Class X Session 2024-25
Subject - Mathematics (Standard)
Sample Question Paper - 18

Time: 3 Hours **Total Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A - E.
2. Section A has 18 multiple choice questions and 2 Assertion-Reason based questions carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case study based questions carrying 4 marks each with subparts of 1, 1, and 2 marks each, respectively.
7. All Questions are compulsory. However, an internal choice in 2 Question of Section B, 2 Questions of Section C and 2 Questions of Section D has been provided. An internal choice has been provided in all the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take π = 22/7 wherever required if not stated.

Section A

Section A consists of 20 questions of 1 mark each.

Choose the correct answers to the questions from the given options. [20]

1. The HCF of 405 and 2520 is ______

A. 40
B. 45
C. 50
D. 55

2. Find the discriminant of an equation $3x^2 - 2x + 8 = 0$.

A. –92
B. 92
C. –96
D. 96

3. In Arithmetic progression, we have a common ______ between the terms.

A. difference
B. ratio
C. sum
D. term

4. Which term of the A.P. 3, 8, 13, 18,... is 78?

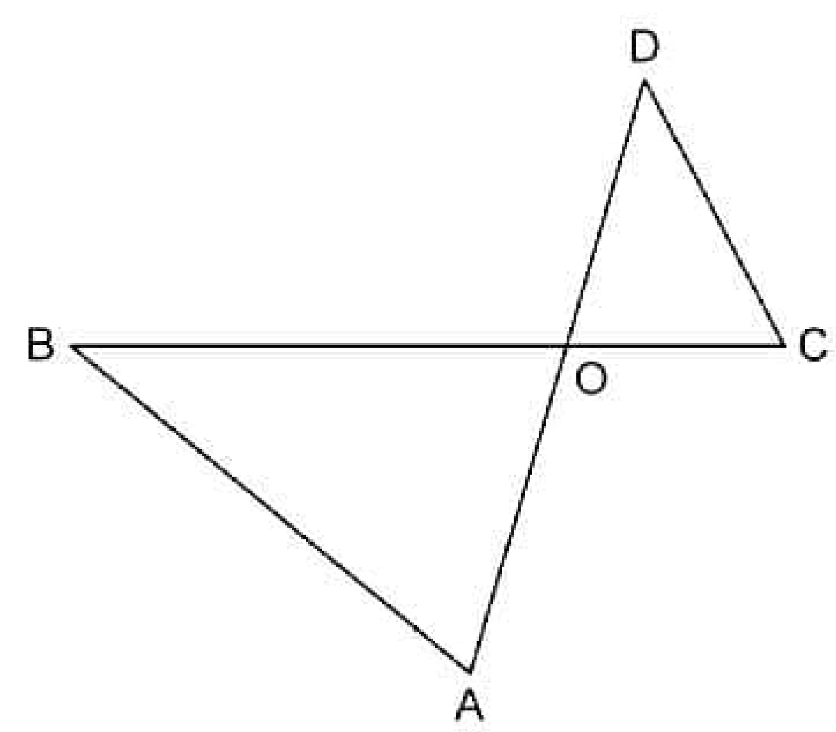
A. 14
B. 16
C. 18
D. 20

5. Points that satisfy a polynomial .....

i. are called its zeros
ii. lie on the graph of polynomial
iii. are called its roots

A. (i) only
B. (i) and (ii)
C. (ii) and (iii)
D. (i), (ii) and (iii)

6. In the given figure, $\Delta OAB \sim \Delta OCD$. If AB = 8 cm, BO = 6.4 cm, OC = 3.5 cm and CD = 5 cm, find DO.



A. 5.6 cm
B. 4 cm
C. 6.4 cm
D. 3.5 cm

7. If cosec θ = 3x and cot $\theta = \frac{3}{x}$, then find the value of $3\left(x^2 - \frac{1}{x^2}\right)$.

A. $\frac{1}{5}$

B. $\frac{1}{4}$

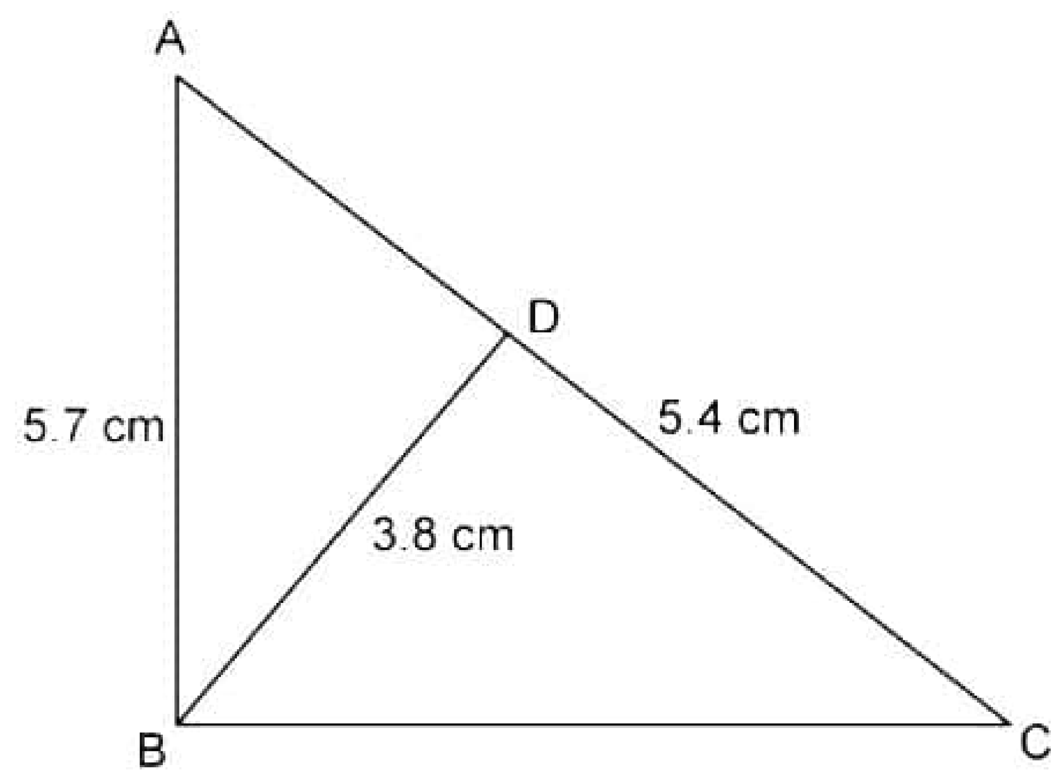
C. $\frac{1}{3}$

D. $\frac{1}{2}$

8. If $2\sin^2\theta - \cos^2\theta = 2$, then find the value of θ.

A. 100°
B. 70°
C. 90°
D. 80°

9. In the given figure, $\angle ABC = 90°$ and $BD \perp AC$. Find BC.

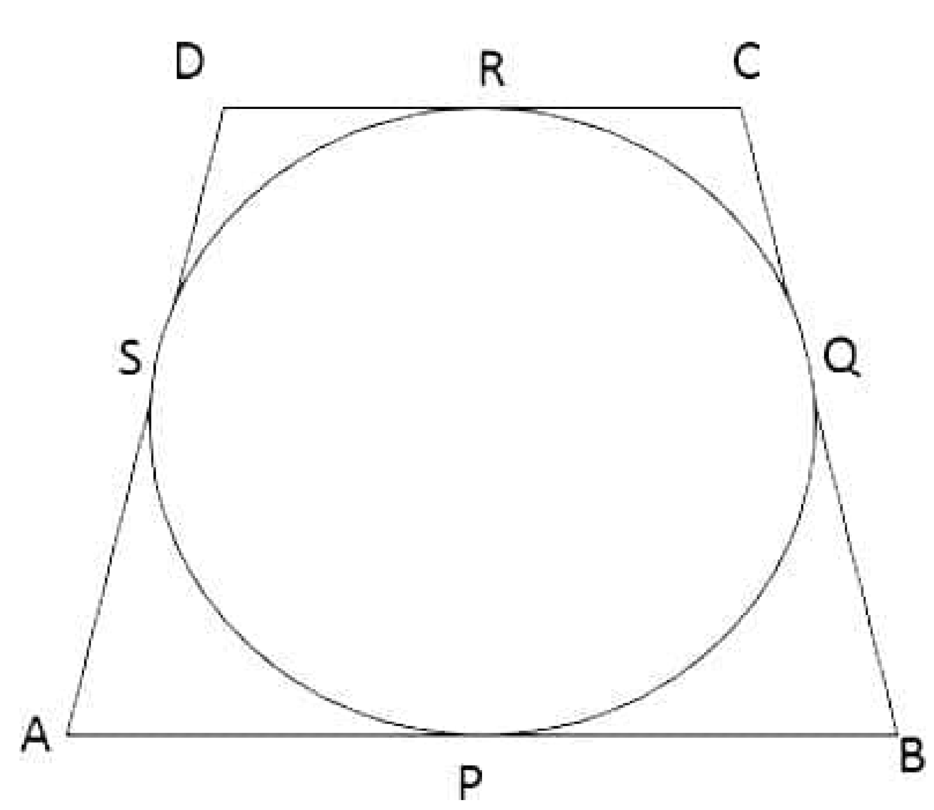


A. 2.1 cm
B. 5.1 cm
C. 7.1 cm
D. 8.1 cm

10. The perimeters of two similar triangles ABC and PQR are 32 cm and 24 cm, respectively. If PQ = 12 cm, then find AB.

A. 12 cm
B. 14 cm
C. 15 cm
D. 16 cm

11. In the given figure, a circle touches all the four sides of a quadrilateral ABCD whose three sides are AB = 6 cm, BC = 7 cm and CD = 4 cm. Find AD.



A. 3 cm
B. 4 cm
C. 5 cm
D. 2 cm

12. Find the area of a sector with radius 7 cm and central angle 90°.

A. 38 cm^2
B. 39 cm^2
C. 38.5 cm^2
D. 37.5 cm^2

13. The volume of a following object with diameter 42 cm is given by

A. 6174π cm^3
B. 6417π cm^3
C. 4617π cm^3
D. 4176π cm^3

14. Compute the modal class from the following series:

Size	Frequency
45–55	7
55–65	12
65–75	17
75–85	30
85–95	32
95–105	6
105–115	10

A. 75-85
B. 105-115
C. 85-95
D. 95-105

15. The length of a chain used as the boundary of a semi-circular park is 90 m. Find the area of the park.

A. 481.25 m^2
B. 481.15 m^2
C. 481.05 m^2
D. 481.35 m^2

16. From the set of numbers −2, −1, 0, 1, 2, the probability that the square of a chosen number is 1 will be

A. $\frac{2}{5}$

B. $\frac{1}{5}$

C. $\frac{1}{2}$

D. $\frac{2}{3}$

17. Cards bearing numbers 1, 3, 5, ..., 35 are kept in a bag. A card is drawn at random from the bag. Find the probability of getting a card bearing a prime number less than 15.

A. $\frac{3}{18}$

B. $\frac{4}{18}$

C. $\frac{5}{18}$

D. $\frac{7}{18}$

18. Find the value of θ if sec θ = cosec θ.

A. 25°

B. 15°

C. 35°

D. 45°

DIRECTION: In the question number 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option

19. Statement A (Assertion): $\sqrt{7}$ is a rational number

Statement R (Reason): Rational numbers can be represented as p/q, where p and q are co-prime and q is not equal to zero.

A. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

B. Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

C. Assertion (A) is true but reason (R) is false.

D. Assertion (A) is false but reason (R) is true.

20. Statement A (Assertion): $\Delta LMN \sim \Delta XYZ$ by AA similarity criterion where $\angle L = \angle X$ and $\angle M = \angle Y$.

Statement R (Reason): Two triangles are similar by AA similarity criterion if only one of the corresponding angles are equal.

A. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

B. Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

C. Assertion (A) is true but reason (R) is false.

D. Assertion (A) is false but reason (R) is true.

Section B

21. The number of fruits of each kind A, B and C are 50, 90 and 110 respectively. In each basket, the equal number of fruits of same kind are to be kept. Find the minimum number of baskets required to accommodate all fruits. [2]

22. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that $\Delta ABE \sim \Delta CFB$. [2]

23. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Find the length of PQ. [2]

24. ΔABC is right angled at B. If $\tan A = \frac{1}{\sqrt{3}}$, find the value of sin A cos C + cos A sin C. [2]

OR

In ΔPQR, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

25. Find the area of a quadrant of a circle whose circumference is 22 cm. [2]

OR

The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Section C

Section C consists of 6 questions of 3 marks each.

26. Prove that $\frac{1}{\sqrt{3}}$ is irrational. [3]

27. If one of the zero of the quadratic polynomial $2x^2 - 3x + p$ is 3, then find its other zero. Also find the value of p. [3]

28. Find two numbers whose sum is 27 and product is 182. [3]

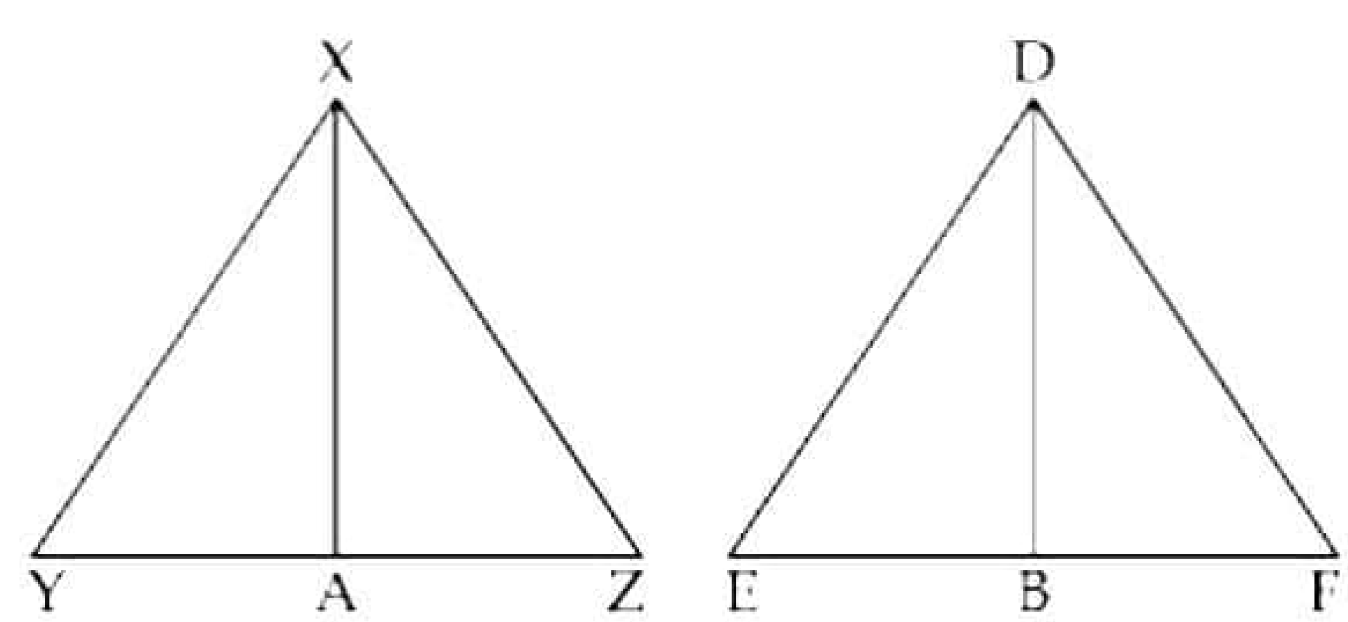
OR

A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was Rs. 750. Find out the number of toys produced on that day.

29. Find the four angles of a cyclic quadrilateral ABCD in which $\angle A = (x + y + 10)°$, $\angle B = (y + 20)°$, $\angle C = (x + y - 30)°$ and $\angle D = (x + y)°$. [3]

OR

In the figure, sides XY and YZ and median XA of a triangle XYZ are proportional to sides DE, EF and median DB of ΔDEF. Show that $\Delta XYZ \sim \Delta DEF$.



30. If $\tan\theta = \frac{1}{\sqrt{7}}$, show that $\frac{(\operatorname{cosec}^2\theta - \sec^2\theta)}{(\operatorname{cosec}^2\theta + \sec^2\theta)} = \frac{3}{4}$. [3]

31. A box contains 20 balls bearing numbers 1, 2, 3, ..., 20, respectively. A ball is taken out at random from the box. What is the probability that the number on the ball is [3]

i. an odd number?
ii. divisible by 2 or 3?
iii. a prime number?

Section D

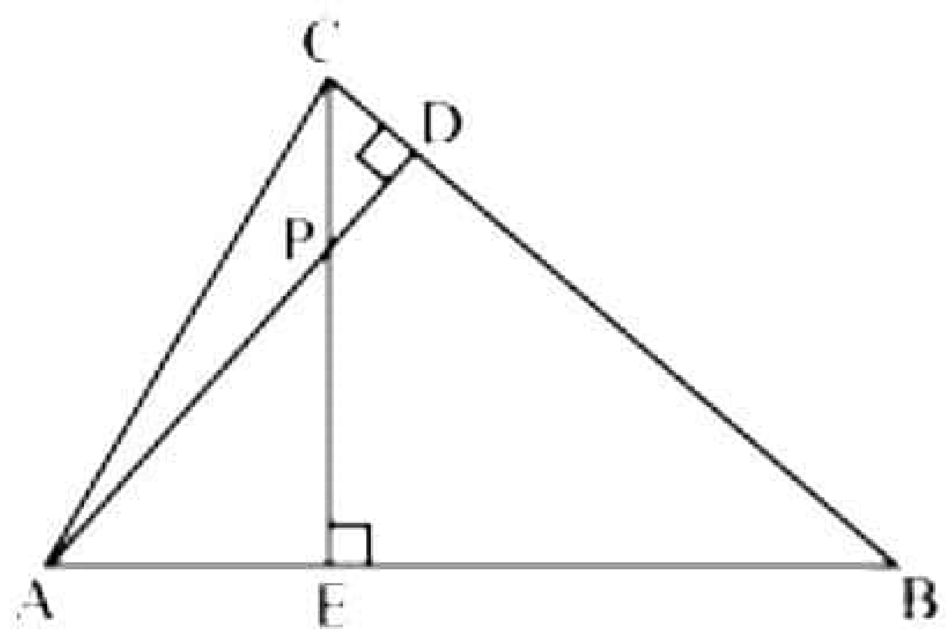
Section D consists of 4 questions of 5 marks each.

32. The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age. [5]

OR

A rectangular field is 16 m long and 10 m wide. There is a path of uniform width all around it with an area of 120 m^2. Find the width of the path. [5]

33. In the figure, altitudes AD and CE of ΔABC intersect each other at the point P. Show that [5]



i. $\Delta AEP \sim \Delta CDP$
ii. $\Delta ABD \sim \Delta CBE$
iii. $\Delta AEP \sim \Delta ADB$
iv. $\Delta PDC \sim \Delta BEC$

34. A tent is in the shape of a right circular cylinder up to a height of 3 m and conical above it. The total height of the tent is 13.5 m, and the radius of its base is 14 m. Find the cost of cloth required to make the tent at the rate of Rs. 80 per square metre. Take $\pi = 22/7$. [5]

OR

A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass. (Use $\pi = 3.14$)

35. The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is Rs. 18. Find the missing frequency f. [5]

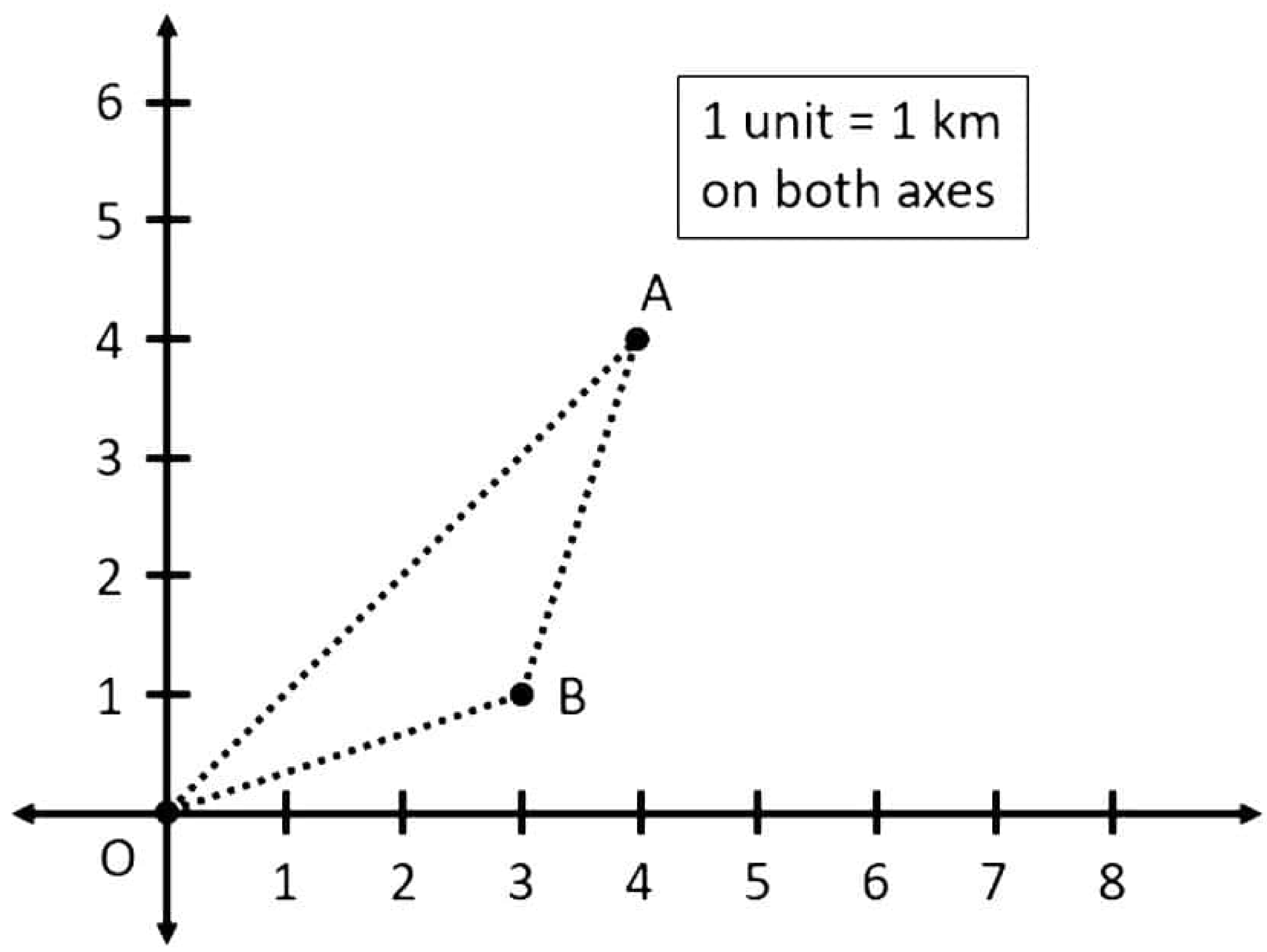
Daily pocket allowance (in Rs.)	11 – 13	13 – 15	15 –17	17 – 19	19 – 21	21 – 23	23 – 25
Number of children	7	6	9	13	f	5	4

Section E

Case study based questions are compulsory.

36. Bus number 735 travels from source O to A, and Bus number 736 travels from Source O to B, then reaches A. The routes taken by both the buses are shown below. Using the details given, answer the following questions.

37.



i. Find the distance covered by Bus No. 735. [1]
ii. Find the distance between the locations B and A. [1]
iii. Find the distance covered by Bus No. 736. [2]

OR

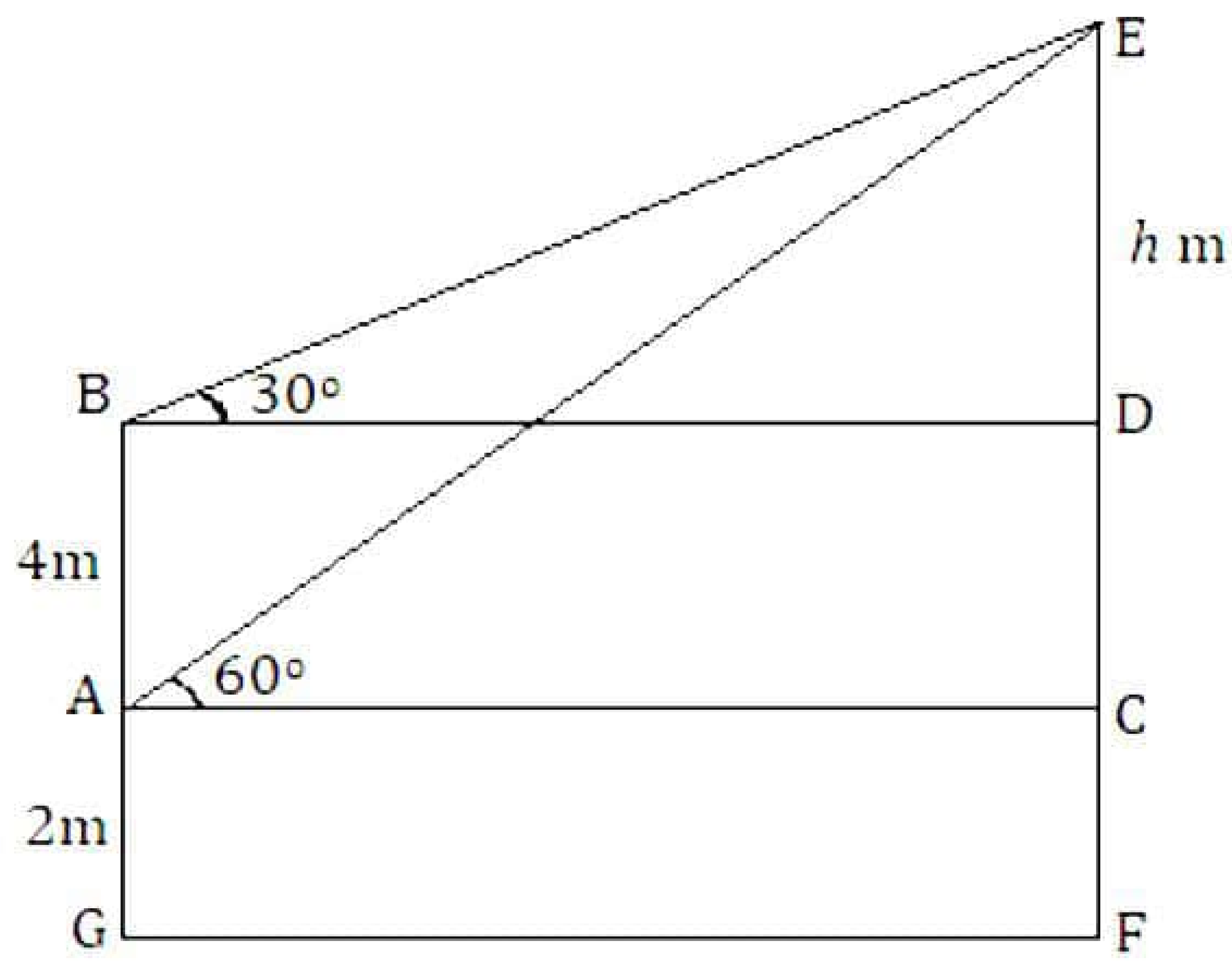
If Bus No. 735 starts from O at 12 pm and reaches location A at 12.15 pm, then find the speed of Bus No. 735. [2]

38. Rakesh is much worried about his upcoming assessment on chapter Arithmetic Progression. He is vigorously practicing for the exam but unable to solve some questions. One of these questions is as shown below.
The 3^{rd} and 9^{th} terms of an A.P. are 4 and −8 respectively.
i. What is the common difference? [1]
ii. What is the first term? [1]
iii. Which term of A.P. is −160? [2]

OR

Which of the following is not a term of the given A.P.? [2]
−123, −100, 0, −200

39. Reema's house has two windows. First is at the height of 2 m above the ground and the second is at the height of 4 m above the first window. Reema and her brother Rishabh are watching outside from the two windows at points A and B respectively. Now, the angles of elevation of an airplane from these windows are observed to be 60° and 30° as shown below.



Based on the above information, answer the following questions.

i. Who is closed to the airplane? [1]

ii. Find an expression for the length of BD in terms of h. [2]

OR

Find the value of h. [2]

iii. If the airplane is moving towards the building, then both the angles of elevation will increase or decrease? [1]

Solution

Section A

1. Correct option: B

Explanation:

On dividing 2520 by 405, quotient = 6, remainder = 90

$\therefore$ 2520 = (405 × 6) + 90

Dividing 405 by 90, quotient = 4, remainder = 45

$\therefore$ 405 = 90 × 4 + 45

Dividing 90 by 45, quotient = 2, remainder = 0

$\therefore$ 90 = 45 × 2

$\therefore$ HCF of 405 and 2520 is 45.

2. Correct option: A

Explanation:

The given equation is $3x^2 - 2x + 8 = 0$.

Comparing it with $ax^2 + bx + c = 0$, we get

a = 3, b = −2, c = 8

Then, discriminant = $b^2 - 4ac = (-2)^2 - 4(3)(8) = 4 - 96 = -92$

3. Correct option: A

Explanation:

In Arithmetic progression we have a common difference between the terms.

4. Correct option: B

Explanation:

For a given A.P., a = 3 and $d = a_2 - a_1 = 8 - 3 = 5$

$a_n = a + (n - 1)d$

$\Rightarrow 78 = 3 + (n - 1)5$

$\Rightarrow 75 = (n - 1)5$

$\Rightarrow n - 1 = 15$

$\Rightarrow n = 16$

Hence, the 16^{th} term of the given A.P. is 78.

5. Correct option: D

Explanation:

Points that satisfy a polynomial are called its roots or zeros and they lie on its graph.

6. Correct option: B

Explanation:

$\Delta OAB \sim \Delta OCD$,

$$\Rightarrow \frac{OA}{OC} = \frac{AB}{CD} = \frac{BO}{DO}$$

$$\Rightarrow \frac{OA}{3.5} = \frac{8}{5} = \frac{6.4}{DO}$$

$$\Rightarrow \frac{6.4}{DO} = \frac{8}{5}$$

$$DO = \frac{6.4 \times 5}{8} = 4 \text{ cm}$$

7. Correct option: C

Explanation:

$$\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$$

$$\Rightarrow (3x)^2 - \left(\frac{3}{x}\right)^2 = 1$$

$$\Rightarrow 9x^2 - \frac{9}{x^2} = 1$$

$$\Rightarrow 9\left(x^2 - \frac{1}{x^2}\right) = 1$$

$$\Rightarrow 3 \times 3\left(x^2 - \frac{1}{x^2}\right) = 1$$

$$\Rightarrow 3\left(x^2 - \frac{1}{x^2}\right) = \frac{1}{3}$$

8. Correct option: C

Explanation:

$$2\sin^2\theta - \cos^2\theta = 2$$

$$\Rightarrow 2(1 - \cos^2 \theta) - \cos^2 \theta = 2$$

$$\Rightarrow 2 - 2\cos^2 \theta - \cos^2 \theta = 2$$

$$\Rightarrow 2 - 3\cos^2 \theta = 2$$

$$\Rightarrow 3\cos^2 \theta = 0$$

$$\Rightarrow \cos^2 \theta = 0$$

$$\Rightarrow \cos^2 \theta = \cos^2 90^\circ$$

$$\Rightarrow \theta = 90^\circ$$

9. Correct option: D
Explanation:
In ΔCBA and ΔCDB,
$\angle CBA = \angle CDB = 90^\circ$
And $\angle C = \angle C$ (Common)
$\Delta CBA \sim \Delta CDB$ (by AA similarity)

$$\Rightarrow \frac{CB}{CD} = \frac{BA}{DB}$$

$$\Rightarrow \frac{BC}{5.4} = \frac{5.7}{3.8}$$

$$\Rightarrow BC = \frac{5.7 \times 5.4}{3.8} = 8.1 \text{ cm}$$

10. Correct option: D
Explanation:
It is given that ΔABC and ΔPQR are similar triangles, so the corresponding sides of both triangles are proportional.

$$\therefore \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta PQR} = \frac{AB}{PQ}$$

$$\Rightarrow \frac{32}{24} = \frac{AB}{12}$$

$$\Rightarrow AB = \frac{32 \times 12}{24} = 16 \text{ cm}$$

11. Correct option: A
Explanation:
The length of tangents drawn from an external point to a circle are equal.
Then,
AP = AS ....(1)
BP = BQ ....(2)
CR = CQ ....(3)
DR = DS ....(4)
Adding (1), (2), (3) and (4), we get
$\therefore$ AP + BP + CR + DR = AS + BQ + CQ + DS
$\Rightarrow$ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)
$\Rightarrow$ AB + CD = AD + BC
$\Rightarrow$ AD = (AB + CD) − BC = {(6 + 4) − 7} cm = 3 cm

12. Correct Option: C
Explanation:

Area of sector of angle $\theta = \dfrac{\theta}{360^\circ} \times \pi r^2$

$$= \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times (7)^2$$

$= 38.5 \text{ cm}^2$

13. Correct option: A

Explanation:

Volume of a given hemispherical object $= \frac{2}{3}\pi r^3 = \frac{2}{3}\pi(21)^3 = 6174\pi\ cm^3$

14. Correct option: C

Explanation:

As the class 85–95 has the maximum frequency, it is the modal class.

15. Correct Option: A

Explanation:

Let the radius of the park be r metres.

Thus, $\pi r + 2r = 90$

$\Rightarrow \frac{22r}{7} + 2r = 90$

$\Rightarrow \frac{36r}{7} = 90$

$\Rightarrow r = \frac{90 \times 7}{36} = 17.5\ m$

Area of semicirular park $= \frac{1}{2}\pi r^2 = \left(\frac{1}{2} \times \frac{22}{7} \times 17.5 \times 17.5\right) m^2 = 481.25\ m^2$

16. Correct option: A

Explanation:

Total numbers = 5

Numbers having square as 1 are −1 and 1.

Therefore, required probability $= \frac{2}{5}$

17. Correct option: C

Explanation:

Total cards kept in a bag = 18

Prime numbers less than 15 are 3, 5, 7, 11, 13.

Therefore, required probability $= \frac{5}{18}$

18. Correct option: D

Explanation:

$\sec\theta = \operatorname{cosec}\theta$

Now, $\sec 45^\circ = \sqrt{2} = \operatorname{cosec} 45^\circ$

$\Rightarrow \theta = 45^\circ$

19. Correct option: D

Explanation:

$\sqrt{7}$ is an irrational number as it can be expressed as a nonterminating and nonrepeating decimal.

Hence, assertion is false.

The statement given in reason is correct and hence, reason is true.

20. Correct option: C

Explanation:

The statement given in assertion is correct and hence, assertion is true.

For two triangles to be similar by AA similarity criterion two angles of one triangle need to be equal to corresponding two angles of another triangle.

Hence, reason is false.

Section B

21. To find minimum number of baskets, we need to first find the maximum and equal number of fruits of same kind to be kept in each basket.

That is, HCF of 50, 90 and 110.

$50 = 2 \times 5 \times 5$

$90 = 2 \times 3 \times 3 \times 5$

$110 = 2 \times 5 \times 11$

Therefore, HCF(50, 90, 110) $= 2 \times 5 = 10$

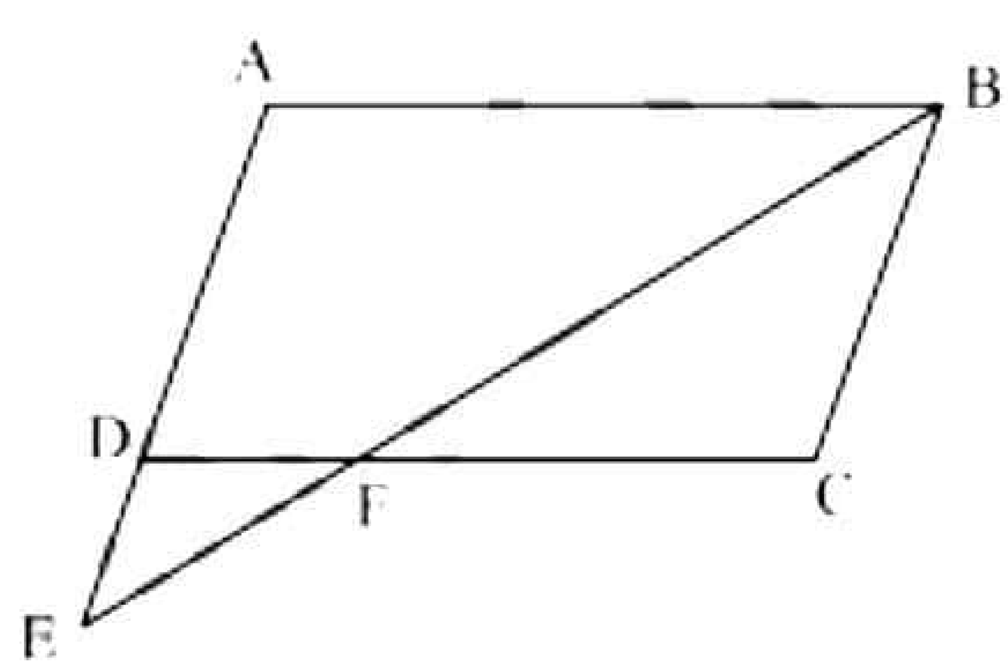
So, minimum number of baskets required

$= \dfrac{50 + 90 + 110}{10}$

$= \dfrac{250}{10}$

$= 25$

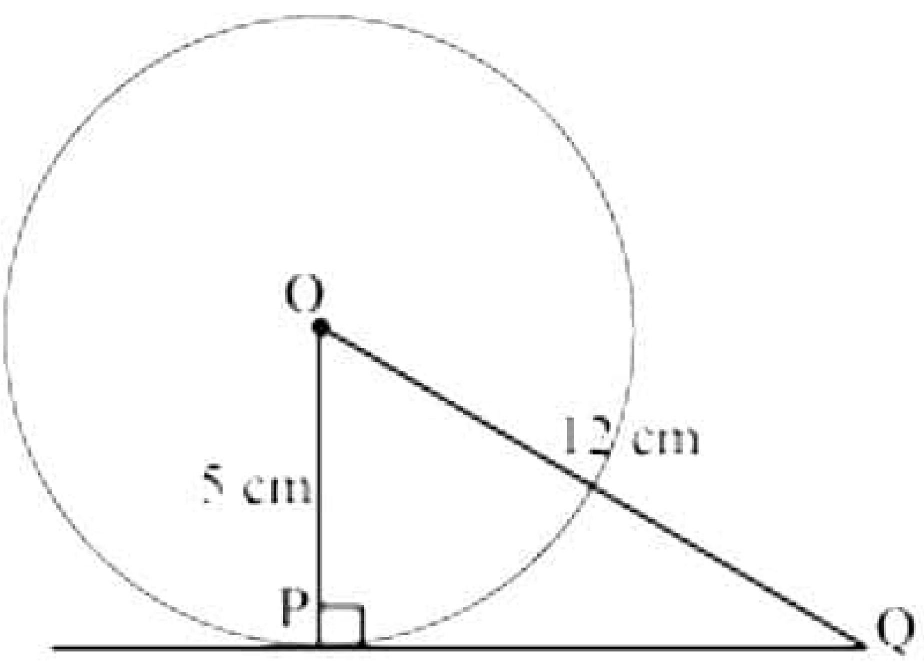
22.



In ΔABE and ΔCFB,

∠A = ∠C	(opposite angles of a parallelogram)
∠AEB = ∠CBF	(Alternate interior angles AE || BC)
∠ABE = ∠CFB	(remaining angle)
Therefore ΔABE ~ ΔCFB	(by AAA rule)

23. Radius is perpendicular to the tangent at the point of contact. So, OP $\perp$ PQ.



Now, applying Pythagoras theorem in ΔOPQ,

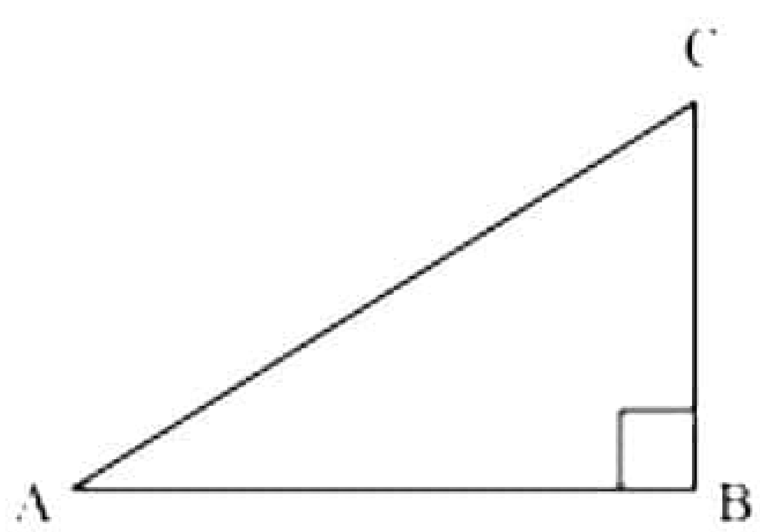
$OP^2 + PQ^2 = OQ^2$

$5^2 + PQ^2 = 12^2$

$PQ^2 = 144 - 25$

$PQ = \sqrt{119}$ cm

24.



$\tan A = \frac{1}{\sqrt{3}}$

$\frac{BC}{AB} = \frac{1}{\sqrt{3}}$

If BC is K, AB will be $\sqrt{3}$ K, where K is a positive integer.

In ΔABC,

$AC^2 = AB^2 + BC^2 = \left(\sqrt{3}\ K\right)^2 + (K)^2 = 3K^2 + K^2 = 4K^2$

AC = 2K

$\sin A = \frac{\text{Side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{K}{2K} = \frac{1}{2}$

$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}K}{2K} = \frac{\sqrt{3}}{2}$

$\sin C = \frac{\text{Side opposite to } \angle C}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}K}{2K} = \frac{\sqrt{3}}{2}$

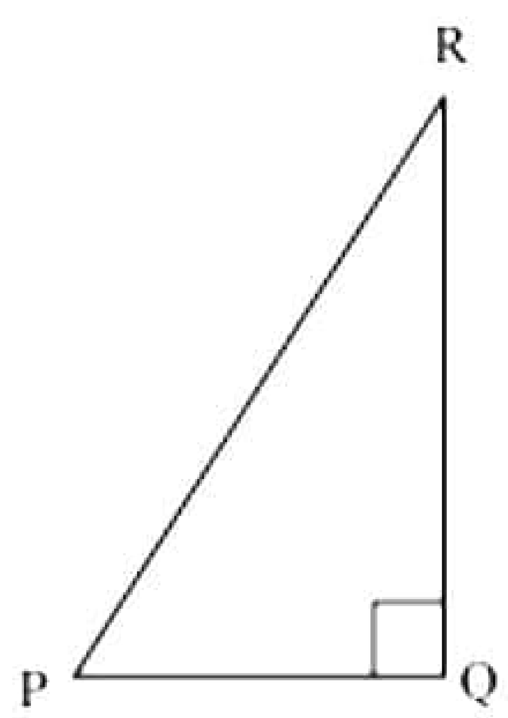
$\cos C = \frac{\text{Side adjacent to } \angle C}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{K}{2K} = \frac{1}{2}$

sin A cos C + cos A sinC

$$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$
$$= \frac{1}{4} + \frac{3}{4}$$
$$= \frac{4}{4}$$
$$= 1$$

OR

Given that PR + QR = 25 cm and PQ = 5 cm
Let PR = x
So, QR = 25 − x



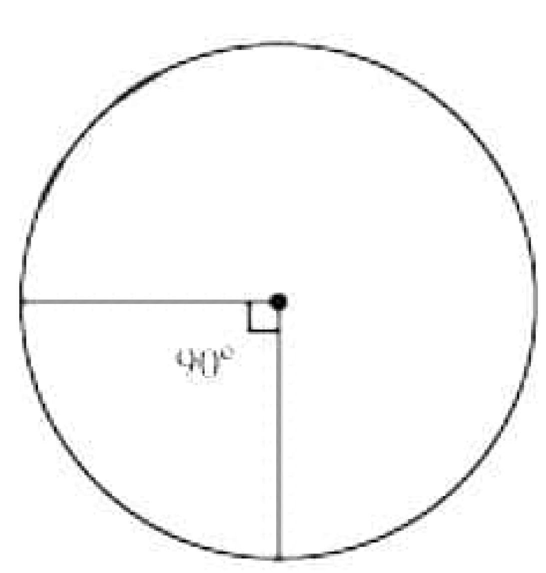
Now applying Pythagoras theorem in ΔPQR,
$PR^2 = PQ^2 + QR^2$
$x^2 = (5)^2 + (25 - x)^2$
$x^2 = 25 + 625 + x^2 - 50x$
$50x = 650$
$x = 13$
So, PR = 13 cm
QR = 25 − 13 = 12 cm

$$\sin P = \frac{\text{Side opposite to } \angle P}{\text{hypotenuse}} = \frac{QR}{PR} = \frac{12}{13}$$

$$\cos P = \frac{\text{side adjacent to } \angle P}{\text{hypotenuse}} = \frac{PQ}{PR} = \frac{5}{13}$$

$$\tan P = \frac{\text{Side opposite to } \angle P}{\text{side adjacent to } \angle P} = \frac{QR}{PQ} = \frac{12}{5}$$

25.



Let the radius of a circle be r.
Circumference = 22 cm
$2\pi r = 22$

$r = \frac{22}{2\pi} = \frac{11}{\pi}$

Quadrant of circle will subtend 90° angle at centre of circle.

Then, area of such quadrant of a circle $= \frac{90°}{360°} \times \pi \times r^2$

$$= \frac{1}{4} \times \pi \times \left(\frac{11}{\pi}\right)^2$$

$$= \frac{121}{4\pi} = \frac{121 \times 7}{4 \times 22}$$

$$= \frac{77}{8} \text{ cm}^2$$

OR

We know the in 1 hour (i.e. 60 minutes), minute hand rotates 360°.

So in 5 minutes, minute hand will rotate $= \frac{360°}{60} \times 5 = 30°$

So area swept by minute hand in 5 minutes will be the area of a sector of 30° in a circle of 14 cm radius.

Area of sector of angle $\theta = \frac{\theta}{360°} \times \pi r^2$

Area of sector of 30° $= \frac{30°}{360°} \times \frac{22}{7} \times 14 \times 14$

$$= \frac{22}{12} \times 2 \times 14$$

$$= \frac{154}{3} \text{ cm}^2$$

So area swept by minute hand in 5 minutes is $\frac{154}{3}$ cm^2.

Section C

26. $\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{1}{3}\sqrt{3}$ ....(1)

If possible, let $\frac{1}{\sqrt{3}}$ be rational.

Then, from (1), it follows that $\frac{1}{3}\sqrt{3}$ is rational.

Let $\frac{1}{3}\sqrt{3} = \frac{a}{b}$ where a and b are non-zero integers having no common factor other than 1.

Then, $\sqrt{3} = \frac{3a}{b}$ ....(2)

But 3a and b are non-zero integers.

$\therefore \frac{3a}{b}$ is rational.

Thus, from (2), it follows that $\sqrt{3}$ is rational.

This contradicts the fact that $\sqrt{3}$ is irrational.

The contradiction arises by assuming that $\frac{1}{\sqrt{3}}$ is rational.

Hence, $\frac{1}{\sqrt{3}}$ is irrational.

27. Let $p(x) = 2x^2 - 3x + p$

If $p(a) = 0$, then it is said that 'a' is a zero of $p(x)$.

Given, 3 is a zero of $p(x)$.

$\therefore\ p(3) = 0$

$2(3)^2 - 3(3) + p = 0$

$18 - 9 + p = 0$

$p = -9$

$\therefore\ p(x) = 2x^2 - 3x - 9$

$= 2x^2 - 6x + 3x - 9$

$= 2x(x - 3) + 3(x - 3)$

$= (x - 3)(2x + 3)$

$= x = 3 \text{ and } x = \frac{-3}{2}$

Thus, the other zero of $p(x)$ is $\frac{-3}{2}$.

28. Let the first number be x.

Then, the second number is $27 - x$.

$x(27 - x) = 182$

$\Rightarrow x^2 - 27x + 182 = 0$

$\Rightarrow x^2 - 13x - 14x + 182 = 0$

$\Rightarrow x(x - 13) - 14(x - 13) = 0$

$\Rightarrow (x - 13)(x - 14) = 0$

Either $x = 13 = 0$ or $x - 14 = 0$

i.e., $x = 13$ or $x = 14$

If first number = 13, then other number = 27 − 13 = 14

If first number = 14, then other number = 27 − 14 = 13

Therefore, the numbers are 13 and 14.

OR

Let the number of toys produced in a day be x.

$\therefore$ Cost of production of each toy = Rs. $(55 - x)$

It is given that, total production of the toys = Rs. 750

$\therefore (55 - x)x = 750$

$\Rightarrow x^2 - 55x + 750 = 0$

$\Rightarrow x^2 - 25x - 30x + 750 = 0$

$\Rightarrow x(x - 25) - 30(x - 25) = 0$

$\Rightarrow (x - 25)(x - 30) = 0$

Either $x - 25 = 0$ or $x - 30 = 0$

i.e., $x = 25$ or $x = 30$

Hence, the number of toys will be either 25 or 30.

29. In a cyclic quadrilateral ABCD,

$\angle A = (x + y + 10)^\circ$, $\angle B = (y + 20)^\circ$, $\angle C = (x + y - 30)^\circ$, $\angle D = (x + y)^\circ$

Then, $\angle A + \angle C = 180^\circ$ and $\angle B + \angle D = 180^\circ$

Now, $\angle A + \angle C = (x + y + 10)^\circ + (x + y - 30)^\circ = 180^\circ$

$\Rightarrow 2x + 2y - 20^\circ = 180^\circ$

$\Rightarrow x + y = 100$ ....(1)

And, $\angle B + \angle D = (y + 20)^\circ + (x + y)^\circ = 180^\circ$

$\Rightarrow x + 2y + 20^\circ = 180^\circ$

$\Rightarrow x + 2y = 160^\circ$ ....(2)

Subtracting (1) from (2), we get $y = 160 - 100 = 60$

and $x = 100 - y = 100 - 60 = 40$

$\angle A = (x + y + 10)^\circ = (100 + 10)^\circ = 110^\circ$

$\angle B = (y + 20)^\circ = (60 + 20)^\circ = 80^\circ$

$\angle C = (x + y - 30)^\circ = (100 - 30)^\circ = 70^\circ$

$\angle D = (x + y)^\circ = 100^\circ$

OR

Given: In ΔXYZ and ΔDEF

$\frac{XY}{DE} = \frac{YZ}{EF} = \frac{XA}{DB}$ ....(1)

To prove: $\Delta XYZ \sim \Delta DEF$

Proof: Since XA and DB are medians,

$2YA = YZ$ and $2EB = EF$ ....(2)

From (1) and (2), we have

$\frac{XY}{DE} = \frac{2YA}{2EB} = \frac{XA}{DB}$

$\Rightarrow \Delta XYA \sim \Delta DEB$ ...(BY SSS rule)

$\Rightarrow \angle Y = \angle E$ ....(3)

Now, in ΔXYZ and ΔDEF,

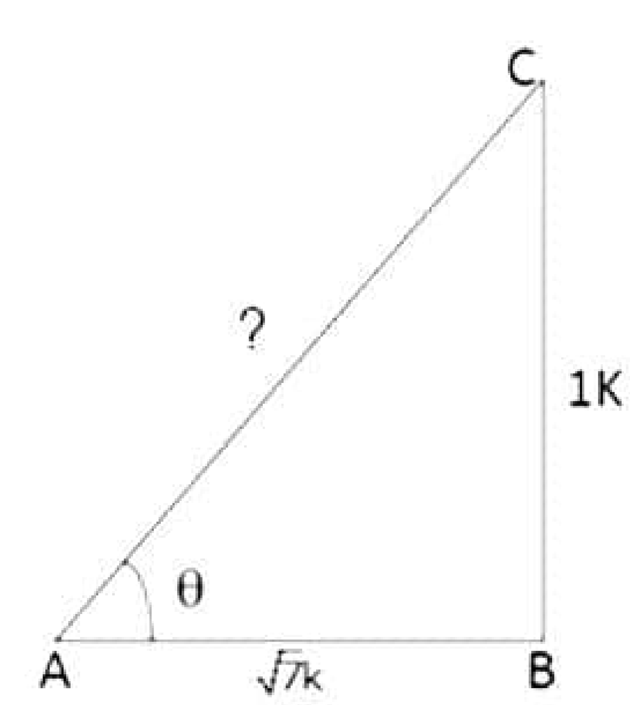
$\frac{XY}{DE} = \frac{YZ}{EF}$ ....[From (1)]

$\angle Y = \angle E$ ....[From (3)]

$\Rightarrow \Delta XYZ \sim \Delta DEF$ ....(BY SAS rule)

30. Let us draw a ΔABC in which $\angle B = 90^\circ$ and $\angle A = \theta$.

Given : $\tan\theta = \frac{1}{\sqrt{7}} = \frac{BC}{AB}$



Let $BC = 1k$ and $AB = \sqrt{7}k$, where k is positive.

In ΔABC, By pythagoras theorem,

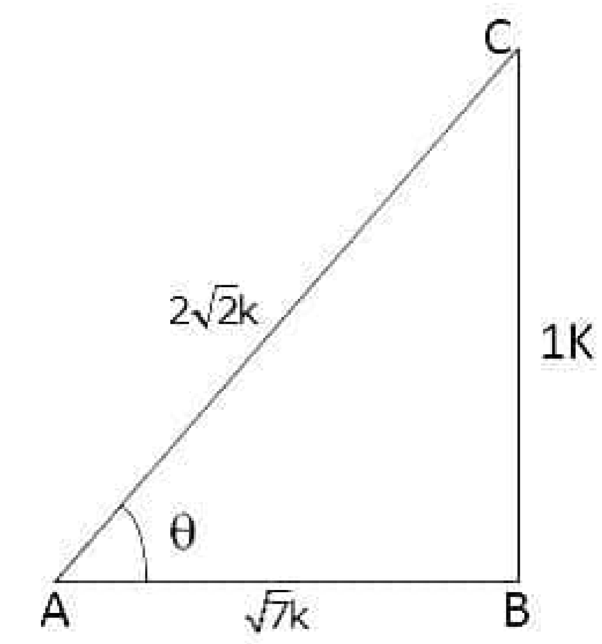
$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = \left(\sqrt{7}k\right)^2 + (1k)^2 = 7k^2 + k^2 = 8k^2$

$\Rightarrow AC = 2\sqrt{2}k$

$$\operatorname{cosec}\theta = \frac{AC}{BC} = \frac{2\sqrt{2}k}{1k} = 2\sqrt{2}$$

$$\sec\theta = \frac{AC}{AB} = \frac{2\sqrt{2}k}{\sqrt{7}k} = \frac{2\sqrt{2}}{\sqrt{7}}$$



$$\frac{\left(\operatorname{cosec}^2\theta - \sec^2\theta\right)}{\left(\operatorname{cosec}^2\theta + \sec^2\theta\right)} = \frac{\left(2\sqrt{2}\right)^2 - \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}{\left(2\sqrt{2}\right)^2 + \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}$$

$$= \frac{8 - \frac{8}{7}}{8 + \frac{8}{7}}$$

$$= \frac{48}{64}$$

$$= \frac{3}{4}$$

31. Total number of balls = 20

i. Odd numbers are 1, 3, 5, 7, 9, 11, 13, 15, 17, 19.
Total no. of odd numbers = 10

$\therefore$ P(getting an odd number) = $\frac{10}{20} = \frac{1}{2}$

ii. Numbers divisible by 2 or 3 are
2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20.
Total no. of numbers divisible by 2 or 3 =13

P(getting a number divisible by 2 or 3) = $\frac{13}{20}$

iii. Prime numbers are 2, 3, 5, 7, 11, 13, 17, 19
Total no. of prime numbers = 8

P(getting a prime number) = $\frac{8}{20} = \frac{2}{5}$

Section D

32. Let the present age of Rehman be x years.

Three years ago, his age was (x − 3) years.

Five years hence, his age will be (x + 5) years.

It is given that the sum of the reciprocals of Rehman's ages 3 years ago and 5 years from now is $\frac{1}{3}$.

$$\therefore \frac{1}{x-3}+\frac{1}{x+5}=\frac{1}{3}$$

$$\frac{x+5+x-3}{(x-3)(x+5)}=\frac{1}{3}$$

$$\frac{2x+2}{(x-3)(x+5)}=\frac{1}{3}$$

$$\Rightarrow 3(2x+2)=(x-3)(x+5)$$

$$\Rightarrow 6x+6=x^2+2x-15$$

$$\Rightarrow x^2-4x-21=0$$

$$\Rightarrow x^2-7x+3x-21=0$$

$$\Rightarrow x(x-7)+3(x-7)=0$$

$$\Rightarrow (x-7)(x+3)=0$$

$$\Rightarrow x=7,-3$$

However, age cannot be negative.

Therefore, Rehman's present age is 7 years.

OR

Let the width of the path be x metres.

Then, Area of the path = 16 × 10 − (16 − 2x)(10 − 2x) = 120

$$\Rightarrow 16\times 10-\left(160-32x-20x+4x^2\right)=120$$

$$\Rightarrow 160-160+32x+20x-4x^2=120$$

$$\Rightarrow -4x^2+52x-120=0$$

$$\Rightarrow x^2-13x+30=0$$

$$\Rightarrow x^2-10x-3x+30=0$$

$$\Rightarrow x(x-10)-3(x-10)=0$$

$$\Rightarrow (x-10)(x-3)=0$$

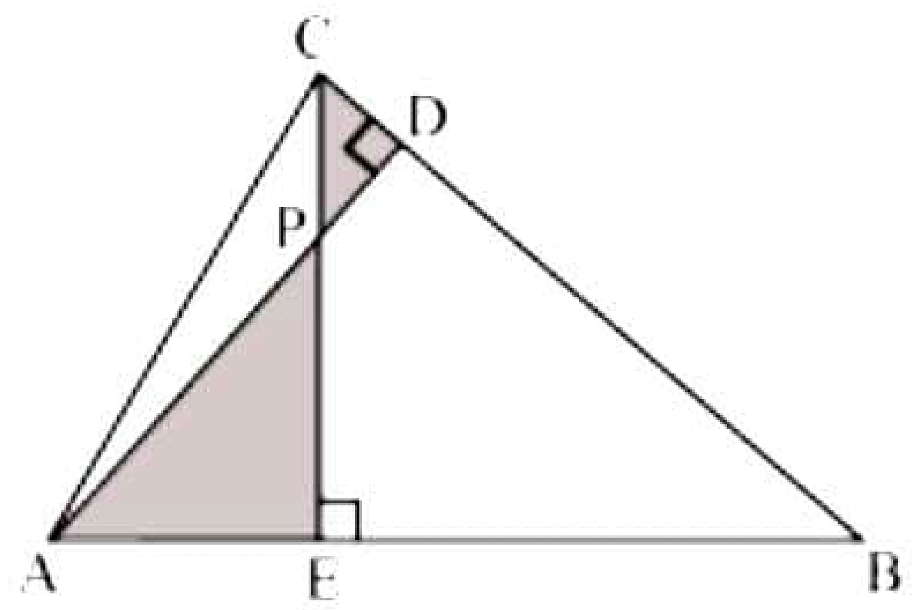
$$\Rightarrow x-10=0 \text{ or } x-3=0$$

$$\Rightarrow x=10 \text{ or } x=3$$

Hence, the required width is 3 metres as x cannot be 10 m since the width of the path cannot be greater than or equal to the width of the field.

33.

i.



In ΔAEP and ΔCDP,

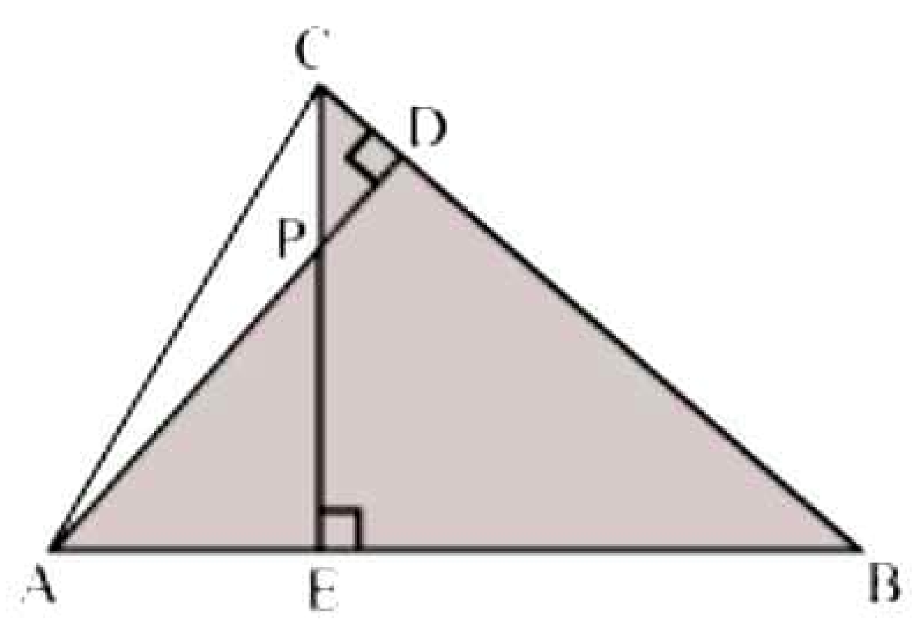
$\angle CDP = \angle AEP = 90^\circ$

$\angle CPD = \angle APE$ ... (vertically opposite angles)

$\angle PCD = \angle PAE$ ... (remaining angle)

Therefore, $\Delta AEP \sim \Delta CDP$ (by AAA rule)

ii.



In ΔABD and ΔCBE,

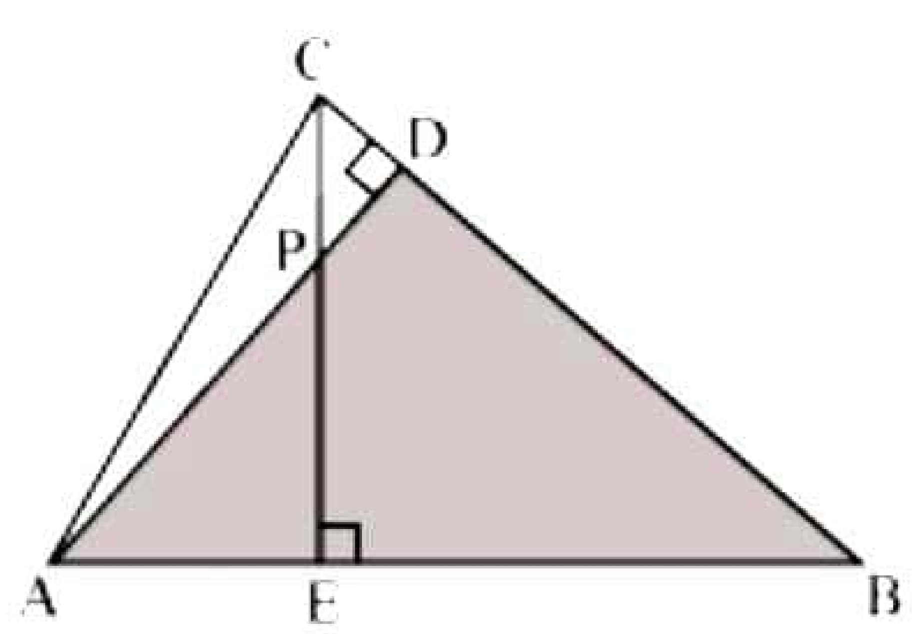
$\angle ADB = \angle CEB = 90^\circ$

$\angle ABD = \angle CBE$ (common angle)

$\angle DAB = \angle ECB$ (remaining angle)

Therefore, $\Delta ABD \sim \Delta CBE$ (by AAA rule)

iii.



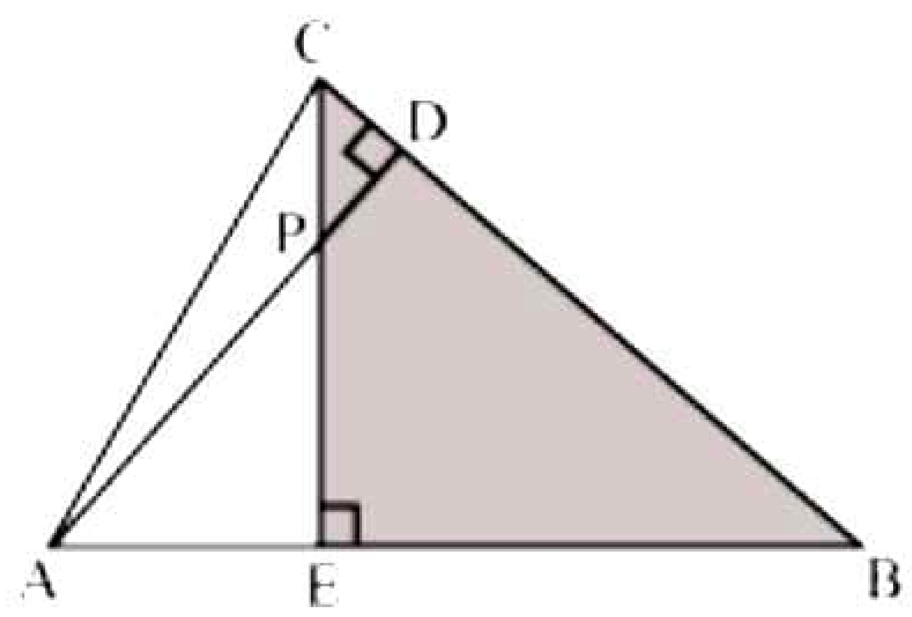
In ΔAEP and ΔADB,

$\angle AEP = \angle ADB = 90^\circ$

$\angle PAE = \angle DAB$ (common angle)

$\angle APE = \angle ABD$ (remaining angle)

Therefore, $\Delta AEP \sim \Delta ADB$ (by AAA rule) iv.



In ΔPDC and ΔBEC

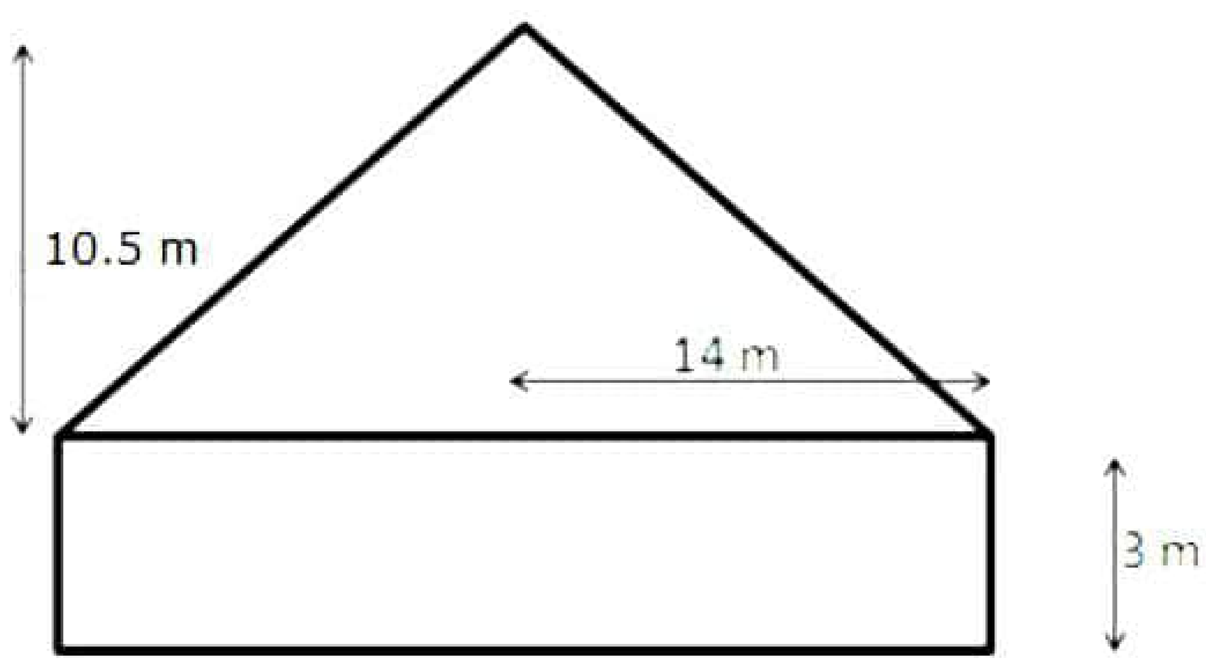
$\angle PDC = \angle BEC = 90°$

$\angle PCD = \angle BCE$ (common angle)

$\angle CPD = \angle CBE$ (remaining angle)

Therefore, $\Delta PDC \sim \Delta BEC$ (by AAA rule)

34.



For cylinder: Radius = 14 m and height = 3 m

For cone: Radius = 14 m and height = 10.5 m

Let l be the slant height of the cone.

$\therefore l^2 = (14)^2 + (10.5)^2 = (196 + 110.25)\text{m}^2 = 306.25 \text{ m}^2$

$\Rightarrow l = \sqrt{306.25}\text{ m} = 17.5 \text{ m}$

Curved surface area of the tent

= (curved surface area of the cylinder + curved surface area of the cone)

$= 2\pi rh + \pi rl$

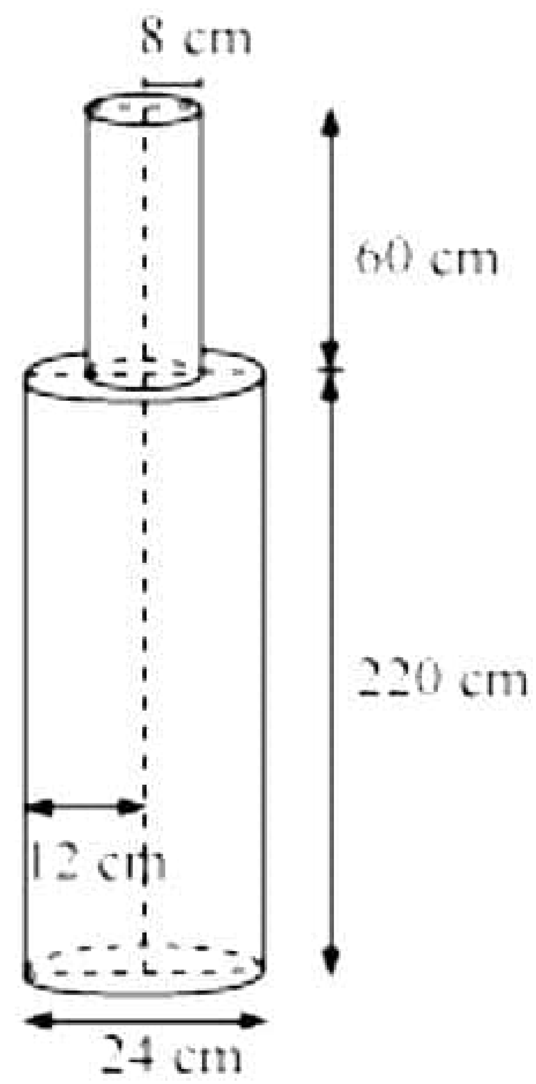
$= \left[\left(2 \times \frac{22}{7} \times 14 \times 3\right) + \left(\frac{22}{7} \times 14 \times 17.5\right)\right]\text{m}^2$

$= (264 + 770)\text{m}^2 = 1034 \text{ m}^2$

Hence, curved surface area of the tent = 1034 m^2

Cost of cloth = Rs. (1034 × 80) = Rs. 82720.

OR



From the figure we have

Height (h_1) of larger cylinder = 220 cm

Radius (r_1) of larger cylinder = $\frac{24}{2} = 12$ cm

Height (h_2) of smaller cylinder = 60 cm

Radius (r_2) of larger cylinder = 8 cm

Total volume of pole = volume of larger cylinder + volume of smaller cylinder

$$= \pi r_1^2 h_1 + \pi r_2^2 h_2$$

$$= \pi(12)^2 \times 220 + \pi(8)^2 \times 60$$

$$= \pi[144 \times 220 + 64 \times 60]$$

$$= 35520 \times 3.14 = 1,11,532.8 \text{ cm}^3$$

Mass of 1 cm^3 iron = 8 gm

Mass of 111532.8 cm^3 iron = 111532.8 × 8 = 892262.4 gm = 892.262 kg.

35. We may find class mark (x_i) for each interval by using the relation.

$$x_i = \frac{\text{upper class limit} + \text{lower class limit}}{2}$$

Given that mean pocket allowance $\bar{x}$ = Rs.18

Now taking 18 as assured mean (a) we may calculate d_i and f_id_i as follows:

Daily pocket allowance (in Rs.)	Number of children f_i	Class mark x_i	$d_i = x_i - 18$	f_id_i
11 - 13	7	12	−6	−42
13 - 15	6	14	−4	−24
15 - 17	9	16	−2	−18
17 - 19	13	18	0	0
19 - 21	f	20	2	2f
21 - 23	5	22	4	20
23 - 25	4	24	6	24
Total	$\sum f_i = 44 + f$			$\sum f_id_i = 2f - 40$

From the table,

$$\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$$

$$18 = 18 + \left(\frac{2f - 40}{44 + f}\right)$$

$$0 = \left(\frac{2f - 40}{44 + f}\right)$$

$$2f - 40 = 0$$

$$2f = 40$$

$$f = 20$$

Hence, the missing frequency f is 20.

Section E

Case study based questions are compulsory.

36.

i. From the graph, the coordinates of points O and A are (0, 0) and (4, 4) respectively.

$\therefore$ Distance covered by bus no. 735 = OA = $\sqrt{(0-4)^2+(0-4)^2} = 4\sqrt{2}$ km

ii. From the graph, the coordinates of points B and A are (3, 1) and (4, 4) respectively.

$\therefore$ Distance between the locations B and A = BA $= \sqrt{(4-3)^2+(4-1)^2} = \sqrt{10}$ km

iii. From the graph, the coordinates of points O and B are (0, 0) and (3, 1) respectively.

$\therefore$ Distance covered by bus no. 736 = O – B – A

= OB + BA

$= \sqrt{(0-3)^2+(0-1)^2} + \sqrt{10}$ km

$= 2\sqrt{10}$ km

OR

Distance between the locations O and A = OA = $4\sqrt{2}$ km

Time taken by bus no. 735 to cover distance OA = 15 minutes $= \frac{1}{4}$ hours

Therefore, speed of the bus no. 735 $= \dfrac{4\sqrt{2}}{1/4}$ km / hr $= 16\sqrt{2}$ km / hr

37.

i. 3^{rd} term = a_3 = a + 2d = 4 ....(1)

9^{th} term = a_9 = a + 8d = −8 ....(2)

Subtracting equation (1) from (2), we get

6d = −12 ⇒ d = −2

So, the common difference is −2.

ii. $d = -2$

And, $a_3 = a + 2d = 4$

$\Rightarrow a = 4 - 2(-2) = 4 + 4 = 8$

iii. Let n^{th} term of A.P. be -160.

Then, $a = 8$, $d = -2$ and $a_n = -160$

Now, $a_n = a + (n - 1)d$

$\Rightarrow -160 = 8 + (n - 1)(-2)$

$\Rightarrow -168 = (n - 1)(-2)$

$\Rightarrow n - 1 = 84$

$\Rightarrow n = 85$

Hence, 85^{th} term of the A.P. is -160.

OR

For a given A.P., $a = 8$ and $d = -2$

Let $a_n = -123$

$\Rightarrow -123 = 8 + (n - 1)(-2)$

$\Rightarrow -131 = -2n + 2$

$\Rightarrow n = 66.5$, which is not possible.

$\Rightarrow -123$ is not a term of the given A.P.

38.

i. Rishabh makes an angle of elevation 30° from the airplane while Reema makes an angle of elevation 60° from the airplane.
Now, whose angle of elevation is smaller is closer to the airplane.
So, Rishabh is closer to airplane and Reema is far from the airplane.

ii. In ΔBDE,

$$\tan 30^o = \frac{DE}{BD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{BD}$$

$$\Rightarrow BD = h\sqrt{3}\ m$$

OR

In ΔACE,

$$\tan 60^o = \frac{CE}{AC}$$

$$\Rightarrow \sqrt{3} = \frac{h+4}{BD}$$

$$\Rightarrow BD = \frac{h+4}{\sqrt{3}}$$

$$\Rightarrow \sqrt{3}h = \frac{h+4}{\sqrt{3}}$$

$\Rightarrow h = 2$ m

iii. If the airplane is moving towards the building, then both the angles of elevation will increase.