 \times 5^2 \times 101$ = 90900

HCF = $3 \times 5 \times 101$ = 1515

22. Given: $\triangle$ABD is a right triangle right angled at A and AC$\perp$ BD.

To Prove: $\frac{AB^2}{AC^2} = \frac{BD}{DC}$



Proof: We know that if a perpendicular is drawn from the vertex of the right angle to the hypotenuse then the triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

So, $\triangle BAD \sim \triangle BCA$ ....(i)

and $\triangle ACB \sim \triangle DCA$ ....(ii)

If two triangles are similar, then the ratio of their corresponding sides are equal.

$\frac{BA}{BC} = \frac{BD}{BA}$ [from (i)]

BA^2 = BC $\times$ BD ....(iii)

Also, $\frac{AC}{DC} = \frac{BC}{AC}$ [from(ii)]

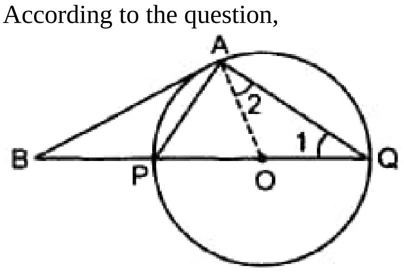
AC^2 = DC $\times$ BC ......(iv)

Hence $\frac{AB^2}{AC^2} = \frac{CB \times BD}{DC \times BC}$

$\frac{AB^2}{AC^2} = \frac{BD}{DC}$

Hence proved.

23. According to the question,



$PAQ = 90^o$ [Angle in semicircle]

$\therefore$ $\angle \text{APQ} + \angle 1 = 90^\circ$ [Sum of acute angels of right angle $\triangle$]

$\Rightarrow \angle APQ + \angle 2 = 90^\circ$ [OA = OQ $\therefore$ $\angle 1 = \angle 2$]

$\Rightarrow \angle \text{APQ} + (\angle \text{BAQ} - \angle \text{BAO}) = 90^\circ$

$\Rightarrow \angle APQ + (105^\circ - 90^\circ) = 90^\circ$ [$\because$ OA $\perp \mathbf{AB}$]

$\Rightarrow \angle \text{APQ} = 90^\circ - 15^\circ = 75^\circ$

24. $\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta} = 2\sec^2\theta$

L.H.S. $= \frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta}$

$= \frac{1+\sin\theta+1-\sin\theta}{(1-\sin\theta)(1+\sin\theta)} = \frac{2}{1-\sin^2\theta}$
$= \frac{2}{\cos^2\theta}\left[\because 1-\sin^2\theta = \cos^2\theta\right]$
$= 2\sec^2\theta$ [$\because \sec(x) = \frac{1}{\cos(x)}$]
= R.H.S. Proved

OR

$\tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta$

R.H.S. = $\tan^2\theta.\sin^2\theta$

= $\tan^2\theta(1 - \cos^2\theta)$ $\left[\because \sin^2\theta = 1-\cos^2\theta\right]$

= $\tan^2\theta - \tan^2\theta\cos^2\theta$

$= \tan^2\theta - \frac{\sin^2\theta}{\cos^2\theta}\cdot\cos^2\theta\left[\because \tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}\right]$

= $\tan^2\theta - \sin^2\theta$= L.H.S.

Hence proved.

25. Let the radius of the large circle be R.

Then, we have

Area of large circle of radius R = Area of a circle of radius 4 cm + Area of circle of radius 3 cm

$\Rightarrow \pi R^2 = \left(\pi\times 4^2 + \pi\times 3^2\right)$

$\Rightarrow \pi\mathrm{R}^2 = (16\pi + 9\pi)$

$\Rightarrow \pi R^2 = 25\pi$

$\Rightarrow$ R^2 = 25

$\Rightarrow$ R = 5 cm

$\Rightarrow$ Diameter = 2R = 10 cm

OR

Radius of the circular piece of cardboard(r) = 3 cm

$\therefore$Two sectors of 90° each have been cut off

$\therefore$We get a semicircular cardboard piece

$\therefore$Perimeter of arc ACB

=$\frac{1}{2}(2\pi r) = \pi r$

= $\frac{22}{7}\times 3 = \frac{66}{7}$ = 9.428 cm

Section C

26. As per question, the required number of books are to be distributed equally among the students of section A or B.

There are 30 students in section A and 28 students in section B.

So, the number of these books must be a multiple of 30 as well as that of 28.

Consequently, the required number is LCM(30, 28).

Now, 30 = 2 $\times$3 $\times$5

and 28 = 2^2 $\times$7 .

$\therefore$ LCM(30, 28) = product of prime factors with highest power

= $2^2\times 3\times 5\times 7$

= $4\times 3\times 5\times 7$

= 420

Hence, the required number of books = 420.

27. p(y) = $7y^2 - \frac{11}{3}y - \frac{2}{3} = \frac{1}{3}(21y^2 - 11y - 2)$

= $\frac{1}{3}$ ($21y^2$ - 14y + 3y - 2)

= $\frac{1}{3}$ [7y (3y - 2) + 1 (3y - 2)]

= $\frac{1}{3}$[(7y + 1)(3y - 2)]

$\therefore$ Zeroes are $\frac{2}{3}$, - $\frac{1}{7}$

Sum of Zeroes = $\frac{2}{3} - \frac{1}{7} = \frac{11}{21}$

$\frac{-b}{a} = \frac{11}{21}$

$\therefore$ sum of zeroes = $\frac{-b}{a}$

Product of Zeroes = $(\frac{2}{3})(-\frac{-1}{7}) = -\frac{2}{21}$

$\frac{c}{a}$ = - $\frac{2}{3}$ ($\frac{1}{7}$) = - $\frac{2}{21}$

∴ Product = $\frac{c}{a}$

28. 1. By Elimination method,

The given system of equation is :

3 x + 4 y = 10 ....(1)

2 x - 2 y = 2 ....(2)

Multiplying equation(2) by 2, we get

4 x - 4 y = 4 ....(2)

Adding equation (1) and equation (3), we get

7 x = 14

∴ $x = \frac{14}{7} = 2$

Substituting this value of x in equation (2), we get

2(2) - 2y = 2

⇒ $4 - 2y = 2$

⇒ $2y = 4 - 2$

⇒ $2y = 2$

⇒ $y = \frac{2}{2} = 1$

So, the solution of the given system of equation is

x = 2, y = 1

2. By Substitution method,

The given system of equation is:

3 x + 4 y = 10.................(1)

2 x - 2 y = 2....................(2)

From equation(1)

$3x = 10 - 4y$

$x = \left(\frac{10-4y}{3}\right)$

Put value of x in equation (2),

$2x - 2y = 2$

$2\left(\frac{10-4y}{3}\right) - 2y = 2$

$\frac{2(10-4y)-2y(3)}{3} = 2$

$20 - 8y - 6y = 6$

$-14y = -14$

y = 1

Putting value of y = 1 in equation (2)

2x - 2 = 2

x = 2

Therefore, x = 2, y = 1 is the solution.

Verification: Substituting x = 2, y = 1, we find that both the equation(1) and (2) are satisfied shown below:

$3x + 4y = 3(2) + 4(1) = 6 + 4 = 10$

$2x - 2y = 2(2) - 2(1) = 4 - 2 = 2$

Hence, the solution is correct.

OR

Let the fraction be $\frac{x}{y}$

Then, according to the question,

$\frac{x+1}{y-1} = 1$ ......(1)

$\frac{x}{y+1} = \frac{1}{2}$ ....(2)

⇒x + 1 = y - 1 ....(3)

2x = y + 1................(4)

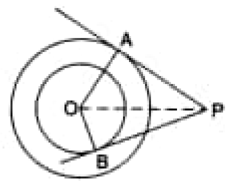
⇒x - y = - 2................(5)

2x - y = 1......................(^)

Substituting equation (5) from equation (6), we get x =3
Substituting this value of x in equation (5), we get
3 - y = -2
⇒ y = 3 + 2
⇒ y = 5
Hence, the required fraction is $\frac{3}{5}$
Verification: Substituting the value of x = 3 and y = 5,
we find that both the equations(1) and (2) are satisfied as shown below:
$\frac{x+1}{y-1} = \frac{3+1}{5-1} = \frac{4}{4} = 1$
$\frac{x}{y+1} = \frac{3}{5+1} = \frac{3}{6} = \frac{1}{2}$
Hence, the solution is correct.

29. We know that the lengths of tangents drawn from an exterior point to a circle are equal
∴AE = AF = x cm (say);
BD = BF = 6 cm;
CD = CE = 8 cm
And so, AB = AF + BF = (x + 6) cm; BC = BD + CD = 14 cm;
CA = CE + AE = (x + 8) cm.
Join OE and OF and also OA, OB and OC.
$\therefore \quad \text{ar}(\triangle ABC) = \text{ar}(\triangle OAB) + \text{ar}(\triangle OBC) + \text{ar}(\triangle OCA)$
$\Rightarrow 63 = \left(\frac{1}{2} \times AB \times OF\right) + \left(\frac{1}{2} \times BC \times OD\right) + \left(\frac{1}{2} \times CA \times OE\right)$
$\Rightarrow 63 = \left\{\frac{1}{2} \times (x+6) \times 3\right\} + \left(\frac{1}{2} \times 14 \times 3\right) + \left\{\frac{1}{2} \times (x+8) \times 3\right\}$
$\Rightarrow \quad 63 = \frac{3}{2} \times (2x + 28) \Rightarrow x = 7$
∴ AB = (x + 6)cm = (7 + 6)cm = 13 cm

OR



We have
$OA \perp AP$ and $OB \perp BP$ [∴the tangent at any point of a circle is perpendicular to the radius through the point of contact].
Join OP.
In right ΔOAP,
we have
OA = 8 cm, AP = 15 cm.
∴$OP^2 = OA^2 + AP^2$ [by Pythagoras' theorem]
⇒ OP$= \sqrt{OA^2 + AP^2}$
$= \sqrt{8^2 + 15^2}\text{cm}$
$= \sqrt{289}\text{cm} = 17\ cm$
In right △OBP,
we have
OB = 5 cm,
OP = 17 cm
∴ $OP^2 = OB^2 + BP^2$ [by Pythagoras' theorem]
⇒ BP $= \sqrt{OP^2 - OB^2}$
$= \sqrt{17^2 - 5^2}\text{cm}$
$= \sqrt{264}\text{cm}$
Thus, the length of BP $= \sqrt{264}\text{cm}$
$= 16.25\text{cm}$ (approx).

30. We have, p = sinθ + cosθ and q = secθ + cosecθ
∴ LHS = q(p^2 -1) = (secθ + cosecθ) {(sinθ + cosθ)^2 - 1 }

= $\left(\frac{1}{\cos\theta}+\frac{1}{\sin\theta}\right)\left\{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1\right\}$
= $\left(\frac{\sin\theta+\cos\theta}{\cos\theta\sin\theta}\right)(1+2\sin\theta\cos\theta-1)$
= $\left(\frac{\sin\theta+\cos\theta}{\cos\theta\sin\theta}\right)(2\sin\theta\cos\theta)=2(\sin\theta+\cos\theta)$ = 2p = RHS

31. Here, the frequency table is given in inclusive form. So, we first transform it into exclusive form by subtracting and adding $\frac{h}{2}$ to the lower and upper limits respectively of each class, where h denotes the difference of lower limit of a class and the upper limit of the previous class.

Here, h = 1 So, $\frac{h}{2}$ = 0.5

Transforming the above table into exclusive form and preparing the cumulative frequency table, we get:-

Weekly wages (in ₹)	**No of workers**	**Cumulative frequency**
59.5-69.5	5	5
69.5-79.5	15	20
79.5-89.5	20	40
89.5-99.5	30	70
99.5-109.5	20	90
109.5-119.5	8	98
		N = Σf_i = 98

We have, N(Total frequency) = 98 Or, $\frac{h}{2}$ = 49

The cumulative frequency just greater than $\frac{h}{2}$ is 70 and the corresponding class is 89.5-99.5. So, 89.5-99.5 is the median class.

Now,

l = 89.5 (lower limit of median class),

h = 10 (length of interval of median class),

f = 30 (frequency of median class)

F = 40 (cumulative frequency of the class just preceding the median class)

Now, Median is given by:-

= l + $\frac{\frac{N}{2}-f}{F}\times$ h

= 89.5 + $\frac{49-40}{30}\times$ 10

= 89.5 + 3 = 92.5

Section D

32. Let number of books the shopkeeper buys = x

Price of each book = Rs $\frac{1200}{x}$

cost of each book when x +10 books are bought = RS $\frac{1200}{x+10}$

According to given question,

$\frac{1200}{x}-\frac{1200}{x+10}$= 20

1200$(\frac{1}{x}-\frac{1}{x+10})$ = 20

$(\frac{1}{x}-\frac{1}{x+10})=\frac{20}{1200}$

$\frac{(x+10)-x}{x(x+10)}=\frac{1}{60}$

x +10- x = $\frac{x^2+10x}{60}$

600 = x^2 + 10x

x^2+ 10x - 600 = 0

Here, it is quadratic equation

x^2 + 30x - 20x - 600 = 0

x(x+30) -20(x+30) = 0

(x+30) (x-20) = 0

either

(x+30) =0 or (x-20) = 0

x = -30 or x= 20

x= -30, is not possible because the number of books can't be negative.

so, number of books = x = 20.

OR

Let the speed of the train be x km/hr for first 54 km and for next 63 km, speed is (x + 6) km/hr.

According to the question

$\frac{54}{x}+\frac{63}{x+6}=3$

$\frac{54(x+6)+63x}{x(x+6)}=3$

or, 54x + 324 + 63x = 3x(x + 6)

or, $117x+324=3x^2+18x$

or, $3x^2-99x-324=0$

or, $x^2-33x-108=0$

or, $x^2-36x+3x-108=0$

or, x(x - 36) + 3(x - 36) = 0

(x - 36)(x + 3) = 0

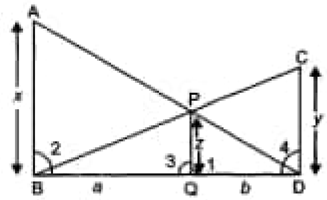
x = 36

x = - 3 rejected.

(as speed is never negative)

Hence First speed of train = 36 km/h

33. Let BQ = a units, DQ = b units



$\because$ PQ $\parallel$ AB $\therefore \angle 1=\angle 2$,

and $\angle ADB=\angle PDQ$

$\therefore$ $\triangle$ADB $\sim$ $\triangle$PDQ

Similarly $\triangle BCD\sim\triangle BPQ$

$\because$ $\triangle ADB\sim\triangle PDQ$

$\therefore$ $\frac{\text{AB}}{\text{PQ}}=\frac{\text{BD}}{\text{DQ}}$

$\frac{x}{z}=\frac{a+b}{b}$

$\frac{x}{z}=\frac{a}{b}+1\Rightarrow\frac{x}{z}-1=\frac{a}{b}$ ..(i)

Also, $\triangle BCD\sim\triangle BPQ$

$\therefore$ $\frac{\text{BD}}{\text{BQ}}=\frac{\text{CD}}{\text{PQ}}\Rightarrow\frac{a+b}{a}=\frac{y}{z}$

$1+\frac{b}{a}=\frac{y}{z}\Rightarrow\frac{b}{a}=\frac{y}{z}-1$

$\Rightarrow$ $\frac{b}{a}=\frac{y-z}{z}\Rightarrow\frac{a}{b}=\frac{z}{y-z}$ ..(ii)

From (i) and (ii)

$\frac{x}{z}-1=\frac{z}{y-z}\Rightarrow\frac{x}{z}=\frac{z}{y-z}+1$

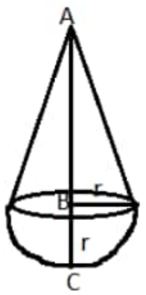
$\frac{x}{z}=\frac{z+y-z}{y-z}$

$\frac{x}{z}=\frac{y}{y-z}\Rightarrow\frac{z}{x}=\frac{y-z}{y}$

$\frac{z}{x}=1-\frac{z}{y}$

$z\left(\frac{1}{x}\right)=z\left(\frac{1}{z}-\frac{1}{y}\right)\Rightarrow\frac{1}{x}=\frac{1}{z}-\frac{1}{y}$

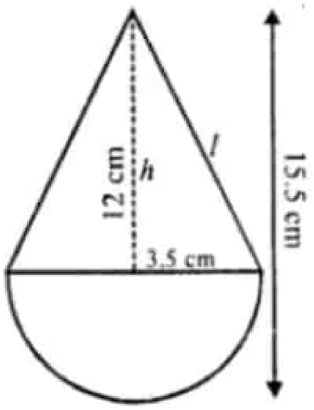
$\Rightarrow$ $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$ (Hence proved)

34. 

From the given figure,
Height (AB) of the cone = AC - BC (Radius of the hemisphere)
Thus, height of the cone = Total height - Radius of the hemisphere
= 9.5 - 3.5
= 6 cm
Volume of the solid = Volume of the cone + Volume of the hemisphere
$= \left(\frac{1}{3}\pi r^2 h\right) + \left(\frac{2}{3}\pi r^3\right)$
$= \frac{1}{3}\pi r^2(h + 2r)$
$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5(6 + 2 \times 3.5)$
$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 13$
= 166.83 cm^3
Thus, total volume of the solid is 166.83 cm^3.

OR

The Radius of the toy (r) = 3.5 cm
Total height of the toy = 15.5 cm
∴ Height of the conical part is = 15.5 - 3.5 = 12 cm
Slant height of the conical part (l)
$= \sqrt{r^2 + h^2} = \sqrt{(3.5)^2 + (12)^2}$
$= \sqrt{12.25 + 144} = \sqrt{156.25} = 12.5\text{cm}$



i. Now total surface area of the toy = curved surface area of conical part + curved surface area of hemipherical part
$= \pi rl + 2\pi r^2 = \pi r(l + 2r)$
$= \frac{22}{7} \times 3.5(12.5 + 2 \times 3.5)$ cm^2
= 11 (12.5 + 7) = 11 × 19.5 cm^2
= 214.5 cm^2
ii. Volume of the toy $= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$
$= \frac{1}{3}\pi r^2(h + 2r)$
$= \frac{1}{3} \times \frac{22}{7}(3.5)^2(12 + 2 \times 3.5)\text{cm}^3$
$= \frac{1}{3} \times \frac{22}{7} \times 12.25(12 + 7)\text{cm}^3$
$= \frac{22}{3} \times 1.75 \times 19\text{cm}^3$
$= \frac{731.5}{3}$ = 243.83 cm^3

35. class 10000 - 15000 has the maximum frequency,
so it is the modal class.

$\therefore$ l = 10000, h = 5000, f = 41, f_1 = 26 and f_2 = 16

Mode = l + $\frac{f-f_1}{2f-f_1-f_2} \times$ h

= 10000 + $\frac{41-26}{2(41)-26-16} \times$ 5000

= 10000 + $\frac{15}{40} \times$ 5000

= 10000 + 1875

= 11875

Section E

36. i. 51, 49, 47, ... 31 AP

d = -2

First 4 terms of AP are: 51, 49, 47, 45 ...

ii. 51, 49, 47, ... 31 AP

d = -2

t_n = a +(n - 1)d

31 = 51 + (n - 1) (-2)

31 = 51 - 2n + 2

31 = 53 - 2n

31 - 53 = -2n

-22 = -2n

n = 11

i.e., he acheived his goal in 11 days.

iii. 51, 49, 47, ... 31 AP

d = -2

t_6 = a + (n - 1)d

= 51 + (6 - 1)(-2)

= 51 + (-10)

= 41 sec

OR

The given AP is

51, 49, 47, 45, 43, 41, 39, 37, 35, 33, 31, 29

$\therefore$ 30 is not in the AP.

37. i. Distance travelled by second bus = 7.2 km

$\therefore$ Total fare = 7.2 $\times$ 15 = ₹08

ii. Required distance = $\sqrt{(2+2)^2+(3+3)^2}$

$= \sqrt{4^2+6^2} = \sqrt{16+36} = 2\sqrt{13}$ km $\approx$ 7.2 km

iii. Required distance = $\sqrt{(3+2)^2+(2+3)^2}$

= $\sqrt{5^2+5^2} = 5\sqrt{2}$ km

OR

Distance between B and C

= $\sqrt{(3-2)^2+(2-3)^2}$ = $\sqrt{1+1} = \sqrt{2}$ km

Thus, distance travelled by first bus to reach to B

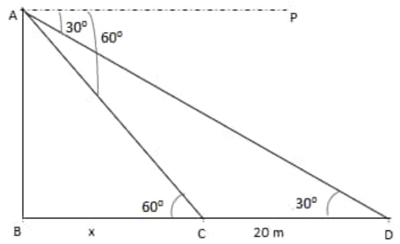
= AC + CB = $5\sqrt{2}+\sqrt{2} = 6\sqrt{2}$ km $\approx$ 8.48 km

and distance travelled by second bus to reach to B

= AB = $2\sqrt{13}$ km = 7.2 km

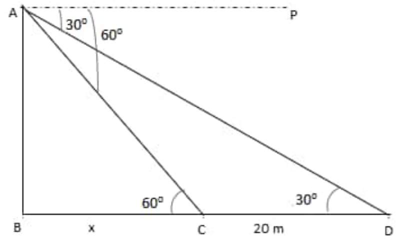
$\therefore$ Distance of first bus is greater than distance of the cond bus, therefore second bus should be chosen.

38. i. The above figure can be redrawn as shown below:



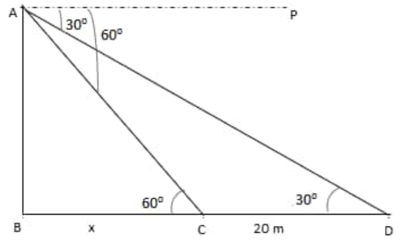
From the figure,
let AB = h and BC = x
In $\triangle$ABC,
tan 60 = $\frac{\mathrm{AB}}{\mathrm{BC}} = \frac{\mathrm{h}}{x}$
$\sqrt{3}$ = $\frac{\mathrm{h}}{x}$
h = $\sqrt{3}$x ...(i)
In $\triangle$ABD,
tan 30 = $\frac{\mathrm{AB}}{\mathrm{BD}} = \frac{\mathrm{h}}{x+20}$
$\frac{1}{\sqrt{3}}$ = $\frac{\sqrt{3}x}{x+20}$ [using (i)]
x + 20 = 3x
x = 10 m

ii. The above figure can be redrawn as shown below:



Height of the building, h = $\sqrt{3}$x = 10$\sqrt{3}$ = 17.32 m

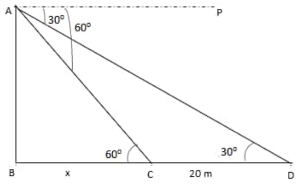
iii. The above figure can be redrawn as shown below:



Distance from top of the building to point D.
In $\triangle$ABD
sin 30^{o} = $\frac{AB}{AD}$
$\Rightarrow AD = \frac{AB}{\sin 30^0}$
$\Rightarrow AD = \frac{10\sqrt{3}}{\frac{1}{2}}$
$\Rightarrow$ AD = $20\sqrt{3}m$

OR

The above figure can be redrawn as shown below:



Distance from top of the building to point C is
In $\triangle$ABC
sin 60^{o} = $\frac{AB}{AC}$
$\Rightarrow AC = \frac{AB}{\sin 60^0}$
$\Rightarrow AC = \frac{10\sqrt{3}}{\frac{\sqrt{3}}{2}}$
$\Rightarrow$ AD = 20 m