Class X Session 2024-25
Subject - Mathematics (Standard)
Sample Question Paper - 15

Time: 3 Hours **Total Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A - E.
2. Section A has 18 multiple choice questions and 2 Assertion-Reason based questions carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case study based questions carrying 4 marks each with subparts of 1, 1, and 2 marks each, respectively.
7. All Questions are compulsory. However, an internal choice in 2 Question of Section B, 2 Questions of Section C and 2 Questions of Section D has been provided. An internal choice has been provided in all the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take π = 22/7 wherever required if not *stated.*

Section A

Section A consists of 20 questions of 1 mark each.

Choose the correct answers to the questions from the given options. [20]

1. The LCM of 12, 15 and 21 is

A. 410
B. 420
C. 440
D. 450

2. Find the roots of $x^2 - 3x - 10$.

A. 5 and -2
B. -5 and 2
C. -5 and -2
D. 5 and 2

3. Find a quadratic polynomial with $\frac{1}{4}, -1$ as the sum and product of its zeroes respectively.

A. $k(4x^2 - x + 4)=0$
B. $k(4x^2 + x - 4)=0$
C. $k(4x^2 - x - 4)=0$
D. $k(4x^2 + x + 4)=0$

4. The lines $5x - 4y + 8 = 0$ and $7x + 6y - 9 = 0$ are

A. parallel
B. intersecting
C. coincident
D. none of above

5. For three terms p, s, q to be in AP...

A. $2p = p + s$
B. $p > s > q$
C. $p < s < q$
D. $2s = p + q$

6. If a point (c, d) lies in the 3rd quadrant, which of the following is true?

A. c is positive and d is negative
B. both c and d are positive
C. both c and d are negative
D. c is negative and d is positive

7. Find the distance between the points (0, 0) and (36, 15).

A. 29
B. 39
C. 49
D. 59

8. A circle can have ________ tangent/tangents.

A. one
B. two
C. four
D. infinite

9. Which of the following is not a test of similarity?

A. SSS
B. SAS
C. AAA
D. SSA

10. Corresponding _______ of similar triangles are equal.

A. Sides
B. Areas
C. Perimeters
D. Angles

11. If $\cot \theta = \frac{7}{8}$, then evaluate $\tan^2 \theta$.

A. 8/7
B. 49/8
C. 49/64
D. 64/49

12. If $\sin A = \frac{3}{4}$, calculate tan A.

A. $\frac{3}{\sqrt{2}}$
B. $\frac{3}{\sqrt{5}}$
C. $\frac{3}{\sqrt{7}}$
D. $\frac{3}{\sqrt{11}}$

13. Raju's teacher checked a question of Raju and found zero errors. So he wrote the following:

"Congratulations Raju! The number of errors in your question is equal to $\cos \theta$".

What will be the value of θ here?

A. 0°
B. 30°
C. 60°
D. 90°

14. Find the area of a sector of a circle with radius 6 cm, if angle of the sector is 60°.

A. $\frac{132}{3}$ cm^2
B. $\frac{132}{2}$ cm^2
C. $\frac{132}{7}$ cm^2
D. $\frac{132}{5}$ cm^2

15. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding minor segment.

A. 28.6 cm^2
B. 26.6 cm^2
C. 24.6 cm^2
D. 22.6 cm^2

16. Time taken by a group of swimmers for different range of ages is being recorded and maintained in a table. Which formula could be used to find the middle-most age?

A. $\frac{\sum fx}{\sum f}$

B. $a + \frac{\sum fd}{\sum f}$

C. $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

D. $l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$

17. Two coins are tossed simultaneously. Find the probability of getting at most one head.

A. 1/4
B. 1/2
C. 3/4
D. None of above

18. Find the mode.

Daily wages (in Rs.)	**Number of workers (f_i)**
100	12
120	14
140	8
160	6
180	10

A. 100
B. 120
C. 160
D. 180

DIRECTION: In the question number 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option

19. Statement A (Assertion): A spherical glass vessel has a cylindrical neck 7 cm long and 4 cm in diameter. The diameter of the spherical part is 21 cm. Hence the quantity of water it can hold is 4939 cm^3. Use = $\pi = \frac{22}{7}$.

Statement R (Reason): Quantity of water it can hold = volume of spherical glass vessel + volume of cylindrical neck

A. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
B. Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
C. Assertion (A) is true but reason (R) is false.
D. Assertion (A) is false but reason (R) is true.

20. Statement A (Assertion): If the solution of system of equations $x - y = 4$ and $x + y = 6$ is $x = p$ and $y = 2q$ then $p = 4$.

Statement R (Reason): A pair of linear equations in two variables can be algebraically solved by elimination method.

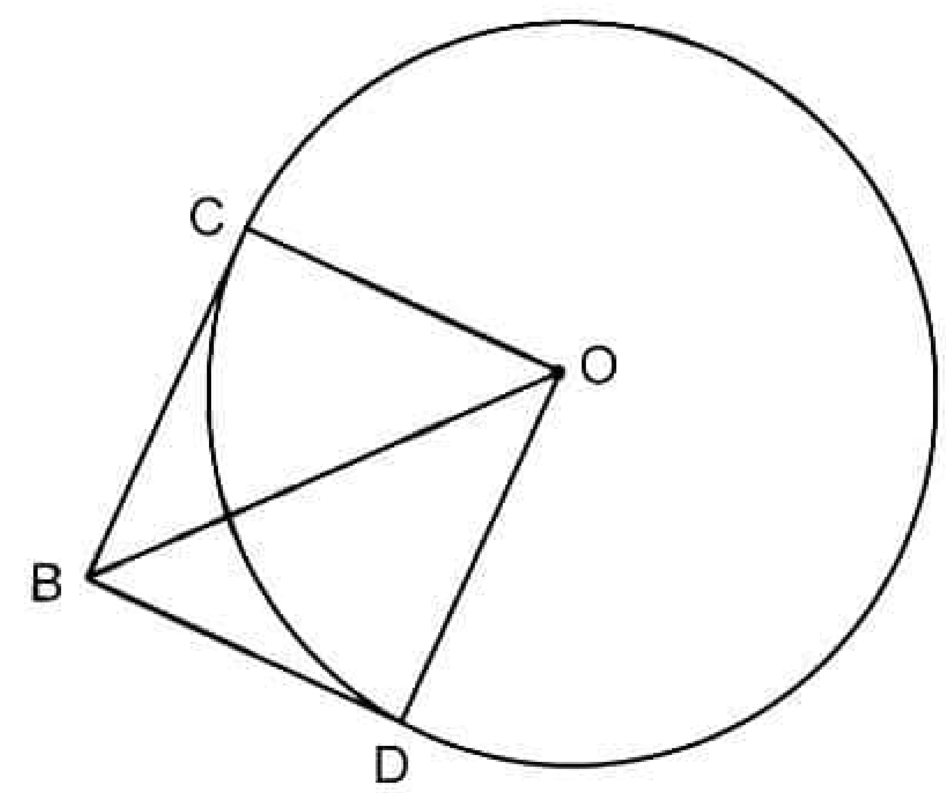
A. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
B. Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
C. Assertion (A) is true but reason (R) is false.
D. Assertion (A) is false but reason (R) is true.

Section B

21. The HCF of 110 and 65 is (42a – 205). Find the value of a. [2]

22. If α, β are zeroes of the polynomial $p(x) = 5x^2 - 6x + 1$, then find the value of $\alpha + \beta + \alpha\beta$. [2]

23. Two tangent segments BC and BD are drawn to a circle with centre O such that $\angle CBD = 120°$. Prove that OB = 2BC. [2]

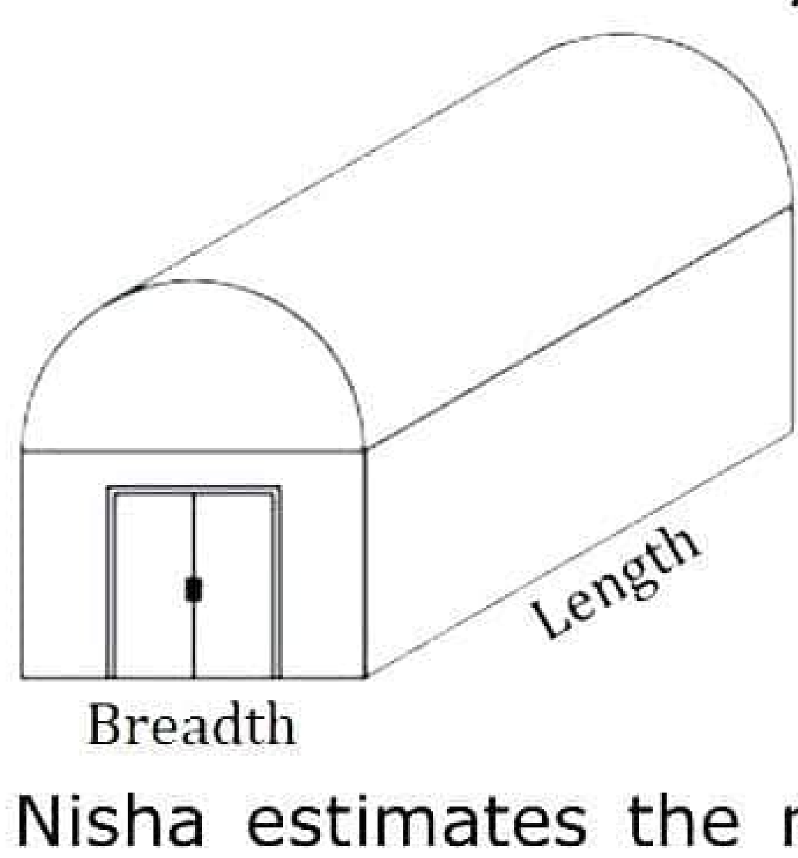


24. Prove that $\sqrt{\frac{1-\cos A}{1+\cos A}} + \sqrt{\frac{1+\cos A}{1-\cos A}} = 2\operatorname{cosec} A$ [2]

OR

If $\sqrt{3}\tan\theta - 1 = 0$, find the value of $\sin^2\theta - \cos^2\theta$.

25. Nisha's mother stored antiques in a box which is in the shape of a cuboid at the bottom and half cylinder at the top as shown below: [2]



Nisha estimates the measures as Length = 3 times height & breadth = 7/5 times height.

According to her mother, the total height of the cuboid is 0.5 m. What is the total surface area of the box? (Take π = 22/7)

OR

A cuboidal jar with height 14 cm is ¾ filled with sugar. If the length and breadth of the jar is 9 cm each, how much more sugar (in kg) can be put in the jar so that it is filled completely?

Section C

Section C consists of 6 questions of 3 marks each.

26. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction, after how many minutes will they meet again at the starting point? [3]

27. Find two numbers whose sum is 27 and product is 182. [3]

28. If α and β are the zeroes of the polynomial $p(x) = 2x^2 + 5x + k$ satisfying the relation $\alpha^2 + \beta^2 + \alpha\beta = 21/4$, then find the value of k. [3]

OR

The monthly incomes of A and B are in the ratio 5 : 4 and their monthly expenditures are in the ratio 7 : 5. If each saves Rs. 3000 per month, then find the monthly income of each. [3]

29. Find the four angles of a cyclic quadrilateral ABCD in which $\angle A = (x + y + 10)^\circ$, $\angle B = (y + 20)^\circ$, $\angle C = (x + y - 30)^\circ$ and $\angle D = (x + y)^\circ$. [3]

OR

AX and DY are the altitudes of two similar triangles ABC and DEF. Prove that AX : DY = AB : DE. [3]

30. If $\cos\theta = \frac{7}{25}$, find the values of all T-ratios of θ. [3]

31. Two dice are thrown simultaneously. What is the probability that [3]

i. 5 will not come up on either of them?
ii. 5 will not come up on at least one?
iii. 5 will come up at both dice?

Section D

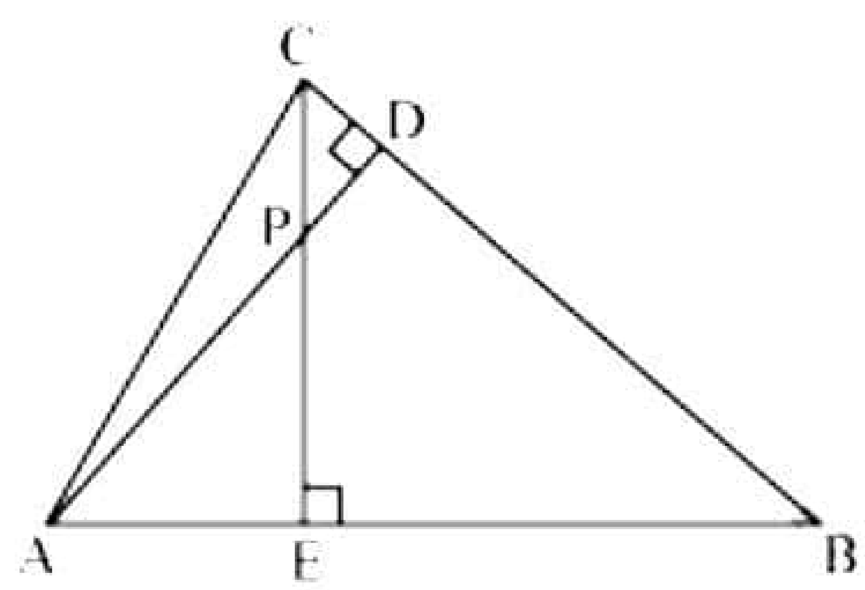
Section D consists of 4 questions of 5 marks each.

32. The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age. [5]

OR

A rectangular field is 16 m long and 10 m wide. There is a path of uniform width all around it with an area of 120 m^2. Find the width of the path. [5]

33. In the figure, altitudes AD and CE of ΔABC intersect each other at the point P. Show that [5]



i. $\Delta AEP \sim \Delta CDP$

ii. $\Delta ABD \sim \Delta CBE$

iii. $\Delta AEP \sim \Delta ADB$

iv. $\Delta PDC \sim \Delta BEC$

34. A tent is in the shape of a right circular cylinder up to a height of 3 m and conical above it. The total height of the tent is 13.5 m, and the radius of its base is 14 m. Find the cost of cloth required to make the tent at the rate of Rs. 80 per square metre. Take $\pi = \frac{22}{7}$. [5]

OR

A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass. (Use $\pi = 3.14$)

35. The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is Rs. 18. Find the missing frequency f. [5]

Daily pocket allowance (in Rs.)	11 – 13	13 – 15	15 –17	17 – 19	19 – 21	21 – 23	23 – 25
Number of children	7	6	9	13	f	5	4

Section E

Case study based questions are compulsory.

36. India is competitively manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production runs. The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6^{th} year and 22600 in 9^{th} year.

Based on the above information, answer the following questions.

i. Find the production during first year. [2]

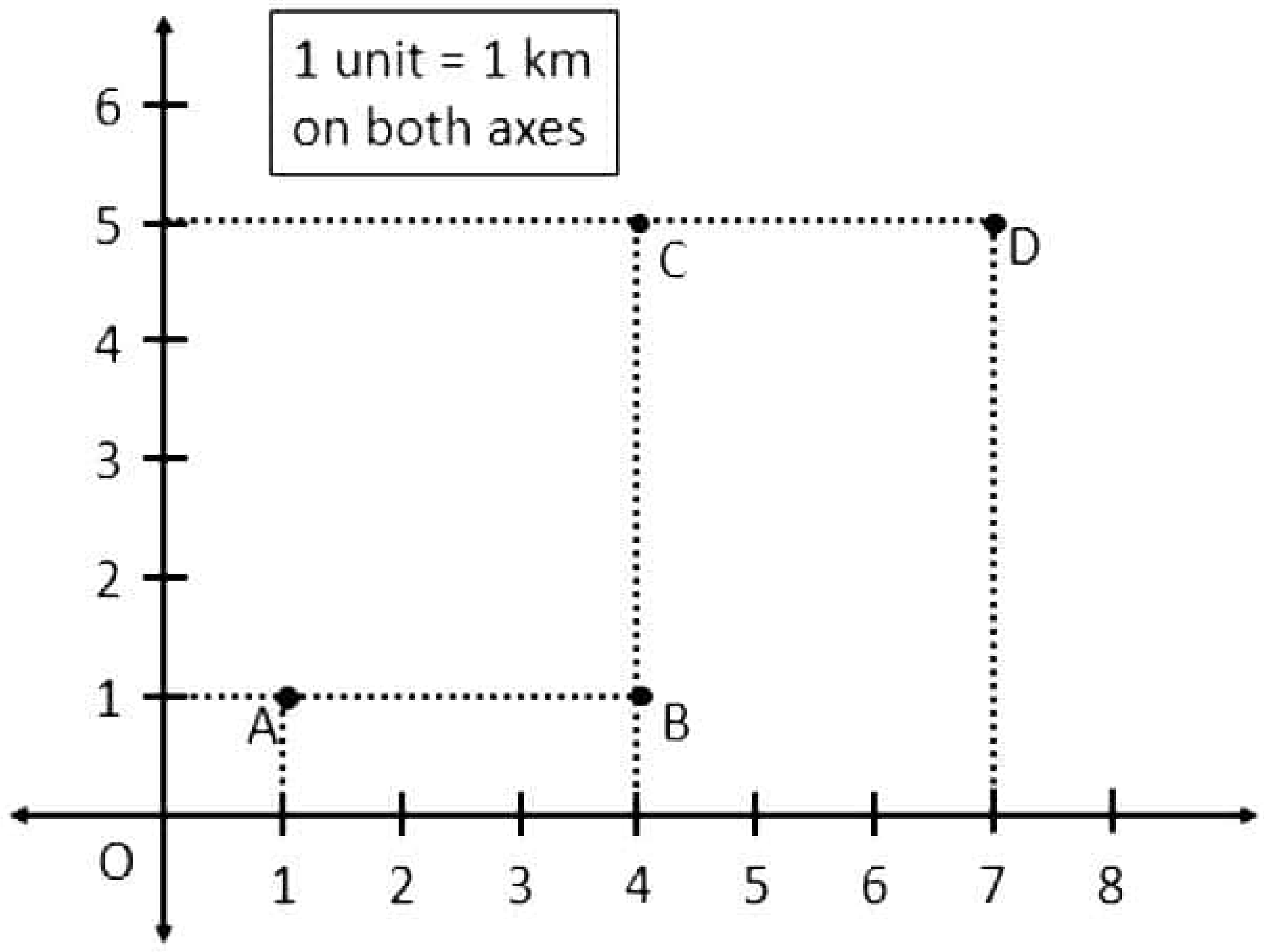
OR

Find the difference in production between two consecutive years. [2]

ii. Find the production during 8^{th} year. [1]

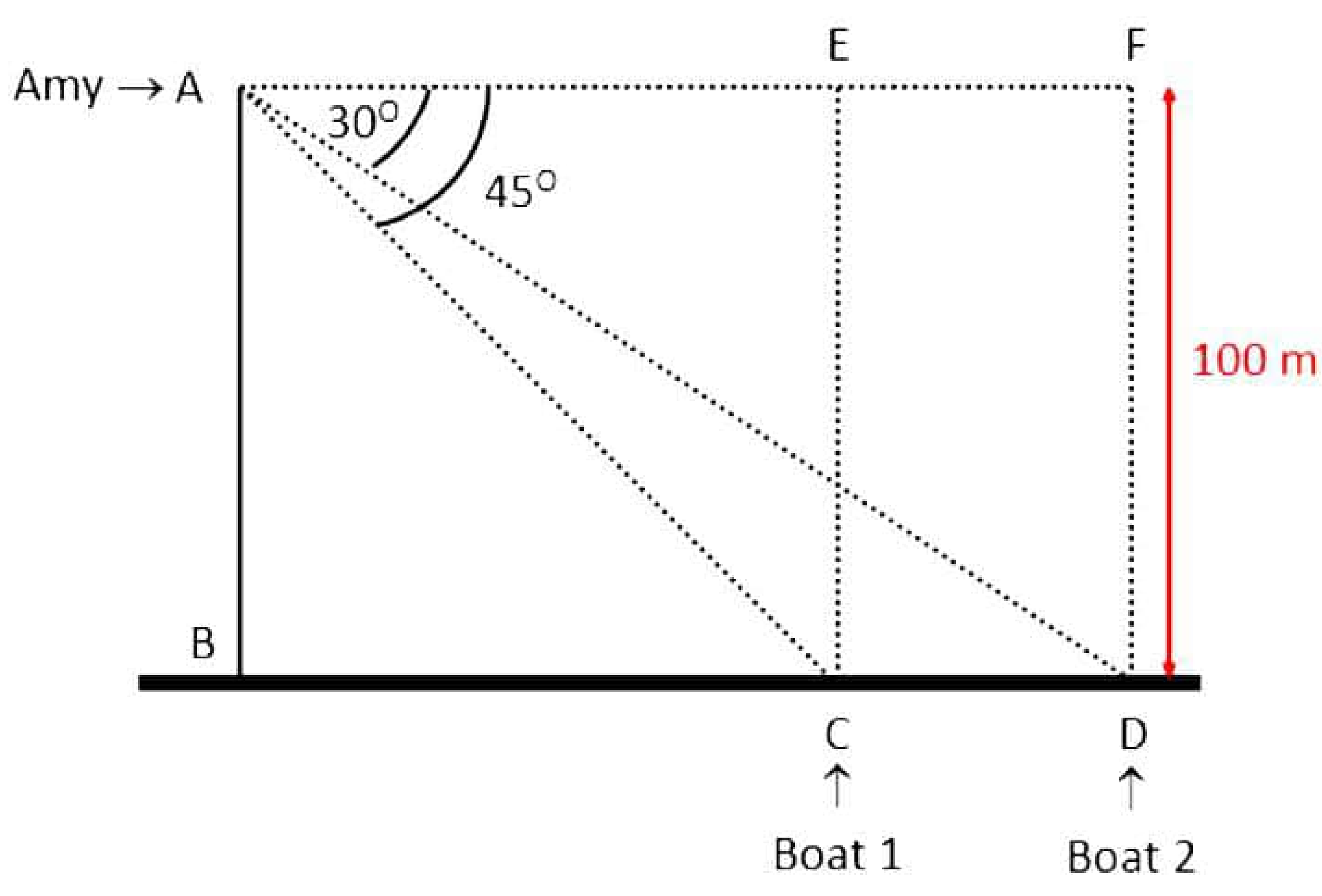
iii. Find the production during first 3 years. [1]

37. Amey runs a grocery store that offers home delivery of fresh groceries to its customers. His store is located at location A as indicated in the graph below. Now, he receives regular orders from the families living in the colonies located at B, C and D. Now, using the data given, answer the following questions.



i.	Find the shortest distance between locations A and C.	[2]
	OR	
	Find the shortest distance between locations B and D.	[2]
ii.	Find the shortest distance between locations B and A.	[1]
iii.	Find the shortest distance between locations C and B.	[1]

38. Amy, with her school friends, visit a lighthouse. On reaching the lighthouse, they went up to the topmost floor of the lighthouse to see the entire area. The lighthouse is 100m tall, and at the top, one can get a beautiful clear view of the vast ocean. Now Amy saw two boats approaching the lighthouse from the same direction. One of the boats is at an angle of depression 45°, and the other is at 30°. Using the given data, answer the following questions.



i.	Find the distance between Boat 1 and the base of the lighthouse.	[2]
	OR	
	Find the distance between Boat 2 and the base of the lighthouse.	[2]
ii.	Find the length of AC.	[1]
iii.	Find the length of AD.	[1]

Solution

Section A

1. Correct option: B
Explanation:
12, 15 and 21
$12 = 2^2 \times 3$
$15 = 3 \times 5$
$21 = 3 \times 7$
$\text{LCM} = 2^2 \times 3 \times 5 \times 7 = 420$

2. Correct option: A
Explanation:
$x^2 - 3x - 10$
$= x^2 - 5x + 2x - 10$
$= x(x-5) + 2(x-5)$
$= (x-5)(x+2)$
$\therefore \quad (x-5)(x+2) = 0$
i.e., x = 5 or x = −2

3. Correct option: C
Explanation:
Let the required polynomial be $ax^2 + bx + c$, and let its zeroes be α and β.
$\alpha + \beta = \frac{1}{4} = \frac{-b}{a}$
$\alpha\beta = -1 = \frac{-4}{4} = \frac{c}{a}$
If $a = 4k$, then $b = -k$, $c = -4k$
Therefore, the quadratic polynomial is $k(4x^2 - x - 4) = 0$, where k is a real number .

4. Correct option: B
Explanation:
$5x - 4y + 8 = 0$
$7x + 6y - 9 = 0$
Comparing these equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

$a_1 = 5,\ b_1 = -4,\quad c_1 = 8$

$a_2 = 7,\ b_2 = 6,\quad c_2 = -9$

$$\frac{a_1}{a_2} = \frac{5}{7}$$

$$\frac{b_1}{b_2} = \frac{-4}{6} = \frac{-2}{3}$$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the given pair of equations intersect at exactly one point.

5. Correct option: D

Explanation:

If p, s, q are in AP,

s = p + d [d = common difference]

q = p + 2d

So, p + q = 2p + 2d = 2(p + d) = 2s

6. Correct option: C

Explanation:

If a point lies in the 3rd quadrant, then its x-coordinate as well as its y-coordinate will be negative.

7. Correct option: B

Explanation:

Distance between two points (x_1, y_1) and (x_2, y_2) $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Then, distance between points $(0,0)$ and $(36,15)$

$$= \sqrt{(36-0)^2 + (15-0)^2} = \sqrt{36^2 + 15^2}$$

$$= \sqrt{1296 + 225} = \sqrt{1521} = 39$$

8. Correct option: D

Explanation:

A circle can have infinite tangents.

9. Correct option: D

Explanation:

SSA is not a test of similarity, the angle should be included between the two sides.

10. Correct option: D

Explanation:

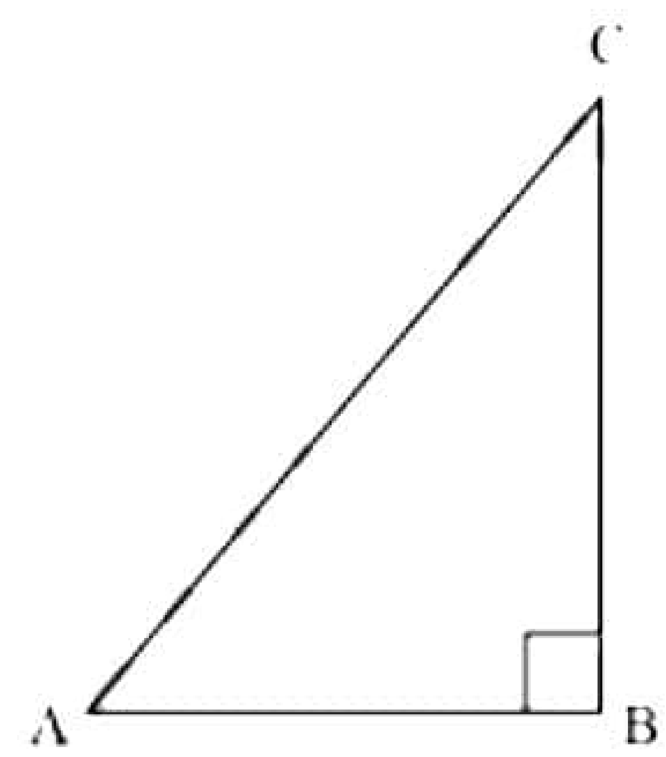
Corresponding angles of similar triangles are equal.

11. Correct option: D
Explanation:

$$\cot^2\theta = (\cot\theta)^2 = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

so $\tan^2\theta = 64/49$

12. Correct option: C
Explanation:
Let ΔABC be a right-angled triangle, right angled at point B.
Given that



$$\sin A = \frac{3}{4}$$

$$\frac{BC}{AC} = \frac{3}{4}$$

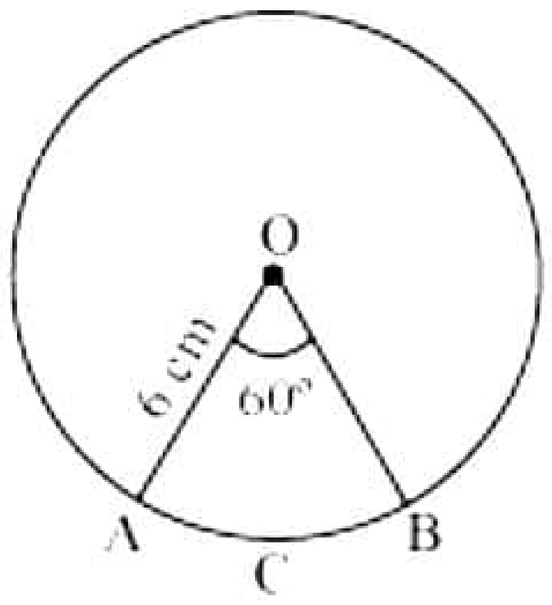
Let BC be 3K.
So AC will be 4K where K is a positive integer.
Now applying Pythagoras theorem in ΔABC,
$AC^2 = AB^2 + BC^2$
$(4K)^2 = AB^2 + (3K)^2$
$16K^2 - 9K^2 = AB^2$
$7K^2 = AB^2$
$AB = \sqrt{7}K$

$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{side adjacent to } \angle A}$$

$$= \frac{BC}{AB} = \frac{3K}{\sqrt{7}K} = \frac{3}{\sqrt{7}}$$

13. Correct option: D
Explanation:
Given,
No. of errors = 0
Also,
No. of errors = $\cos\theta$
Hence,
$\cos\theta = 0$
Therefore,
$\theta = 90^\circ$.

14. Correct option: C
Explanation:



Let OACB be a sector of circle making 60° angle at centre O of circle.

Area of sector of angle $\theta = \frac{\theta}{360^\circ} \times \pi r^2$

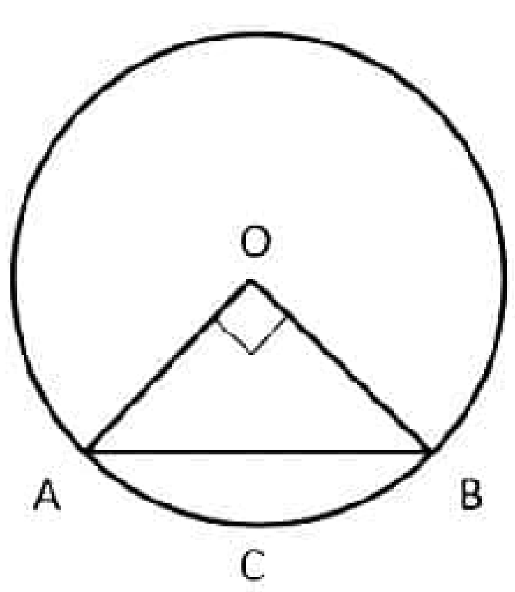
So, area of sector OACB $= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times (6)^2$

$= \frac{1}{6} \times \frac{22}{7} \times 6 \times 6 = \frac{132}{7}$ cm^2

So, area of sector of circle making 60° at centre of circle is $\frac{132}{7}$ cm^2.

15. Correct option: A
Explanation:
Let AB be the chord of a circle subtending 90° angle at centre O of circle.

Area of minor sector OACB $= \frac{90^\circ}{360^\circ} \times \pi r^2 = \frac{1}{4} \times \frac{22}{7} \times 10 \times 10 = \frac{1100}{14} = 78.6$ cm^2



Area of ΔOAB $= \frac{1}{2} \times OA \times OB = \frac{1}{2} \times 10 \times 10 = 50$ cm^2

Area of minor segment ACB = Area of minor sector OACB − Area of ΔOAB = 78.6 − 50 = 28.6 cm^2

16. Correct option: D
Explanation:
For a group of observations, the middle-most value is the median.
So, to find the middle-most age, we must use the formula of median which is

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

17. Correct option: C

Explanation:

When two coins are tossed simultaneously, all possible outcomes are HH, HT, TH, TT.

Total number of possible outcomes = 4

Let E be the event of getting at the most one head.

So, the favourable outcomes are HT, TH, TT.

Number of favourable outcomes = 3

$\therefore$ P(getting at the most 1 head) = P(E) = $\frac{3}{4}$

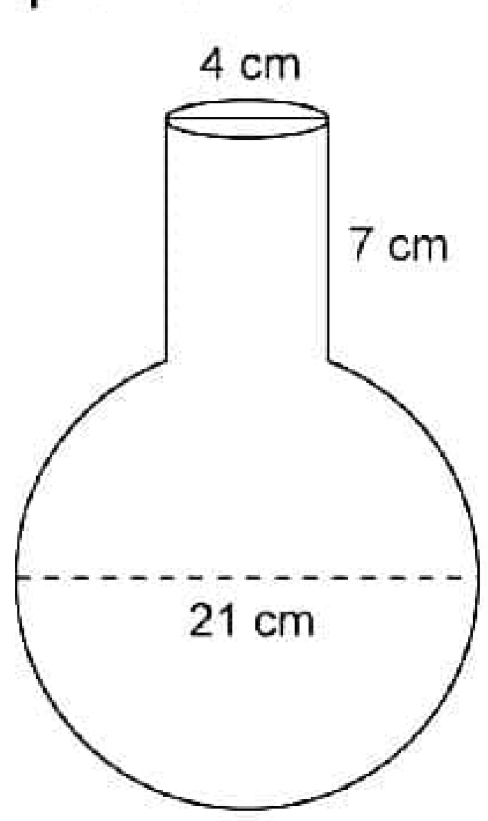
18. Correct option: B

Explanation:

Mode is the observation with the highest frequency, which is 120.

19. Correct option: A

Explanation:



Diameter of the spherical part of vessel = 21 cm

Its radius $= \frac{21}{2}$ cm

Its volume $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2}$

$= 11 \times 21 \times 21 \text{ cm}^3 = 4851 \text{ cm}^3$

Volume of cylindrical part of vessel

$= \pi r^2 h = \frac{22}{7} \times 2 \times 2 \times 7 \text{ cm}^3$

$= 88 \text{ cm}^3$

Now, Quantity of water it can hold = volume of spherical glass vessel + volume of cylindrical neck

Hence, reason (R) is true.

$\therefore$ Quantity of water it can hold = 4851 + 88 = 4939 cm^3.

Thus, assertion (A) and reason (R) are true, and reason (R) is the correct explanation of assertion (A).

20. Correct option: D

Explanation:

The statement given in reason is correct and hence, reason is true.
Given system of equations is $x - y = 4$ and $x + y = 6$.

Substituting $x = p$ and $y = 2q$,

$p - 2q = 4$

$p + 2q = 6$

$\Rightarrow 2p = 10$

$\Rightarrow p = 5$

Hence, assertion is false.

Section B

21. $110 = 2 \times \underline{5} \times 11$

$65 = \underline{5} \times 13$

Therefore, HCF(110, 65) = 5

Then, $42a - 205 = 5$

$42a = 210 \Rightarrow a = 5$

22. $p(x) = 5x^2 - 6x + 1$

Here, $a = 5$, $b = -6$ and $c = 1$

Sum of roots $= \alpha + \beta = -\frac{b}{a} = -\frac{-6}{5} = \frac{6}{5}$

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{1}{5}$

Then, $\alpha + \beta + \alpha\beta = \frac{6}{5} + \frac{1}{5} = \frac{7}{5}$

23. Given: Two tangent segments BC and BD are drawn to a circle with centre O such that $\angle CBD = 120°$.

In ΔOBC,

$\angle OBC = \angle OBD = 60°$.......... The line joining the centre of the circle and the point of contact of tangents from an external point bisect the angle between two tangents.

$\angle OCB = 90°$ (BC is tangent to the circle)

Therefore, $\angle BOC = 30°$

$\therefore \frac{BC}{OB} = \sin 30° = \frac{1}{2}$

$\Rightarrow OB = 2BC$

24.

$$\text{L.H.S} = \sqrt{\frac{1-\cos A}{1+\cos A}} + \sqrt{\frac{1+\cos A}{1-\cos A}}$$

$$= \sqrt{\frac{(1-\cos A)(1-\cos A)}{(1+\cos A)(1-\cos A)}} + \sqrt{\frac{(1+\cos A)(1+\cos A)}{(1-\cos A)(1+\cos A)}}$$

$$= \sqrt{\frac{(1-\cos A)^2}{(1-\cos^2 A)}} + \sqrt{\frac{(1+\cos A)^2}{(1-\cos^2 A)}}$$

$$= \sqrt{\frac{(1-\cos A)^2}{\sin^2 A}} + \sqrt{\frac{(1+\cos A)^2}{\sin^2 A}}$$

$$= \frac{1-\cos A}{\sin A} + \frac{1+\cos A}{\sin A}$$

$$= \frac{1-\cos A+1+\cos A}{\sin A}$$

$$= \frac{2}{\sin A}$$

$$= 2\operatorname{cosec} A$$

$$= \text{R.H.S.}$$

OR

$$\sqrt{3}\tan\theta - 1 = 0$$

$$\Rightarrow \sqrt{3}\tan\theta = 1$$

$$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan\theta = \tan 30^\circ$$

$$\Rightarrow \theta = 30^\circ$$

Now,

$$\sin^2\theta - \cos^2\theta = \sin^2 30 - \cos^2 30$$

$$= \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= \frac{1}{4} - \frac{3}{4}$$

$$= \frac{-2}{4}$$

$$= \frac{-1}{2}$$

25. Height of cuboid (h) = 0.5 m

$\Rightarrow$ Length(l) = 1.5 m & breadth (b) = 0.7 m

$\Rightarrow$ Radius of cylinder (r) = 0.7/2 = 0.35 m

Total surface area of the box = TSA of cuboid – (l × b) + ½ × Total surface area of cylinder

$= 2(lb + bh + lh) - lb + (\pi rh + \pi r^2)$

$= 2(1.5\times0.7 + 0.7\times0.5 + 0.5\times1.5) - 1.5\times0.7 + 22/7 \times 0.35 \times 0.5 + 22/7 \times (0.35)^2$

$= 2(1.05 + 0.35 + 0.75) - 1.05 + 0.55 + 0.385$

$= 4.3 - 1.05 + 0.935$

$= 4.185\ m^2$

Hence, the total surface area of the box = $4.185\ m^2$

OR

Volume of cuboid = length × breadth × height

$= 14 \times 9 \times 9$

$= 1134\ cm^3$

Now, as the jar is ¾ filled with sugar, so $(1/4)^{th}$ volume of sugar will fill the jar completely.

Now, $\frac{1}{4} \times 1134 = 283.5\ cm^3$

But, $283.5\ cm^3 = 0.2835$ kg

So, adding 0.2835 kg of more sugar will fill the jar completely.

Section C

26. It can be observed that Ravi and Sonia do not take the same amount of time. Ravi takes less time than Sonia to complete 1 round of the circular path.
As they are going in the same direction, they will meet again at the same time when Ravi has completed one round of that circular path with respect to Sonia.
i.e., when Sonia completes one round, then Ravi completes 1.5 rounds.
So they will meet first at a time that is a common multiple of the time it takes them to complete one round, i.e., LCM of 18 minutes and 12 minutes.
Now,
$18 = 2 \times 3 \times 3 = 2 \times 3^2$
And, $12 = 2 \times 2 \times 3 = 2^2 \times 3$
LCM of 12 and 18 = product of factors raised to highest exponent = $2^2 \times 3^2 = 36$
Therefore, Ravi and Sonia will meet at the starting point after 36 minutes.

27. Let the first number be x.
Then, the second number is 27 – x.
Now,

$$x(27 - x) = 182$$

$$\Rightarrow x^2 - 27x + 182 = 0$$

$$\Rightarrow x^2 - 13x - 14x + 182 = 0$$

$$\Rightarrow x(x - 13) - 14(x - 13) = 0$$

$$\Rightarrow (x - 13)(x - 14) = 0$$

Either x = 13 = 0 or x – 14 = 0
i.e., x = 13 or x = 14
If first number = 13, then other number = 27 – 13 = 14
If first number = 14, then other number = 27 – 14 = 13
Therefore, the numbers are 13 and 14.

28. Given: $p(x) = 2x^2 + 5x + k$
Sum of the zeroes = $\alpha + \beta = -5/2$
Product of zeroes = $\alpha\beta = k/2$
Given $\alpha^2 + \beta^2 + \alpha\beta = 21/4$
$(\alpha + \beta)^2 - \alpha\beta = 21/4$

$$\left(\frac{-5}{2}\right)^2 - \frac{k}{2} = \frac{21}{4}$$

$$\frac{25}{4} - \frac{k}{2} = \frac{21}{4}$$

$$\frac{k}{2} = 1$$

k = 2

OR

Let the monthly income of A and B be Rs. 5x and Rs. 4x, respectively, and let their expenditures be Rs. 7y and Rs. 5y, respectively.

Then

$5x - 7y = 3000$ ... (1)

$4x - 5y = 3000$ ... (2)

Multiplying (1) by 5 and (2) by 7, we get

$25x - 35y = 15000$ ... (3)

$28x - 35y = 21000$ ... (4)

Subtracting (3) from (4), we get

$3x = 6000 \Rightarrow x = 2000$

Hence,

Income of A = $5x = 5 \times 2000$ = Rs. 10000

Income of B = $4x = 4 \times 2000$ = Rs. 8000

29. In a cyclic quadrilateral ABCD,

$\angle A = (x + y + 10)^\circ$, $\angle B = (y + 20)^\circ$, $\angle C = (x + y - 30)^\circ$, $\angle D = (x + y)^\circ$

We have, $\angle A + \angle C = 180^\circ$ and $\angle B + \angle D = 180^\circ$

$[\because$ ABCD is a cyclic quadrilateral$]$

Now, $\angle A + \angle C = (x + y + 10)^\circ + (x + y - 30)^\circ = 180^\circ$

$\Rightarrow 2x + 2y - 20^\circ = 180^\circ$

$\Rightarrow x + y - 10^\circ = 90^\circ$

$x + y = 100^\circ$ ... (1)

$\angle B + \angle D = (y + 20)^\circ + (x + y)^\circ = 180^\circ$

$\Rightarrow x + 2y + 20^\circ = 180^\circ$

$\Rightarrow x + 2y = 160^\circ$ ... (2)

Subtracting (1) from (2), we get

$y = 160 - 100 = 60$

and $x = 100 - y = 100 - 60 = 40$

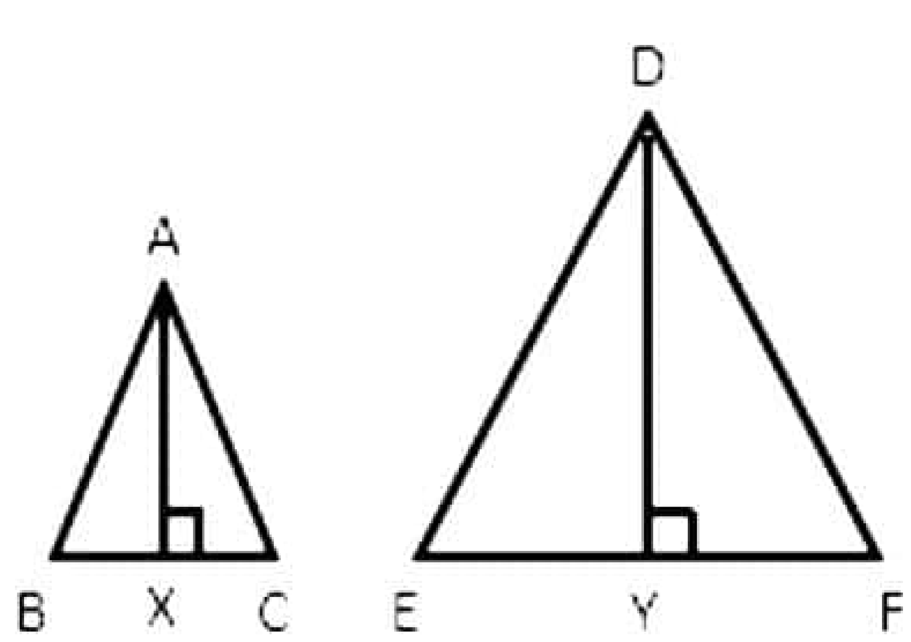
$\angle A = (x + y + 10)^\circ = (100 + 10)^\circ = 110^\circ$

$\angle B = (y + 20)^\circ = (60 + 20)^\circ = 80^\circ$

$\angle C = (x + y - 30)^\circ = (100 - 30)^\circ = 70^\circ$

$\angle D = (x + y)^\circ = 100^\circ$

OR



Given: $\Delta ABC \sim \Delta DEF$

To prove that AX : DY = AB : DE

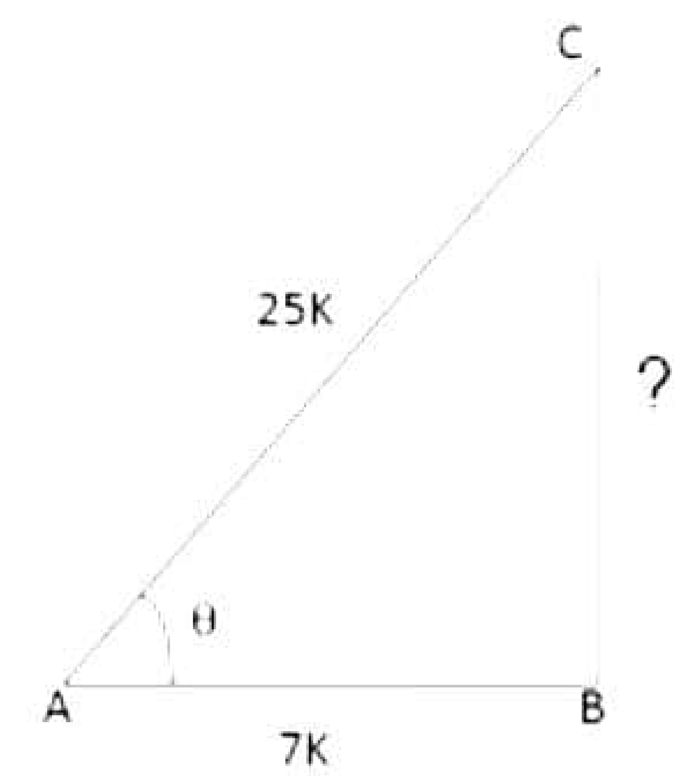
Proof:
In ΔABX and ΔDEY,
∠B = ∠E (corresponding angles)
∠AXB = ∠DYE (Each 90°)
⇒ ΔABX ~ ΔDEY (AA similarity)

$$\frac{AB}{DE} = \frac{BX}{EY} = \frac{AX}{DY}$$

$$\frac{AX}{DY} = \frac{AB}{DE}$$

30. Given : $\cos\theta = \frac{7}{25}$

Let Ab = 7k and AC = 25k, where k is positive
Let us draw a ΔABC in which ∠B = 90° and ∠BAC = θ.
By Pythagoras' theorem, we have



$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow BC^2 = AC^2 - AB^2$$

$$BC^2 = \left[(25k)^2 - (7k)^2\right]$$

$$= \left(625k^2 - 49k^2\right)$$

$$= 576k^2$$

$$\Rightarrow BC = \sqrt{576k^2} = 24k$$

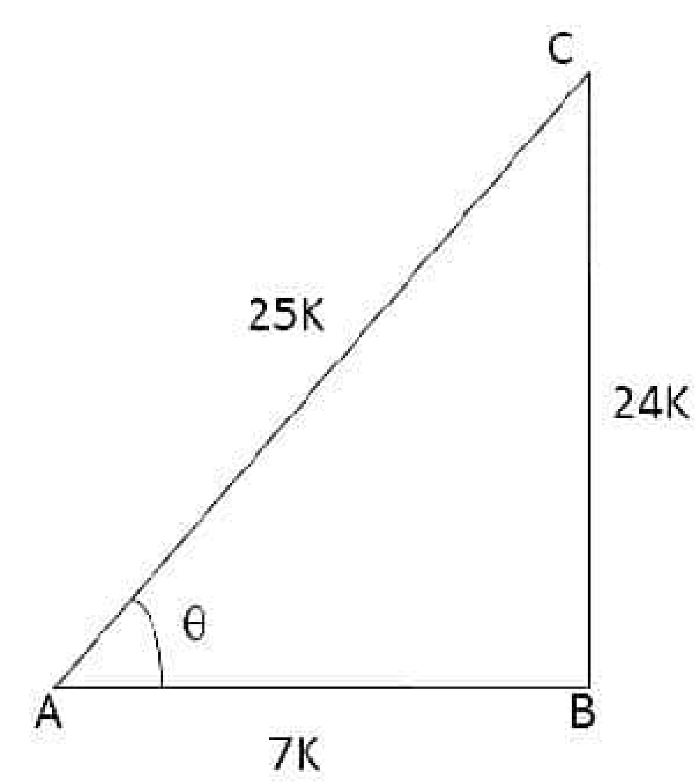
$$\therefore \sin\theta = \frac{BC}{AC} = \frac{24k}{25k} = \frac{24}{25}; \cos\theta = \frac{7}{25}(\text{given})$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \left(\frac{24}{25} \times \frac{25}{7}\right) = \frac{24}{7}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{25}{24}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{25}{7}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{7}{24}$$



31. Two dice are thrown simultaneously.

Total number of outcomes = 6 × 6 = 36

i. 5 will not come up on either of them.
Favorable cases are

(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 1), (2, 2),

(2, 3), (2, 4), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4),

(3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 6), (6, 1),

(6, 2), (6, 3), (6, 4), (6, 6) = 25

$\therefore$ Probability that 5 will not come up on either die $= \frac{25}{36}$

ii. 5 will not come up on at least one.
Favorable cases are
(1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6) = 11

Probability that 5 will come at least once $= \frac{11}{36}$

iii. 5 will come up on both dice.

Favourable case: (5, 5)

$\therefore$ Probability that 5 will come on both dice $= \frac{1}{36}$

Section D

32. Let the present age of Rehman be x years.

Three years ago, his age was (x – 3) years.

Five years hence, his age will be (x + 5) years.

It is given that the sum of the reciprocals of Rehman's ages 3 years ago and 5 years from now is $\frac{1}{3}$.

$$\therefore \frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$$

$$\frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3}$$

$$\frac{2x+2}{(x-3)(x+5)} = \frac{1}{3}$$

$$\Rightarrow 3(2x+2) = (x-3)(x+5)$$

$$\Rightarrow 6x + 6 = x^2 + 2x - 15$$

$$\Rightarrow x^2 - 4x - 21 = 0$$

$$\Rightarrow x^2 - 7x + 3x - 21 = 0$$

$$\Rightarrow x(x-7) + 3(x-7) = 0$$

$$\Rightarrow (x-7)(x+3) = 0$$

$$\Rightarrow x = 7, -3$$

However, age cannot be negative.

Therefore, Rehman's present age is 7 years.

OR

Let the width of the path be x metres.

Then, Area of the path = 16 × 10 – (16 – 2x)(10 – 2x) = 120

$$\Rightarrow 16 \times 10 - \left(160 - 32x - 20x + 4x^2\right) = 120$$

$$\Rightarrow 160 - 160 + 32x + 20x - 4x^2 = 120$$

$$\Rightarrow -4x^2 + 52x - 120 = 0$$

$$\Rightarrow x^2 - 13x + 30 = 0$$

$$\Rightarrow x^2 - 10x - 3x + 30 = 0$$

$$\Rightarrow x(x - 10) - 3(x - 10) = 0$$

$$\Rightarrow (x - 10)(x - 3) = 0$$

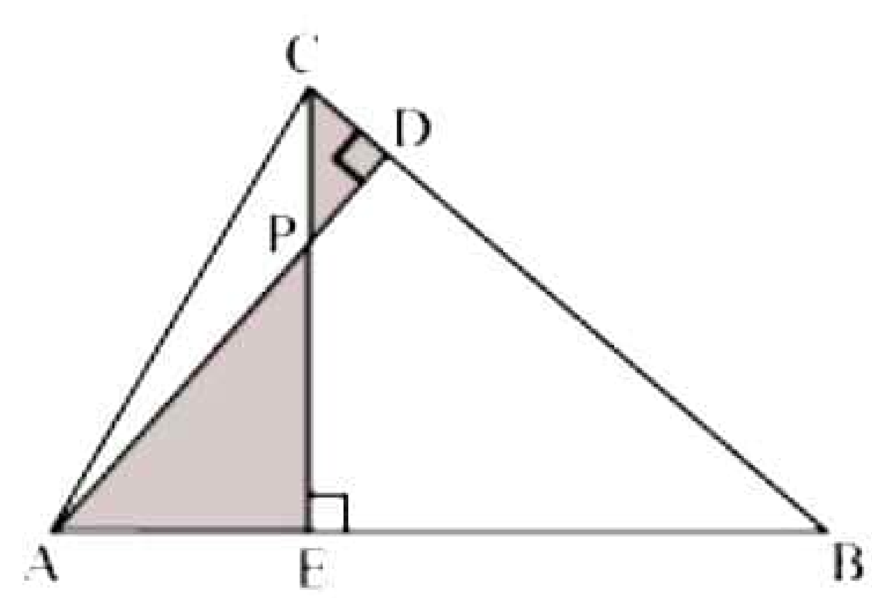
$$\Rightarrow x - 10 = 0 \text{ or } x - 3 = 0$$

$$\Rightarrow x = 10 \text{ or } x = 3$$

Hence, the required width is 3 metres as x cannot be 10 m since the width of the path cannot be greater than or equal to the width of the field.

33.

i.



In ΔAEP and ΔCDP,

$\angle CDP = \angle AEP = 90°$

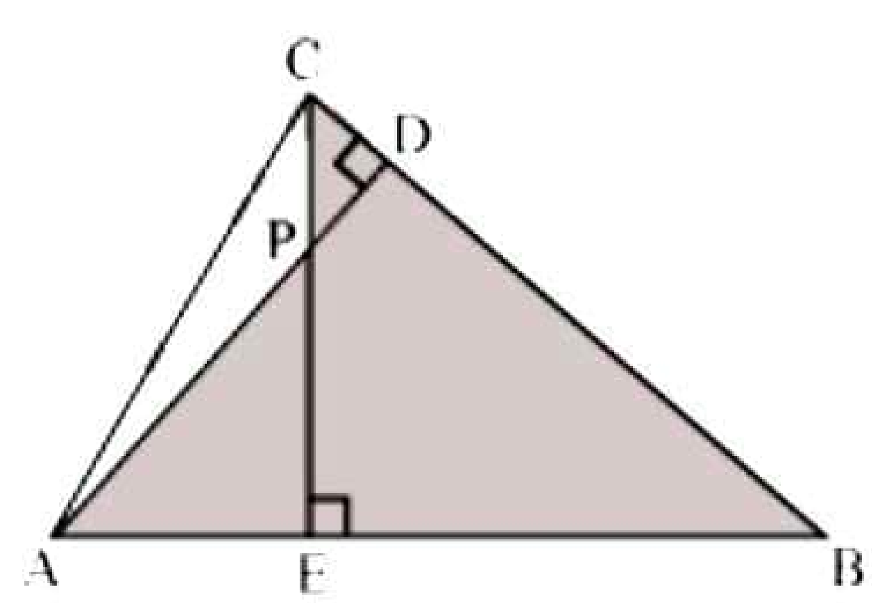
$\angle CPD = \angle APE$ ... (vertically opposite angles)

$\angle PCD = \angle PAE$ ... (remaining angle)

Therefore by AAA rule,

$\Delta AEP \sim \Delta CDP$

ii.



In ΔABD and ΔCBE,

$\angle ADB = \angle CEB = 90°$

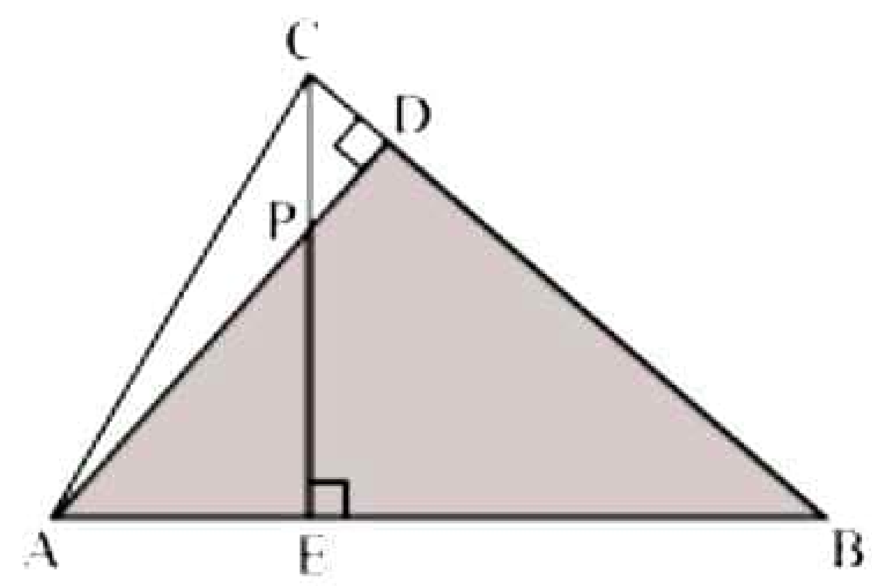
$\angle ABD = \angle CBE$ (common angle)

$\angle DAB = \angle ECB$ (remaining angle)

Therefore by AAA rule,

$\Delta ABD \sim \Delta CBE$

iii.



In ΔAEP and ΔADB,

∠AEP = ∠ADB = 90°

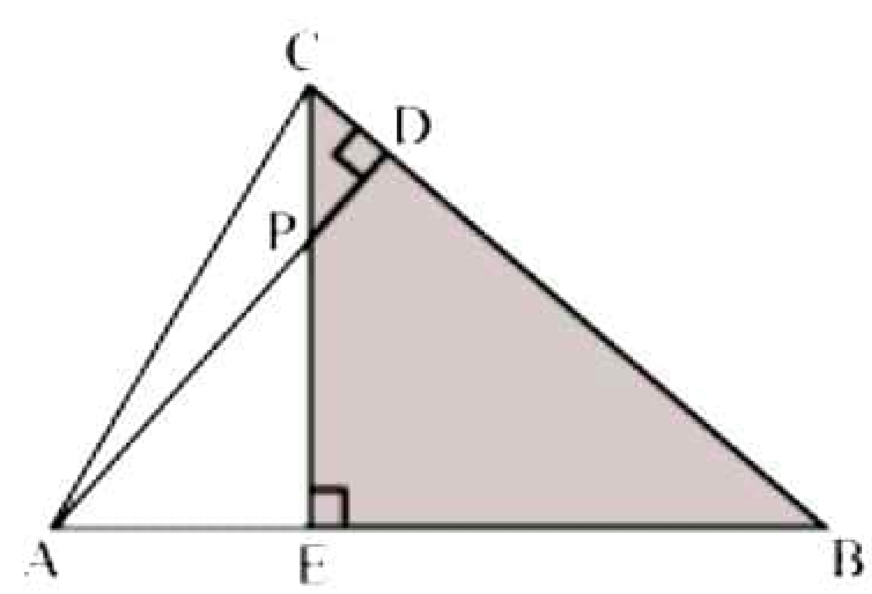
∠PAE = ∠DAB (common angle)

∠APE = ∠ABD (remaining angle)

Therefore, by AAA rule,

ΔAEP ~ ΔADB

iv.



In ΔPDC and ΔBEC

∠PDC = ∠BEC = 90°

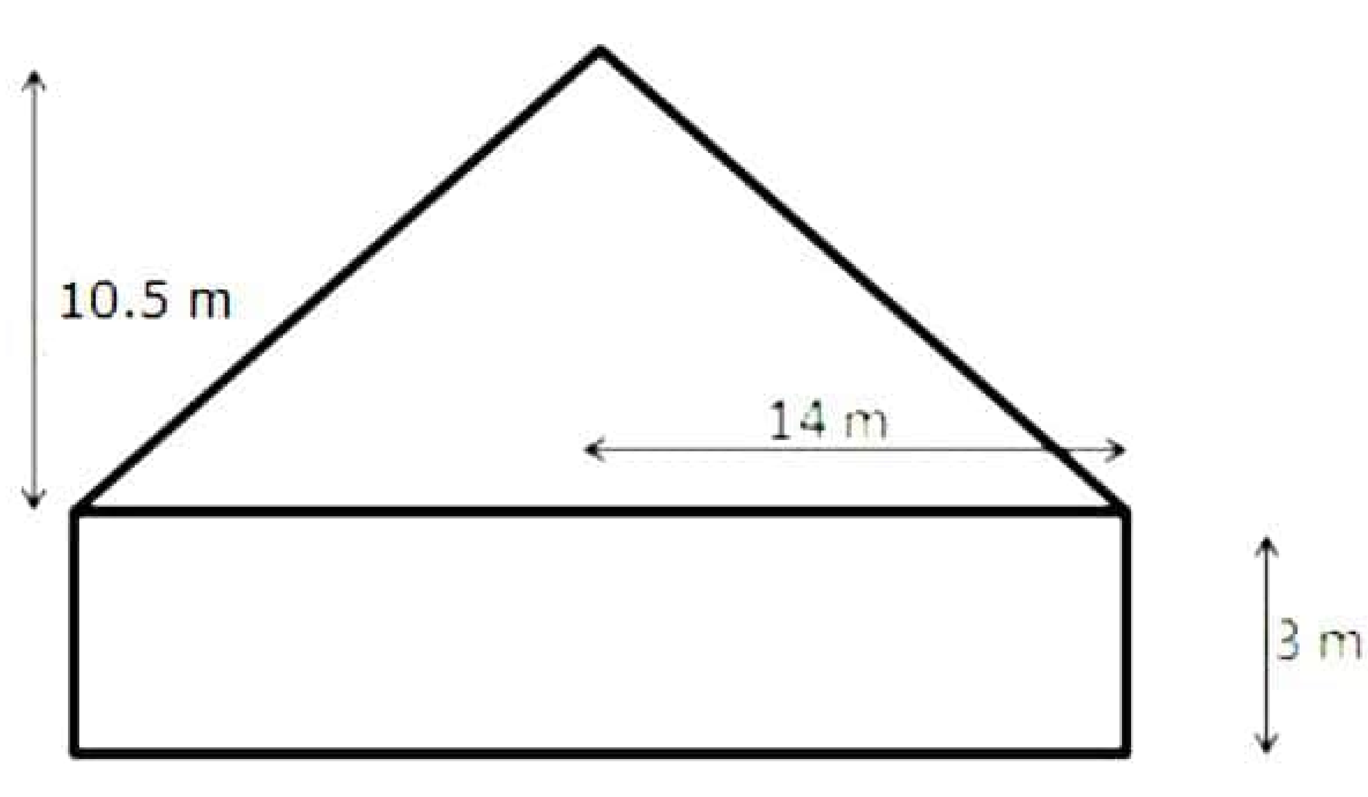
∠PCD = ∠BCE (common angle)

∠CPD = ∠CBE (remaining angle)

Therefore, by AAA rule,

ΔPDC ~ ΔBEC

34.



For cylinder: Radius = 14 m and height = 3 m

For cone: Radius = 14 m and height = 10.5 m

Let l be the slant height of the cone.

$\therefore\ l^2 = (14)^2 + (10.5)^2$

$l^2 = (196 + 110.25)m^2$

$l^2 = 306.25\ m^2$

$l = \sqrt{306.25}\,m$

$= 17.5\ m$

Curved surface area of the tent

= (curved surface area of the cylinder + curved surface area of the cone)

$= 2\pi rh + \pi rl$

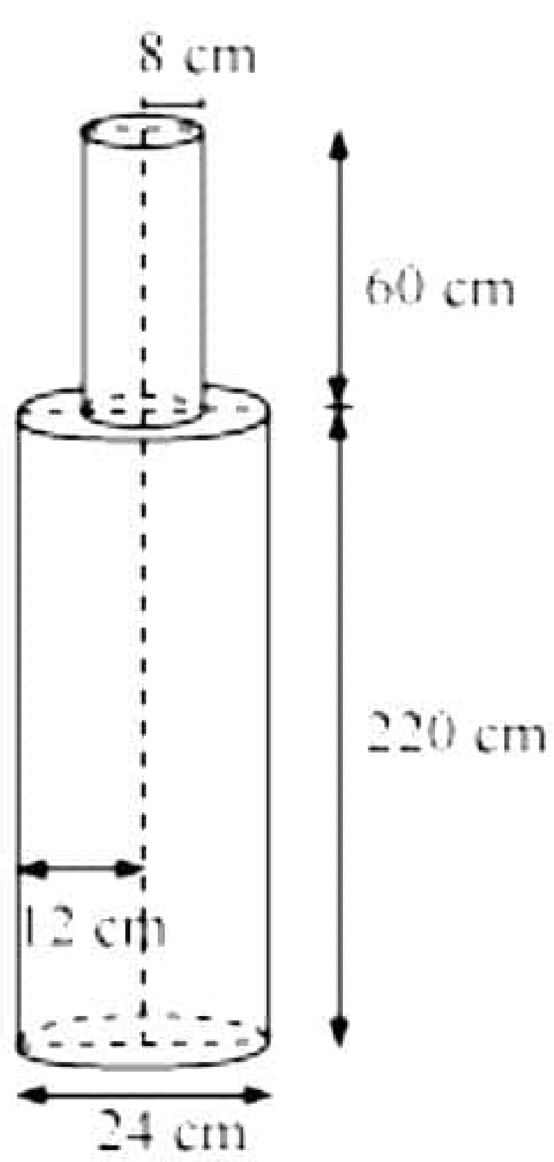
$= \left[\left(2 \times \frac{22}{7} \times 14 \times 3\right) + \left(\frac{22}{7} \times 14 \times 17.5\right)\right] m^2$

$= (264 + 770)m^2 = 1034\ m^2$

Hence, curved surface area of the tent = 1034 m^2

Cost of cloth = Rs. (1034 × 80) = Rs. 82720.

OR



From the figure we have

Height (h_1) of larger cylinder = 220 cm

Radius (r_1) of larger cylinder = $\frac{24}{2}$ = 12 cm

Height (h_2) of smaller cylinder = 60 cm

Radius (r_2) of larger cylinder = 8 cm

Total volume of pole = volume of larger cylinder + volume of smaller cylinder

$= \pi r_1^2 h_1 + \pi r_2^2 h_2$

$= \pi(12)^2 \times 220 + \pi(8)^2 \times 60$

$= \pi[144 \times 220 + 64 \times 60]$

$= 35520 \times 3.14 = 1,11,532.8\ cm^3$

Mass of 1 cm^3 iron = 8 gm

Mass of 111532.8 cm^3 iron = 111532.8 × 8 = 892262.4 gm = 892.262 kg.

35. We may find class mark (x_i) for each interval by using the relation.

$$x_i = \frac{\text{upper class limit} + \text{lower class limit}}{2}$$

Given that mean pocket allowance $\overline{x} = \text{Rs.}18$

Now taking 18 as assured mean (a) we may calculate d_i and f_id_i as follows:

Daily pocket allowance (in Rs.)	**Number of children f_i**	**Class mark x_i**	**$d_i = x_i - 18$**	**f_id_i**
11 - 13	7	12	−6	−42
13 - 15	6	14	−4	−24
15 - 17	9	16	−2	−18
17 - 19	13	18	0	0
19 - 21	f	20	2	2f
21 - 23	5	22	4	20
23 - 25	4	24	6	24
Total	$\sum f_i = 44 + f$			$\sum f_id_i = 2f - 40$

From the table,

$$\overline{x} = a + \frac{\sum f_id_i}{\sum f_i}$$

$$18 = 18 + \left(\frac{2f - 40}{44 + f}\right)$$

$$0 = \left(\frac{2f - 40}{44 + f}\right)$$

$2f - 40 = 0$

$2f = 40$

$f = 20$

Hence, the messing frequency f is 20.

Section E

36.

i. The production of TV sets in a factory increases uniformly by a fixed number every year.
This is an example of an A.P.
Production of TV sets in 6^{th} year = 16000 and
Production of TV sets in 9^{th} year = 22600
n^{th} term of an AP = a + (n − 1)d
⇒ 6^{th} term of an AP = a + (6 − 1)d
⇒ 16000 = a + 5d ….. (i)
Similarly we get, 22600 = a + 8d ….. (ii)
Solving (i) and (ii), we get
d = 2200 and a = 5000
Therefore, the production during first year is 5000.

OR

The production of TV sets in a factory increases uniformly by a fixed number every year.
This is an example of an A.P.
Production of TV sets in 6^{th} year = 16000 and
Production of TV sets in 9^{th} year = 22600
n^{th} term of an AP = a + (n − 1)d
⇒ 6^{th} term of an AP = a + (6 − 1)d
⇒ 16000 = a + 5d ….. (i)
Similarly we get, 22600 = a + 8d ….. (ii)
Solving (i) and (ii), we get
d = 2200 and a = 5000
Therefore, the difference in production between two consecutive years is 2200.

ii. Production during 8^{th} year = a + 7d = 5000 + 7(2200) = 20400

iii. Production during first three years,

$$S_3 = \frac{3}{2}(2 \times 5000 + 2 \times 2200)$$
$$= \frac{3}{2} \times 14400$$
$$= 21600$$

37.

i.

A(1,1) and C(4,5)

$$d(AC) = \sqrt{(4-1)^2 + (5-1)^2} = 5 \text{ km}$$

OR

B(4,1) and D(7,5)

$$d(BD) = \sqrt{(7-4)^2 + (5-1)^2} = 5 \text{ km}$$

ii.

$B(4,1)$ and $A(1,1)$

$$d(BA) = \sqrt{(1-4)^2 + (1-1)^2} = 3 \text{ km}$$

iii.

$C(4,5)$ and $B(4,1)$

$$d(BC) = \sqrt{(4-4)^2 + (5-1)^2} = 4 \text{ km}$$

38.

i. Distance between Boat 1 and the base of the lighthouse = BC

$\angle ACB = \angle EAC = 45°$

$$\tan 45° = \frac{AB}{BC}$$

$$\therefore 1 = \frac{100}{BC}$$

$$\therefore BC = 100 \text{ m}$$

OR

Distance between Boat 2 and the base of the lighthouse = BD

$\angle ADB = \angle EAD = 30°$

$$\tan 30° = \frac{AB}{BD}$$

$$\therefore \frac{1}{\sqrt{3}} = \frac{100}{BD}$$

$$\therefore BD = 100\sqrt{3} \text{ m}$$

ii.

$\angle ACB = \angle EAC = 45°$

$$\sin 45° = \frac{AB}{AC}$$

$$\therefore \frac{1}{\sqrt{2}} = \frac{100}{AC}$$

$$\therefore AC = 100\sqrt{2} \text{ m}$$

iii.

$\angle ADB = \angle EAD = 30°$

$$\sin 30° = \frac{AB}{AD}$$

$$\therefore \frac{1}{2} = \frac{100}{AD}$$

$$\therefore AD = 200 \text{ m}$$