Class X Session 2023-24
Subject - Mathematics (Standard)
Sample Question Paper - 4

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub- parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

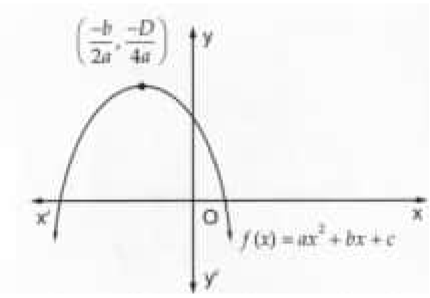
Section A

1. The largest number which divides 245 and 1029 leaving remainder 5 in each case is **[1]**

a) 8 b) 12

c) 4 d) 16

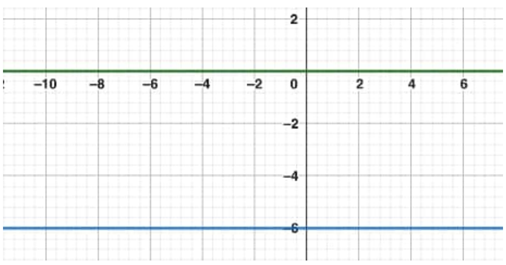
2. If the diagram in Fig. shows the graph of the polynomial f(x) = ax^2 + bx + c, then **[1]**



a) a < 0, b < 0 and c < 0 b) a < 0, b > 0 and c > 0

c) a < 0, b < 0 and c > 0 d) a < 0, b > 0 and c < 0

3. The pair of linear equations y = 0 and y = - 6 has: **[1]**



a) no solution

b) only solution (0, 0)

c) infinitely many solutions

d) a unique solution

4. 500 bananas were divided equally among a certain number of students. If there were 25 more students, each would have received one banana less. Then the number of students is **[1]**

a) 500

b) 125

c) 250

d) 100

5. The first and last terms of an A.P. are 1 and 11. If their sum is 36, then the number of terms will be **[1]**

a) 7

b) 5

c) 8

d) 6

6. The distance of a point from the x-axis is called **[1]**

a) None of these

b) origin

c) abscissa

d) ordinate

7. The ratio in which the line segment joining P(x_1, y_1) and Q(x_2, y_2) is divided by x-axis is **[1]**

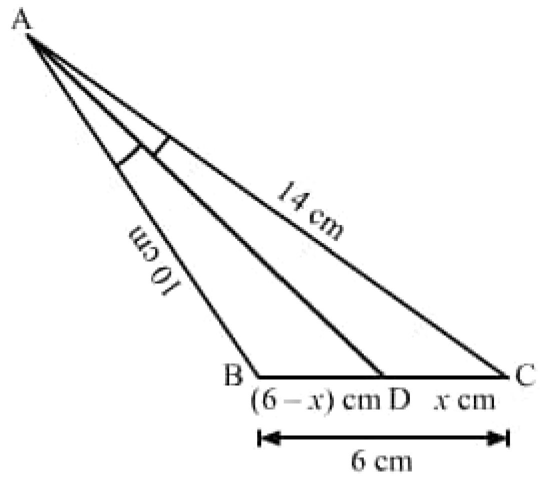
a) $y_1 : y_2$

b) $-y_1 : y_2$

c) $-x_1 : x_2$

d) $x_1 : x_2$

8. In a $\triangle$ABC, it is given that AD is the internal bisector of $\angle$A. If AB = 10 cm, AC = 14 cm and BC = 6 cm, the CD = ? **[1]**



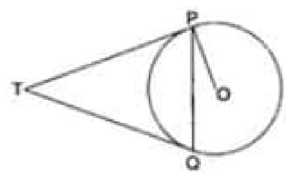
a) 3.5 cm

b) 7 cm

c) 4.8 cm

d) 10.5 cm

9. In the figure, two tangents TP and TQ are drawn to a circle with centre O from an external point T. Then $\angle$PTQ = **[1]**



a) $\angle$OPQ

b) $2\angle$OPQ

c) $4\angle$OPQ

d) $\frac{1}{2}\angle$OPQ

10. From a point Q, the length of tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is **[1]**

a) 8 cm
b) 10 cm
c) 6 cm
d) 7 cm

11. $1 + \frac{\cot^2\alpha}{1+\cos ec\alpha} =$ **[1]**

a) sinα
b) secα
c) cosecα
d) tanα

12. $\frac{\sin\theta}{1-\cot\theta} + \frac{\cos\theta}{1-\tan\theta}$ is equal to **[1]**

a) sinθ + cosθ
b) sinθ - cosθ
c) 0
d) 1

13. The tops of two poles of height 16 m and 10 m are connected by a wire. If the wire makes an angle of 30^o with the horizontal, then the length of the wire is **[1]**

a) 12 m
b) $10\sqrt{3}$ m
c) 16 m
d) 10 m

14. If a chord of a circle of radius 28 cm makes an angle of 90^o at the centre, then the area of the major segment is **[1]**

a) 1456 cm^2
b) 1848 cm^2
c) 392 cm^2
d) 2240 cm^2

15. The length of the minute hand of a clock is 21 cm. The area swept by the minute hand in 10 minutes is **[1]**

a) 252 cm^2
b) 126 cm^2
c) 231 cm^2
d) 210 cm^2

16. From a well-shuffled deck of 52 cards, one card is drawn at random. What is the probability of getting a queen? **[1]**

a) None of these
b) $\frac{4}{39}$
c) $\frac{1}{13}$
d) $\frac{1}{26}$

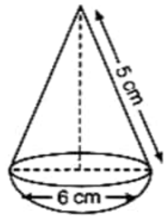
17. The probability of getting a sum of 13 in a single throw of two dice is **[1]**

a) $\frac{5}{6}$
b) $\frac{1}{6}$
c) 0
d) 1

18. The arithmetic mean of 1, 2, 3, 4, ..., n is: **[1]**

a) $\frac{n-1}{2}$
b) $\frac{n(n+1)}{2}$
c) $\frac{n}{2}$
d) $\frac{n+1}{2}$

19. **Assertion (A):** The given figure represents a hemisphere surmounted by a conical block of wood. The diameter of their bases is 6 cm each and the slant height of the cone is 5 cm. The volume of the solid is 196 cm^3 **[1]**



Reason (R): The volume hemisphere is given by $\frac{2}{3}\pi r^3$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Common difference of the AP -5, -1, 3, 7, ... is 4. **[1]**

Reason (R): Common difference of the AP a, a + d, a + 2d, ... is given by d = 2nd term - 1st term.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Find the LCM and HCF of the pairs of integers 336 and 54 and verify that LCM $\times$ HCF = product of the two numbers. **[2]**

22. E and F are points on the sides PQ and PR respectively of a $\triangle$PQR. For PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm case, state whether EF || QR. **[2]**

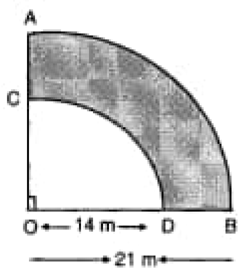
23. From a point P, 10 cm away from the centre of a circle, a tangent PT of length 8 cm is drawn. Find the radius of the circle. **[2]**

24. Simplify: $\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$ **[2]**

OR

If θ be an acute angle and 5 cosecθ = 7, then evaluate sinθ + cos$^2\theta$ -1.

25. ABCD is a flower bed. If OA =21 m and OC = 14 m, find the area of the bed. **[2]**



OR

Find the area of a quadrant of a circle , whose circumference is 22 cm .

Section C

26. Mika exercises every 12 days and Nanu every 8 days. Mika and Nanu both exercised today. How many days will it be until they exercise together again? **[3]**

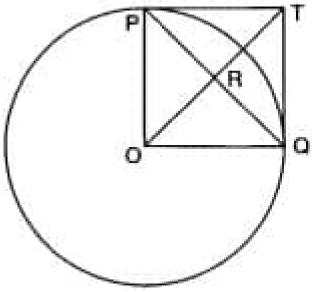
27. Find the zeroes of the polynomial 7y^2 - $\frac{11}{3}y - \frac{2}{3}$ by factorisation method and verify the relationship between the zeroes and coefficient of the polynomial. **[3]**

28. Is the pair of linear equation consistent/inconsistent? If consistent, obtain the solution graphically: x + y = 5, 2x + 2y = 10 **[3]**

OR

If the numerator of a fraction is multiplied by 2 and the denominator is reduced by 5 the fraction becomes $\frac{6}{5}$. And, if the denominator is doubled and the numerator is increased by 8, the fraction becomes $\frac{2}{5}$. Find the fraction.

29. In figure $PO \perp QO$. The tangents to the circle at P and Q intersect at a point T. Prove that PQ and OT are right bisectors of each other. **[3]**



OR

A quadrilateral is drawn to circumscribed a circle. Prove that the sum of opposite sides are equal.

30. If $\sin\theta = \frac{12}{13}$, find the value of $\frac{\sin^2\theta - \cos^2\theta}{2\sin\theta\cos\theta} \times \frac{1}{\tan^2\theta}$. **[3]**

31. Find the mean of the following frequency distribution: **[3]**

Class interval	0-6	6-12	12-18	18-24	24-30
Frequency	6	8	10	9	7

Section D

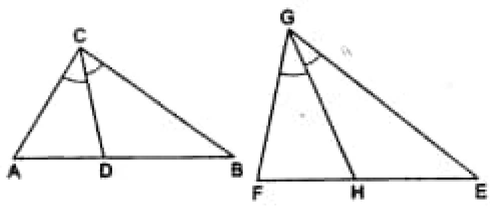
32. If the difference between the radii of the smaller circle and the larger circle is 7 cm and the difference between the areas of the two circles is 1078 sq. cm. Find the radius of the smaller circle. **[5]**

OR

Two pipes running together can fill a tank in $11\frac{1}{9}$ minutes. If one pipe takes 5 minutes more than the other to fill the tank, find the time in which each pipe would fill the tank separately.

33. In the given figure, CD and GH are respectively the bisectors of C and G respectively. If, $\Delta ABC \sim \Delta FEG$, prove that: **[5]**

a. $\triangle ADC \sim \triangle FHG$
b. $\triangle BCD \sim \triangle EGH$
c. $\frac{CD}{GH} = \frac{AC}{FG}$



34. A spherical glass vessel has a cylindrical neck 8 cm long and 1 cm in radius. The radius of the spherical part is 9 cm. Find the amount of water (in litres) it can hold, when filled completely. **[5]**

OR

A solid is in the shape of a right-circular cone surmounted on a hemisphere, the radius of each of them is being 3.5 cm and the total height of solid is 9.5 cm. Find the volume of the solid.

35. Compute the median from the following data: **[5]**

Marks	0 - 7	7 - 14	14 - 21	21 - 28	28 - 35	35 - 42	42 - 49
Number of students	3	4	7	11	0	16	9

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Your friend Varun wants to participate in a 200m race. He can currently run that distance in 51 seconds and with each day of practice it takes him 2 seconds less. He wants to do in 31 seconds.

(i) Write first four terms are in AP for the given situations.

(ii) What is the minimum number of days he needs to practice till his goal is achieved?

OR

Out of 41, 30, 37 and 39 which term is not in the AP of the above given situation?

(iii) How many second takes after 5^{th} days?

37. **Read the text carefully and answer the questions:** **[4]**

Using Cartesian Coordinates we mark a point on a graph by how far along and how far up it is.

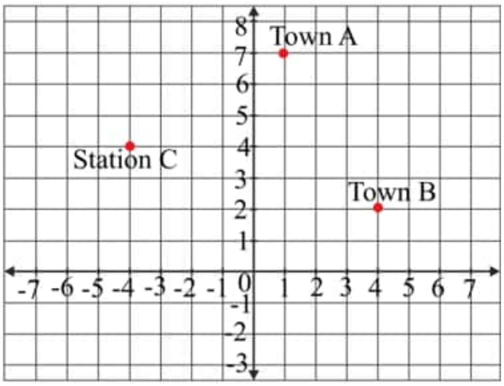
The left-right (horizontal) direction is commonly called X-axis.

The up-down (vertical) direction is commonly called Y-axis.

When we include negative values, the x and y axes divide the space up into 4 pieces.

Read the information given above and below:

Two friends Veena and Arun work in the same office in Delhi. In the Christmas vacations, both decided to go their hometowns represented by Town A and Town B respectively in the figure given below. Town A and Town B are connected by trains from the same station C (in the given figure) in Delhi.



(i) Who will travel more distance to reach their home?

(ii) Find the location of the station.

OR

Find the distance between Town A and Town B.

(iii) Find in which ratio Y-axis divide Town B and Station.

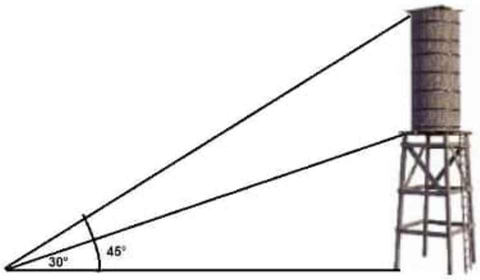
38. **Read the text carefully and answer the questions:** **[4]**

In a society, there are many multistory buildings. The RWA of the society wants to install a tower and a water tank so that all the households can get water without using water pumps.

For this they have measured the height of the tallest building in the society and now they want to install a tower that will be taller than that so that the level of water must be higher than the tallest building in their society. Here is one solution they have found and now they want to check if it will work or not.

From a point on the ground 40 m away from the foot of a tower, the angle of elevation of the top of the tower is

300. the angle of elevation of the top of the water tank is 45^{o}.



(i) What is the height of the tower?

(ii) What is the height of the water tank?

OR

What will be the angle of elevation of the top of the water tank from the place at $\frac{40}{\sqrt{3}}$m from the bottom of the tower.

(iii) At what distance from the bottom of the tower the angle of elevation of the top of the tower is 45^{o}.

Solution

Section A

1.

(d) 16

Explanation: Let us subtract 5 (the remainder) from each number in order to find their HCF.

245 - 5 = 240

1029 - 5 = 1024

Now, Let us find HCF of 240, 1024

1024 = 240 $\times$ 4 + 64

240 = 64 $\times$ 3 + 48

64 = 48 $\times$ 1 + 16

48 = 16 $\times$ 3 + 0

The largest number which divides 245 and 1029 leaving remainder 5 in each case is 16.

2.

(c) a < 0, b < 0 and c > 0

Explanation: Clearly, $f(x) = ax^2 + bx + c$ represent a parabola opening downwards.

Clearly a < 0

Let, y = ax^2 + bx + c cuts y-axis at P which lies on OY.

Putting x = 0 in $y = ax^2 + bx + c$, we get y = c. So the coordinates of P are $(0, c)$.

Clearly, P lies on OY. Therefore c > 0

The vertex $\left(\frac{-b}{2a}, \frac{-D}{4a}\right)$ of the parabola is in the second quadrant.

Therefore, $\frac{-b}{2a} < 0$, b < 0

Therefore a < 0, b < 0 and c > 0.

3. **(a)** no solution

Explanation: Since, we have y = 0 and y = -6 are two parallel lines.

therefore, no solution exists.

4.

(d) 100

Explanation: Let the number of students be x

$\therefore$ Each student would get = $\frac{500}{x}$ bananas

$\therefore$ If there were 25 more students, then each student would get = $\frac{500}{x+25}$ bananas

According to question, $\frac{500}{x} - \frac{500}{x+25}$ = 1

$\Rightarrow \frac{500x+12500-500x}{x(x+25)}$ = 1

$\Rightarrow \frac{12500}{x^2+25x}$ = 1

$\Rightarrow$ x^2 + 25x -12500 = 0

$\Rightarrow$ x^2 + 125x - 100x - 12500 = 0

$\Rightarrow$ x(x + 125) - 100(x + 125) = 0

$\Rightarrow$ (x + 125)(x - 100) = 0

$\Rightarrow$ x + 125 = 0 and x - 100 = 0

$\Rightarrow$ x = -125 and x = 100 [x = -125 is not possible]

Therefore, the number of students is 100

5.

(d) 6

Explanation: Given: $a = 1, l = 11$ and $S_n = 36$

$\therefore S_n = \frac{n}{2}(a + l)$

$\Rightarrow 36 = \frac{n}{2}(1 + 11)$

$\Rightarrow 72 = n \times 12$

$\Rightarrow n = 6$

6.

(d) ordinate

Explanation: The distance of a point from the x-axis is the y (vertical) coordinate of the point and is called ordinate.

7.

(b) $-y_1 : y_2$

Explanation: Let a point A on x-axis divides the line segment joining the points $P(x_1, y_1)$ $Q(x_2, y_2)$ in the ratio $m_1 : m_2$ and let co-ordinates of A be (x, 0)

$\therefore 0 = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \Rightarrow 0 = m_1 y_2 + m_2 y_1$

$\Rightarrow m_1 y_2 = -m_2 y_1 \Rightarrow \frac{m_1}{m_2} = \frac{-y_1}{y_2}$

$\therefore$ Ratio is $-y_1 : y_2$

8. **(a)** 3.5 cm

Explanation: By using angle bisector theore in $\triangle$ABC, we have

$\frac{\mathrm{AB}}{\mathrm{AC}} = \frac{\mathrm{BD}}{\mathrm{DC}}$

$\Rightarrow \frac{10}{14} = \frac{6-x}{x}$

$\Rightarrow 10x = 84 - 14x$

$\Rightarrow 24x = 84$

$\Rightarrow x = 3.5$

Hence, the correct answer is 3.5.

9.

(b) $2\angle \mathrm{OPQ}$

Explanation: Since, tangents from an external point to a circle are equal.

$\therefore$ TP = TQ $\Rightarrow \angle$TQP = $\angle$TPQ [Angle opposite to equal sides]……(i)

Now, since tangent is perpendicular to the radius through the point of contact.

$\therefore \angle$OPT = 90°

$\Rightarrow \angle$OPQ + $\angle$TPQ = 90° $\Rightarrow \angle$TPQ = 90° $- \angle \mathrm{OPQ}$ …….(ii)

In triangle TPQ,

$\angle \mathrm{PTQ} + \angle \mathrm{TPQ} + \angle \mathrm{TQP} = 180^\circ$

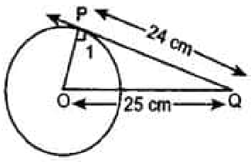
$\Rightarrow \angle \mathrm{PTQ} + 2\angle \mathrm{TPQ} = 180^\circ$ [From eq. (i)]

$\Rightarrow \angle \mathrm{PTQ} + 2\left(90^\circ - \angle \mathrm{OPQ}\right) = 180^\circ$ [From eq. (ii)]

$\Rightarrow \angle \mathrm{PTQ} = 2\angle \mathrm{OPQ}$

10.

(d) 7 cm

Explanation: 

Here $\angle$OPQ = 90° [Tangent makes right angle with the radius at the point of contact]

in right angled triangle OPQ

$\therefore OQ^2 = OP^2 + PQ^2 \Rightarrow (25)^2 = OP^2 + (24)^2$

$\Rightarrow OP^2 = 625 - 576$

$\Rightarrow$OP = 7 cm Therefore, the radius of the circle is 7 cm

11.

(c) cosecα

Explanation: $1 + \frac{\cot^2\alpha}{1+\cos ec\alpha}$

$= 1 + \frac{cosec^2\alpha - 1}{1+\cos ec\alpha}$

$= 1 + \frac{(cosec\,\alpha - 1)(cosec\,\alpha + 1)}{1 + cosec\alpha}$

= 1 + cosecα - 1 = cosec α

12. **(a)** sinθ + cosθ

Explanation: We have, $\frac{\sin\theta}{1-\cot\theta}+\frac{\cos\theta}{1-\tan\theta}=\frac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}}$

$=\frac{\sin\theta\times\sin\theta}{\sin\theta-\cos\theta}+\frac{\cos\theta\times\cos\theta}{\cos\theta-\sin\theta}$

$=\frac{\sin^2\theta}{\sin\theta-\cos\theta}-\frac{\cos^2\theta}{\sin\theta-\cos\theta}$

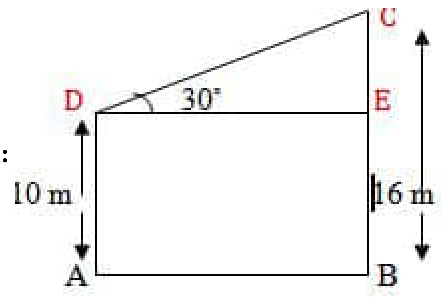
$=\frac{\sin^2\theta-\cos^2\theta}{\sin\theta-\cos\theta}$

$=\frac{(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)}{\sin\theta-\cos\theta}$

$=\sin\theta+\cos\theta$

13. **(a)** 12 m

Explanation:



Given: Two poles BC = 16 m and AD = 10 m

And $\angle\mathrm{CDE}=30^{\circ}$

To find: Length of wire CD = x

$\therefore$ In triangle CDE,

$\sin 30^{\circ}=\frac{\mathrm{CE}}{\mathrm{CD}}$

$\Rightarrow\frac{1}{2}=\frac{\mathrm{BC}-\mathrm{BE}}{\mathrm{CD}}$

$\Rightarrow\frac{1}{2}=\frac{16-10}{x}$

$\Rightarrow\frac{1}{2}=\frac{6}{x}$

$\Rightarrow x=12\ \mathrm{m}$

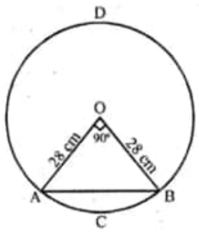
Therefore, the length of the wire is 12 m.

14.

(d) 2240 cm^2

Explanation: A chord AB makes an angle of 90° at the centre

Radius of the circle = 28 cm



Area of minor segment ACB

$=\pi r^2\times\frac{\theta}{360^{\circ}}$ - area of $\triangle AOB$

$=\pi r^2\times\frac{90^{\circ}}{360^{\circ}}-\frac{1}{2}\mathrm{OA}\times\mathrm{OB}$

$=\frac{1}{4}\pi r^2-\frac{1}{2}\times r^2$

$=\frac{1}{4}\times\frac{22}{7}\times 28\times 28-\frac{1}{2}\times 28\times 28$

= 616 - 392

= 224 cm^2

$\therefore$ Area of the major segment ADB

= Area of circle - area of minor segment

$=\pi r^2-224=\frac{22}{7}\times 28\times 28-224$

= 2464 - 224

= 2240 sq. cm

15.

(c) 231 cm^2

Explanation: Area swept by minute hand in 60 minutes = πR^2

Area swept by it in 10 minutes

$= \left(\frac{\pi R^2}{60} \times 10\right) \text{cm}^2 = \left(\frac{22}{7} \times 21 \times 21 \times \frac{1}{6}\right) \text{cm}^2$

= 231 cm^2

16.

(c) $\frac{1}{13}$

Explanation: Number of all possible outcomes = 52.

Number of queens = 4.

$\therefore$P (getting a queen) = $\frac{4}{52} = \frac{1}{13}$

17.

(c) 0

Explanation: Elementary events are

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

$\therefore$ Number of Total outcomes = 36

And Number of possible outcomes (sum of numbers appearing on die is 13) = 0

$\therefore$ Required Probability = $\frac{0}{36} = 0$

18.

(d) $\frac{n+1}{2}$

Explanation: According to question,

Arithmetic Mean = $\frac{1+2+3+...+n}{n}$

= $\frac{\frac{n(n+1)}{2}}{n}$

= $\frac{n+1}{2}$

19.

(d) A is false but R is true.

Explanation: A is false but R is true.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Common difference, d = -1 - 1(-5) = 4

So, both A and R are true and R is the correct explanation of A.

Section B

21. 336 and 54

336 = 2 × 2 × 2 × 2 × 3 × 7 = 2^4 × 3 × 7

54 = 2 × 3 × 3 × 3 = 2 × 3^3

HCF = 2 × 3 = 6

LCM = 2^4 × 3^3 × 7 = 3024

Product of two numbers 336 and 54 = 336 × 54 = 18144

HCF × LCM = 6 × 3024 = 18144

Hence, product of two numbers = HCF × LCM`

22. We have

$\frac{PE}{EQ} = \frac{3.9}{3} = \frac{1.3}{1} \ldots\ldots (I)$

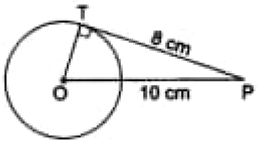
$\frac{PF}{FR} = \frac{3.6}{2.4} = \frac{3}{2} = \frac{1.5}{1} \ldots\ldots (II)$

From (I) and (II),

we get

$\frac{PE}{EQ} \neq \frac{PF}{FR}$

Therefore, EF is not parallel to QR. (By converse of basic proportionality theorem)

23. 

Let O be the centre of the given circle.

Then, OP = 10 cm. Also, PT = 8 cm.

Join OT.

Now, PT is a tangent at T and OT is the radius through the point of contact T.

$\therefore \quad OT \perp PT$

In the right ΔOTP

we have

$OP^2 = OT^2 + PT^2$ [by Pythagoras' theorem]

$\Rightarrow$ OT = $\sqrt{OP^2 - PT^2} = \sqrt{(10)^2 - (8)^2}\text{cm} = \sqrt{36}\text{cm}$ = 6cm.

Hence, the radius of the circle is 6 cm.

24. $\frac{\sin 30^\circ + \tan 45^\circ - cosec 60^\circ}{sec 30^\circ + \cos 60^\circ + cot 45^\circ}$

$\frac{\frac{1}{2}+1-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1} = \frac{\frac{\sqrt{3}+2\sqrt{3}-4}{2\sqrt{3}}}{\frac{4+\sqrt{3}+2\sqrt{3}}{2\sqrt{3}}}$

$= \frac{3\sqrt{3}-4}{3\sqrt{3}+4} = \frac{3\sqrt{3}-4}{3\sqrt{3}+4} \times \frac{3\sqrt{3}-4}{3\sqrt{3}-4}$

$= \frac{(3\sqrt{3}-4)^2}{(3\sqrt{3})^2-(4)^2}$

$= \frac{27+16-24\sqrt{3}}{27-16} = \frac{43-24\sqrt{3}}{11}$

OR

Given, $5 cosec\theta = 7$

or, $cosec\theta = \frac{7}{5}$

or, $\sin\theta = \frac{5}{7} [\because \text{cosec}\theta = \frac{1}{\sin\theta}]$

$\sin\theta + \cos^2\theta - 1 = \sin\theta - (1 - \cos^2\theta)$

$= \sin\theta - \sin^2\theta \, (\because \sin^2\theta + \cos^2\theta = 1)$

$= \frac{5}{7} - \left(\frac{5}{7}\right)^2$

$= \frac{5}{7} - \frac{25}{49}$

$= \frac{35-25}{49} = \frac{10}{49}$

25. We have, OA = R = 21 m and OC = r = 14 m

$\therefore$Area of the flower bed = Area of a quadrant of a circle of radius R - Area of a quadrant of a circle of radius r

$= \frac{1}{4}\pi R^2 - \frac{1}{4}\pi r^2$

$= \frac{\pi}{4}(R^2 - r^2)$

$= \frac{1}{4} \times \frac{22}{7}(21^2 - 14^2)\text{ cm}^2$

$= \left\{\frac{1}{4} \times \frac{22}{7} \times (21+14)(21-14)\right\} m^2$

$= \left\{\frac{1}{4} \times \frac{22}{7} \times 35 \times 7\right\} \text{m}^2$

= 192.5 m^2

OR

Given , Circumference = 22 cm

$\Rightarrow 2\pi r = 22$

$\Rightarrow r = \frac{7}{2} = 3.5cm$

Area of Circle = πr^2 = $\frac{22}{7} \times (3.5)^2 = 38.5cm^2$

Area of quadrant of circle = $\frac{Area\,of\,circle}{4}$

$= \frac{38.5}{4} = 9.625cm^2$

$\therefore$Area of the quadrant of cicle = 9.625 cm^2

Section C

26. This problem can be solved using Least Common Multiple because we are trying to figure out when the soonest (Least) time will be that as the event of exercising continues (Multiple), it will occur at the same time (Common).

L.C.M. of 12 and 8 is 24.

So,

They will exercise together again in 24 days.

27. $7y^2 - \frac{11}{3}y - \frac{2}{3}$

$= \frac{1}{3}(21y^2 - 11y - 2)$

$= \frac{1}{3}(21y^2 - 14y + 3y - 2)$

$= \frac{1}{3}[7y(3y - 2) + 1(3y - 2)]$

$= \frac{1}{3}(3y - 2)(7y + 1)$

$\Rightarrow y = \frac{2}{3}, \frac{-1}{7}$ are zeroes of the polynomial.

If Given polynimoal is $7y^2 - \frac{11}{3}y - \frac{2}{3}$

Then a = 7 , b = $-\frac{11}{3}$ and c = $-\frac{2}{3}$

Sum of zeroes = $\frac{2}{3} + \frac{-1}{7} = \frac{14-3}{21} = \frac{11}{21}$ .... (i)

Also, $\frac{-b}{a} = \frac{-\left(\frac{-11}{3}\right)}{7} = \frac{11}{21}$ .... (ii)

From (i) and (ii)

Sum of zeroes = $\frac{-b}{a}$

Now, product of zeroes = $\frac{2}{3} \times \frac{-1}{7} = \frac{-2}{21}$ .... (iii)

Also, $\frac{c}{a} = \frac{\frac{-2}{3}}{7} = \frac{-2}{21}$ .... (iv)

From (iii) and (iv)

Product of zeroes = $\frac{c}{a}$

28. x + y = 5 ...(1)

2x + 2y = 10 ...(2)

Here, $a_1 = 1$, $b_1 = 1$, $c_1 = -5$

$a_2 = 2$, $b_2 = 2$, $c_2 = -10$

We see that $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, the lines represented by the equations (1) and (2) are coincident.

Therefore, equations (1) and (2) have infinitely many common solutions, i.e., the given pair of linear equations is consistent.

Graphical Representation, we draw the graphs of the equations (1) and (2) by finding two solutions for each if the equations.

These two solutions of the equations (1) and (2) are given below in table 1 and table 2 respectively.

For equation (1) x + y = 5 $\Rightarrow$ y = 5 - x

Table 1 of solutions

x	0	5
y	5	0

For equations (2) x + 2y = 10

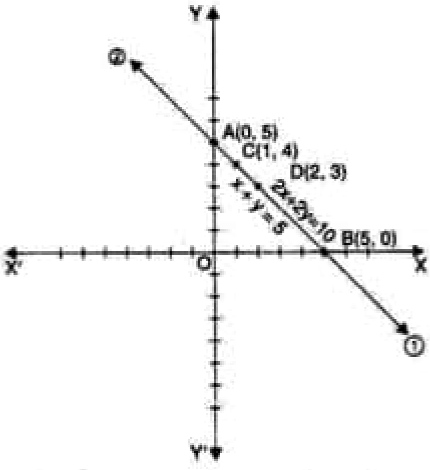
$\Rightarrow$ 2y = 10 - 2x

$\Rightarrow y = \frac{10-2x}{2} \Rightarrow$ y = 5 - x

Table 2 of solutions

x	1	2
y	4	3

We plot the points A(0, 5) and B(5, 0) on a graph paper and join these points to form the line AB representing the equation (1) as shown in the figure, Also, we plot the points C(1, 4) and D (2, 3) on the same graph paper and join these points to form the line CD representing the equation (2) as shown in the same figure.



In the figure we observe that the two lines AB and CD coincide.

OR

Let the numerator be ‘a’ and denominator be ‘b’.

According to the first condition in the question, we have

$\Rightarrow \frac{2a}{b-5} = \frac{6}{5}$

$\Rightarrow 5a - 3b = -15$ ..... (1)

Also, according to the second condition, we have

$\Rightarrow \frac{a+8}{2b} = \frac{2}{5}$

$\Rightarrow 5a - 4b = -40$ ......(2)

Subtracting (2) from (1), gives

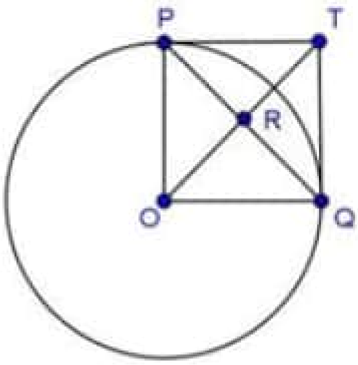
b = 25

Using this value of b in (1) gives

a $= \frac{-15+3b}{5} = \frac{-15+3\times 25}{5} = 12$

So, the required fraction is $\frac{12}{25}$.

29. Given, PO⊥QO and The tangents to the circle at P and Q intersect at a point T.



Consider, $\Delta TPO\ and\ \Delta TQO$

PT = TQ [∵ Tangents from external point are equal in length]

OT = OT [Common]

$\angle TPO = \angle TQO = 90^\circ$

So, by RHS rule, we have

$\Delta TPO \cong \Delta TQO$

$\Rightarrow \angle PTO = \angle QTO$ ...(i) [C.P.C.T.]

Now, In ΔPTR and ΔQTR

PT = TQ [∵ Tangents from external point are equal in length]

$\angle PTO = \angle QTO$ [By equation (i)]

TR = TR [Common]

So, by SAS rule, we have

$\Delta PTR \cong \Delta QTR$

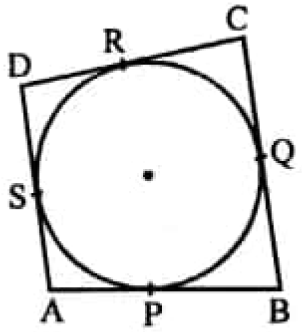
∴ PR = RQ ...(ii)

And, $\angle TRP = \angle TRQ$

But, $\angle TRP + \angle TRQ = 180^{\circ}$
$\Rightarrow 2\angle TRP = 180^{\circ}$
$\Rightarrow \angle TRP = 90^{\circ}$ ...(iii)
Therefore, PQ and OT are right bisectors of each other.

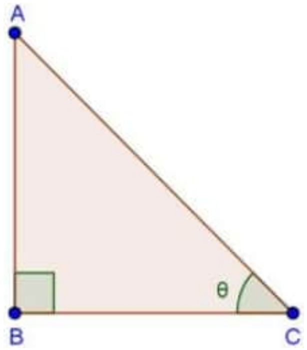
OR

In the figure, quad. ABCD is circumscribed about a circle which touches its sides at P, Q, R and S respectively



To prove : AB + CD = AD + BC
Proof: Tangents drawn from an external point to a circle are equal
AP = AS
BP = BQ
CR = CQ
DR = DS
Adding, we get,
AP + BP + CR + DR = AS + BQ + CQ + DS
$\angle$ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)
$\angle$ AB + CD = AD + BC
Hence AB + CD = AD + BC

30.



Let θ is $\angle C$. Given $\sin\theta = \frac{12}{13} = \frac{AB}{AC}$ ......(1)

Let AB = 12K and AC = 13K ,where K is positive integer.

In ΔABC, By using Pythagoras theorem :-
$AB^2 + BC^2 = AC^2$
Or, $(12K)^2 + BC^2 = (13K)^2$
Or, $144K^2 + BC^2 = 169K^2$
Or, $BC^2 = 169K^2 - 144K^2$
Or, $BC^2 = 25K^2$
$\therefore BC = \sqrt{25K^2} = 5K$
Now,
$\cos\theta = \frac{BC}{AC} = \frac{5K}{13K} = \frac{5}{13}$ ....(2)
$\tan\theta = \frac{AB}{BC} = \frac{12K}{5K} = \frac{12}{5}$ ....(3)
Now,

$\frac{\sin^2\theta-\cos^2\theta}{2\sin\theta\times\cos\theta}\times\frac{1}{\tan^2\theta}$
$=\frac{\left(\frac{12}{13}\right)^2-\left(\frac{5}{13}\right)^2}{2\times\frac{12}{13}\times\frac{5}{13}}\times\frac{1}{\left(\frac{12}{5}\right)^2}$ [from (1),(2) & (3)]
$=\frac{\frac{144}{169}-\frac{25}{169}}{\frac{120}{169}}\times\frac{1}{\frac{144}{25}}$
$=\frac{\frac{144-25}{169}}{\frac{120}{169}}\times\frac{25}{144}$
$=\frac{119}{169}\times\frac{169}{120}\times\frac{25}{144}$
$=\frac{595}{3456}$

31. Calculation of mean:

Class interval	**Mid – value (x_i)**	**f_i**	**f_ix_i**
0 – 6	3	6	18
6 – 12	9	8	72
12 – 18	15	10	150
18 – 24	21	9	189
24 – 30	27	7	189
		Σf_i = 40	Σf_ix_i = 618

We know that, Mean = $\frac{\Sigma f_ix_i}{\Sigma f_i}$
$=\frac{618}{40}$
= 15.45

Section D

32. Let the lengths of the radii of the smaller and larger circles be r cm and R cm respectively.
It is given that,R - r = 7.....(i).
It is also given that the difference between the areas of two circles is 1078 cm^2
$\therefore\quad \pi R^2-\pi r^2=1078$
$\Rightarrow\quad \pi\left(R^2-r^2\right)=1078$
$\Rightarrow\quad \frac{22}{7}(R+r)(R-r)=1078$
$\Rightarrow\quad \frac{22}{7}(R+r)\times 7=1078$
⇒ R + r = 49 ....(ii)
Subtracting (i) from (ii), we get
2r = 42⇒r = 21
Hence, the radius of the smaller circle is of length 21 cm.

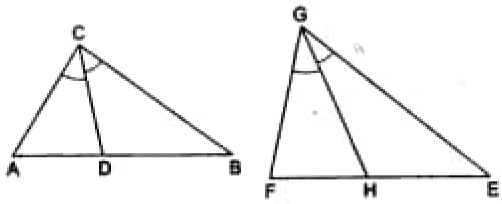
OR

Let time taken by pipe A be x minutes, and time taken by pipe B be x + 5 minutes.
In one minute pipe A will fill $\frac{1}{x}$ tank
In one minute pipe B will fill $\frac{1}{x+5}$ tank
pipes A + B will fill in one minute = $\frac{1}{x}+\frac{1}{x+5}$ tank
Now according to the question.
$\frac{1}{x}+\frac{1}{x+5}=\frac{9}{100}$
or, $\frac{x+5+x}{x(x+5)}=\frac{9}{100}$
or, 100(2x + 5) = 9x(x + 5)
or, $200x+500=9x^2+45x$
or, $9x^2-155x-500=0$
or, $9x^2-180x+25x-500=0$
or,9x(x - 20) + 25(x - 20) = 0
or, (x-20)(9x + 25) = 0
or, $x=20,\frac{-25}{9}$

rejecting negative value, x = 20 minutes

and x + 5 = 25 minutes

Hence pipe A will fill the tank in 20 minutes and pipe B will fill it in 25 minutes.

33. 

Given: $\triangle ABC \sim \triangle FEG$

a. In $\triangle ADC$ and $\triangle FHG$

$\angle ACB = \angle FGE$ (As $\triangle$ABC $\sim$ $\triangle$ FEG)

$\frac{1}{2}\angle \text{ACB} = \frac{1}{2}\angle \text{FGE}$

or $\angle \text{ACD} = \angle \text{FGH}$

and $\angle$CAC =$\angle$GFH [As $\triangle$ABC $\sim$ $\triangle$ FEG]

So, $\triangle ADC \sim \triangle FHG$ (By AA criteria) ------(i)

b. In $\triangle BCD$ and ΔEGH , we have

$\angle DBC = \angle HEG$

$\angle ACB = \angle FGE$ (As $\triangle$ABC $\sim$ $\triangle$ EFG)

$\frac{1}{2}\angle \text{ACB} = \frac{1}{2}\angle \text{FGE}$

or $\angle \text{DCB} = \angle \text{HGE}$

So, $\triangle \text{BCD} \sim \triangle \text{EGH}$ (By AA criteria)

c. From (i), $\triangle ADC \sim \triangle FHG$

So, $\frac{\text{AC}}{\text{FG}} = \frac{\text{CD}}{\text{GH}}$

or $\frac{\text{CD}}{\text{GH}} = \frac{\text{AC}}{\text{FG}}$

Hence proved.

34. The volume of the spherical vessel is

calculated by the given formula

V = $\frac{4}{3}\pi \times r^3$

Now,

V = $\frac{4}{3} \times \frac{22}{7} \times 9 \times 9 \times 9$

V = 3,054.85 cm^3

The volume of the cylinder neck is calculated by the given formula.

V = $\pi \times R^2 \times h$

Now,

V = $\frac{22}{7} \times 1 \times 1 \times 8$

V = 25.14 cm^3

The total volume of the vessel is equal to the volume of the spherical shell and the volume of its cylindrical neck.

3054.85 + 25.14 = 3, 080 cm^3

The total volume of the vessel is 3,080 cm^3.

As we know,

1 L = 1000 cm^3

$\frac{3080}{1000}$ = 3.080 L

Thus, the amount of water (in litres) it can hold is 3.080 L.

OR

From the given figure,
Height (AB) of the cone = AC - BC (Radius of the hemisphere)
Thus, height of the cone = Total height - Radius of the hemisphere
= 9.5 - 3.5
= 6 cm
Volume of the solid = Volume of the cone + Volume of the hemisphere
$= \left(\frac{1}{3}\pi r^2 h\right) + \left(\frac{2}{3}\pi r^3\right)$
$= \frac{1}{3}\pi r^2(h + 2r)$
$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5(6 + 2 \times 3.5)$
$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 13$
= 166.83 cm^3

Thus, total volume of the solid is 166.83 cm^3.

35. Calculation of median:

Marks	**Number of students(f_i)**	**Cumulative frequency**
0 - 7	3	3
7 - 14	4	7
14 - 21	7	14
21 - 28	11	25
28 - 35	0	25
35 - 42	16	41
42 - 49	9	50
	N = Σf_i = 50	

Now,N = 50⇒ $\frac{N}{2}$= 25.
The cumulative frequency just greater than 25 is 41.
So, median class is 35 - 42.
$\therefore l = 35, h = 7, f = 16, c.f. = 25$
Median, M =$l + \left\{h \times \frac{\left(\frac{N}{2} - cf\right)}{f}\right\}$
$= 35 + \left[7 \times \frac{(25-25)}{16}\right]$
$= 35 + 0$
$= 35$
Hence, the required median is 35.

Section E

36. **Read the text carefully and answer the questions:**
Your friend Varun wants to participate in a 200m race. He can currently run that distance in 51 seconds and with each day of practice it takes him 2 seconds less. He wants to do in 31 seconds.

(i) 51, 49, 47, ... 31 AP

d = -2

First 4 terms of AP are: 51, 49, 47, 45 ...

(ii) 51, 49, 47, ... 31 AP

d = -2

$t_n = a + (n - 1)d$

31 = 51 + (n - 1) (-2)

31 = 51 - 2n + 2

31 = 53 - 2n

31 - 53 = -2n

-22 = -2n

n = 11

i.e., he acheived his goal in 11 days.

OR

The given AP is

51, 49, 47, 45, 43, 41, 39, 37, 35, 33, 31, 29

$\therefore$ 30 is not in the AP.

(iii) 51, 49, 47, ... 31 AP

d = -2

$t_6 = a + (n - 1)d$

= 51 + (6 - 1)(-2)

= 51 + (-10)

= 41 sec

37. **Read the text carefully and answer the questions:**

Using Cartesian Coordinates we mark a point on a graph by how far along and how far up it is.

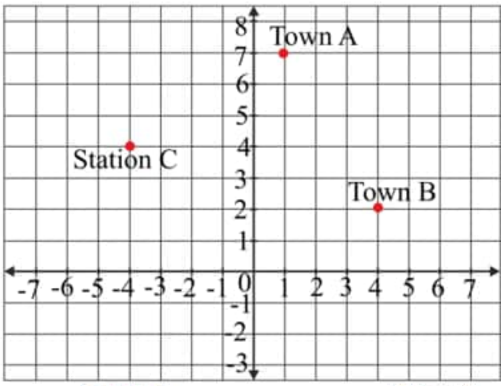
The left-right (horizontal) direction is commonly called X-axis.

The up-down (vertical) direction is commonly called Y-axis.

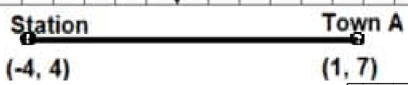
When we include negative values, the x and y axes divide the space up into 4 pieces.

Read the information given above and below:

Two friends Veena and Arun work in the same office in Delhi. In the Christmas vacations, both decided to go their hometowns represented by Town A and Town B respectively in the figure given below. Town A and Town B are connected by trains from the same station C (in the given figure) in Delhi.



(i)



Distance travelled by veena = $\sqrt{1-(-4)^2+(7-4)^2}$

$= \sqrt{5^2+3^2}$

$= \sqrt{25+9}$

$= \sqrt{34}$

Distance travelled by Arun = $\sqrt{(4-(-4))^2+(2-4)^2}$

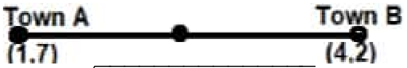
$=\sqrt{64+4}$

$=\sqrt{68}$

$\therefore$ Arun will travel more distance to reach his home.

(ii) Location of station = (-4, 4)

OR

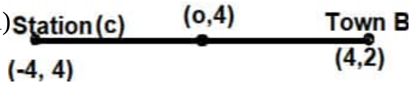


AB = $\sqrt{(4-1)^2+(2-7)^2}$

$=\sqrt{9+25}$

$=\sqrt{34}$

(iii)



Let y-axis divides station (c) and Town B in K : 1

0 = $\frac{4k-4}{k+1}$

4k = 4

k = 1

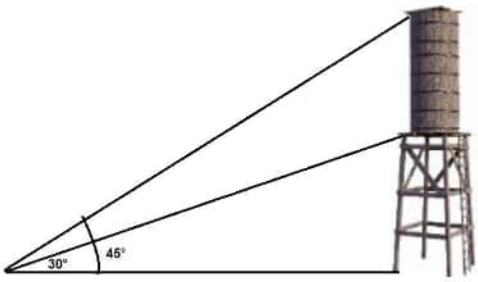
$\therefore$ y-axis divides in 1 : 1

38. **Read the text carefully and answer the questions:**

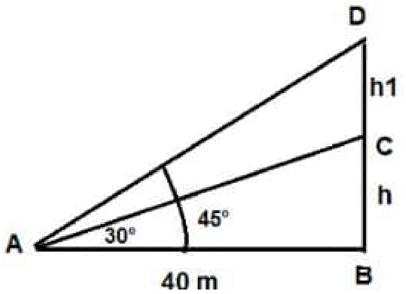
In a society, there are many multistory buildings. The RWA of the society wants to install a tower and a water tank so that all the households can get water without using water pumps.

For this they have measured the height of the tallest building in the society and now they want to install a tower that will be taller than that so that the level of water must be higher than the tallest building in their society. Here is one solution they have found and now they want to check if it will work or not.

From a point on the ground 40 m away from the foot of a tower, the angle of elevation of the top of the tower is 300. the angle of elevation of the top of the water tank is 45°.



(i)



Let BC be the tower of height h and CD be the water tank of height h_1

In ΔABD, we have

$\tan 45^{\circ}=\frac{BD}{AB}$

$\Rightarrow 1=\frac{h+h_1}{40}$

$\Rightarrow h+h_1=40$ ...(1)

In ΔABC, we have

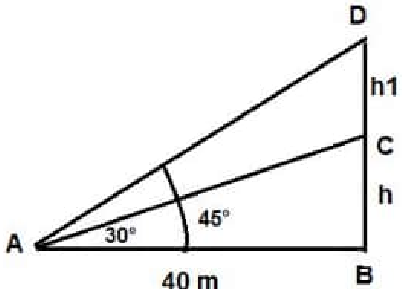
$\tan 30^{\circ}=\frac{BC}{AB}$

$\Rightarrow \quad \frac{1}{\sqrt{3}}=\frac{h}{40}$

$\Rightarrow h = \frac{40}{\sqrt{3}} = \frac{40\sqrt{3}}{3}$ = 23.1 m

Thus height of the tower is 23.1 m.

(ii)



Let BC be the tower of height h and CD be the water tank of height h_1

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$\tan 45^\circ = \frac{BD}{AB}$

$\Rightarrow 1 = \frac{h+h_1}{40}$

$\Rightarrow h + h_1 = 40$ ...(1)

In ΔABC, we have

$\tan 30^\circ = \frac{BC}{AB}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{40}$

$\Rightarrow h = \frac{40}{\sqrt{3}} = \frac{40\sqrt{3}}{3}$ = 23.1 m

Thus height of the tower is 23.1 m.

Substituting the value of h in (1), we have

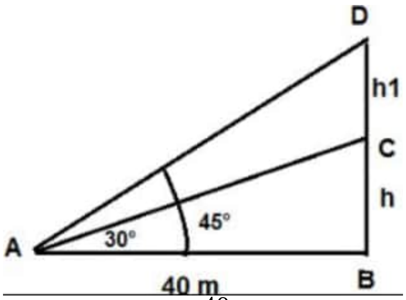
$23.1 + h_1 = 40$

⇒ h_1 = 40 - 23.1

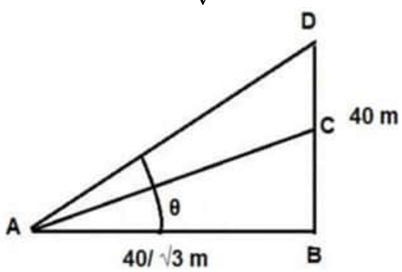
= 6.9 m

Thus height of the tank is 6.9 m.

OR



Given that $AB = \frac{40}{\sqrt{3}}$



In the $\triangle ABD$

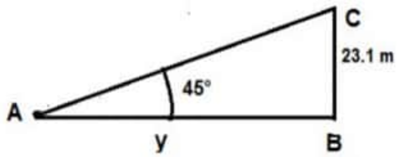
$\cot\theta = \frac{AB}{BD} = \frac{\frac{40}{\sqrt{3}}}{40}$

$\cot\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = 60^\circ$

Hence the angle of elevation would be 60^o.

(iii)



In the $\triangle ABC$ if $\angle CAB = 45^\circ$ then

$\cot 45^\circ = \frac{y}{23.1} = 1$

y = 23.1 m

Thus the angle of elevation will be 45^o at 23.1 m.