Class X Session 2024-25
Subject - Mathematics (Standard)
Sample Question Paper - 9

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment carrying 04 marks each.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

Section A

1. The probability of getting a bad egg in a lot of 400 is 0.035. The number of bad eggs in the lot is **[1]**

   a) 21 b) 7

   c) 28 d) 14

2. If $(a^2 + b^2) x^2 + 2(ac + bd) x + c^2 + d^2 = 0$ has no real roots, then **[1]**

   a) ab = cd b) ad = bc

   c) ad $\neq$ bc d) ac = bd

3. The volume of the largest right circular cone that can be cut out from a cube of edge 4.2 cm is **[1]**

   a) 58.2 cm^3 b) 19.4 cm^3

   c) 9.7 cm^3 d) 77.6 cm^3

4. If one root of the equation $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ is 1, then the other root is ________. **[1]**

   a) $\frac{a(b-c)}{c(a-b)}$ b) $\frac{c(a-b)}{a(b-c)}$

   c) $\frac{b(c-a)}{a(b-c)}$ d) $\frac{a(b-c)}{b(c-a)}$

5. The 11th term of the AP: $-5, \frac{-5}{2}, 0, \frac{5}{2}$,...is **[1]**

   a) –30 b) –20

   c) 30 d) 20

6. The distance between the points (0, 0) and (a - b, a + b) is **[1]**

a) $2\sqrt{\mathrm{ab}}$

b) $\sqrt{2a^2+2b^2}$

c) $2\sqrt{a^2+b^2}$

d) $\sqrt{2a^2+ab}$

7. The zeros of the quadratic polynomial $x^2 + 88x + 125$ are **[1]**

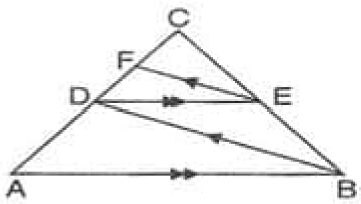
a) both negative

b) both positive

c) both equal

d) one positive and one negative

8. We have, $AB||DE$ and $BD||EF$.Then, **[1]**



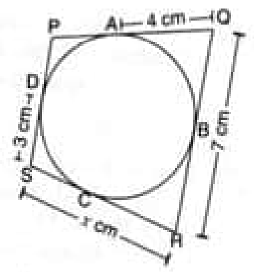
a) $BC^2 = AB\ .\ CE$

b) $AC^2 = BC\ .\ DC$

c) $AB^2 = AC\ .\ DE$

d) $DC^2 = CF \times AC$

9. In the given figure, if AQ = 4 cm, QR = 7 cm, DS = 3 cm, then x is equal to **[1]**



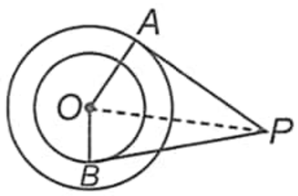
a) 6 cm

b) 10 cm

c) 11 cm

d) 8 cm

10. In the given figure, O is the centre of two concentric circles of radii 5 cm and 3 cm. From an external point P, tangents PA and PB are drawn to these circles. If PA = 12 cm, then PB = **[1]**



a) $3\sqrt{5}$ cm

b) $5\sqrt{2}$ cm

c) $5\sqrt{10}$ cm

d) $4\sqrt{10}$ cm

11. If a sinθ + b cosθ = c, then the value of a cosθ - b sinθ is **[1]**

a) $\sqrt{a^2+b^2-c^2}$

b) $\sqrt{a^2+b^2+c^2}$

c) $\sqrt{a^2-b^2+c^2}$

d) $\sqrt{a^2-b^2-c^2}$

12. If the HCF of 72 and 234 is 18, then the LCM (72, 234) is: **[1]**

a) 936

b) 836

c) 234

d) 324

13. The tops of two towers of heights x and y, standing on a level ground subtend angles of 30° and 60° respectively at the centre of the line joining their feet. Then, x : y is **[1]**

a) 1 : 3

b) 2 : 1

c) 1 : 2

d) 3 :1

14. In a circle of radius 21 cm, an arc subtends an angle of 60^o at the centre. The area of the sector formed by the arc is: **[1]**

a) 231 cm^2

b) 250 cm^2

c) 220 cm^2

d) 200 cm^2

15. A piece of paper in the shape of a sector of a circle (see figure 1) is rolled up to form a right-circular cone (see figure 2). The value of angle θ is: **[1]**

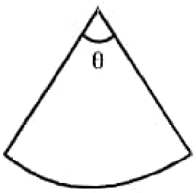


figure 1

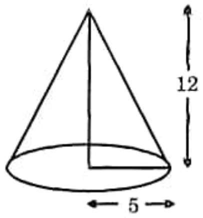


figure 2

a) $\frac{5\pi}{13}$

b) $\frac{6\pi}{13}$

c) $\frac{10\pi}{13}$

d) $\frac{9\pi}{13}$

16. A letter is chosen at random from the letters of the word **ASSOCIATION**. Find the probability that the chosen letter is a vowel. **[1]**

a) $\frac{6}{11}$

b) $\frac{7}{11}$

c) $\frac{5}{11}$

d) $\frac{3}{11}$

17. Two dice are thrown simultaneously. The probability that the product of the numbers appearing on the dice is 7 is **[1]**

a) 7

b) 2

c) 0

d) 1

18. In a data, if l = 40, h = 15, $f_1 = 7$, $f_0 = 3$, $f_2 = 6$, then the mode is **[1]**

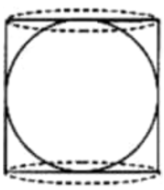
a) 82

b) 62

c) 52

d) 72

19. **Assertion (A):** In the given figure, a sphere is inscribed in a cylinder. The surface area of the sphere is not equal to the curved surface area of the cylinder. **[1]**



Reason (R): Surface area of sphere is $4\pi r^2$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** $a_n - a_{n-1}$ is not independent of n then the given sequence is an AP. **[1]**

Reason (R): Common difference d = a_n - a_{n-1} is constant or independent of n.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

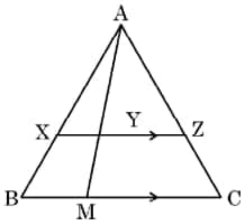
c) A is true but R is false.

d) A is false but R is true.

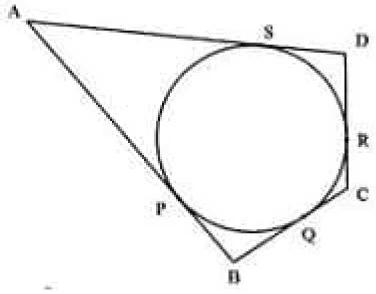
Section B

21. Prove that 2 + $3\sqrt{3}$ is an irrational number. It is given that $\sqrt{3}$ is an irrational number. **[2]**

22. In the given figure, XZ is parallel to BC. AZ = 3 cm, ZC = 2 cm, BM = 3 cm and MC = 5 cm. Find the length of XY. **[2]**



23. A quadrilateral ABCD is drawn to the circumference of a circle. Prove that: AB + CD = AD + BC **[2]**

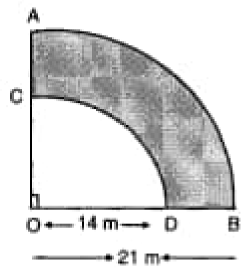


24. If $x = a\cos^3\theta, y = b\sin^3\theta$, prove that $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$ **[2]**

OR

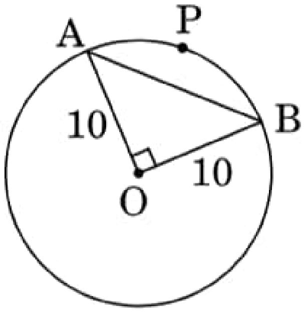
If $\sqrt{2}\sin\theta$ = 1, find the value of $\sec^2\theta$ - $\text{cosec}^2\theta$.

25. ABCD is a flower bed. If OA =21 m and OC = 14 m, find the area of the bed. **[2]**



OR

In Figure, a chord AB of a circle of radius 10 cm subtends a right angle at the centre.



Find

i. Area of sector OAPB

ii. Area of minor segment APB. (Use π = 3.14)

Section C

26. Find the LCM and HCF of 404 and 96 and verify that LCM × HCF = product of the two numbers **[3]**

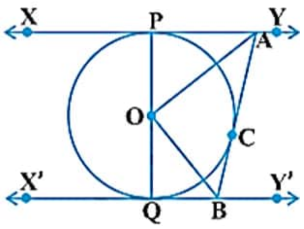
27. The mid-point P of the line segment joining the points A (-10,4) and B (- 2,0) lies on the line segment joining the points C (- 9, - 4) and D (- 4, y). Find the ratio in which P divides CD. Also, find the value of y. **[3]**

28. Had Aarush scored 8 more marks in a Mathematics test, out of 35 marks, 7 times these marks would have been 4 less than square of his actual marks. How many marks did he get in the test? **[3]**

OR

If two pipes function simultaneously, a reservoir will be filled in 12 hours. One pipe fills the reservoir 10 hours faster than the other. How many hours will the second pipe take to fill the reservoir?

29. In Figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersects XY at A and X'Y' at B. Prove that $\angle AOB$ = 90°. **[3]**



OR

If all the sides of a parallelogram touch a circle, show that the parallelogram is a rhombus.

30. Prove that $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$. **[3]**

31. The angle of elevation of the top of a tower from a point A on the ground is 30^{o}. On moving a distance of 20 metre towards the foot of the tower to a point B the angle of elevation increases to 60^{o}. Find the height of the tower and the distance of the tower from the point A. **[3]**

Section D

32. Abdul travelled 300 km by train and 200 km by taxi taking 5 hours 30 minutes. But, if he travels 260 km by train and 240 km by taxi, he takes 6 minutes longer. Find the speed of the train and that of the taxi. **[5]**

OR

The ratio of incomes of two persons is 11 : 7 and the ratio of their expenditures is 9 : 5. If each of them manages to save Rs 400 per month, find their monthly incomes.

33. Find the lengths of the medians of a $\triangle$ABC having vertices at A (0, -1), B (2, 1) and C (0, 3). **[5]**

34. In a cylindrical vessel of radius 10 cm, containing some water, 9000 small spherical balls are dropped which are completely immersed in water which raises the water level. If each spherical ball is of radius 0·5 cm, then find the rise in the level of water in the vessel. **[5]**

OR

A hemispherical depression is cut out from one face of a cubical block of side 7 cm, such that the diameter of the hemisphere is equal to the edge of the cube. Find the surface area of the remaining solid.

35. In an A.P., the n^{th} term is $\frac{1}{m}$ and the m^{th} term is $\frac{1}{n}$. Find (i) $(mn)^{th}$ term, (ii) sum of first (mn) terms. **[5]**

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

An object which is thrown or projected into the air, subject to only the acceleration of gravity is called a projectile, and its path is called its trajectory. This curved path was shown by Galileo to be a parabola. Parabola is represented by a polynomial. If the polynomial to represent the distance covered is,

$p(t) = -5t^2 + 40t + 1.2$

i. What is the degree of the polynomial $p(t) = -5t^2 + 40t + 1.2$? (1)
ii. What is the height of the projectile at the time of 4 seconds after it is launched? (1)
iii. What is the name of the polynomial $p(t) = -5t^2 + 40t + 1.2$ that is classified based on its degree? (2)

OR

What are the factors of the given quadratic equation $p(x) = x^2 - 5x + 6$? (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Under the physical and health education a medical check up program was conducted in a Vidyalaya to improve the health and fitness conditions of the students. Reading of the heights of 50 students was obtained as given in the table below:

Height (in cm)	Number of students
135-140	2
140-145	8
145-150	10
150-155	15
155-160	6
160-165	5
165-170	4

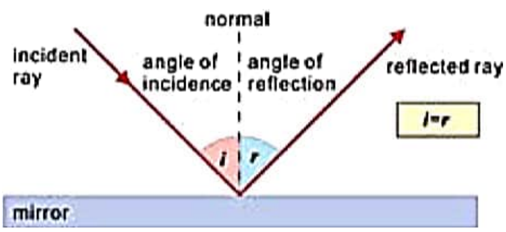
i. Find the lower class limit of the modal class. (1)
ii. Find the median class. (1)
iii. Find the assumed mean of the data. (2)

OR

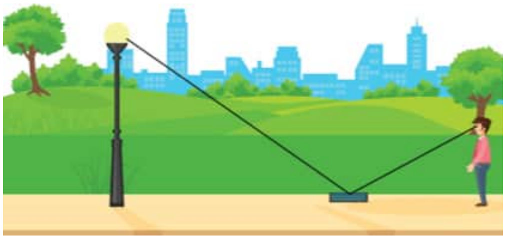
Find the median of the given data. (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

The law of reflection states that when a ray of light reflects off a surface, the angle of incidence is equal to the angle of reflection.



Suresh places a mirror on level ground to determine the height of a pole (with traffic light fired on it). He stands at a certain distance so that he can see the top of the pole reflected from the mirror. Suresh's eye level is 1.5 m above the ground. The distance of Suresh and the pole from the mirror are 1.8 m and 6 m respectively.



i. Which criterion of similarity is applicable to similar triangles? (1)
ii. What is the height of the pole? (1)
iii. If angle of incidence is i, find tan i. (2)

OR

Now Suresh move behind such that distance between pole and Suresh is 13 meters. He place mirror between him and pole to see the reflection of light in right position. What is the distance between mirror and Suresh? (2)

Solution

Section A

1.

(d) 14

Explanation: Probability of getting bad eggs = $\frac{\text{No. of bad eggs}}{\text{Total no. of eggs}}$

$\Rightarrow 0.035 = \frac{\text{No. of bad eggs}}{400}$

$\Rightarrow$ No. of bad eggs = 0.035 $\times$ 400 = 14

2.

(c) ad $\neq$ bc

Explanation: $(a^2 + b^2) x^2 + 2(ac + bd)x + c^2 + d^2 = 0$

Here $A = a^2 + b^2$, $B = 2(ac + bd)$, $C = c^2 + d^2$

$D = B^2 - 4AC = [2(ac + bd)]^2 - 4(a^2 + b^2)(c^2 + d^2)$

$= 4[a^2c^2 + b^2d^2 + 2abcd] - 4[a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2]$

$= 4a^2c^2 + 4b^2d^2 + 8abcd - 4a^2c^2 - 4a^2d^2 - 4b^2c^2 - 4b^2d^2$

$= -4a^2d^2 - 4b^2c^2 + 8abcd$

$= -4(a^2d^2 + b^2c^2 - 2abcd)$

$= -4(ad - bc)^2$

$\because$ Roots are not real

$\therefore D < 0$

$\therefore -4(ad - bc)^2 < 0 \Rightarrow (ad - bc)^2 < 0$

$\Rightarrow ad - bc < 0$ or $ad \neq bc$

3.

(b) 19.4 cm^3

Explanation: R = $\frac{4.2}{2}$

= 2.1cm

h = 4.2 cm

Volume of cone= $\frac{1}{3}\pi r^2 h$

$\frac{1}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 4.2$

= 19.404 cm^3

4.

(b) $\frac{c(a-b)}{a(b-c)}$

Explanation: Given equation is

$a(b - c)x^2 + b(c - a)x + c(a - b) = 0$

Let α be the other root, then

Product of roots $= \alpha \times 1 = \frac{c(a-b)}{a(b-c)}$

$\Rightarrow \alpha = \frac{c}{a}\left(\frac{a-b}{b-c}\right)$

5.

(d) 20

Explanation: First term, a = - 5

Common difference, $d = \frac{5}{2} - 0 = \frac{5}{2}$

n = 11

As we know, nth term of an AP is

$a_n = a + (n - 1)d$

where a = first term

a_n is nth term

d is the common difference

$a_{11} =$ - 5 + (11 - 1)(5/2)

$a_{11} =$ - 5 + 25 = 20

6.

(b) $\sqrt{2a^2 + 2b^2}$

Explanation: distance between the point. (0, 0) and (a - b, a + b) is

= $\sqrt{(a-b-0)^2 + (a+b-0)^2}$

= $\sqrt{(a-b)^2 + (a+b)^2}$

= $\sqrt{a^2 + b^2 - 2ab + a^2 + b^2 + 2ab}$

= $\sqrt{2(a^2 + b^2)}$ = $\sqrt{2a^2 + 2b^2}$ units.

7. **(a)** both negative

Explanation: Given; $x^2 + 88x + 125 = 0$

$D = (88)^2 - 4(1)(125)$

$D = 7244$

Now,

$x = \frac{-(88) \pm \sqrt{7244}}{2(1)}$

$\Rightarrow x = \frac{-88 + 2\sqrt{1811}}{2}$

There roots are $x = -44 + \sqrt{1811}, -44 - \sqrt{1811}$

Which are both negative.

8.

(d) $DC^2 = CF \times AC$

Explanation: In $\triangle$ABC, using Thales theorem,

$\frac{\mathrm{DC}}{\mathrm{AC}} = \frac{\mathrm{CE}}{\mathrm{BC}}$ $[AB||DE]$ ……..(i)

And in triangle BCD, using Thales theorem,

$\frac{\mathrm{CF}}{\mathrm{DC}} = \frac{\mathrm{CE}}{\mathrm{BC}}$ $[BD||EF]$ ……..(ii)

From eq. (i) and (ii), we have

$\frac{\mathrm{DC}}{\mathrm{AC}} = \frac{\mathrm{CF}}{\mathrm{DC}}$

$\Rightarrow DC^2 = CF \times AC$

9. **(a)** 6 cm

Explanation: Here AQ = 4 cm

$\therefore$ QB = AQ = 4 cm [Tangents from an external point]

$\therefore$ BR = 7 - 4 = 3 cm

$\therefore$ BR = CR = 3 cm [Tangents from an external point]

Also SD = SC = 3 cm [Tangents from an external point]

Therefore, x = CS + CR = 3 + 3 = 6 units

10.

(d) $4\sqrt{10}$ cm

Explanation: In right $\triangle$PAO, PA = 12 cm and OA = 5 cm

$\therefore$ By Pythagoras theorem,

$OP^2 = OA^2 + PA^2 = 5^2 + (12)^2 = 25 + 144 = 169$

$\Rightarrow$ OP = $\sqrt{169}$ = 13 cm

In right $\triangle$PBO, $PB^2 = OP^2 - OB^2$

= $13^2 - 3^2$ = 169 - 9 = 160

$\Rightarrow$ PB = $\sqrt{160}$ cm = $4\sqrt{10}$ cm

11. **(a)** $\sqrt{a^2 + b^2 - c^2}$

Explanation: Given: a sinθ + b cosθ = c

Squaring both sides, we get

$\Rightarrow a^2\sin^2\theta + b^2\cos^2\theta + 2ab\sin\theta\cos\theta = c^2$

$\Rightarrow a^2(1 - \cos^2\theta) + b^2(1 - \sin^2\theta) + 2ab\sin\theta\cos\theta = c^2$

$\Rightarrow a^2 - a^2\cos^2\theta + b^2 - b^2\sin^2\theta + 2ab\sin\theta\cos\theta = c^2$

$\Rightarrow a^2\cos^2\theta - b^2\sin\theta - 2ab\sin\theta\cos\theta = a^2 + b^2 - c^2$

$\Rightarrow (a\cos\theta - b\sin\theta)^2 = a^2 + b^2 - c^2$

$\Rightarrow a\cos\theta - b\sin\theta = \sqrt{a^2 + b^2 - c^2}$

12. **(a)** 936

Explanation: LCM (72, 234) = $\frac{(72\times234)}{18}$ = 936

Therefore, the LCM of (72, 234) is 936.

13. **(a)** 1 : 3

Explanation: Let AB and CD be the given pillars and O be the midpoint of AC.

Then, AB = x, CD = y, $\angle AOB = 30°$ and $\angle COD = 60°$.

From right $\triangle OAB$, we have

$\frac{OA}{AB} = \cot 30^\circ \Rightarrow \frac{OA}{x} = \sqrt{3}$

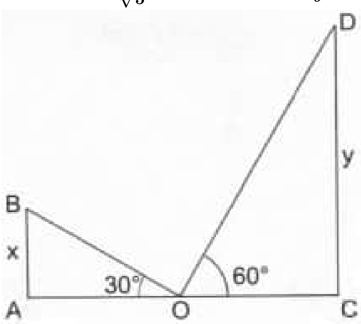
$\Rightarrow$ OA = $x\sqrt{3}$. .....(i)

From right $\triangle OCD$, we have

$\frac{OC}{CD} = \cot 60^\circ \Rightarrow \frac{OC}{y} = \frac{1}{\sqrt{3}} \Rightarrow OC = \frac{y}{\sqrt{3}}$ ....(ii)

But, OA = OC

$\therefore x\sqrt{3} = \frac{y}{\sqrt{3}} \Rightarrow 3x = y \Rightarrow \frac{x}{y} = \frac{1}{3} \Rightarrow$ x : y = 1 : 3



14. **(a)** 231 cm^2

Explanation: The angle subtended by the arc = 60^o

So, area of the sector = $(\frac{60^o}{360^o}) \times \pi r^2$ cm^2

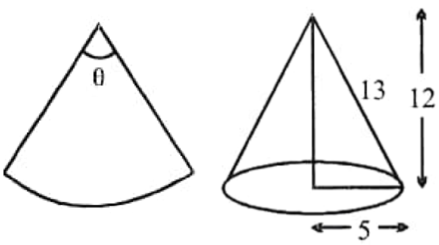
= $(\frac{441}{6}) \times (\frac{22}{7})cm^2$

= 231 cm^2

15.

(c) $\frac{10\pi}{13}$

Explanation:



$\therefore$ Slant height = 13

As, $\theta = \frac{S}{r}$

$\Rightarrow S = r\theta$

$\Rightarrow 2\pi(5) = 13\,\theta$

$\Rightarrow \theta = \frac{10\pi}{13}$

16. **(a)** $\frac{6}{11}$

Explanation: Total number of letters in 'ASSOCIATION' = 11

Vowels are A, O, I, A, I, O, i.e, 6 in numbers.
∴ Probability of getting a vowel = $\frac{6}{11}$

17.
(c) 0
Explanation: Elementary events are
(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)
∴ Number of Total outcomes = 36
And Number of possible outcomes (product of numbers appearing on die is 7) = 0
∴ Required Probability = $\frac{0}{36} = 0$

18.
(c) 52
Explanation: Mode = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$
= 40 + $\frac{7-3}{7\times2-3-6} \times 15$
= 40 + $\frac{4}{5} \times 15$
= 40 + 12
= 52

19.
(d) A is false but R is true.
Explanation: A is false but R is true.

20.
(d) A is false but R is true.
Explanation: We have, common difference of an AP
d = $a_n - a_{n-1}$ is independent of n or constant.
So, A is false but R is true.

Section B

21. Let us assume that 2 + $3\sqrt{3}$ is a rational number
2 + $3\sqrt{3}$ = $\frac{p}{q}$; p, q are integers and q $\neq$ 0
$\Rightarrow \sqrt{3}$ = $\frac{p-2q}{3q}$
RHS is rational but LHS is irrational
∴ Our assumption is wrong. Hence 2 + $3\sqrt{3}$ is an irrational number.

22. Given that,
In the figure the triangle ABC
XZ||BC and the length of AZ = 3 cm, ZC = 2 cm, BM = 3 cm and MC = 5 cm.
From $\triangle$ABC and $\triangle$AXZ
$\angle$AXZ = $\angle$ABC [by corresponding angles]
$\angle$AZX = $\angle$ACB [by corresponding angles]
By basic proportionality theorem $\triangle$ABC and $\triangle$AXZ are similar.
So,
$\frac{YZ}{MC} = \frac{AZ}{ZC}$
$\frac{YZ}{5} = \frac{3}{2}$
YZ = $\frac{15}{2}$
Then,
$\frac{XZ}{BC} = \frac{AZ}{ZC}$
$\frac{XY+YZ}{BM+MC} = \frac{AZ}{ZC}$
$\frac{XY+\frac{15}{2}}{3+5} = \frac{3}{2}$

XY + $\frac{15}{2} = \frac{24}{2}$

XY = $\frac{9}{2}$ = 4.5 cm

23. Let the sides of the quadrilateral ABCD touch the circle at P, Q, R and S. Since, the lengths of the tangents from an external point to a given circle are equal.

    $\therefore$ AP = AS

    $\Rightarrow$ BP = BQ

    CR = CQ

    $\Rightarrow$ DR = DS

    Adding, (AP + BP) + (CR + DR) = (BQ + CQ) + (AS + DS)

    $\Rightarrow$ AB + CD = BC + AD.

    Hence proved

24. We have,

    $x = a\cos^3\theta, y = b\sin^3\theta$

    $\frac{x}{a} = \cos^3\theta \ and \ \frac{y}{b} = \sin^3\theta$

    L.H.S $= \left[\frac{x}{a}\right]^{2/3} + \left[\frac{y}{b}\right]^{2/3}$

    $= \left[\cos^3\theta\right]^{2/3} + \left[\sin^3\theta\right]^{2/3} \ [\because (a^m)^n = a^{m\times n}]$

    $= \cos^2\theta + \sin^2\theta$

    = 1 $\left[\because \cos^2\theta + \sin^2\theta = 1\right]$

    = R.H.S.

    Hence proved.

OR

Given, $\sqrt{2}$sinθ = 1

sin θ = $\frac{1}{\sqrt{2}}$

sin 45°=$\frac{1}{\sqrt{2}}$

$\therefore sin\theta$ = sin45°

$\theta = 45^\circ$

put $\theta = 45^\circ in$

sec$^2\theta$ - cosec^2 θ

= sec^{2}45° - cosec^{2}45°

= $(\sqrt{2})^2 - (\sqrt{2})^2$

=$2 - 2$

= 0

25. We have, OA = R = 21 m and OC = r = 14 m

    $\therefore$Area of the flower bed = Area of a quadrant of a circle of radius R - Area of a quadrant of a circle of radius r

    $= \frac{1}{4}\pi R^2 - \frac{1}{4}\pi r^2$

    $= \frac{\pi}{4}\left(R^2 - r^2\right)$

    $= \frac{1}{4} \times \frac{22}{7}\left(21^2 - 14^2\right) \text{cm}^2$

    $= \left\{\frac{1}{4} \times \frac{22}{7} \times (21 + 14)(21 - 14)\right\} m^2$

    $= \left\{\frac{1}{4} \times \frac{22}{7} \times 35 \times 7\right\} \text{m}^2$

    = 192.5 m^2

OR

i. Area of sector OAPB = $\frac{90^\circ}{360^\circ} \times 3.14 \times 100$ = 78.5 cm^2

ii. Area minor segment APB = Area sector OAPB - Area $\triangle\text{OAB}$

    = $78.5 - \frac{1}{2} \times 100$

    = 28.5 cm^2.

Section C

26. Prime factorisation of 404 and 96 is:

    404 = 2 $\times$ 2 $\times$ 101

    or 404 = 2^2 $\times$ 101

$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

or $96 = 2^5 \times 3$

$\therefore$ HCF(404,96)= 2^2 = 4

LCM(404, 96) = $101 \times 2^5 \times 3$

LCM(404, 96) = 9696

Now we have to verify that,

HCF(404, 96) $\times$LCM(404, 96) = 404$\times$96

Hence, LHS = HCF $\times$ LC M = 4 $\times$ 9696 = 38784

RHS = Product of numbers = 404 $\times$ 96 = 38784

Since, LHS = RHS

$\therefore$ HCF $\times$ LCM = Product of 404 and 96.

Hence verified.

27. Coordinates of the midpoint P of A and B are

$\left(\frac{-10+(-2)}{2}, \frac{4+0}{2}\right) = (-6, 2)$

P lies on the line joining C and D.

Let P(-6, 2) divide C(-9, -4) and D(-4, y) in the ratio of r:1

Using the section formula for the x-coordinate we get

$-6 = \frac{-4r-9}{r+1} \Rightarrow$ -6r - 6 = -4r - 9

$\Rightarrow 2r = 3 \Rightarrow r = \frac{3}{2}$

Hence, P(-6, 2) divides C(-9, -4) and D(-4, y) in the ratio of 3:2

Using the section formula for y-coordinate we get

$-6 = \frac{-4r-9}{r+1} \Rightarrow -6r - 6 = -4r - 9$

$\Rightarrow 2r = 3 \Rightarrow r = \frac{3}{2}$

Hence, P(-6, 2) divides C(-9, -4) and D(-4, y) in the ratio of 3:2

Using the section formula for y-coordinate we get

$2 = \frac{3y-8}{3+2} \Rightarrow 10 = 3y - 8 \Rightarrow 3y = 18$

$\Rightarrow y = 6$

28. Let the actual marks be x

Accroding to the question,

7(x + 8) = x^2 - 4

$\Rightarrow$ 7x + 56 = x^2 - 4

$\Rightarrow$ x^2 - 7x - 60 = 0

$\Rightarrow$ x^2 - 12x + 5x - 60 = 0

$\Rightarrow$ x(x - 12) +5(x - 12) = 0

$\Rightarrow$ (x - 12)(x + 5) = 0

$\Rightarrow$ x - 12 = 0 or x + 5 = 0

$\Rightarrow$ x = 12 or x = -5

$\Rightarrow$ x = 12 ... [$\because$ Marks can't be negative]

Hence, Aarush scored 12 marks in Mathematics test.

OR

Let, the slower pipe takes x hours to fill the reservoir.

Hence, the faster pipe will take (x - 10) hours to fill the reservoir.

Since, the slower pipe takes x hours to fill the reservoir.

$\therefore$ Portion of the reservoir filled by the slower pipe in 1 hour $= \frac{1}{x}$

$\therefore$ Portion of the reservoir filled by the slower pipe in 12 hours $= \frac{1}{x} \times 12 = \frac{12}{x}$.

Now, portion of the reservoir filled by the faster pipe in 1hr =$\frac{1}{x-10}$

$\therefore$ Portion of the reservoir filled by faster pipe in 12 hours $= \frac{1}{x-10} \times 12 = \frac{12}{x-10}$

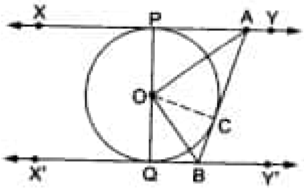
It is given that the reservoir is completely filled in 12 hours by simultaneously operating both pipes.

$\therefore \frac{12}{x} + \frac{12}{x-10} = 1$

$\Rightarrow \frac{12(x-10)+12x}{x(x-10)} = 1$

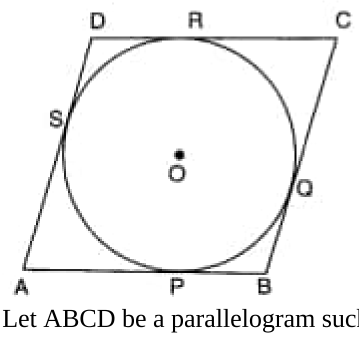
$\Rightarrow \frac{12x-120+12x}{x^2-10x} = 1$
$\Rightarrow x^2 - 10x = 24x - 120$
$\Rightarrow x^2 - 10x - 24x + 120 = 0$
$\Rightarrow x^2 - 34x + 120 = 0$
$\Rightarrow x^2 - 30x - 4x + 120 = 0$
$\Rightarrow$ x(x - 30) - 4(x - 30) = 0
$\Rightarrow$ (x - 30)(x - 4) = 0
$\Rightarrow$ x - 30 = 0. [$\therefore$ x - 4 $\neq$ 0, otherwise (x-10) i.e time taken by faster pipe will become -6 hr i.e. negative, which is not possible]
$\Rightarrow$ x = 30
Hence, the second pipe take 30 hours to fill the reservoir.

29. According to the question, XY and X 'Y ' are x two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersects XY at A and X 'Y ' at B.



In quad. APQB, we have
$\angle APO$ =90°
and $\angle BQO$ = 90° [$\because$ tangent at any point is perpendicular to the radius through the point of contact]
Now, $\angle APO + \angle BQO + \angle QBC + \angle PAC = 360^\circ$
$\Rightarrow \angle PAC + \angle QBC = 360^\circ - (\angle APO + \angle BQO) = 180^\circ$ ...(i)
We have,
$\angle CAO = \frac{1}{2} \angle PAC$
and $\angle CBO = \frac{1}{2} \angle QBC$ [$\because$ tangents from an external point are equally inclined to the line segment joining the centre to that point]
$\therefore \angle \text{CAO} + \angle \text{CBO} = \frac{1}{2} (\angle PAC + \angle QBC) = \frac{1}{2} \times 180^\circ = 90^\circ$ .....(ii)
In $\triangle AOB$, we have
$\angle CAO + \angle AOB + \angle CBO = 180^\circ$
$\Rightarrow 90^\circ + \angle AOB = 180^\circ$
$\Rightarrow \quad \angle AOB = 90^\circ$

OR



Let ABCD be a parallelogram such that its
sides touch a circle with centre O.
We know that the tangents to a circle from an exterior
point are equal in length.
Therefore,AP = AS [From A] ...(i)
BP = BQ [FromB] ...(ii)
CR = CQ [From C] ...(iii)
and , DR = DS [From D] ...(iv)
Adding (i), (ii), (iii) and (iv), we get,
AP + BP + CR + DR = AS + BQ +CQ + DS
$\Rightarrow$ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)
$\Rightarrow$ AB + CD = AD + BC
$\Rightarrow$ 2 AB = 2 BC

⇒ AB = BC

Therefore, AB = BC = CD = AD

Thus, ABCD is a rhombus.

30. LHS = $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

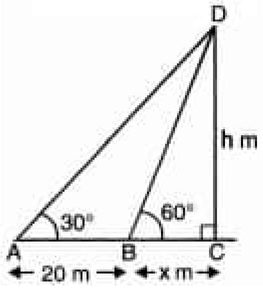
$= \sin^2 A + \operatorname{cosec}^2 A + 2 \sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2 \cos A \sec A$

$= \sin^2 A + \cos^2 A + \operatorname{cosec}^2 A + \sec^2 A + 2 + 2$

$= 1 + \operatorname{cosec}^2 A + \sec^2 A + 4$

$= (1 + \cot^2 A) + (1 + \tan^2 A) + 5$

$= 7 + \tan^2 A + \cot^2 A$ = RHS

31. 

Let height of tower be h m and distance BC be x m

In △ DBC, $\frac{h}{x} = \tan 60^\circ$

⇒ $h = \sqrt{3x}$ .....(i)

$\frac{h}{x+20} = \tan 30^\circ = \frac{1}{\sqrt{3}}$

⇒ $\sqrt{3h} = x + 20$ ....(ii)

Substituting the value of h from eq. (i) in eq. (ii), we get

3x = x + 20

3x - x = 20

Or 2x = 20

⇒ x = 10 m ...(iii)

Again$h = \sqrt{3x}$

or, $h = \sqrt{3} \times 10 = 10\sqrt{3}$

$= 10 \times 1 \cdot 732$

= 17.32 m

[from (i) and (in)]

Hence, height of tower is 17.32 m and distance of

tower from point A is 30 m

Section D

32. Suppose, speed of the train be x km/hr and the speed of taxi be y km/h.

time taken to cover 300 km by the train = $\frac{300}{x}$hours

time taken to cover 200 km by the taxi = $\frac{200}{y}$ hours

Total time taken = $5\frac{30}{60}$ hours $= 5\frac{1}{2}$ hours $= \frac{11}{2}$ hours

$\therefore \frac{300}{x} + \frac{200}{y} = \frac{11}{2}$

$\Rightarrow \frac{600}{x} + \frac{400}{y} = 11$

Put $\frac{1}{x}$ = u and $\frac{1}{y}$ = v

⇒$600u + 400v = 11$ ....(i)

time taken to cover $260\ km$ by the train = $\frac{260}{x}$hours

time taken to cover $240\ km$ by the taxi = $\frac{240}{y}$ hours

Total time taken = $5\frac{36}{60}$ hours $= 5\frac{1}{2}$ hours $= \frac{11}{2}$ hours

⇒$1300u + 1200v = 28$ ....(ii)

Multiplying (i) by 3 and subtracting (ii) from it,

$\Rightarrow 500u = 5 \Rightarrow u = \frac{5}{500} \Rightarrow u = \frac{1}{100}$

Substituting u = $\frac{1}{100}$ in (i), ⇒ v = $\frac{1}{80}$

$\therefore u = \frac{1}{100} \Rightarrow \frac{1}{x} = \frac{1}{100} \Rightarrow x = 100$

$v = \frac{1}{80} \Rightarrow \frac{1}{y} = \frac{1}{80} \Rightarrow y = 80$

$\therefore$ the speed of the train = 100 km/hr

the speed of the taxi = 80 km/hr

OR

Let the incomes of two persons be 11x and 7x.

And the expenditures of two persons be 9y and 5y

$\therefore 11x - 9y = 400$ ....(i)

$7x - 5y = 400$ ....(ii)

Multiplying (i) by 5 and (ii) by 9 and subtracting,

$$\begin{array}{r} 55x - 45y = 2{,}000 \\ 63x - 45y = 3{,}600 \\ - \quad + \quad\quad - \\ \hline \end{array}$$

$-8x = -1600$

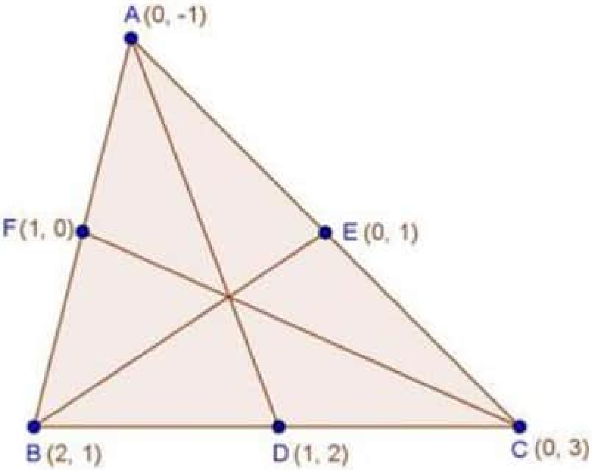
$\therefore -8x = -1600$

$x = \frac{-1{,}600}{-8} = 200$

Therefore, Their monthly incomes are

11 $\times$ 200 = Rs 2200

7 $\times$ 200 = Rs1400

33. 

Let A(0, -1), B(2, 1) and C(0, 3) be the given points.

Let AD, BE and CF be the medians

Coordinates of D are $\left(\frac{2+0}{2}, \frac{1+3}{2}\right) = (1, 2)$

Coordinates of E are $\left(\frac{0}{2}, \frac{3-1}{2}\right) = (0, 1)$

Coordinates of F are $\left(\frac{2+0}{2}, \frac{1-1}{2}\right) = (1, 0)$

Length of median $AD = \sqrt{(1-0)^2 + (2+1)^2} = \sqrt{10}\, units$

Length of median $BE = \sqrt{(2-0)^2 + (1-1)^2} = 2 units$

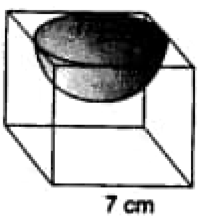
Length of median $CF = \sqrt{(1-0)^2 + (0-3)^2} = \sqrt{10}\, units$

34. Volume of raised water in cylinder = Volume of 9000 spherical balls

$\pi(10)^2$H = 9000 $\times \frac{4}{3} \times \pi \times (0.5)^3$

$\therefore$ H = 15 cm

OR



Edge of the cube, a = 7 cm.

Radius of the hemisphere, $r = \frac{7}{2}$cm.

Surface area of remaining solid

= total surface area of the cube - area of the top of hemispherical part + curved surface area of the hemisphere

$= 6a^2 - \pi r^2 + 2\pi r^2 = 6a^2 + \pi r^2$

$= \left(6 \times 7 \times 7 + \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right) \mathrm{cm}^2$

= (294 + 38.5)cm^2 = 332.5 cm^2.

35. $a_n = \frac{1}{m}$

a + (n - 1)d = $\frac{1}{m}$

$a_m = \frac{1}{n}$

a + (m - 1)d = $\frac{1}{n}$

On solving,

a = $\frac{1}{mn}$

d = $\frac{1}{mn}$

i. $a_{mn} = \frac{1}{mn} + (mn - 1) \times \frac{1}{mn}$

$= \frac{1 + mn - 1}{mn} = 1$

ii. $S_{mn} = \frac{mn}{2}\left(\frac{1}{mn} + 1\right)$

$= \frac{1 + mn}{2}$

Section E

36. i. 2

ii. 81.2 m

iii. quadratic polynomial

OR

(x - 3) and (x - 2)

37. i. The maximum class frequency is 15 belonging to class interval 150-155

∴ 150 - 155 is the modal class

lower limit (l) of modal class =150

ii.

Height (in cm)	frequency	C.F
135-140	2	2
140-145	8	10
145-150	10	20
150-155	15	35
155-160	6	41
160-165	5	46
165-170	4	50
	$\sum fi$ = 50	

$\sum fi$ = 2 + 8 + 10 + 15 + 6 + 5 + 4 =50 =N

$\frac{N}{2} = \frac{50}{2}$ = 25

c.f just greater that $\frac{N}{2}$ i.e, 25 is 35

∴ Median class 150-155

iii.

Height (in cm)	frequency (f_i)	x_i
135-140	2	137.5
140-145	8	142.5
145-150	10	147.5
150-155	15	**152.5**
155-160	6	157.5

160-165	5	162.5
165-170	4	167.5

$x_i = \frac{\text{lower limit}+\text{upper limit}}{2}$

middle term of x_i,is the assumean mean

Hence, Assumed Mean = 152.5

OR

Median = $l\left(\frac{\frac{n}{2}-c.f}{f}\right) \times h$

= 150 + $\left(\frac{25-20}{15}\right) \times 5$

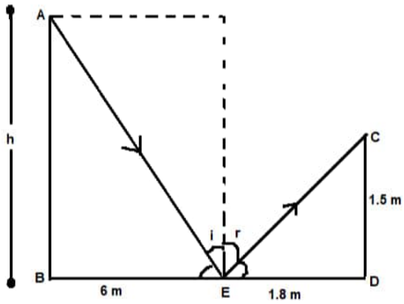
= 150 + $\frac{5}{15} \times 5$

= 150 + $\frac{5}{3}$

= 150 + 1.67

= 151.67

38. i. AA criterion

ii. 

$\triangle ABE \sim \triangle CDE$ (by AA criteria)

$\frac{AB}{CD} = \frac{BE}{DE}$

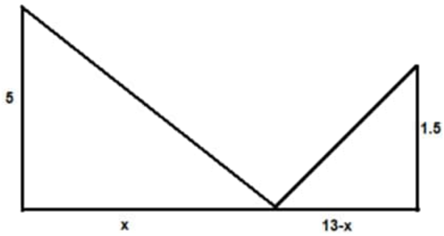
h = $\frac{6\times1.5}{1.8}$

h = 5

i.e., height of pole = 5 m.

iii. tan i = $\frac{6}{5}$

OR



$\frac{1.5}{5} = \frac{13-x}{x}$

1.5x = 65 - 5x

6.5x = 65

x = $\frac{65}{6.5}$

= 10

$\therefore$ distance of Suresh from mirror

= 13 - x

= 13 - 10

= 3 m