Class X Session 2024-25
Subject - Mathematics (Standard)
Sample Question Paper - 10

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment carrying 04 marks each.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

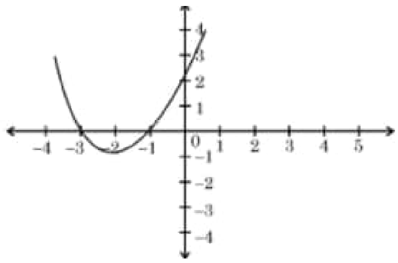
Section A

1. The total number of factors of a prime number is: **[1]**

a) 2 b) 1

c) 3 d) 0

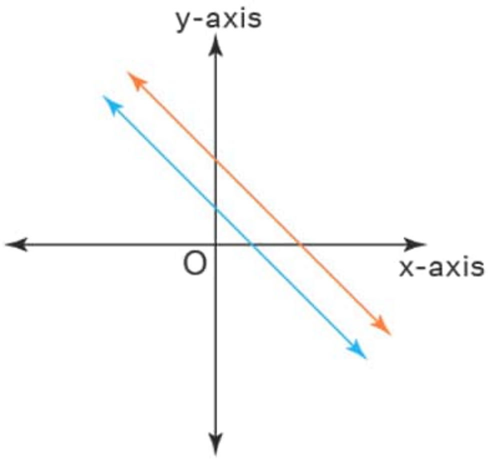
2. In the figure, the graph of the polynomial p(x) is given. The number of zeroes of the polynomial is: **[1]**



a) 2 b) 1

c) 0 d) 3

3. A system of linear equations is said to be inconsistent if it has **[1]**



a) one solution
b) at least one solution
c) two solutions
d) no solution

4. The value of λ for which $(x^2 + 4x + \lambda)$ is a perfect square, is **[1]**

a) 1
b) 16
c) 4
d) 9

5. How many terms are there in the A.P. given below? **[1]**
14, 19, 24, 29, ..., 119

a) 22
b) 21
c) 18
d) 14

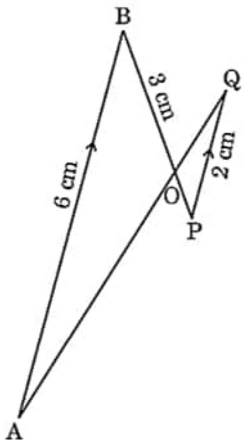
6. The diameter of a circle is of length 6 cm. If one end of the diameter is (-4, 0), the other end on x-axis is at: **[1]**

a) (6, 0)
b) (0, 2)
c) (4, 0)
d) (2, 0)

7. The mid-point of the line-segment AB is P(0, 4). If the coordinates of B are (-2, 3) then the co-ordinates of A are **[1]**

a) (2, 9)
b) (-2, -5)
c) (2, 5)
d) (-2, 11)

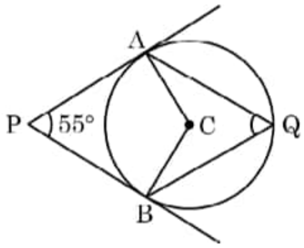
8. In the given figure, AB || PQ. If AB = 6 cm, PQ = 2 cm and OB = 3 cm, then the length of OP is: **[1]**



a) 1 cm
b) 4 cm
c) 9 cm
d) 3 cm

9. In the given figure, PA and PB are tangents from external point P to a circle with centre C and Q is any point on **[1]**

the circle. Then the measure of $\angle AQB$ is



a) 90^o

b) 125^o

c) $62\frac{1}{2}^o$

d) 55^o

10. From an external point Q, the length of tangent to a circle is 12 cm and the distance of Q from the centre of circle is 13 cm. The radius of circle (in cm) is **[1]**

a) 10

b) 5

c) 7

d) 12

11. The value of $\frac{\sin 90^\circ+\cos 60^\circ}{\sec 45^\circ+\tan 45^\circ}$ is: **[1]**

a) $\frac{3}{2}(\sqrt{2}-1)$

b) $\frac{1+\sqrt{3}}{\sqrt{2}+1}$

c) $\frac{3}{2}(\sqrt{2}+1)$

d) 1

12. ($3 \sin^2 30^o$ - $4 \cos^2 60^o$) is equal to: **[1]**

a) $-\frac{1}{4}$

b) $\frac{5}{4}$

c) $-\frac{3}{4}$

d) $-\frac{9}{4}$

13. A ladder 15 m long just reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall, then the height of the wall is **[1]**

a) $15\sqrt{3}$ m

b) 15 m

c) $\frac{15\sqrt{3}}{2}$ m

d) $\frac{15}{2}$ m

14. The area of the sector of a circle with radius 6 cm which subtends an angle of 60^o at the centre of the circle is: **[1]**

a) $\frac{152}{7}$ cm^2

b) $\frac{132}{7}$ cm^2

c) $\frac{142}{7}$ cm^2

d) $\frac{122}{7}$ cm^2

15. Pankaj has a motorcycle with wheels of diameter 91 cm. There are 22 spokes in the wheel. Find the length of arc between two adjoining spokes. **[1]**

a) 13 cm

b) 26 cm

c) 15 cm

d) 18 cm

16. A bag contains 100 cards numbered 1 to 100. A card is drawn at random from the bag. What is the probability that the number on the card is a perfect cube? **[1]**

a) $\frac{1}{20}$

b) $\frac{1}{25}$

c) $\frac{7}{100}$

d) $\frac{3}{50}$

17. A number is chosen from the numbers 1, 2, 3 and denoted as x, and a number is chosen from the numbers 1, 4, 9 and denoted as y. Then $P(xy < 9)$ is: **[1]**

a) $\frac{7}{9}$

b) $\frac{5}{9}$

c) $\frac{3}{9}$

d) $\frac{1}{9}$

18. If the difference of mode and median of a data is 24, then the difference of median and mean of the same data is: **[1]**

a) 8

b) 12

c) 34

d) 24

19. **Assertion (A):** A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water is holds, a child finds its volume to be 345 cm^3. **[1]**

Reason (R): To calculate the volume of vessel the expression used here is $v = \pi r^2 h + \frac{4}{3}\pi r^3$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The 11th term of an AP is 7, 9, 11, 13 is 67. **[1]**

Reason (R): If s_n is the sum of first n terms of an AP then its nth term an is given by $a n = s_n + s_{n-1}$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

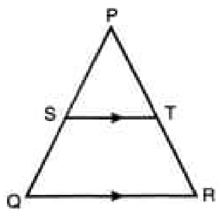
c) A is true but R is false.

d) A is false but R is true.

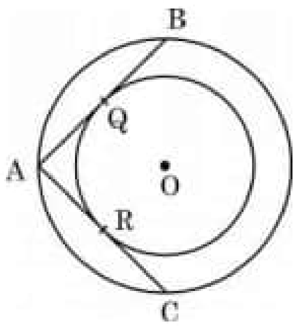
Section B

21. Find H.C.F. and L.C.M. of 56 and 112 by prime factorisation method. **[2]**

22. In the given figure, in a triangle PQR, ST || QR and $\frac{PS}{SQ} = \frac{3}{5}$,PR = 28 cm, find PT. **[2]**



23. In Fig., there are two concentric circles with centre O. If ARC and AQB are tangents to the smaller circle from the point A lying on the larger circle, find the length of AC, if AQ = 5 cm. **[2]**



24. Find the value of : sin 30°. cos 60° + cos 30°. sin 60°. **[2]**

Is it equal to sin 90° or cos 90°?

OR

If a $\cos\theta$ + b $\sin\theta$ = m and a $\sin\theta$ - b $\cos\theta$ = n, prove that $a^2 + b^2 = m^2 + n^2$

25. Find the area of a quadrant of a circle whose circumference is 22 cm. **[2]**

OR

What is the diameter of a circle whose area is equal to the sum of the areas of two circles of diameters 10 cm and 24 cm.

Section C

26. Find the greatest number that will divide 445, 572 and 699 leaving remainders 4, 5 and 6 respectively. **[3]**

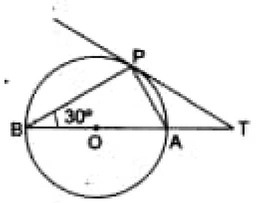
27. Find the quadratic polynomial, sum and product of whose zeroes are -1 and -20 respectively. Also find the zeroes of the polynomial so obtained. **[3]**

28. If the last term of an A.P. of 30 terms is 119 and the 8th term from the end (towards the first term) is 91, then find the common difference of the A.P. Hence, find the sum of all the terms of the A.P. **[3]**

OR

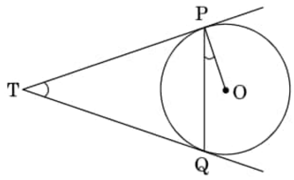
The sum of 5th and 9th terms of an A.P. is 72 and the sum of 7th and 12th terms is 97 .Find the A.P.

29. In the given figure, O is the centre of the circle and TP is the tangent to the circle from an external point T. If $\angle PBT$ = 30°, prove that BA: AT = 2:1. **[3]**



OR

Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that ∠PTQ = 2∠OPQ.



30. Prove that $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$ **[3]**

31. If the median of the following frequency distribution is 32.5. Find the values of f_1 and f_2. **[3]**

Class	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	Total
Frequency	f_1	5	9	12	f_2	3	2	40

Section D

32. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If, the total cost of production on that day was ₹ 90, find the number of articles produced and the cost of each article. **[5]**

OR

If (- 5) is a root of the quadratic equation $2x^2 + px + 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, then find the values of p and k.

33. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the further time taken by the car to reach the foot of the tower from this point. **[5]**

34. A solid is in the shape of a right-circular cone surmounted on a hemisphere, the radius of each of them being 7 cm and the height of the cone is equal to its diameter. Find the volume of the solid. **[5]**

OR

A tent is in the shape of a right circular cylinder up to a height of 3 m and then a right circular cone, with a maximum height of 13.5 m above the ground. Calculate the cost of painting the inner side of the tent at the rate of ₹ 2 per square metre, if the radius of the base is 14 m.

35. A survey regarding the heights (in cm) of 50 girls of X of a school was conducted and the following data was obtained: **[5]**

Height (in cm)	120 - 130	130 - 140	140 - 150	150 - 160	160 - 170	**Total**
Number of girls	2	8	12	20	8	**50**

Find the mean and mode of the above data.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

Two schools **P** and **Q** decided to award prizes to their students for two games of Hockey ₹ x per student and Cricket ₹ y per student. School **P** decided to award a total of ₹ 9,500 for the two games to 5 and 4 students respectively; while school **Q** decided to award ₹ 7,370 for the two games to 4 and 3 students respectively.

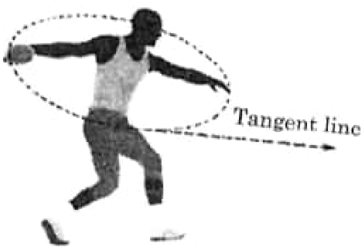
i. Represent the following information algebraically (in terms of x and y). (1)
ii. What is the prize amount for hockey? (1)
iii. Prize amount on which game is more and by how much? (2)

OR

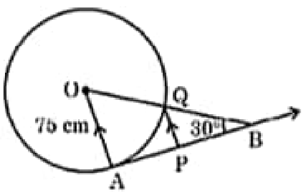
What will be the total prize amount if there are 2 students each from two games? (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

The discus throw is an event in which an athlete attempts to throw a discus. The athlete spins anti-clockwise around one and a half times through a circle, then releases the throw. When released, the discus travels along tangent to the circular spin orbit.



In the given figure, AB is one such tangent to a circle of radius 75 cm. Point O is centre of the circle and $\angle ABO = 30^{o}$. PQ is parallel to OA.



i. find the length of AB. (1)
ii. find the length of OB. (1)
iii. find the length of AP. (2)

OR

find the length of PQ. (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Using Cartesian Coordinates we mark a point on a graph by how far along and how far up it is.

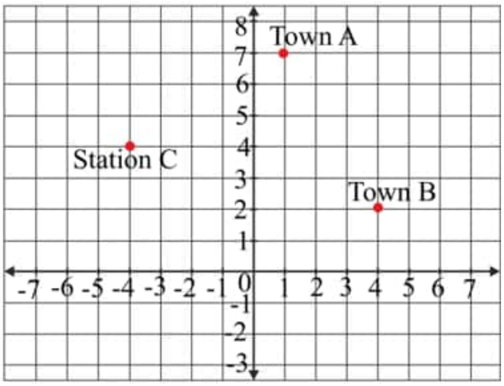
The left-right (horizontal) direction is commonly called X-axis.

The up-down (vertical) direction is commonly called Y-axis.

When we include negative values, the x and y axes divide the space up into 4 pieces.

Read the information given above and below:

Two friends Veena and Arun work in the same office in Delhi. In the Christmas vacations, both decided to go their hometowns represented by Town A and Town B respectively in the figure given below. Town A and Town B are connected by trains from the same station C (in the given figure) in Delhi.



i. Who will travel more distance to reach their home? (1)
ii. Find the location of the station. (1)
iii. Find in which ratio Y-axis divide Town B and Station. (2)

OR

Find the distance between Town A and Town B. (2)

Solution

Section A

1. **(a)** 2

   Explanation: The total number of factors of a prime number = 2 i.e. 1 and itself

2. **(a)** 2

   Explanation: 2

   The number of zeroes is 2 as the graph does cut the x-axis 2 times.

3.

   (d) no solution

   Explanation: A system of linear equations is said to be inconsistent if it has no solution means two lines are running parallel and not cutting each other at any point.

4.

   (c) 4

   Explanation: For a quadratic equation to be a perfect square

   D = 0

   b^2 - 4ac = 0

   $(4)^2$ - 4(1)(λ) = 0

   16 - 4λ = 0

   16 = 4λ

   λ = 4

5. **(a)** 22

   Explanation: Given, a_1 = 14, t_n = 119

   d = a_2 - a_1 = 19 - 14 = 5

   t_n = a_1 + (n - 1)d

   119 = 14 + (n - 1)5

   119 - 14 = 5n - 5

   105 + 5 = 5n

   110 = 5n

   n = 22

6.

   (d) (2, 0)

   Explanation: Let the other point on x axis is (x, 0)

   By distance formula

   $\sqrt{(x+4)^2+(0-0)^2}=(6)$

   x + 4 = 6

   x = 2

   Hence the point is (2, 0)

7.

   (c) (2, 5)

   Explanation: Let cordinate of A(x,y)

   Then cordinate of mid point are $\left[\frac{(x-2)}{2},\frac{(y+3)}{2}\right]$

   On comparing the cordinates of mid points

   $\frac{(x-2)}{2}=0$

   x = 2

   $\frac{(y+3)}{2}=4$

y = 5

Cordinates of A are (2, 5).

8. **(a)** 1 cm

Explanation: In $\triangle$ABO and $\triangle$QPO

$\angle$BAO = $\angle$PQO (by alt. angle)

$\angle$AOB = $\angle$QOP (vert. oppo. angle)

$\therefore \triangle$ABO $\sim \triangle$QPO (by AA Similarity)

$\therefore \frac{AB}{QP} = \frac{OB}{OP}$

$\frac{6}{2} = \frac{3}{OP}$

OP = 1cm

9.

(c) $62\frac{1}{2}^o$

Explanation: Given, $\angle$APB = 55^o

$\therefore \angle$ACB = 180^o - 55^o = 125^o ...($\because \angle$APB and $\angle$ACB are supplementary angles)

Now, as we know that

Angle subtended by an arc at the centre = 2 $\times$ angle subtended by arc at any point on the remaining part of the circle

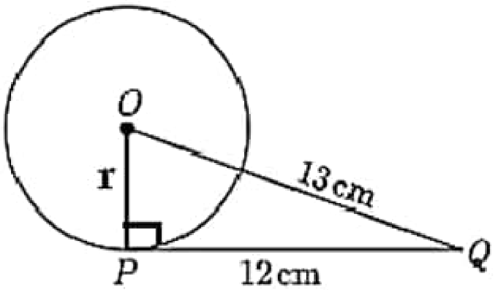
$\therefore 125^o = 2 \times \angle$AQB

$\Rightarrow \angle$AQB = $\frac{125}{2}$

= 62.5^o or $62\frac{1}{2}^o$

10.

(b) 5

Explanation: 

$\because$ We know that

PQ $\perp$ OP

$\therefore \triangle QPO$ is right angled $\triangle$

$\therefore$ By python theory.

$QO^2 = QP^2 + OP^2$

$(13)^2 = (12)^2 + OP^2$

r^2 = 169 - 144.

$r^2 = 25$

r = 5 cm

11. **(a)** $\frac{3}{2}(\sqrt{2}-1)$

Explanation: = $\frac{\sin 90^\circ + \cos 60^\circ}{\sec 45^\circ + \tan 45^\circ}$

$= \frac{1+\frac{1}{2}}{\sqrt{2}+1}$

$= \frac{\frac{3}{2}}{\sqrt{2}+1}$

upon rationalization

$= \frac{3}{2} \times \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$

$= \frac{3}{2} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$

$= \frac{3}{2}(\sqrt{2}-1)$

12. **(a)** $-\frac{1}{4}$

Explanation: (3 $\sin^2 30^o$ - 4 $\cos^2 60^o$)

$\Rightarrow 3 \times \left(\frac{1}{2}\right)^2 - 4 \times \left(\frac{1}{2}\right)^2$

$\Rightarrow -\frac{1}{4}$

13.

(d) $\frac{15}{2}$ m

Explanation:

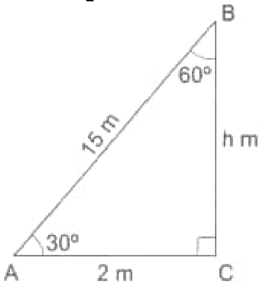
Let AB be the ladder and BC be the wall

Then, $\angle$ABC = 60° $\Rightarrow$ $\angle$CAB = (90° - 60°) = 30°

Let BC = h m. then,

$\frac{BC}{AB} =$ = sin 30° $\Rightarrow \frac{h}{15} = \frac{1}{2}$

$\Rightarrow$ h = $\frac{15}{2}$



14.

(b) $\frac{132}{7}$ cm^2

Explanation: Angle of the sector is 60°

Area of sector = $(\frac{\theta}{360^o}) \times \pi r^2$

$\therefore$ Area of the sector with angle 60^o = $(\frac{60^o}{360^o}) \times \pi r^2$ cm^2

= $(\frac{36}{6})\pi$ cm^2

= 6 $\times$ $(\frac{22}{7})$ cm^2

= $\frac{132}{7}$ cm^2

15. **(a)** 13 cm

Explanation: Radius of wheel = $\frac{91}{2}$ cm

Angle between two adjoining spokes, $\theta = \frac{360^\circ}{22}$

$\therefore$ Length of arc = $\frac{\theta}{360^\circ} \times 2\pi r$

= $\frac{360^\circ}{360^\circ \times 22} \times 2 \times \frac{22}{7} \times \frac{91}{2}$ = 13 cm

16.

(b) $\frac{1}{25}$

Explanation: n(S) = 100

E = {1, 8, 27, 64}

n(E) = 4

the probability of drawing a number on the card that is a cube is

P(E) = $\frac{4}{100}$ = $\frac{1}{25}$

17.

(b) $\frac{5}{9}$

Explanation: Numbers x = 1, 2, 3 and y = 1, 4, 9

Now xy = {1, 4, 9, 2, 8, 18, 3, 12, 27} = 9

$\therefore$ n = 9

and xy < 9 are 1, 2, 3, 4, 8

$\therefore$ m = 5

$\therefore$ P(xy < 9) = $\frac{5}{9}$

18.

(b) 12

Explanation: Given,

mode - median = 24

median - mean = ?

we know that,

mode = 3 median - 2 mean

mode = median + 2 median - 2 mean

mode - median = 2 median - 2 mean

24 = 2 (median - mean)

median - mean = $\frac{24}{2}$ = 12

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

20.

(c) A is true but R is false.

Explanation: A is true but R is false.

Section B

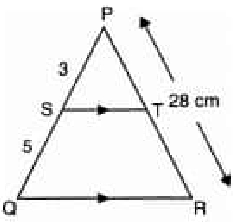
21. Using the factor tree we have,

$56 = 2^3 \times 7$ and $112 = 2^4 \times 7$

Hence HCF is $2^3 \times 7 = 56$ and

LCM is $2^4 \times 7 = 112$

22. According to the question,



ST $\parallel$ QR

$\therefore \frac{PS}{PQ} = \frac{PT}{PR}$ (By BPT)

Given, $\frac{PS}{SQ} = \frac{3}{5}$ and PR= 28 cm

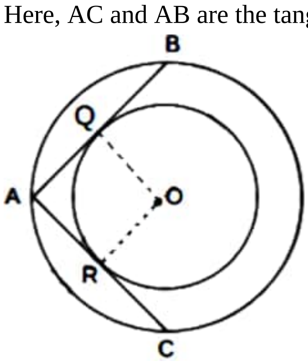
PQ = 3 + 5 = 8

or, $\frac{PS}{PQ} = \frac{PT}{PR}$

or, $\frac{3}{8} = \frac{PT}{28}$

$\therefore PT = \frac{3 \times 28}{8}$ = 10.5 cm

23. Here, AC and AB are the tangents from external point A to the smaller circle.



$\therefore$ AC = AB

Now, AB is the chord of bigger circle and OQ is the perpendicular bisector of chord AB.

$\therefore$ AQ = QB

or, AB = 2AQ

or, AB = 2(5) = 10 cm ...[$\because$ Given AQ = 5 cm]

or, AC = 10 cm

24. $sin 30^\circ . cos 60^\circ + cos 30^\circ . sin 60^\circ$

$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{1}{4} + \frac{3}{4} = \frac{1+3}{4}$

$= \frac{4}{4}$

= 1$= sin 90^\circ = cos 90^\circ$

OR

Given,

R.H.S = $m^2 + n^2$

= $(a \cos\theta + b \sin\theta)^2 + (a \sin\theta - b \cos\theta)^2$ $[\sin ce,\ m = a\ \cos\theta + b\ \sin\theta\ and\ n = a\ \sin\theta - b\ \cos\theta]$

= $(a^2\cos^2\theta + b^2\sin^2\theta + 2ab \cos\theta \sin\theta) + (a^2\sin^2\theta + b^2\cos^2\theta - 2ab \sin\theta \cos\theta)$ $\left[\because (a \pm b)^2 = a^2 + b^2 \pm 2ab\right]$

= $a^2 (\cos^2\theta + \sin^2\theta) + b^2 (\sin^2\theta + \cos^2\theta) = a^2 + b^2$ = L.H.S $\left[\because \sin^2\theta + \cos^2\theta = 1\right]$

therefore, $m^2 + n^2 = a^2 + b^2$

Hence proved.

25. Let the radius of the circle be r cm.

Then, circumference of the circle = 2πr cm

According to the question,

2πr = 22

$\Rightarrow 2 \times \frac{22}{7} \times \mathrm{r} = 22$

$\Rightarrow \mathrm{r} = \frac{22\times7}{2\times22} \Rightarrow \mathrm{r} = \frac{7}{2}\mathrm{cm}$

For a quadrant of a circle,

Area = $\frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$

$= \frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{8}\mathrm{cm}^2$

OR

Let the radius of the large circle be R.

Then, we have

Area of large circle of radius R = Area of a circle of radius 5 cm+ Area of a circle of radius 12 cm

$\Rightarrow \pi R^2 = \left(\pi \times 5^2 + \pi \times 12^2\right)$

$\Rightarrow \pi R^2 = (25\pi + 144\pi)$

$\Rightarrow \pi R^2 = 169\pi$

$\Rightarrow R^2 = 169$

$\Rightarrow$ R = 13 cm

$\Rightarrow$ Diameter = 2R

= 26 cm

Section C

26. We have to find the greatest number that divides 445, 572 and 699 and leaves remainders of 4, 5 and 6 respectively. This means when the number divides 445, 572 and 699, it leaves remainders 4, 5 and 6. It means that

445 - 4 = 441,

572 - 5 = 567

and 699 - 6 = 693

are completely divisible by the required number.

For the highest number which divides the above numbers we need to calculate HCF of 441, 567 and 693 .

Therefore, the required number is the H.C.F. of 441, 567 and 693 Respectively.

First, consider 441 and 567.

By applying Euclid’s division lemma, we get

567 = 441 $\times$ 1 + 126

441 = 126 $\times$ 3 + 63

126 = 63 $\times$ 2 + 0.

Therefore, H.C.F. of 441 and 567 = 63

Now, consider 63 and 693

again we have to apply Euclid’s division lemma, we get

693 = 63 $\times$ 11 + 0.

Therefore, H.C.F. of 441, 567 and 693 is 63

Hence, the required number is 63. 63 is the highest number which divides 445,572 and 699 will leave 4,5 and 6 as remainder respectively.

27. Sum of the zeroes: (2 + β) = (-1)

Product of the zeroes : 2β = -20

So, required Quadratic polynomial

= $\left[x^2 + (\alpha + \beta)x + 2\beta\right]$

= $\left[x^2 + (-1)x + (-20\right]$

= $x^2 - x - 20$

$\Rightarrow$ $x^2 - x - 20$ = 0 is the polynomial

28. Given, last term, l = 119

No. of terms in A.P. = 30

8th term from the end = 91

Let d be a common difference and assume that the first term of A.P. is 119 (from the end)

Since the n_{th} term of AP is

an = l + (n - 1)d

$\therefore$ a8 = 119 + (8 - 1)d

$\Rightarrow$ 91 = 119 + 7d

$\Rightarrow$ 7d = 91 - 119

$\Rightarrow$ 7d = -28

$\Rightarrow$ d = -4

Now, this common difference is from the end of A.P.

So, the common difference from the beginning = -d

= (-4) = 4

Thus, a common difference for the A.P. is 4.

Now, using the formula

I = a + (n - 1)d

$\Rightarrow$ 119 = a + (30 - 1)4

$\Rightarrow$ 119 = a + 29 $\times$ 4

$\Rightarrow$ 119 = a + 116

$\Rightarrow$ a = 119 - 116

$\Rightarrow$ a = 3

Hence, using the formula for the sum of n terms of an A.P.

i.e., $S_n = \frac{n}{2}$[2a + (n - 1)d]

S30 = $\frac{30}{2}$[2 $\times$ 3 + (30 - 1) $\times$ 4]

= 15(6 + 29 $\times$ 4)

= 15 $\times$ 122

= 1830

Therefore, the sum of 30 terms of an A.P. is 1830

OR

It is given that the sum of 5^{th} and 9^{th} terms of an A.P. is 72 and the sum of 7th and 12th terms is 97.

Let "a "be the first term and 'd' be the common difference of the Arithmetic progression .

It is given that $a_5 + a_9$ = 72 and, $a_7 + a_{12}$ = 97

$\Rightarrow$ (a + 4d) + (a + 8d) = 72 and, (a + 6d) + (a + 11 d) = 97.

Therefore, we have

$\Rightarrow$ 2a + 12d = 72 ...(1)

$\Rightarrow$ 2a + 17d = 97...(2)

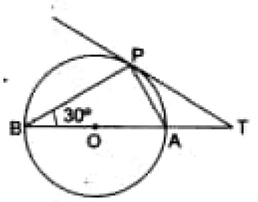
Subtracting (1) from (2), we get

2a+17d-2a-12d =97-72

$\implies$ 5d = 25

$\Rightarrow$ d = 5
Putting d = 5 in (1), we get
2a + 60 = 72 $\Rightarrow$ 2a = 12 $\Rightarrow$ a = 6
Therefore,a = 6 and d = 5
Hence, the Arithmetic Progression a,a+d,a+2d ..... is 6, 11,16, 21, 26,......

29. O is the centre of the circle and TP is the tangent to the circle from an external point T.



From figure,
AB is the diameter
Since, angle in a semicircle is a right angle
$\angle APB = 90^{\circ}$
By using alternate segment theorem
We have $\angle APB = \angle PAT = 30^{\circ}$
Now, in $\triangle$APB
$\angle$BAP + $\angle$APB + $\angle$ABP $= 180^{\circ}$ (Angle sum property of triangle)
$\angle$BAP = 180° - 90° - 30° = 60°
Now, $\angle$BAP = $\angle$APT + $\angle$PTA (Exterior angle property)
60° = 30° + $\angle$PTA
$\angle PTA = 60^{\circ} - 30^{\circ} = 30^{\circ}$
We know that sides opposite to equal angles are equal
AP = AT
In right triangle ABP,
sin 30° = $\frac{\mathrm{AT}}{\mathrm{BA}}$
$\Rightarrow$ $\frac{1}{2}$ = $\frac{\mathrm{AT}}{\mathrm{BA}}$
$\therefore BA : AT = 2 : 1$

OR

TP = TQ ...(length of tangents drawn from external points)
$\therefore$ $\angle$TQP = $\angle$TPQ (angles oppo to equal sides are equal)
OP $\perp$ TP ($\because$ at point of contact radius and tangent are $\perp$r)
$\angle$OPT = 90^{o}
$\angle$OPQ + $\angle$CPQ = 90^{o}
$\angle$TPQ = 90 - $\angle$OPQ
Now, In $\triangle$PTQ
$\angle$TPQ + $\angle$PTQ + $\angle$QTP = 180^{o}
90^{o} - $\angle$OPQ + 90 - $\angle$OPQ + $\angle$PTQ = 180^{o}
$\angle$PTQ = 2$\angle$OPQ
Proved.

30. First, we will show that, $\frac{1+\tan^2 A}{1+\cot^2 A}$ = $\tan^2$ A
LHS = $\frac{1+\tan^2 A}{1+\cot^2 A}$ = $\frac{1+\tan^2 A}{1+\frac{1}{\tan^2 A}}$ = $\frac{1+\tan^2 A}{\frac{1+tan^2 A}{\tan^2 A}}$
= $(1+tan^2 A) \times \frac{tan^2 A}{1+tan^2 A}$
= $\tan^2$ A = RHS ....(i)
Now, we will show that, $\left(\frac{1-\tan A}{1-\cot A}\right)^2$ = $\tan^2$ A
LHS = $\left(\frac{1-\tan A}{1-\cot A}\right)^2$ = $\left(\frac{1-\tan A}{1-\frac{1}{tan A}}\right)^2$ = $\left(\frac{1-\tan A}{\frac{tan A-1}{tan A}}\right)^2$
= $\left[(1-\tan A) \times (\frac{\tan A}{-(1-\tan A)})\right]^2$

= $(\tan A)^2 = \tan^2 A$ = RHS ...(ii)

Hence, from (i) and (ii),

$\frac{1+\tan^2 A}{1+\cot^2 A} = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

Hence proved.

31. Let f_1 and f_2 be the frequencies of class intervals 0 - 10 and 40 - 50.

$f_1 + 5 + 9 + 12 + f_2 + 3 + 2 = 40$

$\Rightarrow f_1 + f_2 = 9$

Median is 32.5 which lies in 30 - 40, so the median class is 30 - 40.

l = 30, h = 10, f = 12, N = 40 and c = $f_1 + 5 + 9 = (f_1 + 14)$

Now, median $= l + \left[h \times \frac{\left(\frac{N}{2}-c\right)}{f}\right]$

$\Rightarrow \quad 32.5 = \left[30 + \left(10 \times \frac{20-f_1-14}{12}\right)\right]$

$= \left[30 + \left(10 \times \frac{6-f_1}{12}\right)\right]$

$= \left[30 + \left(\frac{30-5f_1}{6}\right)\right]$

$\frac{30-5f_1}{6} = 2.5$

$30 - 5f_1 = 15$

$5f_1 = 15 \Rightarrow f_1 = 3$

$f_1 = 3$ and $f_2 = (9 - 3) = 6$

Section D

32. Let cost of production of each article be Rs x

We are given total cost of production on that particular day = Rs 90

Therefore, total number of articles produced that day = 90/x

According to the given conditions,

$x = 2\left(\frac{90}{x}\right) + 3$

$\Rightarrow x = \frac{180}{x} + 3$

$\Rightarrow x = \frac{180+3x}{x}$

$\Rightarrow x^2 = 180 + 3x$

$\Rightarrow x^2 - 3x - 180 = 0$

$\Rightarrow x^2 - 15x + 12x - 180 = 0$

$\Rightarrow x(x - 15) + 12(x - 15) = 0$

$\Rightarrow (x - 15)(x + 12) = 0 \Rightarrow x = 15, -12$

Cost cannot be in negative, therefore, we discard $x = -12$

Therefore, $x = Rs$ 15 which is the cost of production of each article.

Number of articles produced on that particular day = $\frac{90}{15} = 6$

OR

Since (-5) is a root of given quadratic equation $2x^2 + px + 15 = 0$,then,

$2(-5)^2 + p(-5) - 15 = 0$

$50 - 5p - 15 = 0$

$5p = 35 \Rightarrow p = 7$

Now $p\left(x^2 + x\right) + k = 0$ has equal roots

$px^2 + px + k = 0$

So $(b)^2 - 4ac = 0$

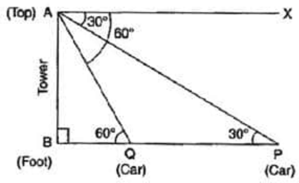
$(p)^2 - 4p \times k = 0$

$(7)^2 - 4 \times 7 \times k = 0$

28k = 49

$k = \frac{49}{28} = \frac{7}{4}$

hence p = 7 and $k = \frac{7}{4}$

33. 

In right triangle ABP,

$\tan 30^\circ = \frac{\text{AB}}{\text{BP}}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\text{AB}}{\text{BP}}$

BP = AB$\sqrt{3}$ .... (i)

In right triangle ABQ,

$tan\ 60^0\ = \frac{AB}{BQ}$

$\Rightarrow \sqrt{3} = \frac{AB}{BQ}$

$\Rightarrow_{BQ} = \frac{AB}{\sqrt{3}}$....... (ii)

$\because$ PQ = BP - BQ

$\therefore$ PQ = AB$\sqrt{3} - \frac{AB}{\sqrt{3}} = \frac{3AB - AB}{\sqrt{3}} = \frac{2AB}{\sqrt{3}}$ = 2BQ [From eq. (ii)]

$\Rightarrow$ BQ = $\frac{1}{2}$PQ

$\because$ Time taken by the car to travel a distance PQ = 6 seconds.

$\therefore$ Time taken by the car to travel a distance BQ, i.e. $\frac{1}{2}$PQ = $\frac{1}{2} \times$ 6 = 3 seconds.

Hence, the further time taken by the car to reach the foot of the tower is 3 seconds.

34. Given, radius of cone = radius of hemisphere

= r

= 7 cm

Height of cone (h) = 2 $\times$ radius

= 2 $\times$ 7

= 14 cm

Volume of solid = Volume of cone + Volume of hemisphere

Volume of solid (V) = $\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi r^2(2r) + \frac{2}{3}\pi r^3$ ...($\because$ h = 2r)

$= \frac{2}{3}\pi r^3 + \frac{2}{3}\pi r^3$

$= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$

$= \frac{4312}{3}$

= 1437.33 cm^3

OR

Height of the cylinder = 3 m.

Total height of the tent above the ground = 13.5 m

height of the cone = (13.5 - 3)m = 10.5 m

Radius of the cylinder = radius of cone = 14 m

Curved surface area of the cylinder = $2\pi rh\text{m}^2 = \left(2 \times \frac{22}{7} \times 14 \times 3\right)\text{m}^2 = 264\text{m}^2$

$\therefore \quad l = \sqrt{r^2 + h^2} = \sqrt{14^2 + (10.5)^2} = \sqrt{196 + 110.25} = \sqrt{306.25} = 17.5$

$\therefore$ Cured surface area of the cone = $\pi rl = \left(\frac{22}{7} \times 14 \times 17.5\right)\text{m}^2 = 770\text{m}^2$

Let S be the total area which is to be painted. Then,

S = Curved surface area of the cylinder + Curved surface area of the cone

$\Rightarrow$ S = (264 + 770) m^2 = 1034 m^2

Hence, Cost of painting = S $\times$ Rate = ₹ (1034×2) = ₹ 2068

35.

Height (in cm)	**No. of girls**	x_i	u_i	$f_i u_i$
120 - 130	2	125	-2	-4
130 - 140	8	135	-1	-8

140 - 150	12	145 = a	0	0
150 - 160	20	155	1	20
160 - 170	8	165	2	16
Total	**50**			**24**

Mean = 145 + $\frac{24}{50} \times 10$

= 149.8

∴ mean height is 149.8 cm

Modal class is 150 - 160

Mode = 150 + $\frac{(20-12)}{(2\times20-12-8)} \times 10$

= 154

∴ modal height is 154 cm

Section E

36. i. Given, prize amount for Hockey ₹ x and ₹ y for cricket per student

∴ Algebraic equations are

5x + 4y = 9500 ...(i)

and 4x + 3y = 7370 ...(ii)

ii. Given, prize amount for Hockey ₹ x and ₹ y for cricket per student

∴ Algebraic equations are

5x + 4y = 9500 ...(i)

and 4x + 3y = 7370 ...(ii)

Multiply by 3 in equation (i) and by 4 in equation (ii)

15x + 12y = 28,500 ...(iii)

16x + 12y = 29480 ...(iv)

On subtracting equation (iii) from equation (iv), we get

x = 980

∴ Prize amount for hockey = ₹ 980

iii. Given, prize amount for Hockey ₹ x and ₹ y for cricket per student

∴ Algebraic equations are

5x + 4y = 9500 ...(i)

and 4x + 3y = 7370 ...(ii)

Now, put this value in equation (i), we get

5 $\times$ 980 + 4y = 9500

$\Rightarrow$ 4y = 9500 - 4900 = 4600

$\Rightarrow$ y = 1150

∴ Prize amount for cricket = ₹ 1150

Difference = 1150 - 980 = 170

∴ Prize amount for cricket is ₹ 170 more than hockey.

OR

Total prize amount for 2 students each from two games

= 2x + 2y

= 2(x + y)

= 2(980 + 1150)

= 2 $\times$ 2130

= ₹ 4260

37. i. tan30^o = $\frac{1}{\sqrt{3}} = \frac{75}{\mathrm{AB}}$

$\Rightarrow$ AB = $75\sqrt{3}$ cm

ii. sin 30^o = $\frac{1}{2} = \frac{75}{OB}$

$\Rightarrow$ OB = 150 cm

iii. QB = 150 - 75 = 75 cm

$\Rightarrow$ Q is mid point of OB

Since PQ || AO therefore P is mid pint of AB

Hence AP = $\frac{75\sqrt{3}}{2}$ cm.

OR

QB = 150 - 75 = 75 cm

Now, $\triangle$BQP $\sim$ $\triangle$BOA

$\Rightarrow \frac{QB}{OB} = \frac{PQ}{OA}$

$\Rightarrow \frac{1}{2} = \frac{PQ}{75}$

$\Rightarrow$ PQ = $\frac{75}{2}$ cm

38. i.

Distance travelled by veena = $\sqrt{1-(-4)^2+(7-4)^2}$

$= \sqrt{5^2+3^2}$

$= \sqrt{25+9}$

$= \sqrt{34}$

Distance travelled by Arun = $\sqrt{(4-(-4))^2+(2-4)^2}$

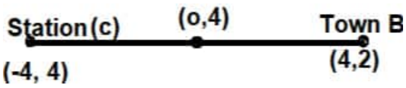
$= \sqrt{64+4}$

$= \sqrt{68}$

$\therefore$ Arun will travel more distance to reach his home.

ii. Location of station = (-4, 4)

iii.



Let y-axis divides station (c) and Town B in K : 1

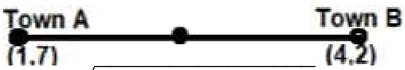
0 = $\frac{4k-4}{k+1}$

4k = 4

k = 1

$\therefore$ y-axis divides in 1 : 1

OR



AB = $\sqrt{(4-1)^2+(2-7)^2}$

$= \sqrt{9+25}$

$= \sqrt{34}$