Class X Session 2024-25
Subject - Mathematics (Standard)
Sample Question Paper - 20

Time: 3 Hours **Total Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A - E.
2. Section A has 18 multiple choice questions and 2 Assertion-Reason based questions carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take π = 22/7 wherever required if not stated.

Section A
Section A consists of 20 questions of 1 mark each.

Choose the correct answers to the questions from the given options. [20]

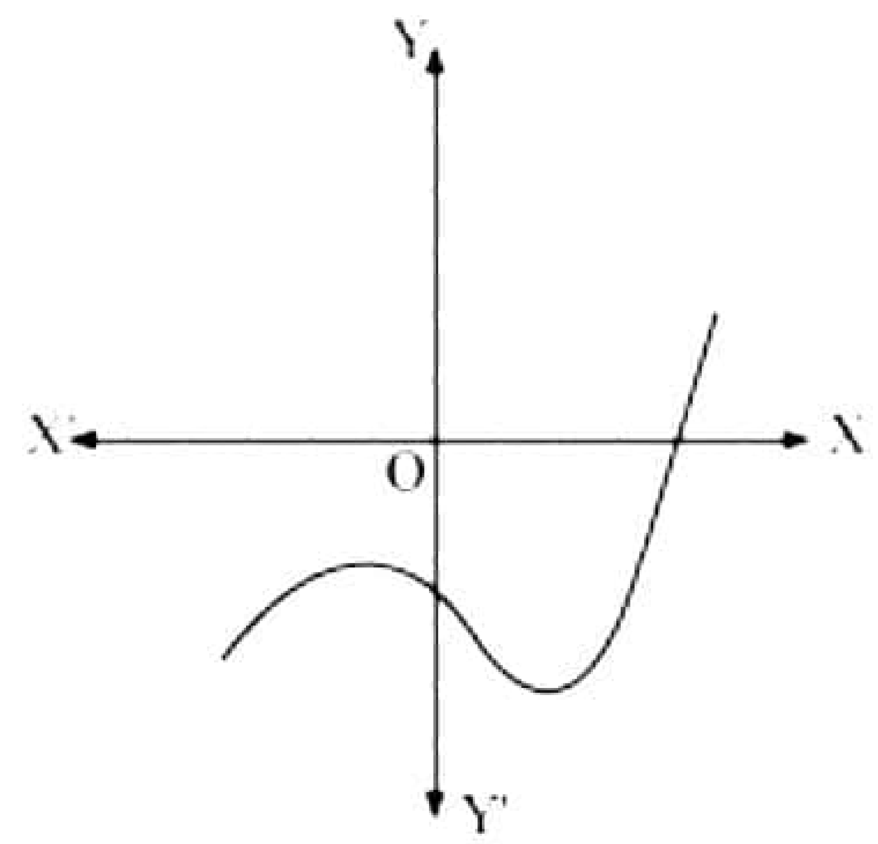
1. Find LCM of 336 and 54.

A. 3042
B. 3024
C. 3204
D. 3044

2. Find the discriminant of equation: $3x^2 - 2x + 8 = 0$

A. −94
B. 94
C. 92
D. −92

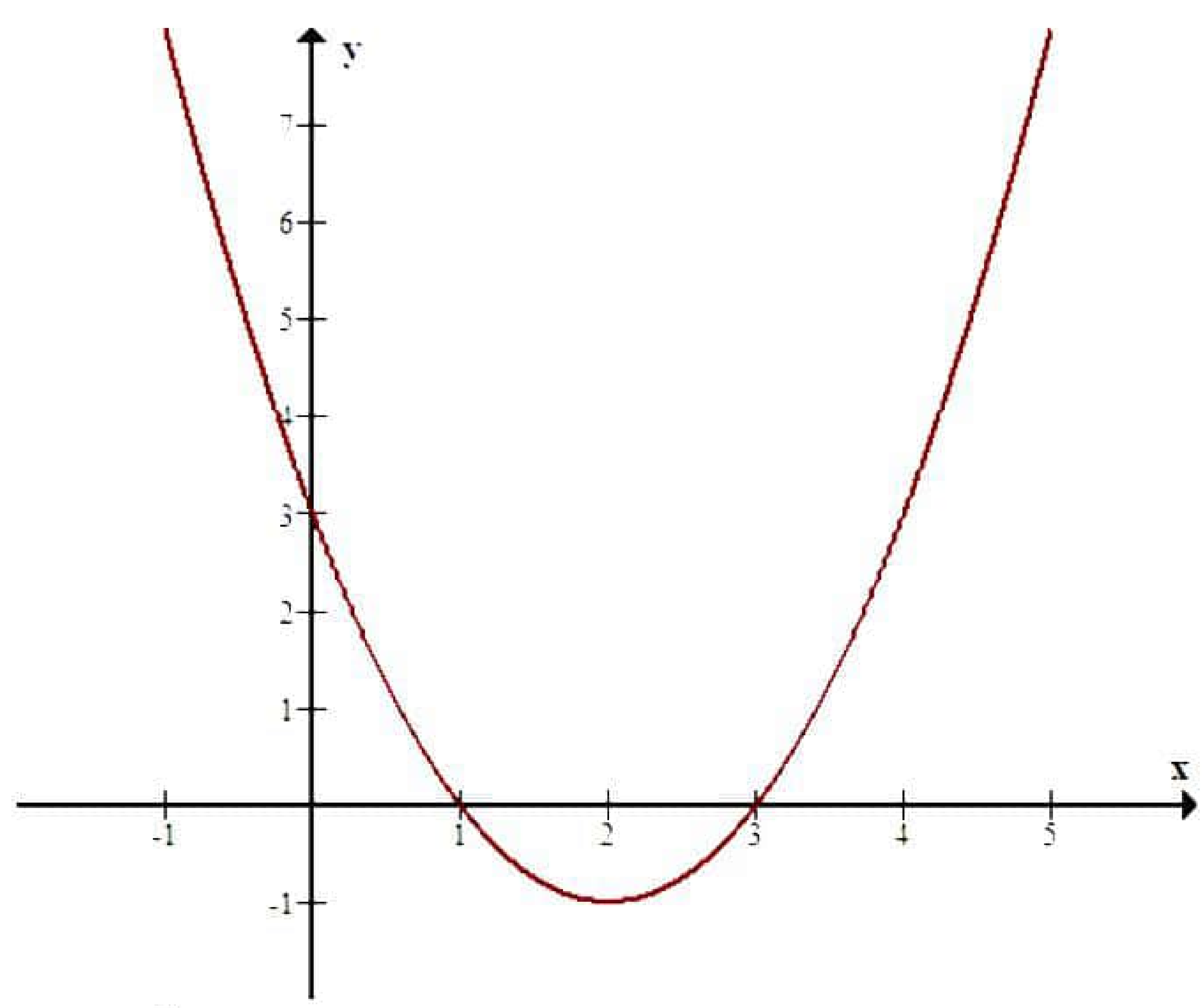
3. The graph of y = p(x) is given in the following figure for some polynomial p(x). Find the number of zeroes of p(x).



A. 2
B. 1
C. 0
D. 3

4. The sum of two numbers is 18, and the difference between them is 2. Find the largest out of the two.
A. 4
B. 6
C. 8
D. 10

5. The algebraic expression of a polynomial representing the following parabola is



A. $x^2 + 4x + 3$
B. $x^2 + 4x - 3$
C. $x^2 - 4x - 3$
D. $x^2 - 4x + 3$

6. Find the coordinates of the point equidistant from three given points A(5, 3), B(5, −5) and C(1, −5).

A. (2, −1)
B. (3, −1)
C. (4, −1)
D. (5, −1)

7. If sum of the zeros of a cubic polynomial $ax^3 + (-7x^2) + (-13x) + (d)$ is $\frac{7}{5}$ and product of zeroes is 1, then the values of 'd' and 'a' are

A. 5, 5
B. −5, 5
C. 5, −5
D. −5, −5

8. The perimeters of two similar triangles ABC and PQR are 32 cm and 24 cm, respectively. If PQ = 12 cm, then find AB.

A. 24 cm
B. 32 cm
C. 12 cm
D. 16 cm

9. Following is not a test of similarity.

A. SSS
B. SAS
C. AAA
D. SSA

10. If $\Delta ABC \sim \Delta DEF$ such that 2AB = DE and BC = 6 cm, find EF.

A. 10 cm
B. 12 cm
C. 1 cm
D. 4 cm

11. Find the value of θ if $\tan\theta = \cot\theta$

A. 30°
B. 60°
C. 45°
D. 90°

12. If $2\sin^2\theta - \cos^2\theta = 2$, then find the value of θ.

A. 100°
B. 70°
C. 90°
D. 80°

13. Ravi and Raju are standing at an equal distance from a Pole, but on the opposite sides. Both are looking at the top of the pole. The angle of elevation of Ravi's eye sight from the horizontal is 45°, and that of Raju is 30°.

If so, then which of the following statements is true?

A. Raju is taller than Ravi
B. Ravi is taller than Raju
C. Both Raju and Ravi have same heights
D. Data is insufficient to compare the heights of Raju and Ravi

14. Find the area of a sector with radius 7 cm and central angle 90°.

A. 38 cm^2
B. 39 cm^2
C. 38.5 cm^2
D. 37.5 cm^2

15. The total surface area of a right circular cylinder is given by

A. $2\pi r(r + h)$
B. $2\pi r(r - h)$
C. $2r(r + h)$
D. $\pi r(r + h)$

16. Find the modal class from the following table:

Size	Frequency
45-55	7
55-65	12
65-75	17
75-85	30
85-95	32
95-105	6
105-115	10

A. 75-85
B. 85-95
C. 95-105
D. 105-115

17. Cards bearing numbers 1, 3, 5,..., 35 are kept in a bag. A card is drawn at random from the bag. Find the probability of getting a card bearing a prime number less than 15.

A. 3/18
B. 4/18
C. 5/18
D. 7/18

18. Netra has a total of 361 songs in her playlist out of which 165 are Hindi, 87 are Punjabi and 109 are English. She starts listening to music by choosing the first song from Hindi category. She will continue listening if the next song which will be played automatically is Punjabi. What is the probability that Netra will continue listening to music?

A. 87/361
B. 87/360
C. 109/361
D. 109/360

DIRECTION: In the question number 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option.

19. **Statement A (Assertion)**: From a solid cylinder whose height and radius are 8 cm and 6 cm, a conical cavity of height 8 cm and of base radius 6 cm is hollowed out. Hence the volume of the remaining solid will be 192 cm^3.

Statement R (Reason): Volume of remaining solid = Volume of the cylinder - Volume of the cone removed

A. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
B. Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
C. Assertion (A) is true but reason (R) is false.
D. Assertion (A) is false but reason (R) is true.

20. **Statement A (Assertion)**: $\frac{4}{5}$, a, 2 are three consecutive terms of an A.P. only if $a = \frac{7}{5}$.

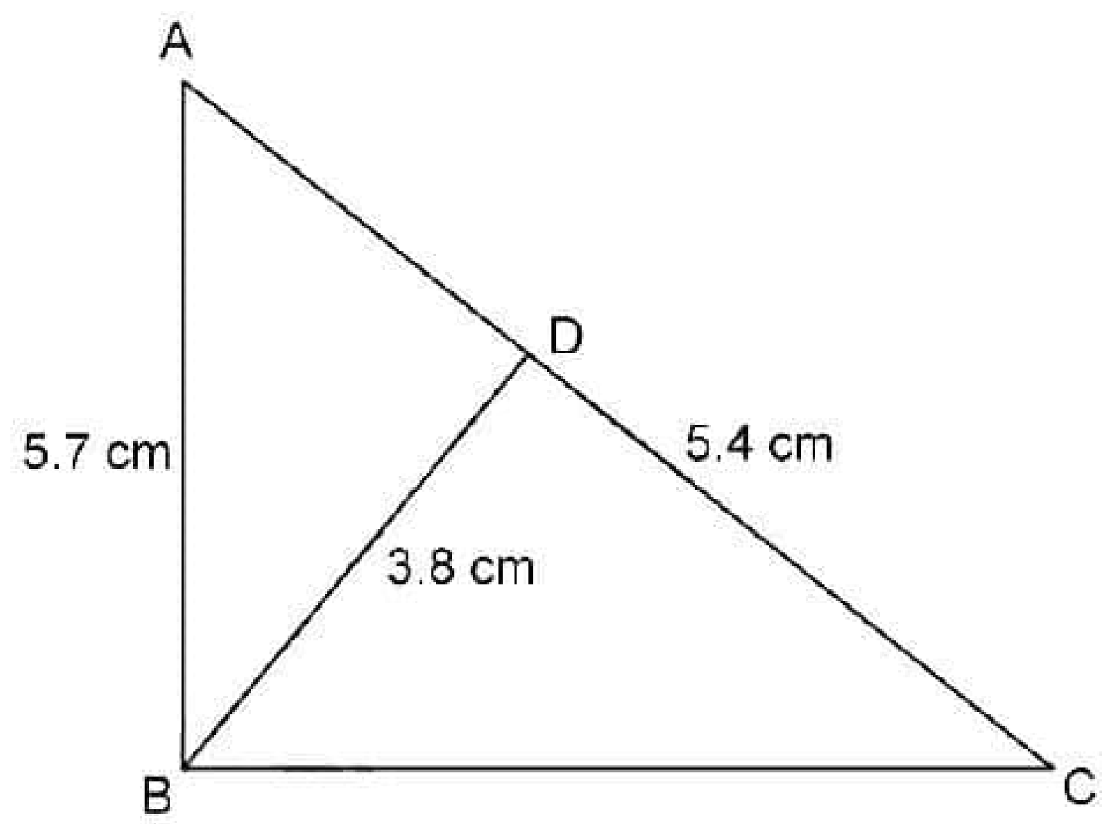
Statement R (Reason): If p, q and r are in A.P. then $q - p = r - q$.

A. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
B. Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
C. Assertion (A) is true but reason (R) is false.
D. Assertion (A) is false but reason (R) is true.

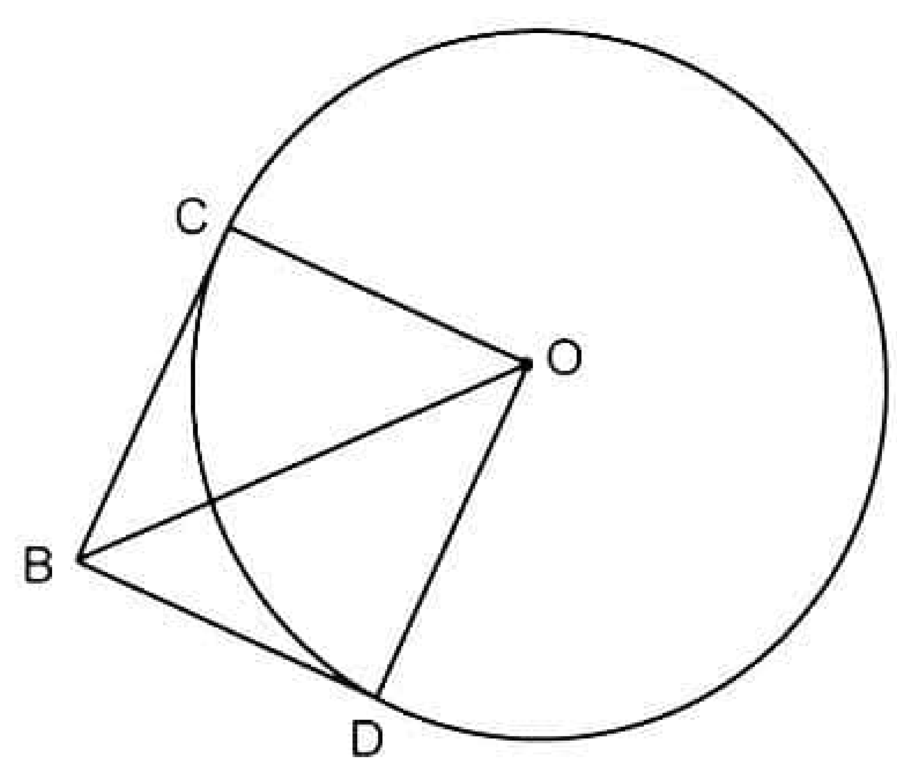
Section B

21. Find LCM and HCF of 120 and 144 by fundamental theorem of arithmetic. [2]

22. In the given figure, $\angle ABC = 90°$ and $BD \perp AC$. If AB = 5.7 cm, BD = 3.8 cm and CD = 5.4 cm, find BC. [2]



23. Two tangent segments BC and BD are drawn to a circle with centre O such that $\angle CBD = 120°$. Prove that OB = 2BC. [2]



24. If $\sin A = \frac{3}{4}$, calculate cos A and tan A. [2]

OR

Given 15 cot A = 8. Find sin A and sec A

25. Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°. [2]

OR

Find the area of a quadrant of a circle whose circumference is 22 cm.

Section C

Section C consists of 6 questions of 3 marks each.

26. In a seminar, the number of participants in Hindi, English and Mathematics is 60, 84 and 108, respectively. Find the minimum number of rooms required, if in each room the same number of participants are to be seated and all of them being in the same subject. [3]

27. If the 7^{th} term of an A.P. is 27 and the 13^{th} term is 16 more than the 9^{th} term, find the A.P. [3]

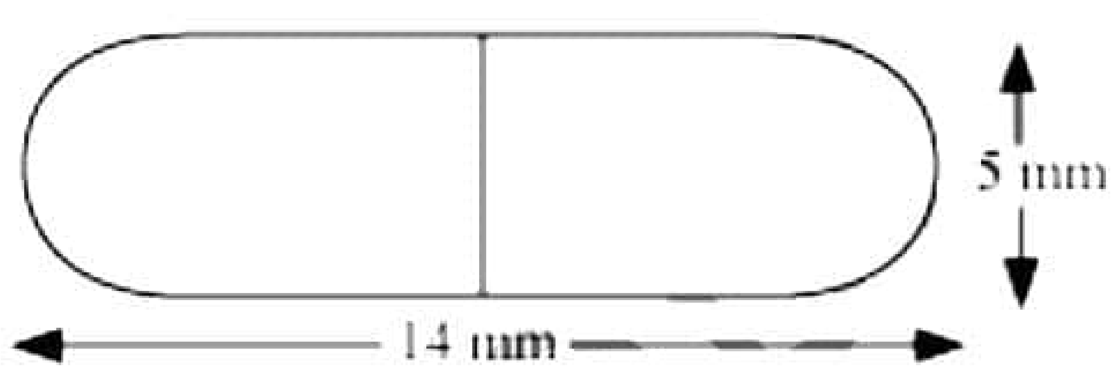
28. If the 3^{rd} and the 9^{th} terms of an A.P. are 4 and −8 respectively. Which term of this A.P. is zero? [3]

OR

A man invested an amount at 12% per annum simple interest and another amount at 10% per annum simple interest. He received an annual interest of Rs. 1145. But if he had interchanged the amounts invested, he would have received Rs. 90 less. What amounts did he invest at the different rates?

29. In ΔABC, right-angled at B, it is given that AB = 12 cm and BC = 5 cm. Find the value of (i) cos A, (ii) cosec A, (iii) cos C and (iv) cosec C. [3]

30. A medicine capsule is in the shape of cylinder with two hemispheres stuck to each of its ends (see the given figure). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area. [3]



OR

From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.

31. A box contains 20 balls bearing numbers 1, 2, 3, ..., 20, respectively. A ball is taken out at random from the box. What is the probability that the number on the ball is [3]

i. an odd number?
ii. divisible by 2 or 3?
iii. a prime number?
iv. not divisible by 10?

Section D

Section D consists of 4 questions of 5 marks each.

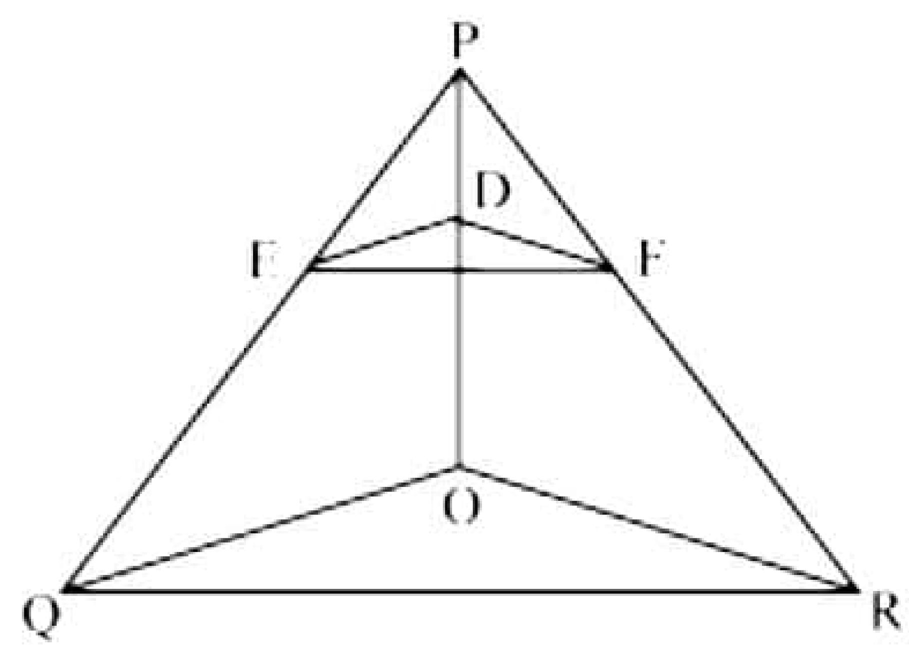
32. Solve the following quadratic equation: [5]

$mnx^2 - (m + n)^2x + (m + n)^2 = 0$

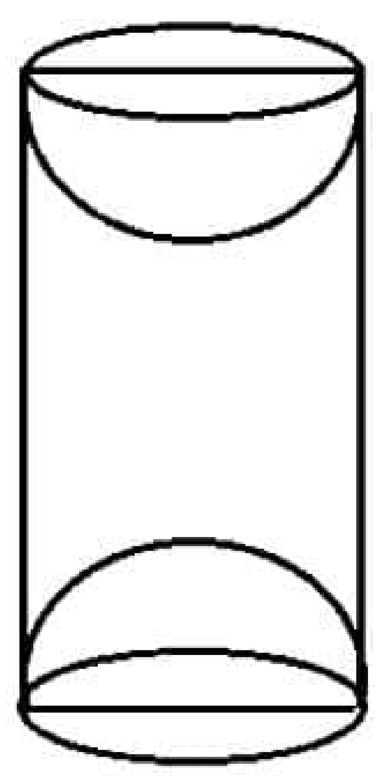
OR

A rectangular field is 16 m long and 10 m wide. There is a path of uniform width all around it with an area of 120 m^2. Find the width of the path.

33. In figure, DE || OQ and DF || OR, show that EF || QR. [5]



34. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in the figure. If the height of the cylinder is 12 cm and its base is of diameter 14 cm, find the total surface area of the wooden article. Also, find the volume of the wood left in the article. Use $\pi = 22/7$. [5]



OR

A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs 600 per m^2.

35. In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes. [5]

Number of mangoes	50–52	53–55	56–58	59–61	62–64
Number of boxes	15	110	135	115	25

Find the mean number of mangoes kept in a packing box to nearest whole number.

Section E

Case study based questions are compulsory.

36. Your elder brother wants to buy a car and plans to take loan from a bank for his car. He repays his total loan of Rs. 118000 by paying every month starting with the first instalment of Rs. 1000. If he increases the instalment by Rs. 100 every month, then answer the following questions.

i. Find the amount paid by him in 30^{th} instalment. [1]

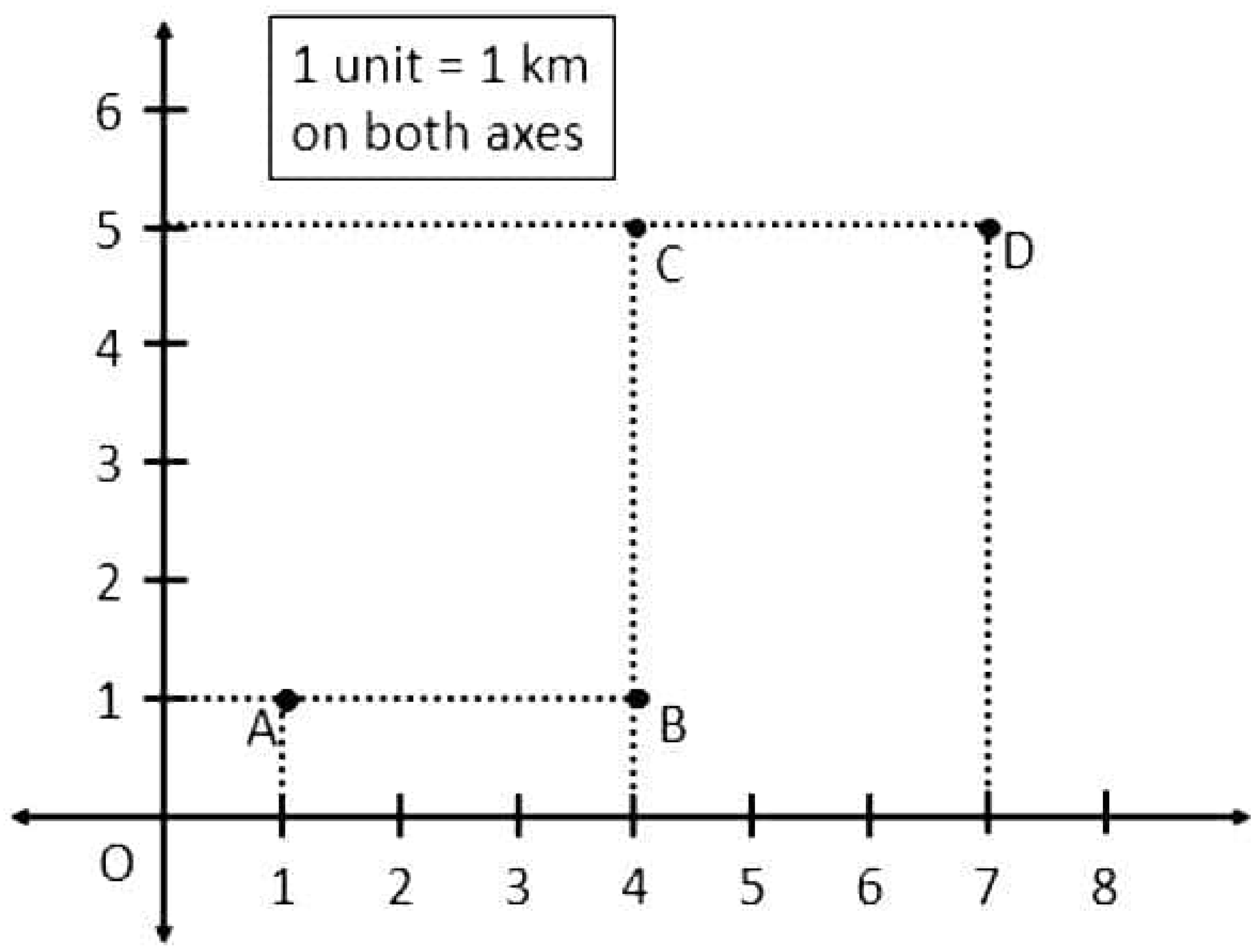
ii. Find the amount to be paid in the 40^{th} instalment. [1]

iii. What is the ratio of the 19^{th} instalment to the 28^{th} instalment? [2]

OR

Find the total amount paid by him in 30 instalments.

37. Amey runs a grocery store that offers home delivery of fresh groceries to its customers. His store is located at location A as indicated in the graph below. Now, he receives regular orders from the families living in the colonies located at B, C and D. Now, using the data given, answer the following questions.



i. Find the shortest distance between locations A and C. [2]

OR

Find the shortest distance between locations B and D.

ii. Find the shortest distance between locations B and A. [1]

iii. Find the shortest distance between locations C and B. [1]

38. There are two poles of equal height on either side of the road. Each pole has one hoarding on it. A car is standing on the road at point A. From A, the angle of elevation of the top of the poles are 60° and 30° respectively.

If height of each pole is 30 m, then answer the following questions.

i. Find the distance between the left pole and point A. [2]

OR

Find the Distance between the right pole and point A.

ii. Find the width of the road. [1]

iii. Angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level is known as [1]

Solution

Section A

1. Correct Option: B
Explanation:
Prime factorization of the numbers:
$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$
$54 = 2 \times 3 \times 3 \times 3$
LCM (336, 54)
$= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7$
$= 3024$

2. Correct option: D
Explanation:
The given equation is $3x^2 - 2x + 8 = 0$
Comparing it with $ax^2 + bx + c = 0,$ we get
$a = 3, b = -2, c = 8$
$\therefore D = \left(b^2 - 4ac\right) = \left[(-2)^2 - (4 \times 3 \times 8)\right]$
$= (4 - 96) = -92$

3. Correct option: B
Explanation:
The graph of p(x) intersects the x-axis at only 1 point.
So, the number of zeroes is 1.

4. Correct Option: D
Explanation:
Let the two numbers be x and y, hence
$x + y = 18$ ..... (i)
$x - y = 2$ ..... (ii)
From (i) and (ii), we get
$2x = 20 \Rightarrow x = 10$
Substituting $x = 10$ in equation (i), we get
$10 + y = 18 \Rightarrow y = 8$

5. Correct Option: D

Explanation:

The parabola intersects X-axis at 1 and 3.

Therefore, 1 and 3 are the zeroes of polynomial representing given parabola.

Then, polynomial = x^2 – (sum of zeroes)x + product of zeroes

$= x^2 - (1 + 3)x + (1 \times 3)$

$= x^2 - 4x + 3$

6. Correct Option: B

Explanation:

Let the required point be P(x, y), then

PA = PB = PC

The points A, B, C are (5, 3), (5, −5) and (1, −5), respectively.

$\Rightarrow PA^2 = PB^2 = PC^2$

$\Rightarrow PA^2 = PB^2$ and $PB^2 = PC^2$

$PA^2 = PB^2$

$\Rightarrow (5 - x)^2 + (3 - y)^2 = (5 - x)^2 + (-5 - y)^2$

$\Rightarrow 25 + x^2 - 10x + 9 + y^2 - 6y = 25 + x^2 - 10x + 25 + y^2 + 10y$

$\Rightarrow -6y - 10y = 25 - 9$

$\Rightarrow -16y = 16$

$\Rightarrow y = -1$

and $PB^2 = PC^2$

$\Rightarrow (5 - x)^2 + (-5 - y)^2 = (1 - x)^2 + (-5 - y)^2$

$\Rightarrow 25 + x^2 - 10x + 25 + y^2 + 10y = 1 + x^2 - 2x + 25 + y^2 + 10y$

$\Rightarrow -10x + 2x = -24$

$\Rightarrow -8x = -24$

$\Rightarrow x = \dfrac{-24}{-8} = 3$

$\Rightarrow x = 3$

Hence, the coordinates of the point are (3, −1).

7. Correct Option: B

Explanation:

Given cubic polynomial is $ax^3 + (-7x^2) + (-13x) + (d)$.

Now, Sum of the zeros $= \dfrac{7}{5} = -\dfrac{b}{a}$

$\Rightarrow \dfrac{7}{5} = -\dfrac{(-7)}{a}$

$\Rightarrow a = 5$

And, product of zeroes = 1

$\Rightarrow 1 = -\frac{d}{a} = -\frac{d}{5}$

$\Rightarrow d = -5$

8. Correct option: D

Explanation:

It is given that ΔABC and ΔPQR are similar triangles, so the corresponding sides of both triangles are proportional.

So, $\frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta PQR} = \frac{AB}{PQ}$

Then, $\frac{32}{24} = \frac{AB}{12} \Rightarrow AB = \frac{32 \times 12}{24} = 16 \text{ cm}$

9. Correct option: D

SSA is not a test of similarity, the angle should be included between the two sides.

10. Correct Option: B

Explanation:

$\Delta ABC \sim \Delta DEF$

$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF}$

$\Rightarrow \frac{1}{2} = \frac{6}{EF}$

$\Rightarrow EF = 12 \text{ cm}$

11. Correct option: C

Explanation:

Since, tan 45° = cot 45°

$\Rightarrow \theta = 45°$

12. Correct option: C

Explanation:

$2\sin^2\theta - \cos^2\theta = 2$

$\Rightarrow 2(1 - \cos^2\theta) - \cos^2\theta = 2$

$\Rightarrow 2 - 2\cos^2\theta - \cos^2\theta = 2$

$\Rightarrow 2 - 3\cos^2\theta = 2$

$\Rightarrow 3\cos^2\theta = 0$

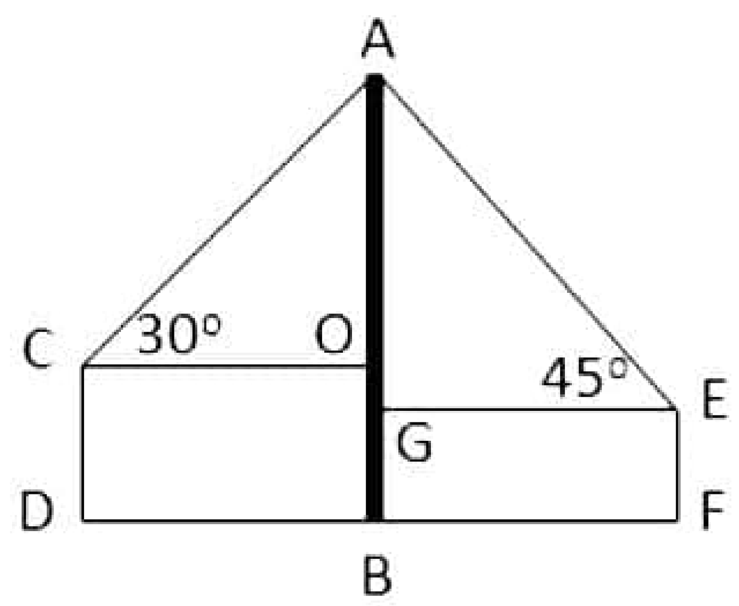
$\Rightarrow \cos^2\theta = 0$

$\Rightarrow \cos^2\theta = \cos^2 90°$

$\Rightarrow \theta = 90°$

13. Correct option: A

Explanation:



In the figure above,

CD represents the height of Raju.

EF represents the height of Ravi.

AB represents the height of pole.

Now, in respective triangles, we have

$$\tan 30° = \frac{AO}{CO} \text{ and } \tan 45° = \frac{AG}{GE}$$

$$\therefore \frac{1}{\sqrt{3}} = \frac{AO}{CO} \text{ and } 1 = \frac{AG}{GE}$$

Now, CO = GE

$$\therefore \frac{1}{\sqrt{3}} = \frac{AO}{CO} \text{ and } 1 = \frac{AG}{CO}$$

$\therefore CO = \sqrt{3}AO$ and $CO = AG$

$\therefore AG = \sqrt{3}AO$

$\therefore AG > AO$

$\therefore AB - GB > AB - OB$

$\therefore OB > GB \Rightarrow CD > EF$

∴ Raju's height > Ravi's height

14. Correct Option: C

Explanation:

Area of a sector of angle $\theta = \frac{\theta}{360°} \times \pi r^2 = \frac{90°}{360°} \times \frac{22}{7} \times (7)^2 = 38.5 \text{ cm}^2$

15. Correct option: A

Explanation:

The total surface area of a right circular cylinder is given by $2\pi rh + 2\pi r^2$

$= 2\pi r(r + h)$

16. Correct option: B

Explanation:

As the class 85–95 has the maximum frequency, it is the modal class.

17. Correct option: C

Explanation:

There are 18 cards having numbers 1, 3, 5, ..., 35 kept in a bag.

Prime numbers less than 15 are 3, 5, 7, 11, 13.

There are 5 numbers.

∴ Probability that a card drawn bears a prime number less than 15 = $\frac{5}{18}$

18. Correct option: B

Explanation:

One Hindi song is already played.

So, there are 360 songs left from which 1 song will be played automatically.

Total outcomes = 360

There are only 87 Punjabi songs.

Favorable outcomes = 87

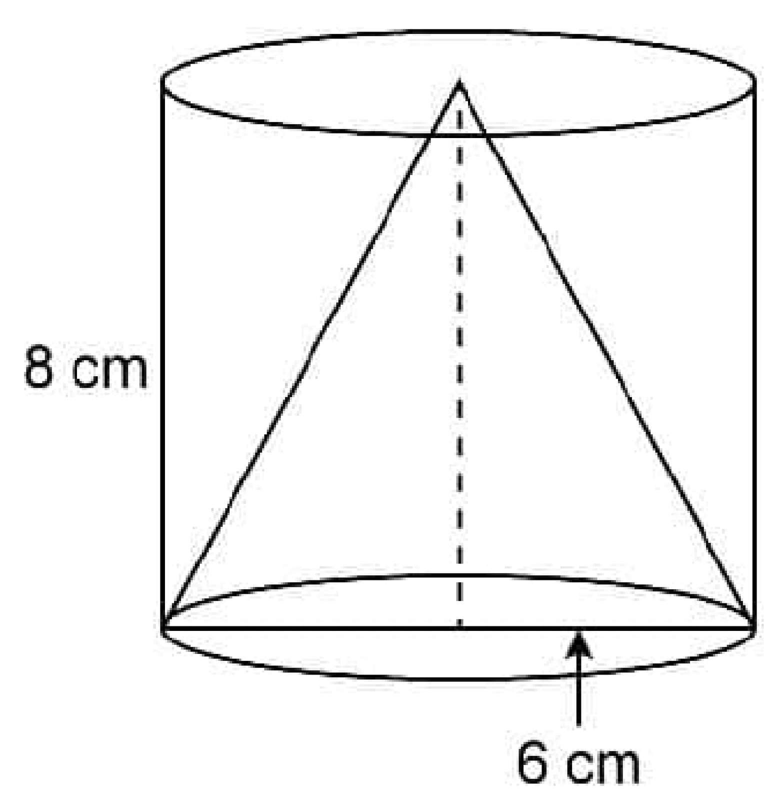
So, the required probability = $\frac{87}{360}$

19. Correct Option: D

Explanation:

Radius of the cylinder = 6 cm

Height of the cylinder = 8 cm



Volume of the cylinder

$= \pi r^2 h$ cu. units

$= \pi \times 6 \times 6 \times 8 \text{ cm}^3$

$= 288\pi \text{ cm}^3$

Volume of the cone removed

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 6 \times 6 \times 8 \text{ cm}^3$

$= 96\pi \text{ cm}^3$

Volume of the remaining solid = Volume of the cylinder − Volume of the cone removed

Hence, reason (R) is true.
Then, volume of the remaining solid
$= 288\pi - 96\pi$
$= 192\pi\ cm^3$
Hence, assertion (A) is false, but reason (R) is true.

20. Correct Option: A
Explanation:
$\frac{4}{5}$, a, 2 are in A.P.
We know that, if p, q and r are in A.P then q − p = r − q.
So, the reason is true.
$\therefore\ a - \frac{4}{5} = 2 - a$
$\Rightarrow 2a = 2 + \frac{4}{5} = \frac{14}{5} \Rightarrow a = \frac{7}{5}$
Hence, the assertion is true and reason is the correct explanation of assertion.

Section B

21. $120 = 2^3 \times 3 \times 5$

$144 = 2^4 \times 3^2$

$LCM = 2^4 \times 3^2 \times 5 = 720$

$HCF = 2^3 \times 3 = 24$

22. Given that AB = 5.7 cm, BD = 3.8 cm and CD = 5.4 cm.

In ΔCBA and ΔCDB,

$\angle CBA = \angle CDB = 90°$

And $\angle C = \angle C$ (Common)

$\Delta CBA \sim \Delta CDB$ (by AA similarity)

$\Rightarrow \frac{CB}{CD} = \frac{BA}{DB}$

$\Rightarrow \frac{BC}{5.4} = \frac{5.7}{3.8}$

$\Rightarrow BC = \frac{5.7 \times 5.4}{3.8} = 8.1 \text{ cm}$

Hence, BC = 8.1 cm.

23. Given: Two tangent segments BC and BD are drawn to a circle with centre O such that $\angle CBD = 120°$.

In ΔOBC,

$\angle OBC = \angle OBD = 60°$.......... The line joining the centre of the circle and the point of contact of tangents from an external point bisect the angle between two tangents.

$\angle OCB = 90°$ (BC is tangent to the circle)

Therefore, $\angle BOC = 30°$

$\therefore \frac{BC}{OB} = \sin 30° = \frac{1}{2}$

$\Rightarrow OB = 2BC$

24. Let ΔABC be a right-angled triangle.

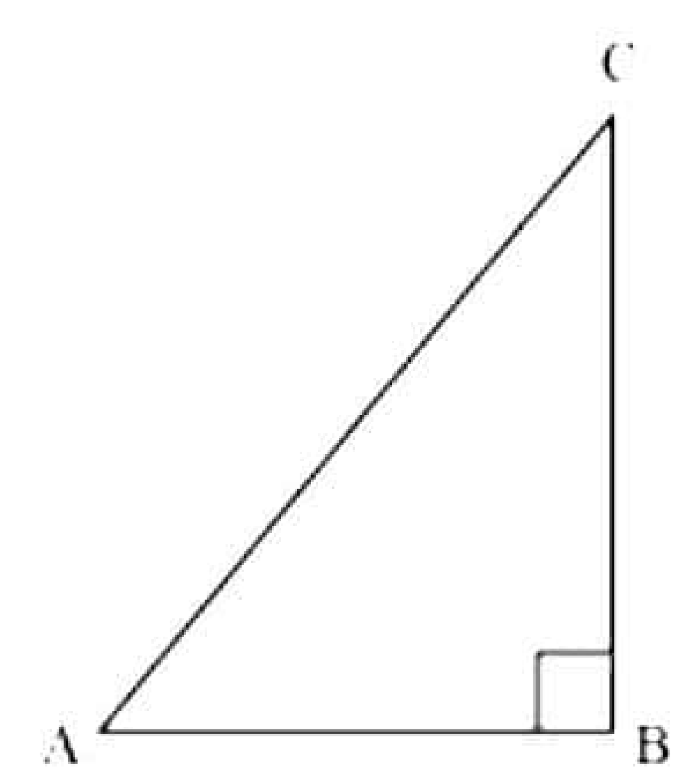
Given that

$\sin A = \frac{3}{4} \Rightarrow \frac{BC}{AC} = \frac{3}{4}$

Let BC be 3K.

Then, AC = 4K, where K is a positive integer.

Now applying Pythagoras theorem in ΔABC,



$AC^2 = AB^2 + BC^2$

$(4K)^2 = AB^2 + (3K)^2$

$16K^2 - 9K^2 = AB^2$

$7K^2 = AB^2$

$AB = \sqrt{7}K$

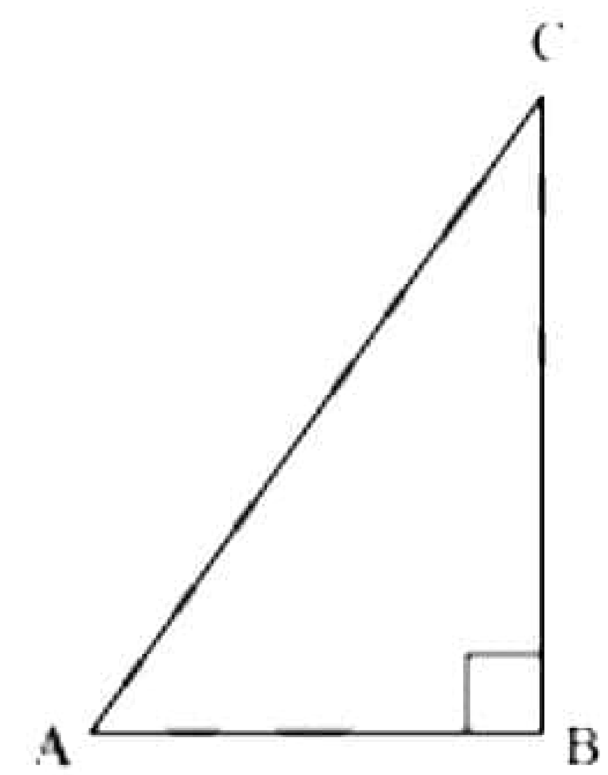
$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{7}K}{4K} = \frac{\sqrt{7}}{4}$$

$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{side adjacent to } \angle A} = \frac{BC}{AB} = \frac{3K}{\sqrt{7}K} = \frac{3}{\sqrt{7}}$$

OR

Consider a right angled triangle, right angled at B

$$\cot A = \frac{\text{Side adjacent to } \angle A}{\text{Side opposite to } \angle A} = \frac{AB}{BC}$$



Given, $\cot A = \frac{8}{15}$

$$\Rightarrow \frac{AB}{BC} = \frac{8}{15}$$

Let AB be 8K.

Then, BC = 15K, where K is a positive integer.

Now applying Pythagoras theorem in ΔABC,

$AC^2 = AB^2 + BC^2$

$= (8K)^2 + (15K)^2$

$= 64K^2 + 225K^2$

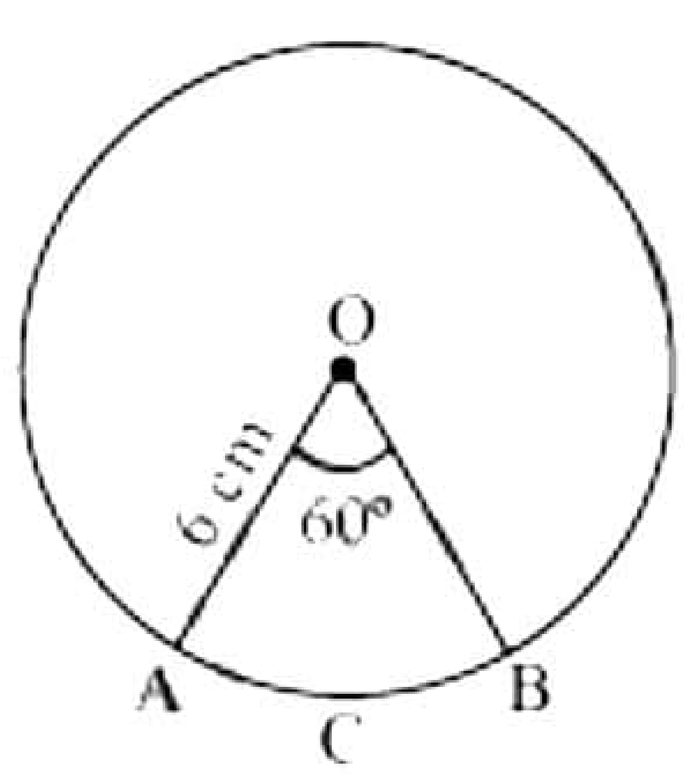
$= 289K^2$

$\Rightarrow AC = 17K$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{15K}{17K} = \frac{15}{17}$$

$$\sec A = \frac{\text{hypotenuse}}{\text{Side adjacent to } \angle A} = \frac{AC}{AB} = \frac{17K}{8K} = \frac{17}{8}$$

25.



Let OACB be a sector of circle making 60° angle at centre O of the circle.

Area of sector of angel $\theta = \frac{\theta}{360^\circ} \times \pi r^2$

So, area of sector OACB $= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times (6)^2$

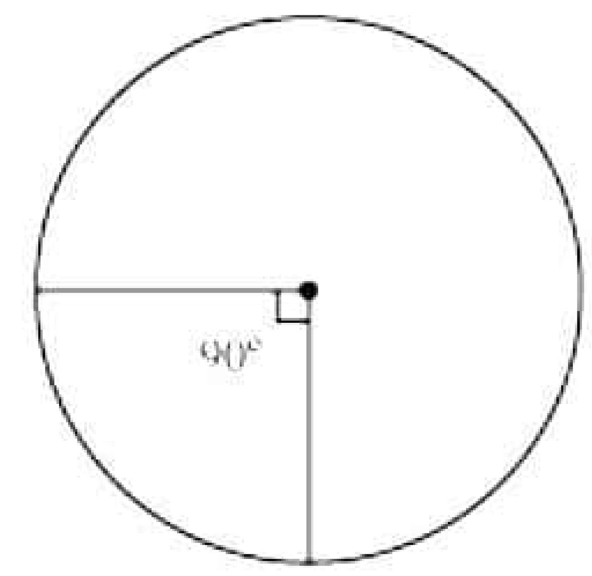
$$= \frac{1}{6} \times \frac{22}{7} \times 6 \times 6 = \frac{132}{7} \text{ cm}^2$$

So, the area of sector of circle making 60° at the centre of a circle is $\frac{132}{7}$ cm².

OR

Let the radius of a circle be r.

Circumference = 22 cm



$2\pi r = 22$

$r = \frac{22}{2\pi} = \frac{11}{\pi}$

Quadrant of circle will subtend 90° angle at the centre of a circle.

So area of such quadrant of circle $= \frac{90°}{360°} \times \pi \times r^2$

$$= \frac{1}{4} \times \pi \times \left(\frac{11}{\pi}\right)^2$$

$$= \frac{121}{4\pi} = \frac{121 \times 7}{4 \times 22}$$

$$= \frac{77}{8} \text{ cm}^2$$

Section C

26. To find the minimum number of rooms required, we first find the maximum number of participants which can be accommodated in each room such that the number of participants in each room is the same.

This can be determined by finding the HCF of 60, 84 and 108.

$60 = 2^2 \times 3 \times 5$

$84 = 2^2 \times 3 \times 7$

$108 = 2^2 \times 3^3$

Then, HCF $= 2^2 \times 3 = 12$

So, the minimum number of rooms required $= \dfrac{\text{Total number of participants}}{12}$

$= \dfrac{60 + 84 + 108}{12}$

$= 21$

27. Let 'a' be the first term and 'd' be the common difference of required A.P.

Now, $a_7 = 27$

$\Rightarrow a + 6d = 27$ ....(1)

And $a_{13} = 16 + a_9$

$\Rightarrow a + 12d = 16 + a + 8d$

$\Rightarrow 4d = 16$

$\Rightarrow d = 4$

Substituting d = 4 in equation (1),

$a + 6(4) = 27$

$\Rightarrow a + 24 = 27$

$\Rightarrow a = 3$

Therefore, required AP is 3, 7, 11, 15, 19, 23,.......

28. Given that,

$a_3 = 4$

$a_9 = -8$

We know that,

$a_n = a + (n - 1)d$

$a_3 = a + (3 - 1)d$

$4 = a + 2d$ ....(1)

$a_9 = a + (9 - 1) d$

$-8 = a + 8d$ ....(2)

On subtracting equation (1) from (2), we obtain

$-12 = 6d$

$d = -2$

From equation (1), we obtain

$4 = a + 2(-2)$

$\Rightarrow a = 8$

Let n^{th} term of this A.P. be zero.

$a_n = a + (n - 1)d$

$0 = 8 + (n - 1)(-2)$

$0 = 8 - 2n + 2$

$2n = 10$

$n = 5$

Hence, 5^{th} term of this A.P. is 0.

OR

Let the amounts invested at 12% and 10% be Rs. x and Rs. y, respectively. Then

SI on Rs. x at 12% p.a. for 1 year = $\frac{x \times 12 \times 1}{100} = \frac{3x}{25}$

SI on Rs. y at 10% p.a. for 1 year = $\frac{y \times 10 \times 1}{100} = \frac{y}{10}$

Total SI = Rs. 1145

$\frac{3x}{25} + \frac{y}{10} = 1145$

$\Rightarrow 6x + 5y = 57250$ ....(1)

SI of Rs. x at 10% p.a. for 1 year = $Rs.\left(\frac{x \times 10 \times 1}{100}\right) = Rs.\frac{x}{10}$

SI of Rs. y at 12% p.a. for 1 year = $Rs.\left(\frac{y \times 12 \times 1}{100}\right) = Rs.\frac{3}{25}y$

Total SI = Rs. (1145 − 90) = 1055

$\frac{x}{10} + \frac{3}{25}y = 1055$

$\Rightarrow 5x + 6y = 52750$ ....(2)

Multiplying equation (1) by 6 and equation (2) by 5, we get

36x + 30y = 343500 ....(3)

25x + 30y = 263750 ....(4)

Subtracting equation (4) from equation (3), we get

11x = 79750

⇒ x = 7250

Putting x = 7250 in equation (1), we get

6(7250) + 5y = 57250

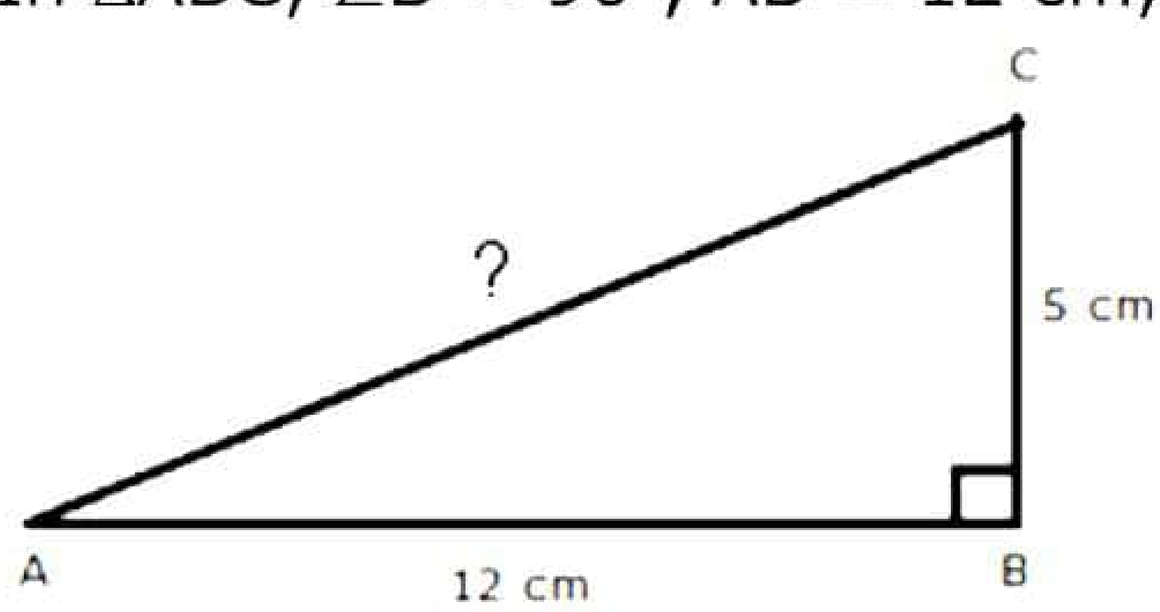
⇒ 43500 + 5y = 57250

⇒ 5y = 13750

⇒ y = 2750

Hence, the amount invested at 12% is Rs. 7250 and that at 10% is Rs. 2750.

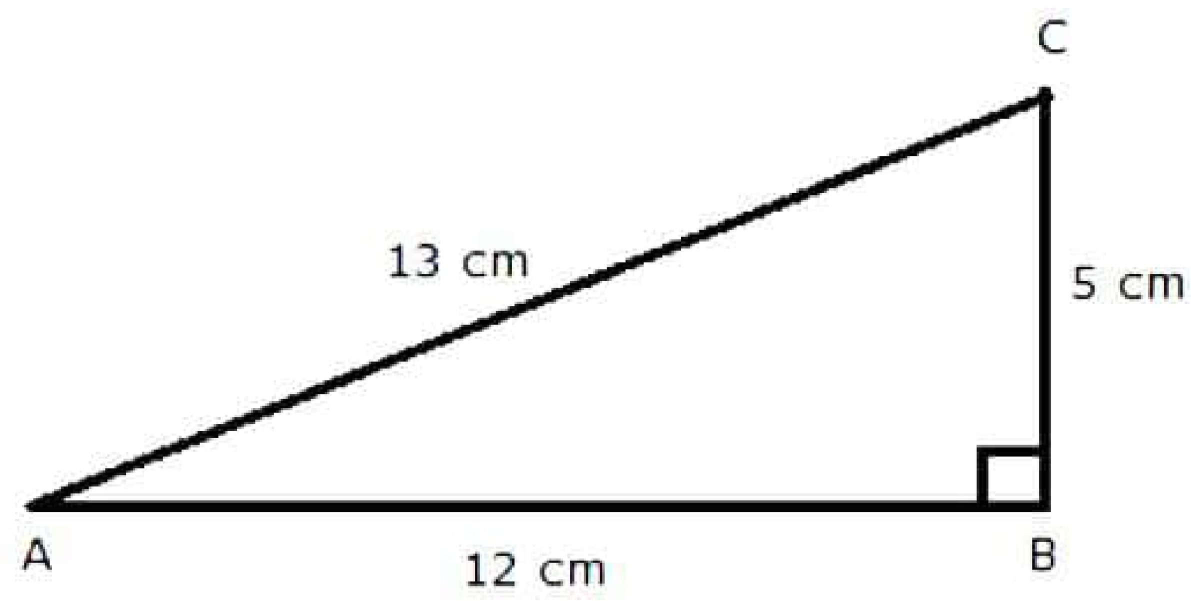
29. In ΔABC, ∠B = 90°, AB = 12 cm, BC = 5 cm



By Pythagoras' theorem, we have

$$AC^2 = (AB)^2 + (BC)^2$$
$$= \left[(12)^2 + (5)^2\right]$$
$$= (144 + 25)\,cm^2$$
$$= 169\,cm^2$$

$\Rightarrow AC = \sqrt{169}\ cm = 13\,cm$



For T-ratio of $\angle A$, we have

Base = AB = 12 cm

Perpendicular = BC = 5 cm and

Hypotenuse = AC = 13 cm

(i) $\cos A = \frac{AB}{AC} = \frac{12}{13}$

(ii) $\operatorname{cosec} A = \frac{AC}{BC} = \frac{13}{5}$

For T-ratio of $\angle C$, we have

Base = BC = 5 cm

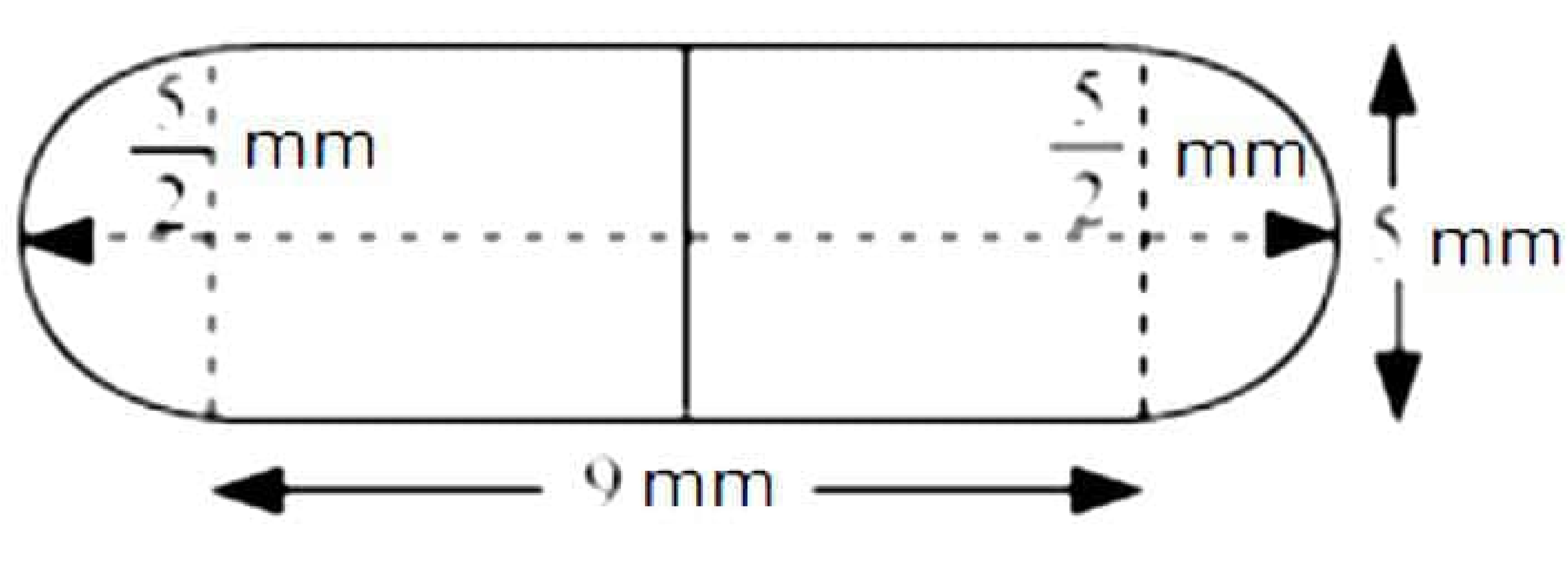
Perpendicular = AB = 12 cm and

Hypotenuse = AC = 13cm

(iii) $\cos C = \frac{BC}{AC} = \frac{5}{13}$

(iv) $\operatorname{cosec} C = \frac{AC}{AB} = \frac{13}{12}$

30.



Radius (r) of cylindrical part = radius (r) of hemispherical part

$= \frac{\text{diameter of the capsule}}{2} = \frac{5}{2}$ mm

Length of cylindrical part (h) = length of the entire capsule – 2 × r

= 14 – 5

= 9 mm

Surface area of capsule = 2 × CSA of hemispherical part + CSA of cylindrical part.

$$= 2 \times 2\pi r^2 + 2\pi rh$$

$$= 4\pi\left(\frac{5}{2}\right)^2 + 2\pi\left(\frac{5}{2}\right)(9)$$

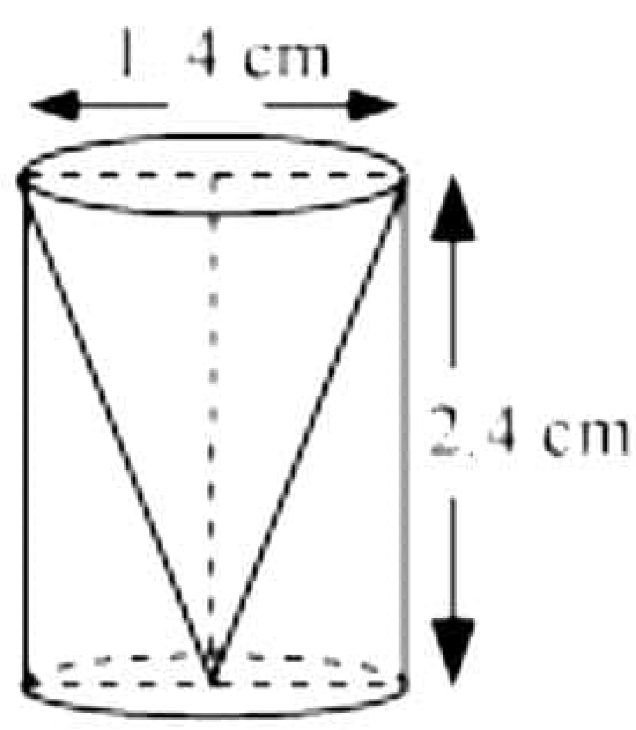
$$= 25\pi + 45\pi$$

$$= 70\pi \text{ mm}^2$$

$$= 70 \times \frac{22}{7}$$

$$= 220 \text{ mm}^2$$

OR



Given that

Height (h) of the conical part = Height (h) of the cylindrical part = 2.4 cm

Diameter of the cylindrical part = 1.4 cm

So, radius (r) of the cylindrical part = 0.7 cm

Slant height (l) of conical part $= \sqrt{r^2 + h^2}$

$$= \sqrt{(0.7)^2 + (2.4)^2}$$

$$= \sqrt{0.49 + 5.76}$$

$$= \sqrt{6.25}$$

$$= 2.5 \text{ cm}$$

Total surface area of the remaining solid

= CSA of cylindrical part + CSA of conical part + Area of cylindrical base

$$= 2\pi rh + \pi rl + \pi r^2$$

$$= 2 \times \frac{22}{7} \times 0.7 \times 2.4 + \frac{22}{7} \times 0.7 \times 2.5 + \frac{22}{7} \times 0.7 \times 0.7$$

= 10.56 + 5.50 + 1.54

= 17.60 cm^2

≈ 18 cm^2

Hence, the total surface area of the remaining solid to the nearest cm^2 is 18 cm^2.

31. Total number of balls = 20

i. Odd numbers are 1, 3, 5, 7, 9, 11, 13, 15, 17, 19.

Total no. of odd numbers = 10

$\therefore$P(getting an odd number) = $\frac{10}{20} = \frac{1}{2}$

ii. Numbers divisible by 2 or 3 are

2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20.

Total no. of numbers divisible by 2 or 3 =13

P(getting a number divisible by 2 or 3) = $\frac{13}{20}$

iii. Prime numbers are 2, 3, 5, 7, 11, 13, 17, 19

Total no. of prime numbers = 8

P(getting a prime number) = $\frac{8}{20} = \frac{2}{5}$

iv. Numbers divisible by 10 are 10, 20.

Number of numbers divisible by 10 = 2

$\therefore$P(getting a number not divisible by 10) = $\left(1 - \frac{2}{20}\right) = \frac{18}{20} = \frac{9}{10}$

Section D

32. Given quadratic equation is $mnx^2 - (m + n)^2x + (m + n)^2 = 0$.

Comparing it with standard form $ax^2 + bx + c = 0$, we have

$a = mn$, $b = -(m + n)^2$ and $c = (m + n)^2$

Using quadratic formula,

$$x = \frac{-\left[-(m+n)^2\right] \pm \sqrt{\left[-(m+n)^2\right]^2 - 4(mn)(m+n)^2}}{2(mn)}$$

$$= \frac{(m+n)^2 \pm \sqrt{(m+n)^4 - 4(mn)(m+n)^2}}{2mn}$$

$$= \frac{(m+n)^2 \pm \sqrt{(m+n)^2\left[(m+n)^2 - 4mn\right]}}{2mn}$$

$$= \frac{(m+n)^2 \pm \sqrt{(m+n)^2(m-n)^2}}{2mn}$$

$$= \frac{(m+n)^2 \pm (m+n)(m-n)}{2mn}$$

$$= \frac{(m+n)\left[(m+n) \pm (m-n)\right]}{2mn}$$

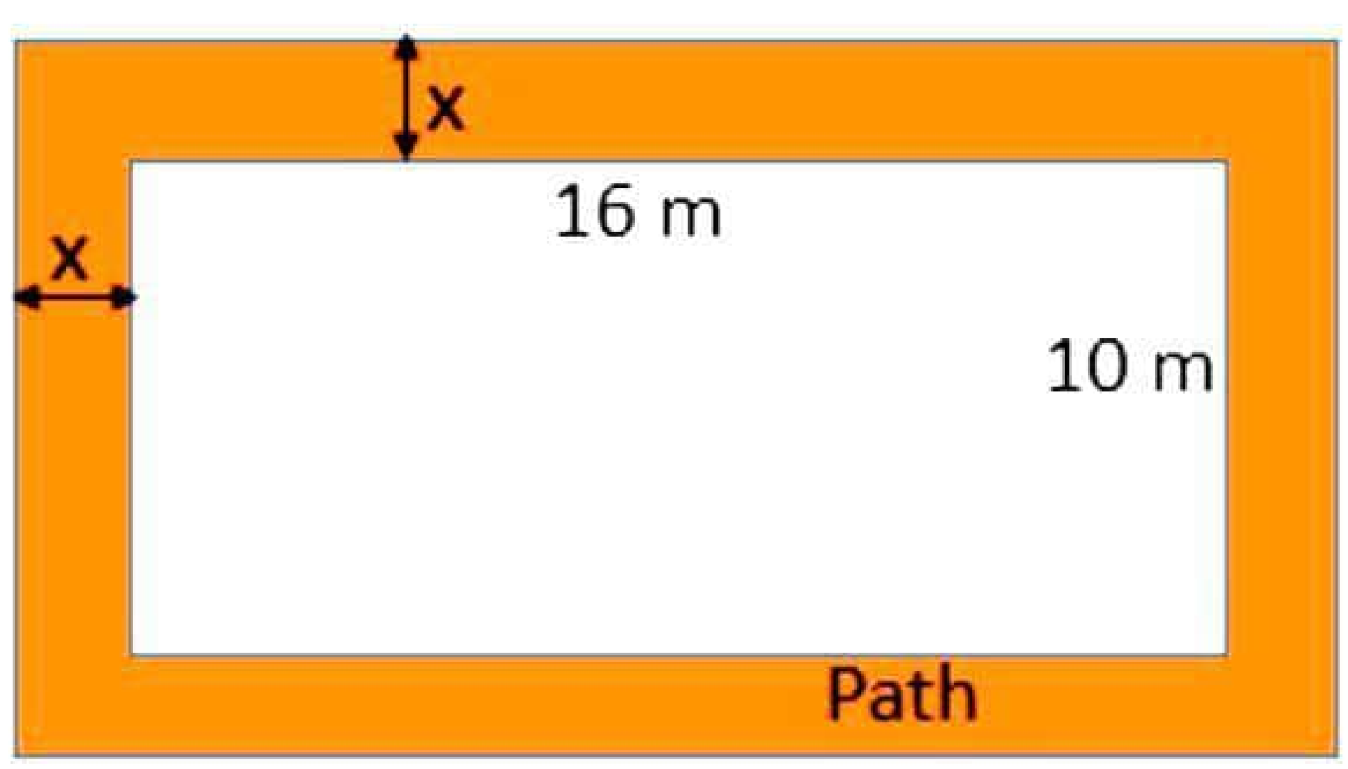
$$\therefore x = \frac{(m+n)}{2mn} \times 2m \text{ or } x = \frac{(m+n)}{2mn} \times 2n$$

$$\Rightarrow x = \frac{(m+n)}{n} \text{ or } x = \frac{(m+n)}{m}$$

OR

Let the width of the path be x metres.

Given, area of the path = 120 m^2



So, we have

$(16 + 2x)(10 + 2x) - 16 \times 10 = 120$

$\Rightarrow (160 + 32x + 20x + 4x^2) - 160 = 120$

$\Rightarrow 52x + 4x^2 = 120$

$\Rightarrow 13x + x^2 = 30$

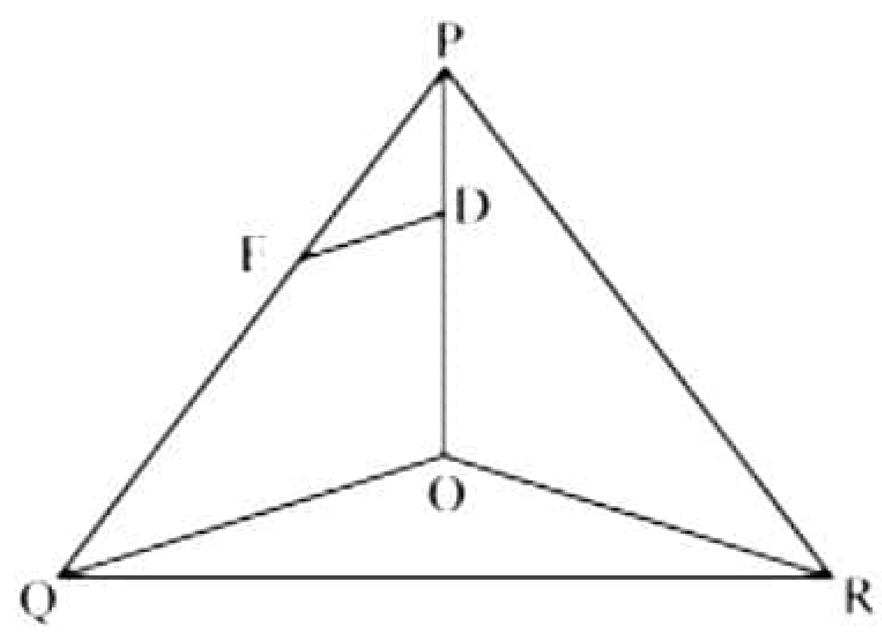
$\Rightarrow x^2 + 13x - 30 = 0$

$\Rightarrow x^2 + 15x - 2x - 30 = 0$

$\Rightarrow x(x + 15) - 2(x + 15) = 0$

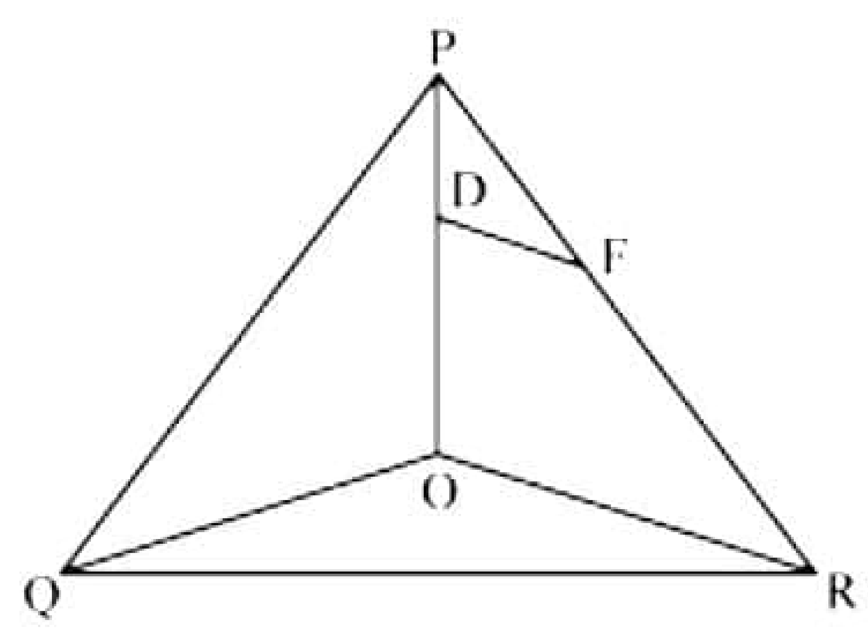
⇒ (x + 15) = 0 or (x − 2) = 0
⇒ x = −15 or x = 2
But, width cannot be negative.
⇒ x ≠ −15
∴ x = 2
Hence, the width of the path is 2 m.

33.



In △POQ, DE || OQ

$\therefore \frac{PE}{EQ} = \frac{PD}{DO}$ ....(1) [By Thales Theorem]

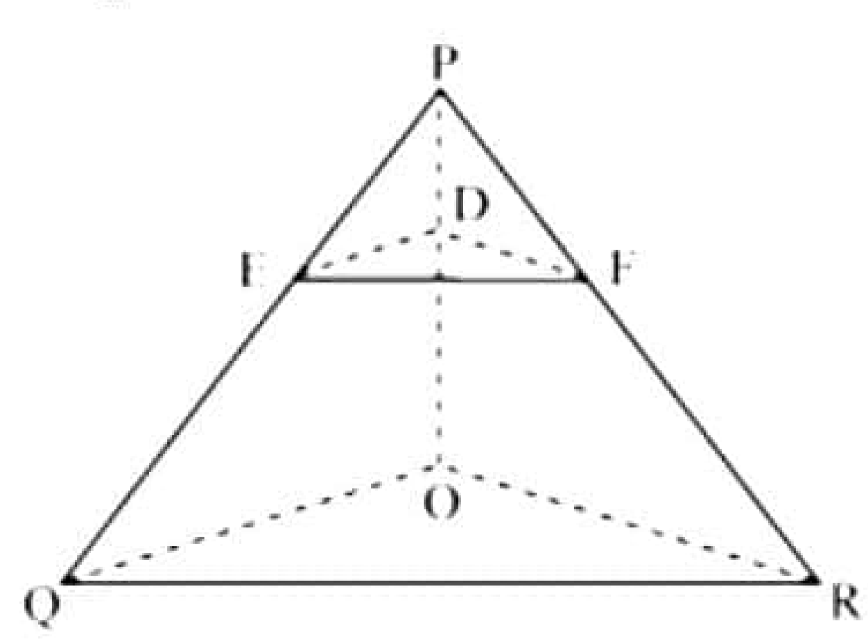


In △POR, DF || OR

$\therefore \frac{PF}{FR} = \frac{PD}{DO}$ ....(2) [By Thales Theorem]

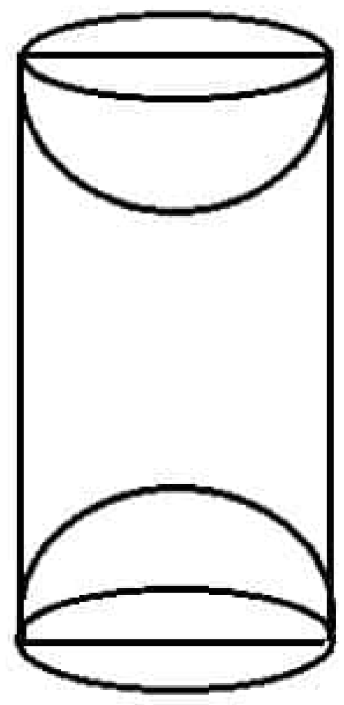
From equations (1) and (2),

$\frac{PE}{EQ} = \frac{PF}{FR}$



Then, by converse of Thales theorem,
EF || QR.

34.



Radius of a wooden cylinder = 7 cm

Height of the wooden cylinder = 12 cm

Lateral surface area of cylinder = $2\pi rh$

$= 2\pi \times 7 \times 12\ cm^2$

$= 168\pi\ cm^2$

Radius of the hemisphere = 7 cm

Surface area of two hemispheres = $2 \times 2\pi r^2$

$= 2 \times 2\pi \times 7 \times 7\ cm^2$

$= 196\pi\ cm^2$

Total surface area of the wooden article = $(168 + 196)\pi\ cm^2$

$= 364\pi\ cm^2$

$= 364 \times \frac{22}{7}\ cm^2$

$= 1144\ cm^2$

Further, the volume of the cylinder = $\pi r^2 h$

$= \pi \times 7 \times 7 \times 12\ cm^2$

$= 588\pi\ cm^3$

Volume of two hemispheres = $2 \times \frac{2}{3}\pi r^3$

$= 2 \times \frac{2}{3}\pi \times 7^3\ cm^3$

$= 457.33\pi\ cm^3$

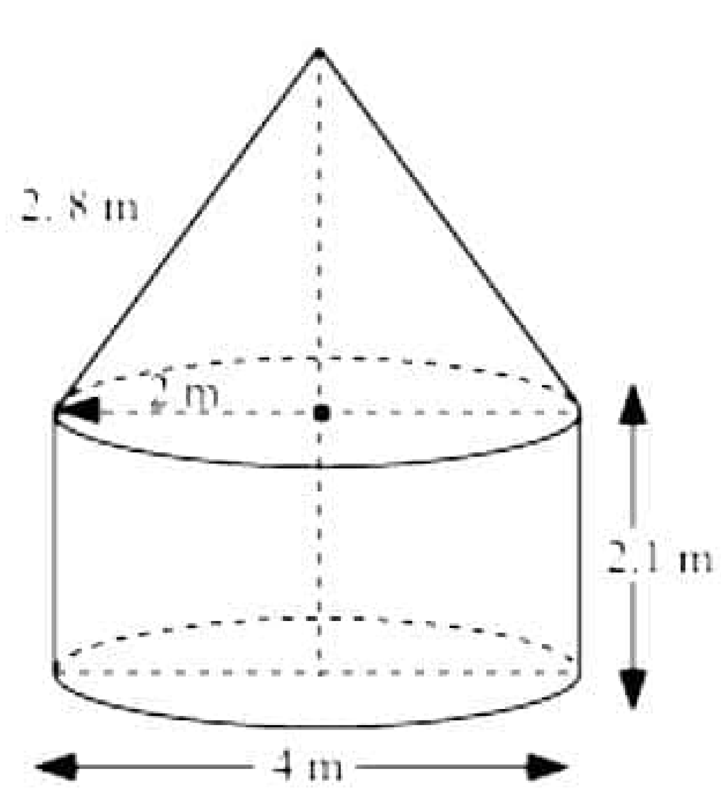
Volume of the wood left in the article = $(588 - 457.33)\pi\ cm^3$

$= 130.67\pi\ cm^3$

$= 130.67 \times \frac{22}{7}\ cm^3$

$= 410.67\ cm^3$

OR



Given that
Height (h) of the cylindrical part = 2.1 m
Diameter of the cylindrical part = 4 m
So, radius of the cylindrical part = 2 m
Slant height (l) of conical part = 2.8 m
Area of canvas used = CSA of conical part + CSA of cylindrical part

$$= \pi rl + 2\pi rh$$
$$= \pi \times 2 \times 2.8 + 2\pi \times 2 \times 2.1$$
$$= 2\pi[2.8 + 4.2]$$
$$= 2 \times \frac{22}{7} \times 7$$
$$= 44 \text{ m}^2$$

Now, cost of 1 m^2 canvas = Rs. 600
Then, cost of 44 m^2 canvas = 44 × 600 = Rs. 26,400
So, it will cost Rs. 26,400 for making a tent.

35.

Number of mangoes	Number of boxes (f_i)
50−52	15
53−55	110
56−58	135
59−61	115
62−64	25

We may observe that class intervals are not continuous. There is a gap of 1 between two class intervals. So we have to add $\frac{1}{2}$ to upper class limit and subtract $\frac{1}{2}$ from lower class limit of each interval.

And class mark (x_i) may be obtained by using the relation

$$x_i = \frac{\text{upper class limit} + \text{lower class limit}}{2}$$

Now taking 57 as assumed mean (A), we may calculate d_i, u_i, f_iu_i as follows:

Classes	Frequency f_i	Class-mark x_i	$d_i = x_i - A$ $= x_i - 57$	$u_i = \frac{x_i - 57}{h}$ $= \frac{d_i}{h}$	f_iu_i
49.5−52.5	15	51	−6	−2	−30
52.5−55.5	110	54	−3	−1	−110
55.5−58.5	135	57	0	0	0
58.5−61.5	115	60	3	1	115
61.5−64.5	25	63	6	2	50
Total	$\sum f_i = 400$				$\sum f_iu_i = 25$

Class size (h) of this data = 3

$$\text{Mean} = A + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$$

$$= 57 + \left(\frac{25}{400}\right) \times 3$$

$$= 57 + \frac{3}{16}$$

$= 57 + 0.1875$

$= 57.1875$

≈ 57

Therefore, the mean number of mangoes kept in a packing box is 57.

Section E

36.

i. Here, $a = 1000$ and $d = 100$

This is an A.P.

Therefore, $a_{30} = a + (30 - 1)d$

$\Rightarrow a_{30} = 1000 + 29(100) = 1000 + 2900 =$ Rs. 3900

ii. $a_{40} = a + 39d = 1000 + 3900 =$ Rs. 4900

iii. Here, $a_{19} = 1000 + 1800 = 2800$

and $a_{28} = 1000 + 2700 = 3700$

$2800:3700 = 28:37$

OR

Here, $a = 1000$ and $d = 100$

$$S_{30} = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{30}{2}\left[2(1000) + (30-1)100\right] = \text{Rs. } 73500$$

37.

i. From the graph, the coordinates of points A and C are (1, 1) and (4, 5) respectively.

$\therefore d(AC) = \sqrt{(4-1)^2 + (5-1)^2} = 5$ km

OR

From the graph, the coordinates of points B and D are (4, 1) and (7, 5) respectively.

$\therefore d(BD) = \sqrt{(7-4)^2 + (5-1)^2} = 5$ km

ii. From the graph, the coordinates of points B and A are (4, 1) and (1, 1) respectively.

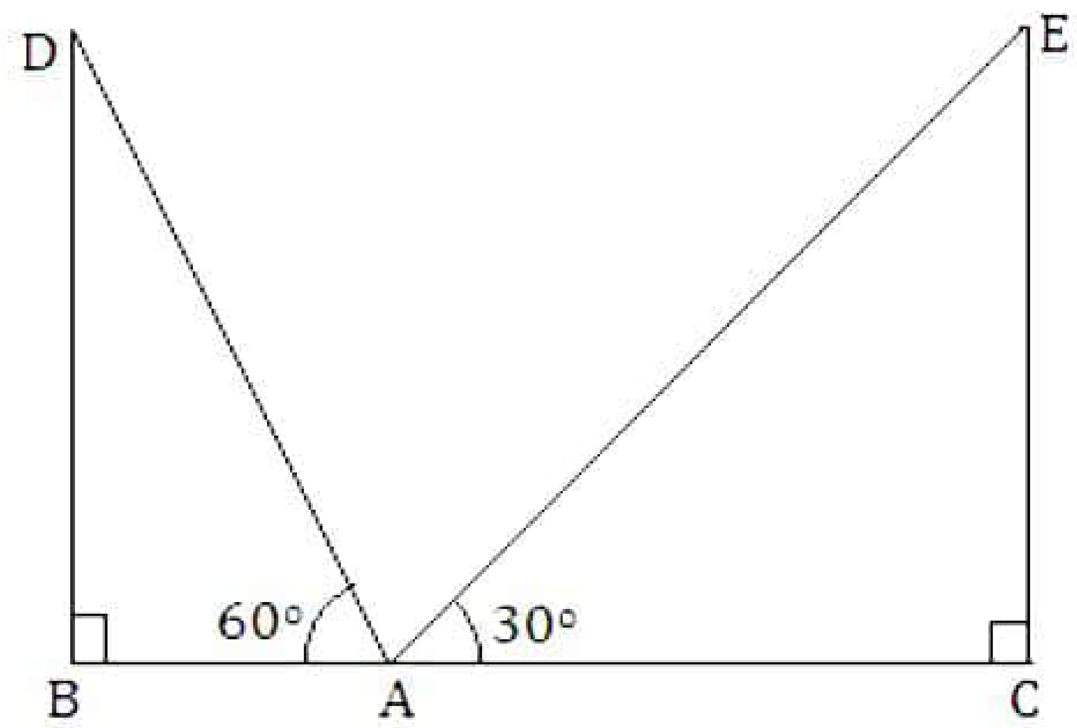
$\therefore d(BA) = \sqrt{(1-4)^2 + (1-1)^2} = 3$ km

iii. From the graph, the coordinates of points B and C are (4, 1) and (4, 5) respectively.

$\therefore d(BC) = \sqrt{(4-4)^2 + (5-1)^2} = 4$ km

38.

i. Let BD and CE be the two poles.



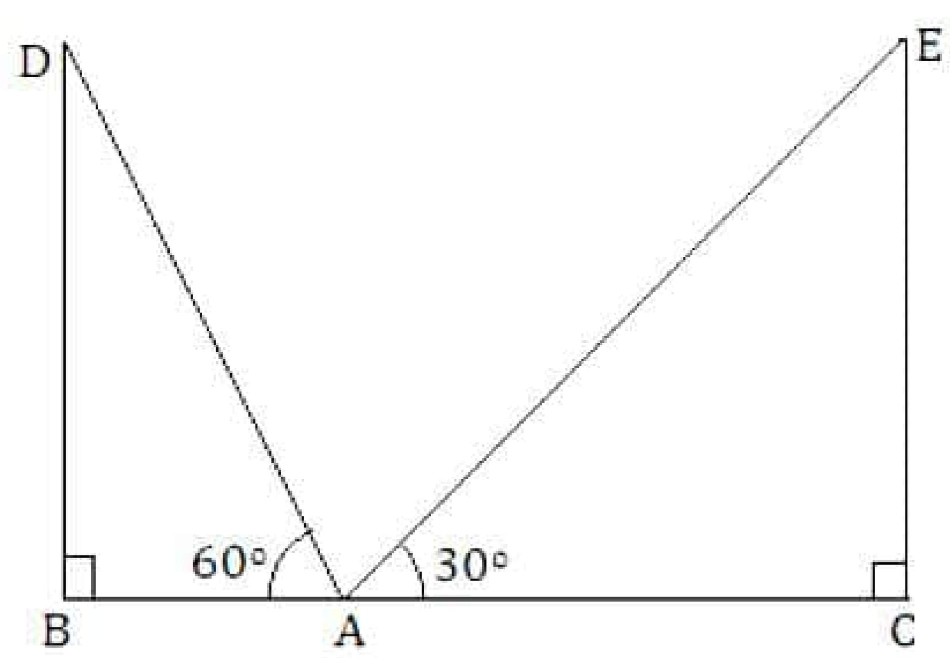
In ΔABD, we have

$$\tan 60^{o} = \frac{BD}{AB}$$

$$\Rightarrow \sqrt{3} = \frac{30}{AB}$$

$$\Rightarrow AB = 10\sqrt{3}\ m$$

OR



In ΔACE, we have

$$\tan 30^{o} = \frac{EC}{AC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{30}{AC}$$

$$\Rightarrow AC = 30\sqrt{3}\ m$$

ii. Width of the road = BC = BA + AC = $10\sqrt{3} + 30\sqrt{3} = 40\sqrt{3}$ m

iii. Angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level is known as the angle of elevation.