Class X Session 2024-25
Subject - Mathematics (Standard)
Sample Question Paper - 17

Time: 3 Hours **Total Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A - E.
2. Section A has 18 multiple choice questions and 2 Assertion-Reason based questions carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case study based questions carrying 4 marks each with subparts of 1, 1, and 2 marks each, respectively.
7. All Questions are compulsory. However, an internal choice in 2 Question of Section B, 2 Questions of Section C and 2 Questions of Section D has been provided. An internal choice has been provided in all the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.

Section A

Section A consists of 20 questions of 1 mark each.

Choose the correct answers to the questions from the given options. [20]

1. Which of the following is an irrational number?

   A. $\sqrt{4}$

   B. $\sqrt{9}$

   C. $\sqrt{7}$

   D. $3\sqrt{25}$

2. Find the roots of the following quadratic equation:

   $x^2 - \left(1 + \sqrt{2}\right)x + \sqrt{2} = 0$

   A. -1 and $\sqrt{2}$

   B. 1 and $\sqrt{2}$

   C. 1 and $-\sqrt{2}$

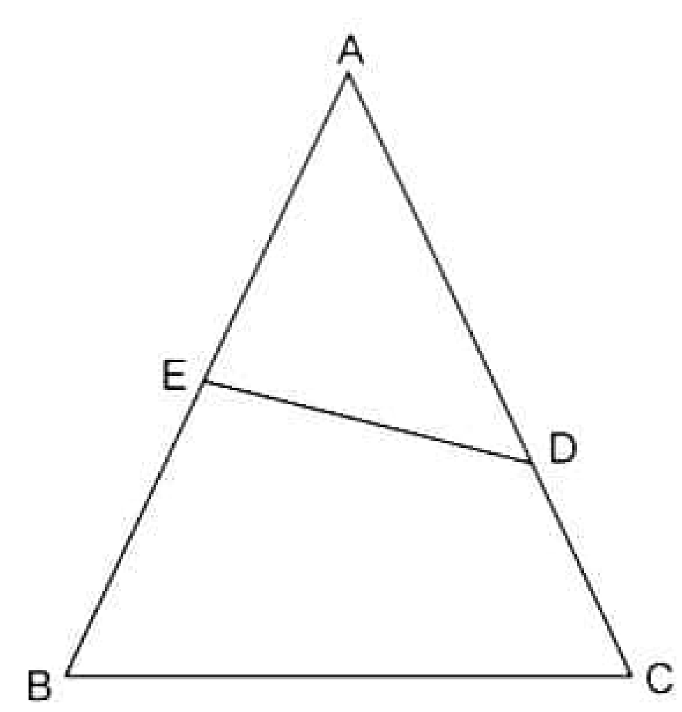
   D. -1 and $-\sqrt{2}$

3. The common difference of the A.P. 2, −1, −4, −7… is
   A. 2
   B. −2
   C. −3
   D. 3

4. In ΔABC, $\angle A = x°$, $\angle B = (3x)°$ and $\angle C = y°$. If $3y - 5x = 30$, then the triangle is ___________
   A. an equilateral triangle
   B. an isosceles triangle
   C. a right-angle triangle, right-angled at $\angle C$.
   D. a right-angle triangle, right-angled at $\angle B$.

5. If the points A(4, 3) and B(x, 5) lie on a circle with centre O(2, 3), find the value of x.
   A. 2
   B. 3
   C. 4
   D. 5

6. $\Delta ADE \sim \Delta ABC$. Also, if AD = 3.8 cm, AE = 3.6 cm, BE = 2.1 cm and BC = 4.2 cm, then find DE.



   A. 2.8 cm
   B. 3.8 cm
   C. 1.8 cm
   D. 5.8 cm

7. Find the value of θ if $\tan\theta = \cot\theta$.
   A. 30°
   B. 60°
   C. 45°
   D. 90°

8. If ΔABC is right-angled at C, then find the value of cos (A + B).

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{2}$

C. 1

D. 0

9. The perimeters of two similar triangles ABC and PQR are 32 cm and 24 cm, respectively. If PQ = 12 cm, then find AB.

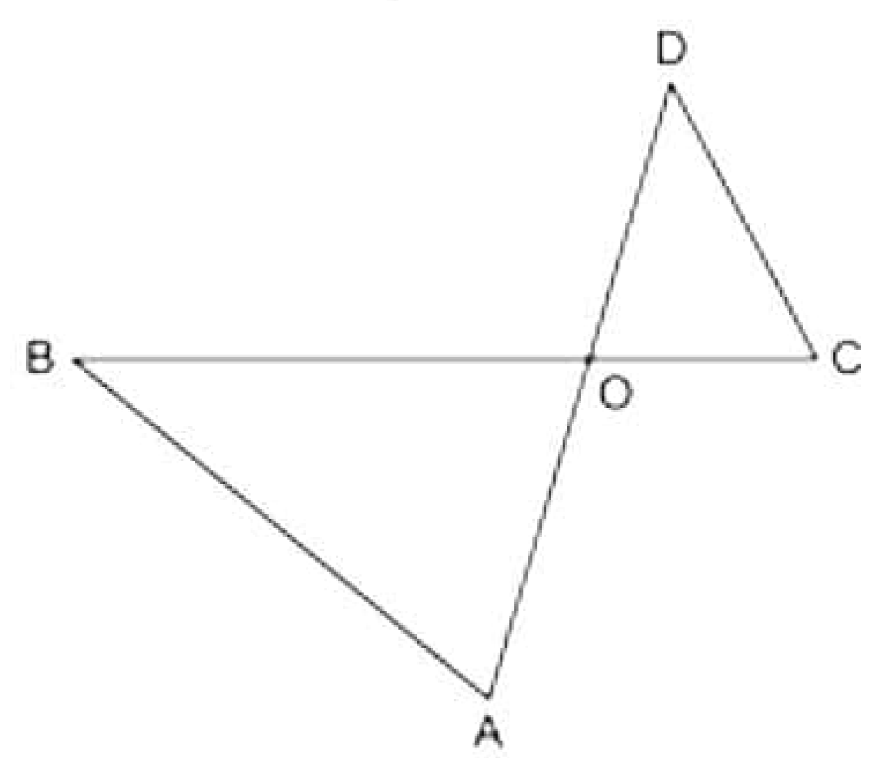
A. 10 cm

B. 16 cm

C. 12 cm

D. 14 cm

10. In the given figure, $\Delta OAB \sim \Delta OCD$. If AB = 8 cm, BO = 6.4 cm, OC = 3.5 cm and CD = 5 cm, find DO.



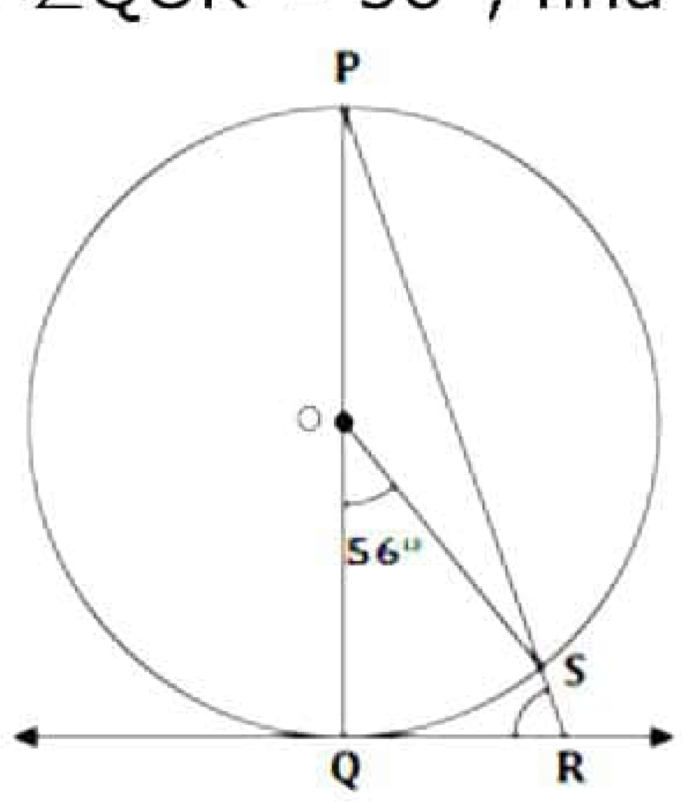
A. 8 cm

B. 6 cm

C. 2 cm

D. 4 cm

11. In the given figure, PQ is a diameter of a circle with centre O and QR is a tangent. If $\angle QOR = 56°$, find $\angle QRS$.



A. 42°

B. 52°

C. 62°

D. 72°

12. Perimeter of a sector of a circle with radius r, is given by

A. $\frac{\theta}{360^\circ} \times \pi r + 2r$

B. $\frac{\theta}{360^\circ} \times 2\pi r + r$

C. $\frac{\theta}{360^\circ} \times 2\pi r$

D. $\frac{\theta}{360^\circ} \times 2\pi r + 2r$

13. Volume of a cone of diameter 8.4 cm and height 6 cm is

A. 53.28π cm^3
B. 35.28π cm^3
C. 45.36π cm^3
D. 52.39π cm^3

14. Find the median of data: 3, 11.5, 5, 2.1, 6, 8.92, 7.

A. 5
B. 4.5
C. 6
D. 5.5

15. Find the area of a sector having central angle 60° and radius 7 cm.

A. 26.57 cm^2
B. 26.77 cm^2
C. 25.77 cm^2
D. 25.67 cm^2

16. Find the modal class from the following table:

Size	Frequency
45-55	7
55-65	12
65-75	17
75-85	30
85-95	32
95-105	6
105-115	10

A. 75-85
B. 85-95
C. 95-105
D. 105-115

17. 250 lottery tickets were sold and there are 5 prizes on these tickets. If Kunal has purchased one lottery ticket, what is the probability that he wins a prize?

A. $\frac{1}{50}$

B. $\frac{1}{25}$

C. $\frac{1}{5}$

D. 1

18. If $2\sin^2\theta - \cos^2\theta = 2$, find the value of θ.

A. 45°

B. 30°

C. 60°

D. 90°

DIRECTION: In the question number 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option

19. **Statement A (Assertion)**: $\sqrt{5}$ is an irrational number.

Statement R (Reason): Irrational numbers can't be represented as p/q form, where p and q are co-prime and q is not zero.

A. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

B. Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

C. Assertion (A) is true but reason (R) is false.

D. Assertion (A) is false but reason (R) is true.

20. **Statement A (Assertion):** If $2^p \times 3^q \times 7^r$ is the prime factorisation of 252, then $p + q + r = 5$

Statement B (Reason): Prime factorisation of 252 = $2^3 \times 3^2 \times 7$

A. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

B. Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

C. Assertion (A) is true but reason (R) is false.

D. Assertion (A) is false but reason (R) is true.

Section B

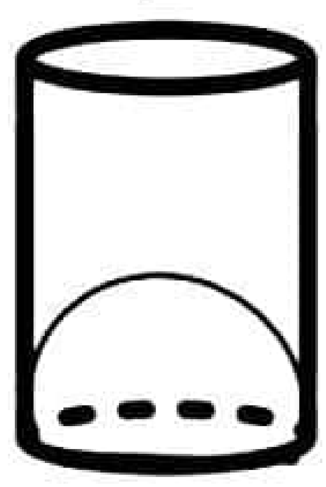
Section B consists of 5 questions of 2 marks each.

21. The area of a rectangle gets reduced by 8 m^2 when its length is reduced by 5 m and its breadth is increased by 3 m. If we increase the length by 3 m and breadth by 2 m, then the area is increased by 74 m^2. Find the length and breadth of the rectangle. [2]

22. P and Q are the points on the sides AB and AC, respectively, of ΔABC. If AP = 2 cm, PB = 4 cm, AQ = 3 cm and QC = 6 cm, show that BC = 3PQ. [2]

23. Out of the two concentric circles, the radius of the outer circle is 5 cm and the chord AC of length 8 cm is a tangent to the inner circle. Find the radius of the inner circle. [2]

24. Monali was serving her guests using glasses whose shape is as shown below. Since the bottom of the glass has hemispherical raised portion, it reduces the capacity of the glass. If the height of the glass is 12 cm and its inner diameter is 6 cm, find the volume of the juice that can be filled in the glass. (Use π = 3.14) [2]



OR

A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

25. Prove that: $\tan A - \cot A = \dfrac{2\sin^2 A - 1}{\sin A \cos A}$ [2]

OR

Prove that $\dfrac{1 - \sin A}{1 + \sin A} = (\sec A - \tan A)^2$

Section C

Section C consists of 6 questions of 3 marks each.

26. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point? [3]

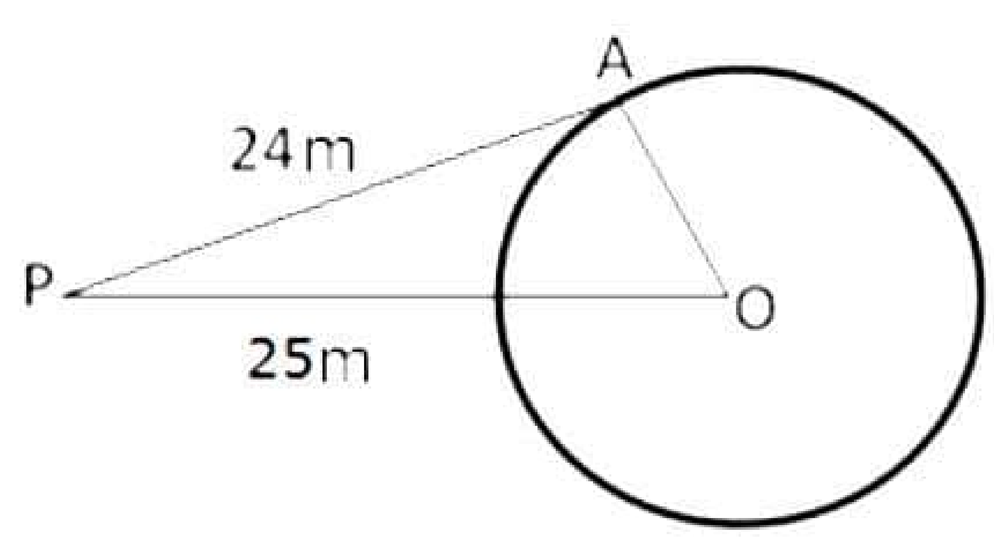
27. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs. 90, find the number of articles produced and the cost of each article. [3]

28. The coach of a cricket team buys 7 bats and 6 balls for Rs. 3800. Later, he buys 3 bats and 5 balls for Rs. 1750. Find the cost of each bat and each ball. [3]

OR

Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

29. Arjun is standing at a point P, which is 25 m away from the centre (O) of a circular park, and the length of a road from the point P to the gate of the park (A) is 24 m. [3]



Find the distance from the centre of the park to the gate.

OR

If two tangents inclined at an angle of 60° are drawn to a circle of radius 3 cm, then find the length of each tangent.

30. Express the trigonometric ratios sin A, sec A and tan A in terms of cot A. [3]

31. A box contains 20 balls bearing numbers 1, 2, 3, ..., 20, respectively. A ball is taken out at random from the box. What is the probability that the number on the ball is [3]

i. an odd number?
ii. divisible by 2 or 3?
iii. a prime number?
iv. not divisible by 10?

Section D

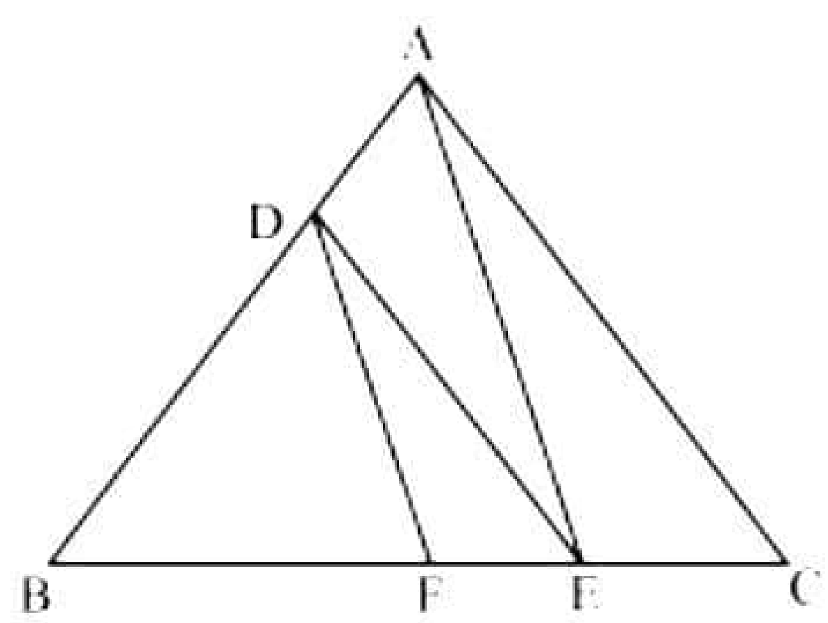
Section D consists of 4 questions of 5 marks each.

32. In a class test, the sum of Kamal's marks in Mathematics and English is 40. Had he got 3 marks more in Mathematics and 4 marks less in English, the product of the marks would have been 360. Find his marks in two subjects separately. [5]

OR

A passenger train takes 2 hours less for a journey of 300 km if its speed is increased by 5 km/hr from its usual speed. Find its usual speed.

33. In the figure, DE || AC and DF || AE. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$. [5]



34. A gulabjamun, when ready for eating, contains sugar syrup of about 30% of its volume. How much syrup would be found in 45 such gulabjamuns, each shaped like a cylinder with two hemispherical ends, if the complete length of each of them is 5 cm and its diameter is 2.8 cm? [5]

OR

Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. if each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made.

35. Thirty women were examined in a hospital by a doctor and the number of heart beats per minute were recorded and summarized as follows. Fine the mean heart beats per minute for these women, choosing a suitable method. [5]

Number of heart beats per minute	65 - 68	68 - 71	71 -74	74 - 77	77 - 80	80 - 83	83 - 86
Number of women	2	4	3	8	7	4	2

Section E

Case study based questions are compulsory.

36. It is festival time, and smartphone companies have come up with no-cost EMI plans to sell their flagship models. Now Aarushi always wanted to buy a flagship smartphone, so she decided to take advantage of this offer. She buys a smartphone on EMI of Rs. 1000 per month. Now, she pays Rs. 1000 for the first month and decides to make the subsequent payment in such a manner that the current month's payment will always be Rs. 100 more than the previous month. Now, using the information given, answer the following questions.

i. Find the amount paid by her in 30^{th} month. [1]

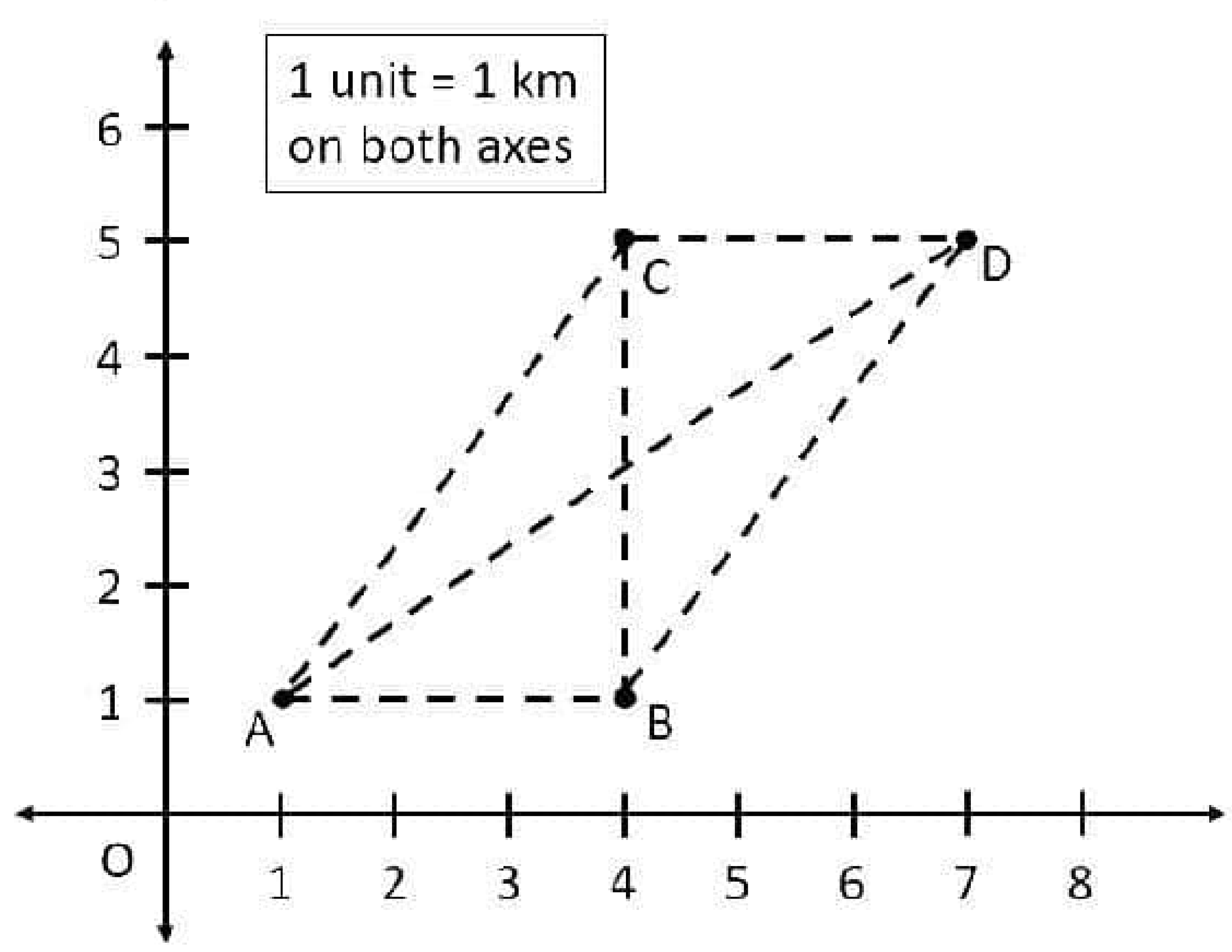
ii. For a particular month, Aarushi pays Rs. 4900 as an instalment; find which month this is. [1]

iii. Find the ratio of the payment made in 19^{th} month to the 28^{th} month. [2]

OR

Find the total amount paid by Aarushi in 30 months.

37. Amey runs a grocery store that offers home delivery of fresh groceries to its customers. His store is located at location A as indicated in the map below. Now, he receives regular orders from the families living in the colonies located at locations B, C and D. Given below is the map containing the location of the store, colonies and the roads connecting them. Use the given information to answer the below questions.



i. Find the shortest distance between locations A and C. [2]

OR

Find the shortest distance between locations B and D.

ii. If point X lies at the midpoint of line segment joining A and B, then find its co-ordinates. [1]

iii. If Amey needs to travel back from location C to location B, then find the shortest distance covered by him. [1]

38. Harsh is standing between two poles having a height 10 m and 15 m. Now, the angle of elevation from the point where Harsh is standing to the top of the 10 m pole is 45°, whereas the angle of elevation from the same point to the top of 15 m pole is 60°. Using the given data, answer the following questions.

i. Draw a neat labelled figure to show the above situation diagrammatically. [1]

ii. Find the distance between Harsh and 15 m pole. [1]

iii. Find the minimum rope length required to tie the top of 10 m pole to point where Harsh is standing. [2]

OR

If a ladder is kept touching the point where Harsh is standing and the top of 15 m pole, then find the length of the ladder required.

Solution

Section A

1. Correct option: C

Explanation:

$\sqrt{7}$ is an irrational number as it cannot be written in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$.

2. Correct option: B

Explanation:

$x^2 - \left(1 + \sqrt{2}\right)x + \sqrt{2} = 0$

$\Rightarrow x^2 - x - \sqrt{2}x + \sqrt{2} = 0$

$\Rightarrow x\left(x - 1\right) - \sqrt{2}\left(x - 1\right) = 0$

$\Rightarrow \left(x - 1\right)\left(x - \sqrt{2}\right) = 0$

$\Rightarrow \left(x - 1\right) = 0$ or $x - \sqrt{2} = 0$

$\Rightarrow x = 1$ or $x = \sqrt{2}$

Hence, 1 and $\sqrt{2}$ are the roots of the given equation.

3. Correct option: C

Explanation:

Common difference = −1 − 2 = −3

4. Correct option: D

Explanation:

By angle-sum property,

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow x + 3x + y = 180$

$\Rightarrow 4x + y = 180$ ....(1)

Also, $3y - 5x = 30$

$\Rightarrow -5x + 3y = 30$ ....(2)

Multiplying equation (1) by 3, we get

$12x + 3y = 540$ ....(3)

Subtracting equation (2) from (3), we get

$17x = 510$

$\Rightarrow x = 30$

Putting x = 30 in equation (1), we get

$4(30) + y = 180$

$\Rightarrow y = 60$

Hence, $\angle A = 30°$, $\angle B = 3(30°) = 90°$, $\angle C = 60°$

So, the triangle is a right angle triangle, right-angled at $\angle B$.

5. Correct option: A

Explanation:

The points A(4, 3) and B(x, 5) lie on a circle with centre is O(2,3).

$\Rightarrow OA = OB$ (radii of the same circle)

$\Rightarrow OA^2 = OB^2$

$\Rightarrow (2 - 4)^2 + (3 - 3)^2 = (2 - x)^2 + (3 - 5)^2$

$\Rightarrow 4 = 4 - 4x + x^2 + 4$

$\Rightarrow x^2 - 4x + 4 = 0$

$\Rightarrow (x - 2)^2 = 0$

$\Rightarrow (x - 2) = 0$

$\Rightarrow x = 2$

6. Correct option: A

Explanation:

$\therefore \Delta ADE \sim \Delta ABC$ (given)

$\Rightarrow \frac{AD}{AB} = \frac{DE}{BC}$

$\Rightarrow \frac{AD}{AE + EB} = \frac{DE}{BC}$

$\Rightarrow \frac{3.8}{(3.6 + 2.1)} = \frac{DE}{4.2}$

$\Rightarrow \frac{3.8}{5.7} = \frac{DE}{4.2}$

$\Rightarrow DE = 2.8$ cm

7. Correct option: C

Explanation:

$\tan 45° = \cot 45° = 1$

$\Rightarrow \theta = 45°$

8. Correct option: D

Explanation:

ΔABC is right-angled at C

Now, $\angle A + \angle B + \angle C = 180°$

$\Rightarrow A + B + 90° = 180°$

$\Rightarrow A + B = 90°$

$\Rightarrow \cos (A + B) = \cos 90° = 0$

9. Correct option: B

Explanation:

It is given that ΔABC and ΔPQR are similar triangles, so the corresponding sides of both the triangles are proportional.

$$\therefore \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta PQR} = \frac{AB}{PQ}$$

$$\Rightarrow \frac{32}{24} = \frac{AB}{12}$$

$\Rightarrow AB = 16$ cm

10. Correct option: D

Explanation:

$\Delta OAB \sim \Delta OCD$,

$$\Rightarrow \frac{OA}{OC} = \frac{AB}{CD} = \frac{BO}{DO}$$

$$\Rightarrow \frac{OA}{3.5} = \frac{8}{5} = \frac{6.4}{DO}$$

$$\Rightarrow \frac{6.4}{DO} = \frac{8}{5}$$

$\Rightarrow DO = 4$ cm

11. Correct option: C

Explanation:

$\angle QOR = 56°$

$$\Rightarrow \angle QPR = \frac{1}{2}\angle QOR = \frac{1}{2} \times 56° = 28°$$

In right-angled ΔPQR,

$\angle QPR + \angle PQR + \angle QRP = 180°$

$\Rightarrow 28° + 90° + \angle QRP = 180°$

$\Rightarrow \angle QRP = 62°$

That is, $\angle QRS = 62°$

12. Correct option: D

Explanation:

Perimeter of a sector of a circle with radius r $= \dfrac{\theta}{360} \times 2\pi r + 2r$

13. Correct option: B

Explanation:

Volume of the cone $= \dfrac{1}{3}\pi r^2 h = \dfrac{1}{3} \times \pi \times 4.2 \times 4.2 \times 6 = 35.28\pi \text{ cm}^3$

14. Correct option: C

Explanation:

Arranging data in ascending order:

2.1, 3, 5, 6, 7, 8.92, 11.5

Number of observations = 7

So, $\left(\frac{n+1}{2}\right)^{th}$ term is the median.

$\Rightarrow \left(\frac{7+1}{2}\right)^{th}$ = 4^{th} term = 6 is the median.

15. Correct option: D

Explanation:

Area of a sector $= \frac{\theta}{360^\circ} \times \pi r^2 = \left[\frac{60}{360} \times \frac{22}{7} \times 7^2\right]$ $cm^2 = 25.67\ cm^2$

16. Correct option: B

Explanation:

As the class 85–95 has the maximum frequency, it is the modal class.

17. Correct option: A

Explanation:

Number of lottery tickets = 250

Number of prize tickets = 5

So, the probability of winning a prize $= \frac{5}{250} = \frac{1}{50}$

18. Correct option: D

Explanation:

$2\sin^2\theta - \cos^2\theta = 2$

$\Rightarrow 2(1-\cos^2\theta) - \cos^2\theta = 2$

$\Rightarrow 2 - 2\cos^2\theta - \cos^2\theta = 2$

$\Rightarrow 2 - 3\cos^2\theta = 2$

$\Rightarrow 3\cos^2\theta = 0$

$\Rightarrow \cos^2\theta = 0$

$\Rightarrow \cos^2\theta = \cos^2 90^\circ$

$\Rightarrow \theta = 90^\circ$

19. Correction option: A

Explanation:

The statements given in both assertion and reason are correct and hence, both assertion and reason are true and reason is the correct explanation for assertion.

20. Correct option: C

Explanation:

Prime factorisation of 252 = $2^2 \times 3^2 \times 7$

Hence, reason is false.

Then, $p + q + r = 2 + 2 + 1 = 5$

Hence, assertion is true.

Section B

21. Let the length and breadth of a rectangle be x metres and y metres respectively.

Then, area = xy m^2

Now, $(x - 5)(y + 3) = (xy - 8)\ m^2$

$\Rightarrow xy + 3x - 5y - 15 = xy - 8$

$\Rightarrow 3x - 5y = 7$ ....(1)

And $(x + 3)(y + 2) = (xy + 74)\ m^2$

$\Rightarrow xy + 2x + 3y + 6 = xy + 74$

$\Rightarrow 2x + 3y = 68$ ....(2)

Multiplying equation (1) by 3 and equation (2) by 5, we get

$9x - 15y = 21$ ....(3)

$10x + 15y = 340$ ....(4)

Adding equations (3) and (4), we get

$19x = 361$

$\Rightarrow x = \frac{361}{19} = 19$

Putting x = 19 in equation (3), we get

$9(19) - 15y = 21$

$\Rightarrow 171 - 15y = 21$

$\Rightarrow 15y = 150$

$\Rightarrow y = 10$

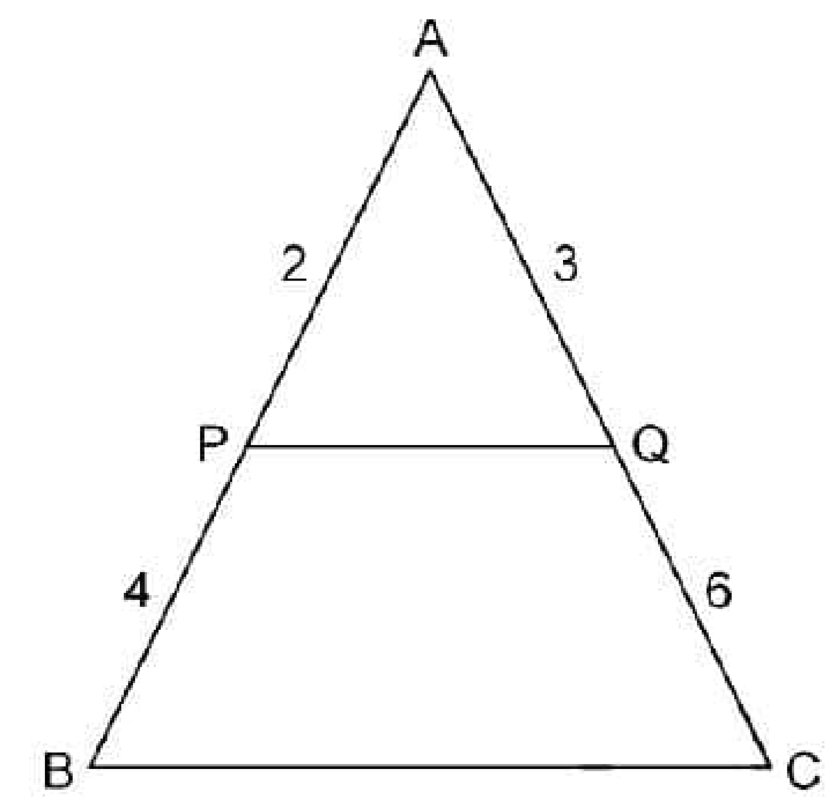
Hence, the length and breadth of a rectangle are 19 m and 10 m respectively.

22. Given that P is a point on AB, then

AB = AP + PB = (2 + 4) cm = 6 cm

Also, Q is a point on AC, then

AC = AQ + QC = (3 + 6) cm = 9 cm



$\therefore \quad \frac{AP}{AB} = \frac{2}{6} = \frac{1}{3}$

and $\frac{AQ}{AC} = \frac{3}{9} = \frac{1}{3}$

$\Rightarrow \quad \frac{AP}{AB} = \frac{AQ}{AC}$

Thus, in ΔAPQ and ΔABC

$\angle A = \angle A$ (common)

And $\frac{AP}{AB} = \frac{AQ}{AC}$

$\therefore \Delta APQ \sim \Delta ABC$ (by SAS similarity)

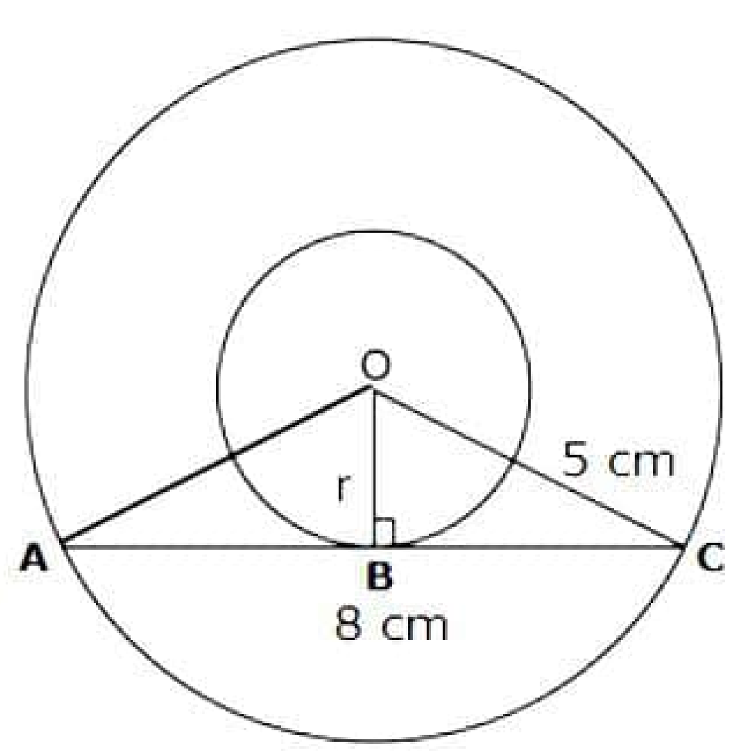
$\Rightarrow \frac{AP}{AB} = \frac{PQ}{BC} = \frac{AQ}{AC}$

$\therefore \frac{PQ}{BC} = \frac{AQ}{AC}$

$\Rightarrow \frac{PQ}{BC} = \frac{3}{9} = \frac{1}{3}$

$\Rightarrow$ BC = 3 PQ

23.



Let O be the centre of the concentric circles.

Now, $\angle OBC = 90^\circ$ (tangent is perpendicular to the radius of a circle)

AC is a chord of the outer circle.

The perpendicular drawn from the centre of a circle to a chord bisects the chord.

$\Rightarrow$ AC = 2BC $\Rightarrow$ BC = 4 cm

In ΔOBC, by Pythagoras theorem,

$OC^2 = OB^2 + BC^2$

$\Rightarrow 5^2 = r^2 + 4^2$

$\Rightarrow r^2 = 9$ cm

$\Rightarrow r = 3$ cm

Hence, the radius of the inner circle is 3 cm.

24. Inner diameter of the glass = 6 cm

Then, inner radius of the glass = 3 cm

Apparent capacity of the glass = $\pi r^2 h$

$= \pi(3 \times 3 \times 12)\ cm^3$

$= 108\pi\ cm^3$

Volume of the hemispherical part $= \frac{2}{3}\pi r^3$

$= \frac{2}{3}\pi \times 3 \times 3 \times 3\ cm^3$

$= 18\pi\ cm^3$

Therefore, volume of the juice that can be filled in the glass

= Apparent capacity of the glass – Volume of the hemispherical part

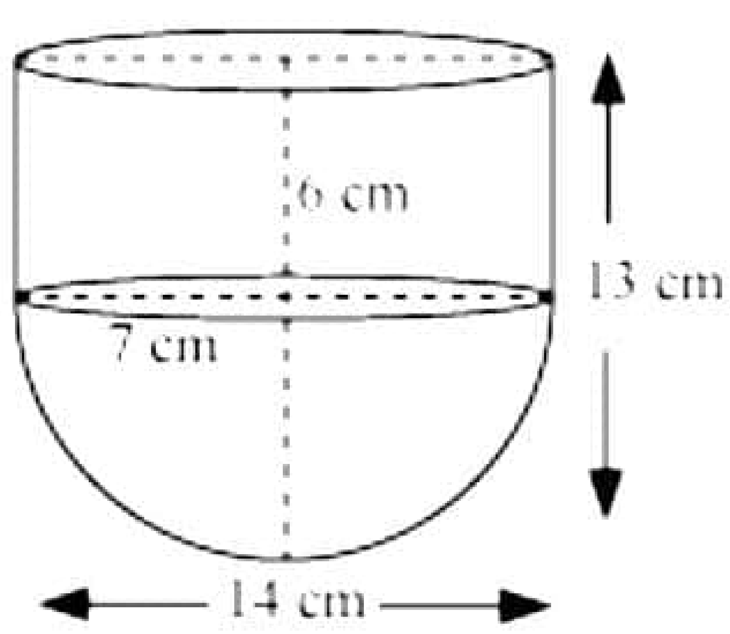
$= (108\pi - 18\pi)\ cm^3$

$= 90\pi\ cm^3$

$= (90 \times 3.14)\ cm^3$

$= 282.6\ cm^3$

OR



Radius of a cylindrical part and hemispherical part, r = 7 cm

Height of hemispherical part = r = 7 cm

Height of cylindrical part, h = 13 – 7 = 6 cm

Inner surface area of the vessel

= CSA of cylindrical part + CSA of hemispherical part

$= 2\pi rh + 2\pi r^2$

$= 2 \times \frac{22}{7} \times 7 \times 6 + 2 \times \frac{22}{7} \times 7 \times 7$

$= 44(6 + 7)$

$= 44 \times 13$

$= 572\ cm^2$

25.

$$\text{L.H.S.} = \tan A - \cot A$$
$$= \frac{\sin A}{\cos A} - \frac{\cos A}{\sin A}$$
$$= \frac{\sin^2 A - \cos^2 A}{\cos A \sin A}$$
$$= \frac{\sin^2 A - \left(1 - \sin^2 A\right)}{\cos A \sin A}$$
$$= \frac{2\sin^2 A - 1}{\sin A \cos A}$$
$$= \text{R.H.S.}$$

OR

$$\text{L.H.S.} = \frac{1 - \sin A}{1 + \sin A}$$
$$= \frac{1 - \sin A}{1 + \sin A} \times \frac{1 - \sin A}{1 - \sin A}$$
$$= \frac{\left(1 - \sin A\right)^2}{1 - \sin^2 A}$$
$$= \frac{\left(1 - \sin A\right)^2}{\cos^2 A}$$
$$= \left(\frac{1 - \sin A}{\cos A}\right)^2$$
$$= \left(\frac{1}{\cos A} - \frac{\sin A}{\cos A}\right)$$
$$= \left(\sec A - \tan A\right)^2$$
$$= \text{R.H.S.}$$

Section C

26. It can be observed that Ravi and Sonia do not take same amount of time. Ravi takes lesser time than Sonia for completing 1 round of the circular path.

As they are going in the same direction, they will meet again at the same time when Ravi will have completed 1 round of that circular path with respect to Sonia. i.e. When Sonia completes one round then Ravi completes 1.5 rounds.

So they will meet first time at the time which is a common multiple of the time taken by them to complete 1 round i.e. LCM of 18 and 12.

Now, $18 = 2 \times 3 \times 3 = 2 \times 3^2$

And, $12 = 2 \times 2 \times 3 = 2^2 \times 3$

LCM of 12 and 18 = product of factors raised to highest exponent

$= 2^2 \times 3^2$

$= 36$

Therefore, Ravi and Sonia will meet together at the starting point after 36 minutes.

27. Let the number of articles produced be x.
Therefore, cost of production of each article = Rs. (2x + 3)
Given, the total cost of production is Rs. 90.

$\therefore x(2x+3) = 90$

$\Rightarrow 2x^2 + 3x - 90 = 0$

$\Rightarrow 2x^2 + 15x - 12x - 90 = 0$

$\Rightarrow x(2x+15) - 6(2x+15) = 0$

$\Rightarrow (2x+15)(x-6) = 0$

Either 2x + 15 = 0 or x − 6 = 0, i.e., $x = \dfrac{-15}{2}$ or x = 6

As the number of articles produced can only be a positive integer, therefore, x can only be 6.
Hence, number of articles produced = 6
Cost of each article = Rs. (2 × 6 + 3) = Rs. 15

28. Let the cost of a bat and a ball be Rs. x and Rs. y respectively.
According to the given information,

$7x + 6y = 3800 \qquad (1)$

$3x + 5y = 1750 \qquad (2)$

From (1), we obtain

$y = \dfrac{3800 - 7x}{6} \qquad (3)$

Substituting this value in equation (2), we obtain

$3x + 5\left(\dfrac{3800 - 7x}{6}\right) = 1750$

$3x + \dfrac{9500}{3} - \dfrac{35x}{6} = 1750$

$3x - \dfrac{35x}{6} = 1750 - \dfrac{9500}{3}$

$\dfrac{18x - 35x}{6} = \dfrac{5250 - 9500}{3}$

$-\dfrac{17x}{6} = \dfrac{-4250}{3}$

$-17x = -8500$

$x = 500 \qquad (4)$

Substituting this in equation (3), we obtain

$y = \dfrac{3800 - 7 \times 500}{6} = \dfrac{300}{6} = 50$

Hence, the cost of a bat is Rs. 500 and that of a ball is Rs. 50.

OR

Let the present age of Jacob be x years and the age of his son be y years.
According to the given information,

$(x+5)=3(y+5)$

$x-3y=10 \qquad (1)$

$(x-5)=7(y-5)$

$x-7y=-30 \qquad (2)$

From (1), we obtain

$x=3y+10 \qquad (3)$

Substituting this value in equation (2), we obtain

$3y+10-7y=-30$

$-4y=-40$

$y=10 \qquad (4)$

Substituting this value in equation (3), we obtain

$x=3\times10+10=40$

Hence, the present age of Jacob is 40 years whereas the present age of his son is 10 years.

29. PA is the tangent to the circle with centre O, such that PO = 25 m, PA = 24 m.
In $\triangle PAO$, $\angle A = 90^\circ$ (tangent $\perp$ radius)
By Pythagoras' theorem,

$PO^2 = PA^2 + AO^2$

$OA^2 = PO^2 - PA^2$

$= 25^2 - 24^2$

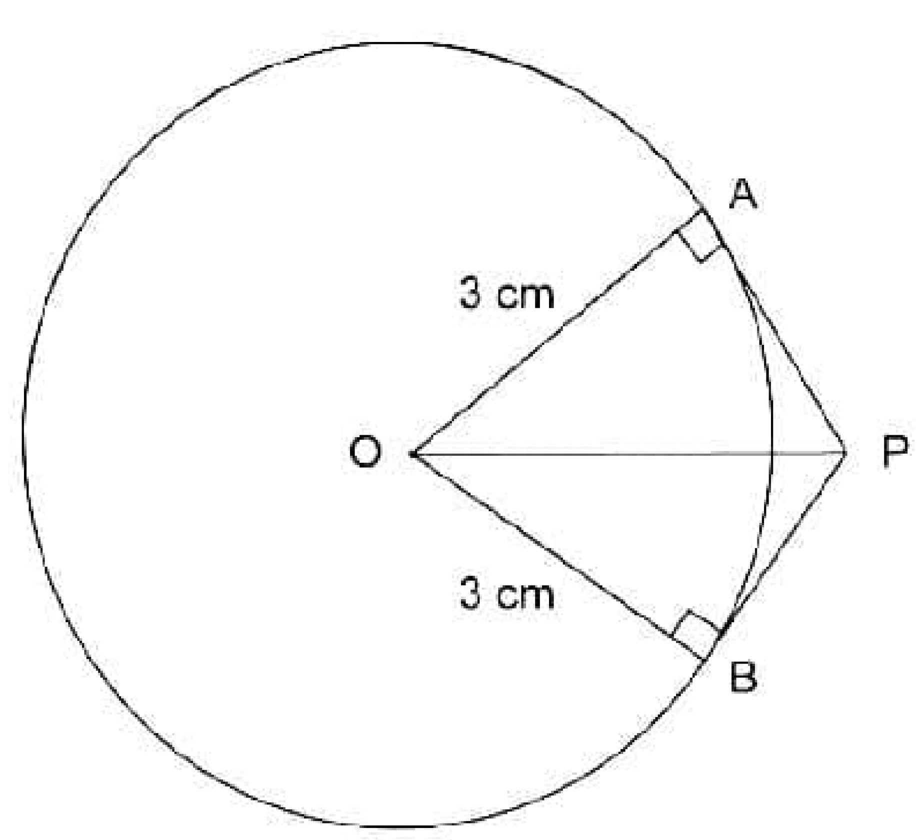
$= (25 - 24)(25 + 24)$

$= 49$

OA = 7 m

Hence, the distance from the centre of the park to the gate is 7 m.

OR



Let P be the external point and PA and PB be the tangents such that,
$\angle APB = 60^\circ$.
Now OA and OB are the radii of the circle.
$\therefore$ OA = OB = 3 cm

Also we know that the tangents drawn from an external point are equally inclined to the line joining the point to the centre.

$\Rightarrow \angle OPA = \angle BPO = \dfrac{\angle APB}{2} = \dfrac{60^\circ}{2} = 30^\circ$

Now, in $\triangle OAP$, $\angle OPA = 30^\circ$

$\Rightarrow \tan 30^\circ = \dfrac{OA}{AP}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{3}{AP}$

$\Rightarrow AP = 3\sqrt{3}$ cm $= BP$

Hence, the length of each tangent is $3\sqrt{3}$ cm.

30. We know that

$\operatorname{cosec}^2 A = 1 + \cot^2 A$

$\dfrac{1}{\operatorname{cosec}^2 A} = \dfrac{1}{1+\cot^2 A}$

$\sin^2 A = \dfrac{1}{1+\cot^2 A}$

$\sin A = \pm \dfrac{1}{\sqrt{1+\cot^2 A}}$

But $\sqrt{1+\cot^2 A}$ will be always positive as we are adding two positive quantities.

So, $\sin A = \dfrac{1}{\sqrt{1+\cot^2 A}}$

We know that $\tan A = \dfrac{\sin A}{\cos A}$

But $\cot A = \dfrac{\cos A}{\sin A}$

So, $\tan A = \dfrac{1}{\cot A}$

Also $\sec^2 A = 1 + \tan^2 A$

$= 1 + \dfrac{1}{\cot^2 A}$

$= \dfrac{\cot^2 A + 1}{\cot^2 A}$

$\sec A = \dfrac{\sqrt{\cot^2 A + 1}}{\cot A}$

31. Total number of balls = 20

i. Odd numbers are 1, 3, 5, 7, 9, 11, 13, 15, 17, 19.

Total no. of odd numbers = 10

$\therefore$ P(getting an odd number) $= \dfrac{10}{20} = \dfrac{1}{2}$

ii. Numbers divisible by 2 or 3 are
2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20.
Total no. of numbers divisible by 2 or 3 =13

P(getting a number divisible by 2 or 3) = $\frac{13}{20}$

iii. Prime numbers are 2, 3, 5, 7, 11, 13, 17, 19
Total no. of prime numbers = 8

P(getting a prime number) = $\frac{8}{20} = \frac{2}{5}$

iv. Numbers divisible by 10 are 10, 20.
Number of numbers divisible by 10 = 2

$\therefore$P(getting a number not divisible by 10) = $\left(1-\frac{2}{20}\right) = \frac{18}{20} = \frac{9}{10}$

Section D

32. Let the marks obtained by Kamal in Mathematics and English be x and y.

$\therefore x + y = 40 \quad ....(1)$

and $(x + 3)(y - 4) = 360 \quad ....(2)$

From $(1), y = 40 - x$

Putting value of y in (2)

$(x+3)(40-x-4)=360$

$\Rightarrow(x+3)(36-x)=360$

$\Rightarrow 36x-x^2+108-3x=360$

$\Rightarrow -x^2+33x-252=0$

$\Rightarrow x^2-33x+252=0$

$\Rightarrow x^2-21x-12x+252=0$

$\Rightarrow x(x-21)-12(x-21)=0$

$\Rightarrow(x-21)(x-12)=0$

When $x-21=0$, $x=21$

when $x-12=0$, $x=12$

For $x=21$,

$y=40-21=19$

For $x=12$,

$y=40-12=28$

The marks obtained by Kamal in Mathematics and English, respectively, are 21 and 19 or 12 and 28.

OR

Let x km/hr be the usual speed of the passenger train.

Then, time taken to travel 300 km $=\dfrac{300}{x}$ hours

When speed is (x + 5) km/hr, the time taken to travel 300 km $=\dfrac{300}{x+5}$ hours

$\therefore \dfrac{300}{x}-\dfrac{300}{x+5}=2$

$\Rightarrow \dfrac{1}{x}-\dfrac{1}{x+5}=\dfrac{2}{300}=\dfrac{1}{150}$

$\Rightarrow \dfrac{x+5-x}{x(x+5)}=\dfrac{1}{150}$

$\Rightarrow \dfrac{5}{x(x+5)}=\dfrac{1}{150}$

$\therefore x(x+5)=750$ or $x^2+5x-750=0$

$\Rightarrow x^2+30x-25x-750=0$

$\Rightarrow x(x+30)-25(x+30)=0$ or $(x+30)(x-25)=0$

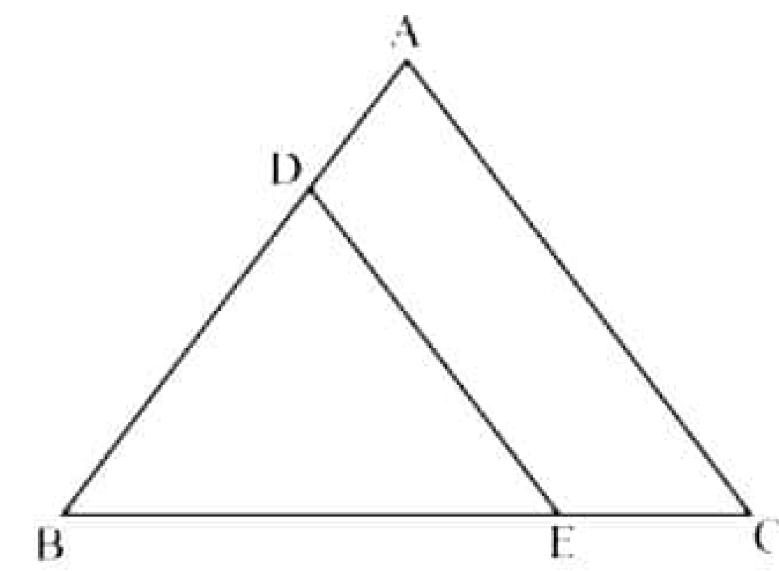
$\therefore x+30=0, x=-30$, but x cannot be negative

$\therefore x-25=0$, $x=25$

Therefore, the usual speed of the passenger train is 25 km/hr.

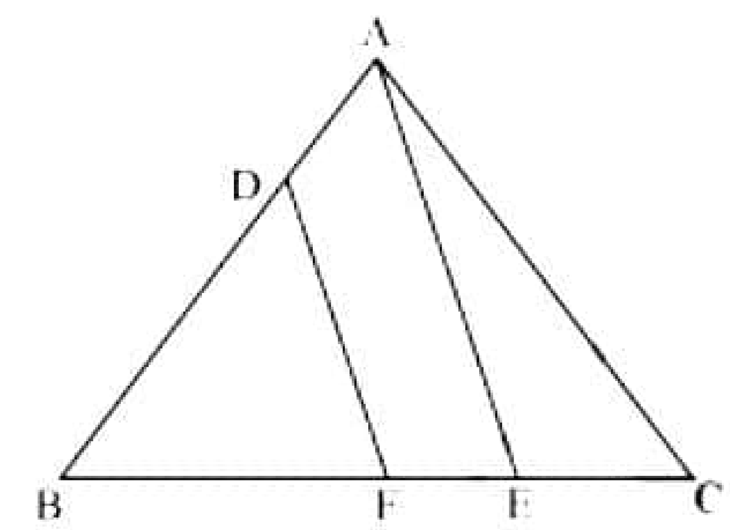
33. In ΔABC, DE || AC.

$\therefore \frac{BD}{DA} = \frac{BE}{EC}$ ....(i)



In ΔBAE, DF || AE.

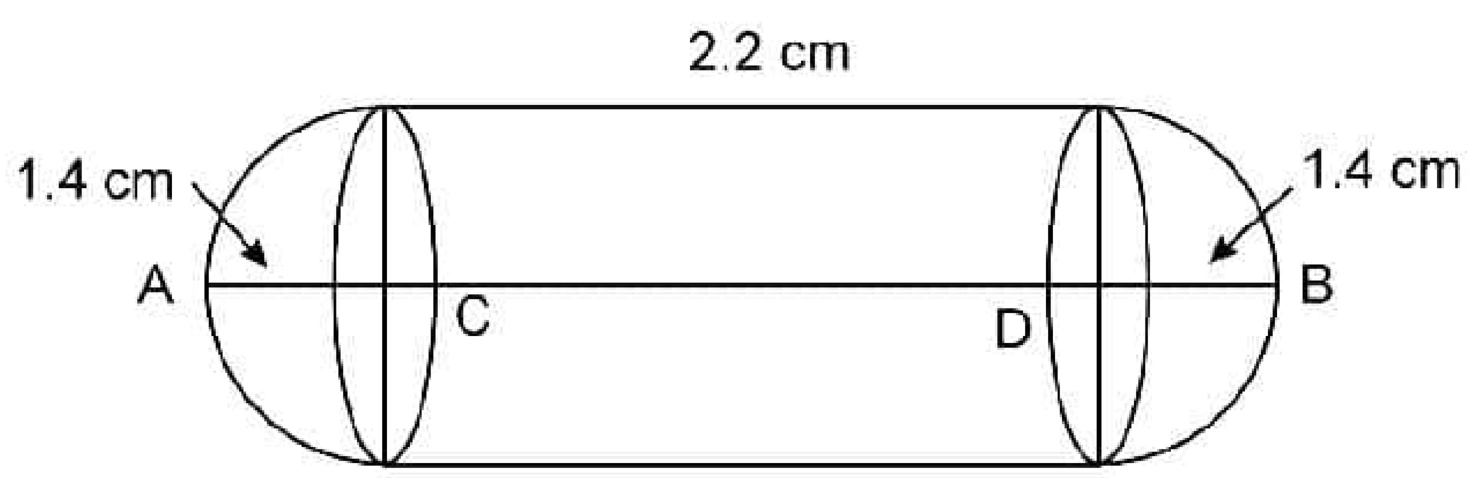
$\therefore \frac{BD}{DA} = \frac{BF}{FE}$ ....(ii)



From (i) and (ii),

$\frac{BE}{EC} = \frac{BF}{FE}$

34.



Diameter of a cylindrical gulab jamun = 2.8 cm

$\Rightarrow$ Radius = 1.4 cm

Total height of the gulab jamun = AC + CD + DB = 5 cm

$\therefore$ 1.4 + CD + 1.4 = 5

2.8 + CD = 5

CD = 2.2 cm

$\therefore$ Height of the cylindrical part h = 2.2 cm

$\therefore$ Volume of 1 gulab jamun

= Volume of the cylindrical part + Volume of two hemispherical parts

$$= \pi r^2 h + \frac{2}{3}\pi r^3 + \frac{2}{3}\pi r^3$$

$$= \pi r^2 h + \frac{4}{3}\pi r^3 = \pi r^2 \left(h + \frac{4}{3}r\right)$$

$$= \frac{22}{7} \times 1.4 \times 1.4 \times \left(2.2 + \frac{4}{3} \times 1.4\right)$$

$$= 22 \times 0.2 \times 1.4 \times (2.2 + 1.87)$$

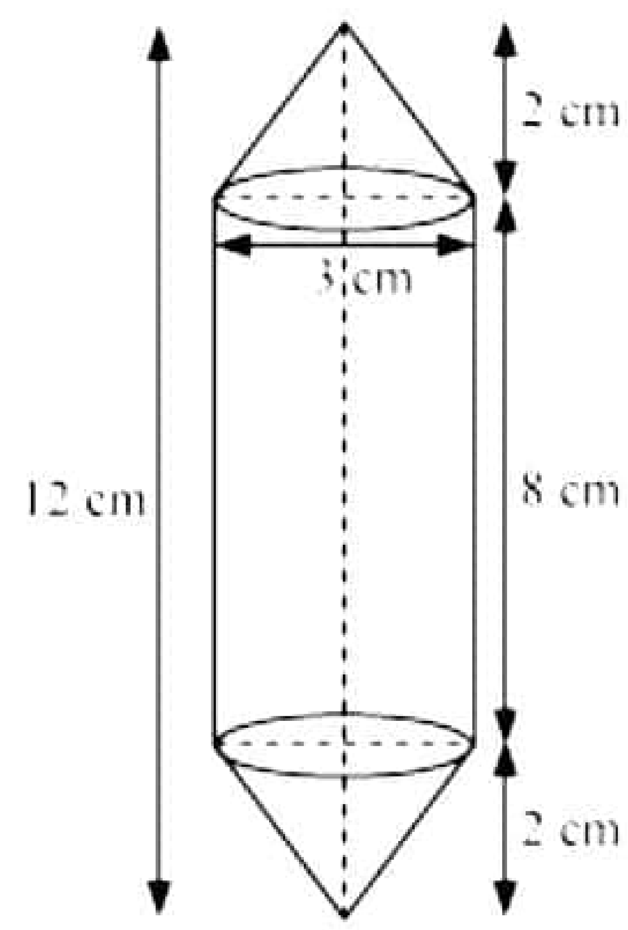
$$= 4.4 \times 1.4 \times 4.07 = 25.07 cm^3$$

$\therefore$ Volume of 45 gulab jamuns = 45 × 25.07 cm^3

Quantity of syrup = 30% of volume of gulab jamuns

$= 0.3 \times 45 \times 25.07 = 338.46\ cm^3$

OR



From the figure Height (h_1) of each conical part = 2 cm

Height (h_2) of cylindrical part = 12 – 2 × height of conical part

= 12 – 2 × 2 = 8 cm

Radius (r) of cylindrical part = radius of conical part = $\frac{3}{2}$ cm

Volume of air present in the model = volume of cylinder + 2 × volume of cone

$$= \pi r^2 h_2 + 2 \times \frac{1}{3}\pi r^2 h_1$$

$$= \pi\left(\frac{3}{2}\right)^2 (8) + 2 \times \frac{1}{3}\pi\left(\frac{3}{2}\right)^2 (2)$$

$$= \pi \times \frac{9}{4} \times 8 + \frac{2}{3}\pi \times \frac{9}{4} \times 2$$

$$= 18\pi + 3\pi$$

$$= 21\pi$$

$$= 21 \times \frac{22}{7}$$

$$= 66 \text{ cm}^3$$

35. Class size h of this data = 3

Now taking 75.5 as assumed mean (a) we may calculate di, u_i, f_iu_i as following.

Number of heart beats per minute	Number of women f_i	x_i	$d_i = x_i - 75.5$	$u_i = \frac{x_i - 75.5}{h}$	f_iu_i
65 – 68	2	66.5	−9	−3	−6
68 – 71	4	69.5	−6	−2	−8
71 – 74	3	72.5	−3	−1	−3
74 – 77	8	75.5	0	0	0
77 – 80	7	78.5	3	1	7
80 – 83	4	81.5	6	2	8
83 – 86	2	84.5	9	3	6
Total	30				4

Here, $\sum fi = 30$ and $\sum f_i u_i = 4$

$$\text{Mean } \bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$$

$$= 75.5 + \left(\frac{4}{30}\right) \times 3$$

$$= 75.5 + 0.4$$

$$= 75.9$$

So, the mean hear beats per minute for these women are 75.9 beats per minute.

Section E

36.

i. Here, $a = 1000$ and $d = 100$

This is an A.P.

Therefore, $a_{30} = a + (30 - 1)d$

$\Rightarrow a_{30} = 1000 + 29(100) = 1000 + 2900 =$ Rs. 3900

ii. $a_n = a + (n - 1)d$

$4900 = 1000 + 100n - 100$

$4000 = 100n$

$\therefore n = 40$

So it is 40^{th} month.

iii. Here, $a_{19} = 1000 + 1800 = 2800$ and $a_{28} = 1000 + 2700 = 3700$

$a_{19} : a_{28} = 2800 : 3700 = 28 : 37$

OR

Here, $a = 1000$ and $d = 100$

$$S_{30} = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{30}{2}\left[2(1000) + (30-1)100\right] = \text{Rs.}73500$$

37.

i.

$A(1,1)$

$C(4,5)$

$$d(AC) = \sqrt{(4-1)^2 + (5-1)^2} = 5 \text{ km}$$

OR

$B(4,1)$

$D(7,5)$

$d(BD) = \sqrt{(7-4)^2 + (5-1)^2} = 5 \text{ km}$

ii.

$B(4,1)$

$A(1,1)$

Using mid-point formula, we have

$X = \left(\frac{4+1}{2}, \frac{1+1}{2}\right)$

$X = (2.5, 1)$

iii.

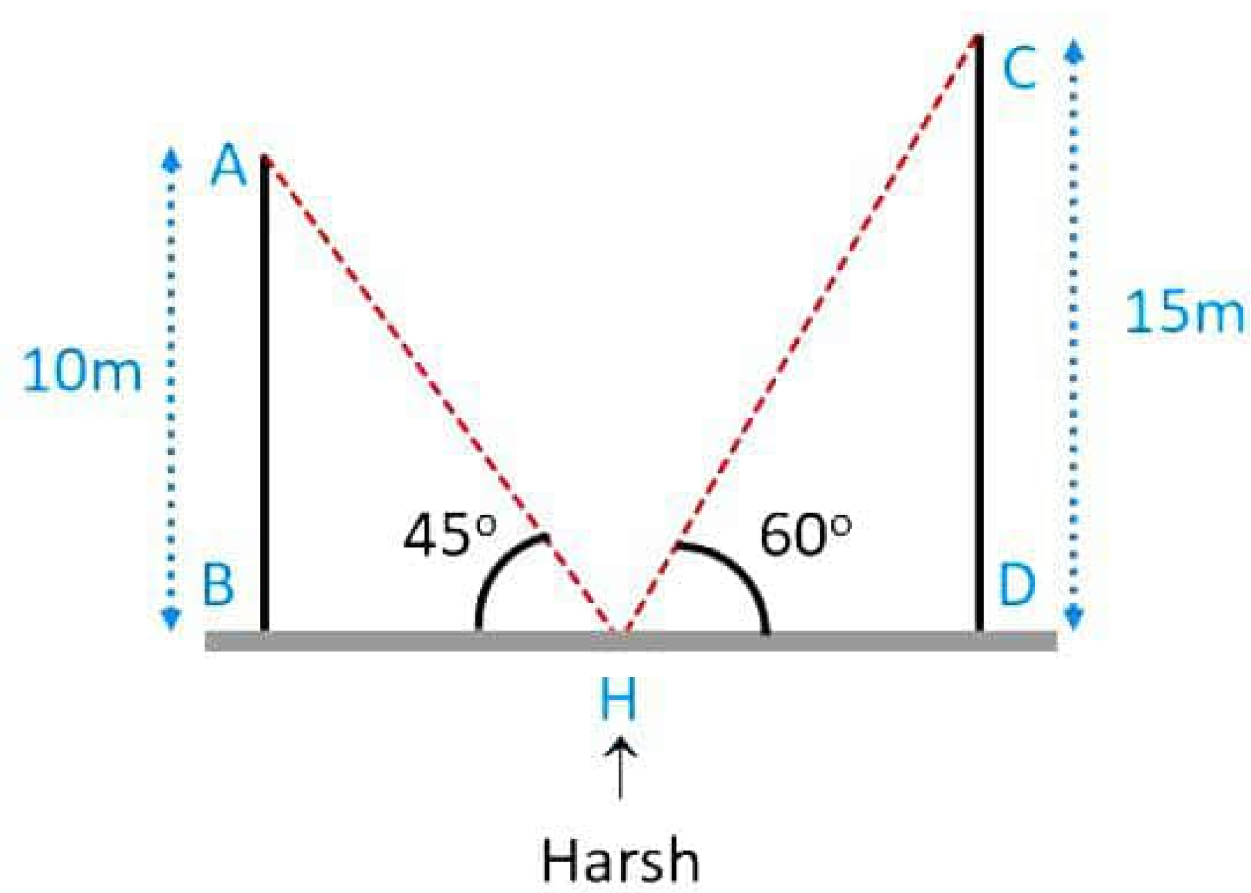
$C(4,5)$

$B(4,1)$

$d(BC) = \sqrt{(4-4)^2 + (5-1)^2} = 4 \text{ km}$

38.

i.



ii.

In ΔCDH,

$\tan 60° = \frac{15}{DH}$

$\therefore \sqrt{3} = \frac{15}{DH}$

$\therefore DH = 5\sqrt{3} \text{ m}$

iii.

In ΔABH

$$\sin 45^\circ = \frac{10}{AH}$$

$$\therefore \frac{1}{\sqrt{2}} = \frac{10}{AH}$$

$$\therefore AH = 10\sqrt{2} \text{ m}$$

OR

In ΔCDH,

$$\sin 60^\circ = \frac{15}{HC}$$

$$\therefore \frac{\sqrt{3}}{2} = \frac{15}{HC}$$

$$\therefore HC = 10\sqrt{3} \text{ m}$$