Class X Session 2023-24
Subject - Mathematics (Standard)
Sample Question Paper - 6

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub- parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

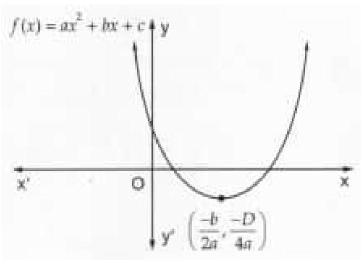
Section A

1. The HCF of two numbers is 27 and their LCM is 162. If one of the numbers is 54, what is the other number? **[1]**

a) 9 b) 81

c) 45 d) 36

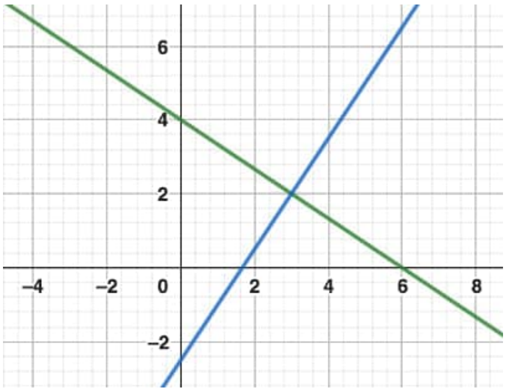
2. Figure show the graph of the polynomial f(x) = ax^2 + bx + c for which **[1]**



a) a > 0, b < 0 and c > 0 b) a < 0, b < 0 and c < 0

c) a < 0, b > 0 and c > 0 d) a > 0, b > 0 and c < 0

3. If 2x + 3y = 12 and 3x - 2y = 5 then **[1]**



a) x = 3, y = 2

b) x = 2, y = -3

c) x = 2, y = 3

d) x = 3, y = -2

4. If the equation $9x^2 + 6kx + 4 = 0$ has equal roots then k = ? **[1]**

a) -2 or 0

b) 0 only

c) 2 or 0

d) 2 or -2

5. If the first term of an A.P. is a and n^{th} term is b, then its common difference is **[1]**

a) $\frac{b-a}{n}$

b) $\frac{b-a}{n-1}$

c) $\frac{b-a}{n+1}$

d) $\frac{b+a}{n-1}$

6. The point on the x-axis which is equidistant from points (-1, 0) and (5, 0) is **[1]**

a) (0, 3)

b) (2, 0)

c) (3, 0)

d) (0, 2)

7. If $\left(\frac{a}{2}, 4\right)$ is the midpoint of the line segment joining the points A (-6 ,5) and B(-2, 3) then the value of a is **[1]**

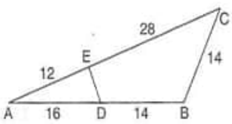
a) 3

b) 4

c) -8

d) -4

8. In the given figure if $\triangle AED \sim \triangle ABC$, then DE is equal to **[1]**



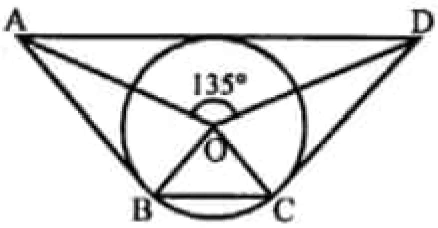
a) 7.5 cm.

b) 5.6 cm.

c) 6.5 cm.

d) 5.5 cm.

9. In the given figure, If ∠AOD = 135° then ∠BOC is equal to **[1]**



a) 45°

b) 25°

c) 52.5°

d) 62.5°

10. Quadrilateral ABCD is circumscribed to a circle. If AB = 6 cm, BC = 7 cm and CD = 4 cm then the length of AD is **[1]**

a) 6 cm

b) 4 cm

c) 7 cm

d) 3 cm

11. (cosec θ - sin θ) (sec θ - cos θ) (tan θ + cot θ) is equal **[1]**

a) 1

b) 0

c) none of these

d) -1

12. If cot A + $\frac{1}{\cot A}$ = 2 then cot^{2}A + $\frac{1}{\cot^2 A}$ = **[1]**

a) 1

b) -1

c) 2

d) 0

13. If the angles of elevation of the top of a tower from two points distant a and b from the base and in the same straight line with it are complementary, then the height of the tower is **[1]**

a) ab

b) $\frac{a}{b}$

c) $\sqrt{ab}$

d) $\sqrt{\frac{a}{b}}$

14. A piece of paper in the shape of a sector of a circle (see figure 1) is rolled up to form a right-circular cone (see figure 2). The value of angle θ is : **[1]**

figure 1

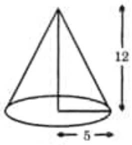


figure 2

a) $\frac{5\pi}{13}$

b) $\frac{10\pi}{13}$

c) $\frac{9\pi}{13}$

d) $\frac{6\pi}{13}$

15. The area of a sector of a circle with radius 6 cm if the angle of the sector is 60^o. **[1]**

a) $\frac{152}{7}$

b) $\frac{132}{7}$

c) $\frac{142}{7}$

d) $\frac{122}{7}$

16. The probability that it will rain on a particular day is 0.76. The probability that it will not rain on that day is **[1]**

a) 0.24

b) 0.76

c) 0

d) 1

17. Which of the following cannot be the probability of a event? **[1]**

a) $\frac{17}{16}$

b) $\frac{1}{3}$

c) 0.1

d) 3%

18. Consider the following frequency distribution: **[1]**

Class	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	3	9	15	30	18	5

The modal class is

a) 50-60

b) 20-30

c) 10-20

d) 30-40

19. **Assertion (A):** In a solid hemisphere of radius 10 cm, a right cone of same radius is removed out. The surface **[1]**

area of the remaining solid is 570.74 cm^2 [Take π = 3.14 and $\sqrt{2}$ = 1.4]

Reason (R): **Reason (R):** Expression used here to calculate Surface area of remaining solid = Curved surface area of hemisphere + Curved surface area of cone

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The constant difference between any two terms of an AP is commonly known as common difference. **[1]**

Reason (R): The common difference of 2, 4, 6, 8 this A.P. is 2.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

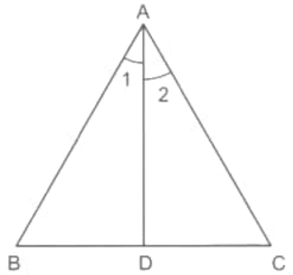
c) A is true but R is false.

d) A is false but R is true.

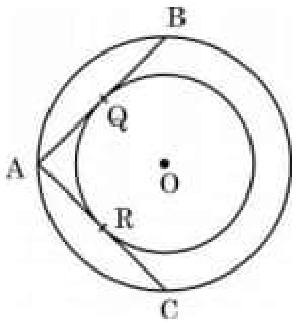
Section B

21. Prove that $4+\sqrt{2}$ is irrational. **[2]**

22. In Fig. ΔABC is a triangle such that $\frac{AB}{AC}=\frac{BD}{DC}, \angle B=70^{\circ}, \angle C=50^{\circ}$ Find ∠BAD. **[2]**



23. In Fig., there are two concentric circles with centre O. If ARC and AQB are tangents to the smaller circle from the point A lying on the larger circle, find the length of AC, if AQ = 5 cm. **[2]**



24. Prove the trigonometric identity: **[2]**

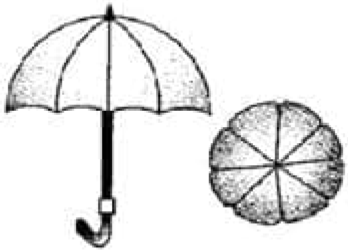
(secθ + cosθ) (secθ - cosθ) = $\tan^2\theta$ + $\sin^2\theta$

OR

Prove the trigonometric identity:

$\sin^2$A.$\cos^2$B - $\cos^2$A.$\sin^2$B = $\sin^2$A - $\sin^2$B

25. An umbrella has 8 ribs which are equally spaced (see figure). Assuming umbrella to be a flat circle of radius 45 cm, Find the area between the two consecutive ribs of the umbrella. **[2]**



OR

The radius of a circle is 17.5 cm. Find the area of the sector enclosed by two radii and an arc 44 cm in length.

Section C

26. Every year the Model School celebrates its Sports Day on 4th September. For this, a lot of activities are arranged in the school premises. To avoid inconvenience, the school fixed days for game practice. After every 4 days, the karate team meets at the School's sports ground, the skating team meets after 3 days and the yoga team meets after 2 days. All the groups met on 7th May. Determine the last day when they will meet before the function. **[3]**

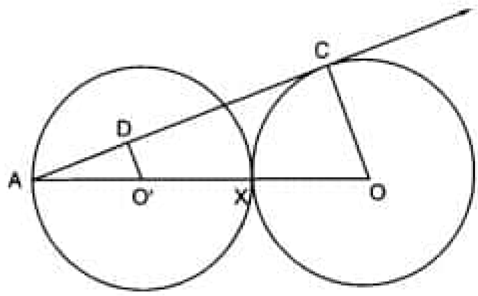
27. Find the zeroes of the given quadratic polynomials and verify the relationship between the zeroes and their coefficients x^2 - 6. **[3]**

28. A two-digit number is 4 times the sum of its digits and twice the product of the digits. Find the number. **[3]**

OR

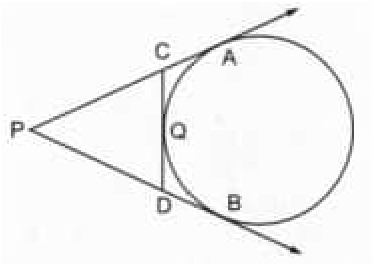
A man has only 20 paisa coins and 25 paisa coins in his purse. If he has 50 coins in all totalling to ₹ 11.25, how many coins of each kind d38196oes he have?

29. Equal circles with centres O and O' touch each other at X as shown in figure. OO' produced to meet a circle with centre O', at A. AC is a tangent to the circle whose centre is O. O' D is perpendicular to AC. Find the value of $\frac{DO'}{CO}$. **[3]**



OR

In figure, PA and PB are tangents to the circle drawn from an external point P. CD is a third tangent touching the circle at Q. If PB = 10 cm and CQ = 2 cm, what is the perimeter of $\triangle$PCD?



30. If 5 tan a = 4, show that $\frac{5\sin\theta-3\cos\theta}{5\sin\theta+2\cos\theta}=\frac{1}{6}$ **[3]**

31. If the median of the following frequency distribution is 46, find the missing frequencies. **[3]**

Variable	10-20	20-30	30-40	40-50	50-60	60-70	70-80	Total
Frequency	12	30	?	65	?	25	18	229

Section D

32. If the equation $\left(1+m^2\right)x^2+2mcx+\left(c^2-a^2\right)=0$ has equal roots, prove that $c^2=a^2\left(1+m^2\right)$ **[5]**

OR

A truck covers a distance of 150 km at a certain average speed and then covers another 200 km at an average speed which is 20 km per hour more than the first speed. If the truck covers the total distance in 5 hours, then find the first speed of the truck.

33. O is the point of intersection of the diagonals AC and BD of a trapezium ABCD with AB || DC. Through O, a line segment PQ is drawn parallel to AB meeting AD in P and BC in Q. Prove that PO = QO. **[5]**

34. A well, whose diameter is 7m, has been dug 22.5 m deep and the earth dugout is used to form an embankment around it. If the height of the embankment is 1.5 m, find the width of the embankment. **[5]**

OR

From a cubical piece of wood of side 21 cm, a hemisphere is carved out in such a way that the diameter of the hemisphere is equal to the side of the cubical piece. Find the surface area and volume of the remaining piece.

35. In a hospital, the ages of diabetic patients were recorded as follows. Find the median age. **[5]**

Age (in years)	0 - 15	15 - 30	30 - 45	45 - 60	60 - 75
Number of patients	5	20	40	50	25

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Students of a school thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of class I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII. There are three sections of each class.

(i) Find total number of trees planted by primary 1 to 5 class students?

(ii) Find the total number of trees planted by the students of the school.

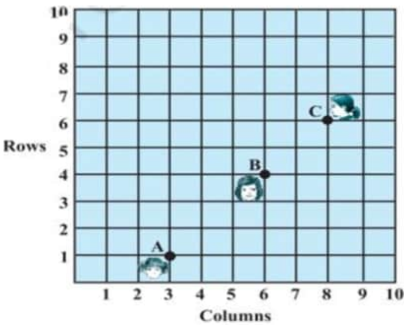
OR

Find the total no of trees planted by class 12th students.

(iii) Find the total number of trees planted by class 10th student.

37. **Read the text carefully and answer the questions:** **[4]**

There is a function in the school. Anishka, Bhawna and Charu are standing in a rectangular ground at points A, B and C respectively as shown in the figure. They are ready to perform an aerobic dance.



(i) How far is Charu from y-axis?

(ii) Find distance between Anishka and Bhawna.

OR

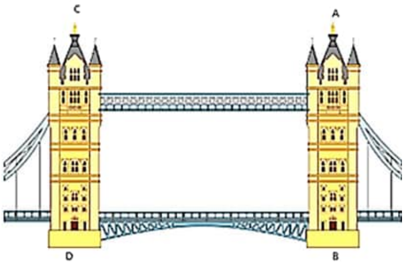
Is A, B and C lies in a straight line?

(iii) Check whether AB + BC = AC?

38. **Read the text carefully and answer the questions:** **[4]**

Tower Bridge is a Grade I listed combined bascule and suspension bridge in London, built between 1886 and 1894, designed by Horace Jones and engineered by John Wolfe Barry. The bridge is 800 feet (240 m) in length and consists of two bridge towers connected at the upper level by two horizontal walkways, and a central pair of bascules that can open to allow shipping.

In this bridge, two towers of equal heights are standing opposite each other on either side of the road, which is 80 m wide. During summer holidays, Neeta visited the tower bridge. She stood at some point on the road between these towers. From that point between the towers on the road, the angles of elevation of the top of the towers was 60^o and 30^o respectively.



(i) Find the distances of the point from the base of the towers where Neeta was standing while measuring the height.

(ii) Neeta used some applications of trigonometry she learned in her class to find the height of the towers without actually measuring them. What would be the height of the towers she would have calculated?

OR

Find the distance between Neeta and top tower CD?

(iii) Find the distance between Neeta and top of tower AB?

Solution

Section A

1.

(b) 81

Explanation: Let the two numbers be x and y.

It is given that:

x = 54

HCF = 27

LCM = 162

We know,

x $\times$ y = HCF $\times$ LCM

$\Rightarrow$ 54 $\times$ y = 27 $\times$ 162

$\Rightarrow$ 54y = 4374

$\Rightarrow$ $\therefore y = \frac{4374}{54} = 81$

2. **(a)** a > 0, b < 0 and c > 0

Explanation: Clearly, $f(x) = ax^2 + bx + c$ represent a parabola opening upwards.

Therefore, $a > 0$

The vertex of the parabola is in the fourth quadrant, therefore b < 0

$y = ax^2 + bx + c$ cuts Y axis at P which lies on OY.

Putting x = 0 in $y = ax^2 + bx + c$, we get y = c.

So the coordinates of P is (0, c).

Clearly, P lies on OY. $\Rightarrow$ c>0

Hence, a>0, b<0 and c>0

3. **(a)** x = 3, y = 2

Explanation: We have:

2x + 3y = 12 …(i)

3x – 2y = 5 …(ii)

Now, by multiplying (i) by 2 and (ii) by 3 and then adding them we get:

4x + 9x = 24 + 15

13x = 39

x = $\frac{39}{13}$ = 3

Now putting the value of x in (i), we get

2 × 3 + 3y = 12

$\therefore$ y = $\frac{12-6}{3}$ = 2

4.

(d) 2 or -2

Explanation: Since the roots are equal, we have D = 0.

$\therefore$ $36k^2 - 4 \times 9 \times 4 = 0 \Rightarrow 36k^2 = 144 \Rightarrow k^2 = 4 \Rightarrow k = 2$ or -2.

5.

(b) $\frac{b-a}{n-1}$

Explanation: In the given A.P.

First term = a and nth term = b

$\therefore$ a + (n - 1)d = b

$\Rightarrow$(n - 1)d = b - a

$\Rightarrow d = \frac{b-a}{n-1}$

6.

(b) (2, 0)

Explanation: Let the required point be P(x, 0). Then,
$PA^2 = PB^2 \Rightarrow (x+1)^2 = (x-5)^2$
$\Rightarrow x^2 + 2x + 1 = x^2 - 10x + 25$
$\Rightarrow 12x = 24 \Rightarrow x = 2$
So, the required point is P(2, 0).

7.

(c) -8

Explanation: $\frac{a}{2} = \frac{(-6-2)}{3} = -4 \Rightarrow a = -8$

8.

(b) 5.6 cm.

Explanation: Since $\triangle AED \sim \triangle ABC$
$\therefore \frac{\text{AE}}{\text{AB}} = \frac{\text{DE}}{\text{BC}}$
$\Rightarrow \frac{12}{16+14} = \frac{\text{DE}}{14}$
$\Rightarrow$ DE = $\frac{12\times14}{30} = \frac{84}{15}$ = 5.6 cm

9. **(a)** 45°

Explanation: In the given figure, ∠AOD = 135°

We know that if a circle is inscribed in a quadrilateral, the opposite sides subtend supplementary angles.

∠AOD + ∠BOC = 180°

135° + ∠BOC = 180°

⇒∠BOC = 180° – 135° = 45°

10.

(d) 3 cm

Explanation: A quadrilateral ABCD is circumscribed to a circle with centre O.

AB = 6 cm, BC = 7 cm, CD = 4 cm, AD = 7 cm

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ABCD circumscribed to a circle.

AB + CD = BC + AD

⇒ 6 + 4 = 7 + AD

⇒ 10 = 7 + AD

AD = 10 - 7 = 3 cm

11. **(a)** 1

Explanation: We have, $(cosec\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$
$= \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$
$= \frac{1-\sin^2\theta}{\sin\theta} \times \frac{1-\cos^2\theta}{\cos\theta} \times \frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$
$= \frac{\cos^2\theta}{\sin\theta} \times \frac{\sin^2\theta}{\cos\theta} \times \frac{1}{\sin\theta\cos\theta}$
$= \frac{\sin^2\theta\cos^2\theta}{\sin^2\theta\cos^2\theta} = 1$

12.

(c) 2

Explanation: Given: cot A + $\frac{1}{\text{cot A}}$ = 2

Squaring both sides, we get

⇒ $\cot^2$A + $\frac{1}{\cot^2\text{A}}$ + 2 × cot A × $\frac{1}{\text{cotA}}$ = 4

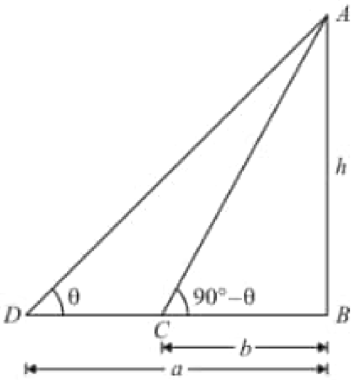
⇒ $\cot^2$A + $\frac{1}{\cot^2\text{A}}$ = 2

13.

(c) $\sqrt{ab}$

Explanation:

Let h be the height of tower AB.



Given that: angle of elevation of top of the tower are $\angle$D = θ and $\angle$C = 90^o - θ.Distance BC = b and BD = a

Here, we have to find the height of tower.

So we use trigonometric ratios.

In a triangle ABC,

$\Rightarrow \tan C = \frac{AB}{BC}$

$\Rightarrow \tan(90^\circ - \theta) = \frac{h}{b}$

$\Rightarrow \cot\theta = \frac{h}{b}$

Again in a triangle *ABD*,

tan D = $\frac{AB}{BD}$

$\Rightarrow \tan\theta = \frac{h}{a}$

$\Rightarrow \frac{1}{\cot\theta} = \frac{h}{a}$

$\Rightarrow \frac{b}{h} = \frac{h}{a}$ [Put cot $\theta = \frac{h}{b}$

$\Rightarrow$ h^2 = ab

$\Rightarrow$ h = $\sqrt{ab}$

14.

(b) $\frac{10\pi}{13}$

Explanation: $\frac{10\pi}{13}$

15.

(b) $\frac{132}{7}$

Explanation: Angle of the sector is 60°

Area of sector = $(\frac{\theta}{360^o}) \times \pi r^2$

$\therefore$ Area of the sector with angle 60^o = $(\frac{60^o}{360^o}) \times \pi r^2$ cm^2

= $(\frac{36}{6})\pi$ cm^2

= 6 $\times$ $(\frac{22}{7})$ cm^2

= $\frac{132}{7}$ cm^2

16. **(a)** 0.24

Explanation: Given: P (It will rain on a particular day) = 0.76

$\therefore$ P (It will not rain on a particular day) = 1 - P (It will rain particular day)

= 1 - 0.76 = 0.24

17. **(a)** $\frac{17}{16}$

Explanation: Since, probability of an event always lies between 0 and 1.

Probability of any event cannot be more than 1 or negative as $\frac{17}{16} > 1$

18.

(d) 30-40

Explanation: Class having maximum frequency is the modal class.

Here, maximum frequency = 30

Hence, the modal class is 30 - 40.

19.

(d) A is false but R is true.

Explanation: A is false but R is true.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

Section B

21. Let us assume that $4+\sqrt{2}$ is rational. Then, there exist positive co-primes a and b such that

$4+\sqrt{2}=\frac{a}{b}$

$\sqrt{2}=\frac{a}{b}-4$

$\sqrt{2}=\frac{a-4b}{b}$

As a - 4b and b are integers.

So, $\frac{a-4b}{b}$ is a rational number .

But $\sqrt{2}$ is not rational number .

Since a rational number cannot be equal to an irrational number . Our assumption that $4+\sqrt{2}$ is a rational number is wrong .

Hence, $4+\sqrt{2}$ is irrational.

22. According to question it is given that

$\frac{AB}{AC}=\frac{BD}{DC}$ Also, ∠B = 70°, ∠C = 180°

Since we know that, if a line through one vertex of a triangle divides the opposite side in the ratio of the other two sides, then the line bisects the angle at the vertex.

$\therefore \angle 1=\angle 2$

In △ABC we know that sum of angles of a Δ is equal to 180°

$\angle A+\angle B+\angle C=180^{\circ}$

$\Rightarrow \angle A+70^{\circ}+50^{\circ}=180^{\circ}$

$\Rightarrow \angle A=180^{\circ}-120^{\circ}=60^{\circ}$

$\Rightarrow \angle 1+\angle 2=60^{\circ}$

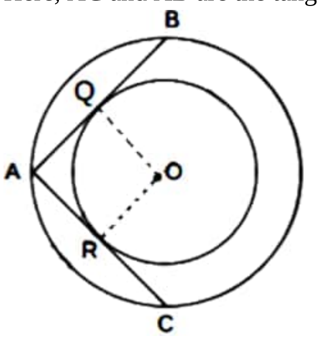
$\Rightarrow \angle 1+\angle 1=60^{\circ}\ [\because \angle 1=\angle 2]$

$\Rightarrow 2\angle 1=60^{\circ}$

$\Rightarrow \angle 1=30^{\circ}$

$\therefore \angle BAD=30^{\circ}$

23. Here, AC and AB are the tangents from external point A to the smaller circle.



∴ AC = AB

Now, AB is the chord of bigger circle and OQ is the perpendicular bisector of chord AB.

∴ AQ = QB

or, AB = 2AQ

or, AB = 2(5) = 10 cm ...[∵ Given AQ = 5 cm]

or, AC = 10 cm

24. LHS

$=(\sec\theta+\cos\theta)(\sec\theta-\cos\theta)$

Using identity (a + b)(a - b) = a^2 - b^2

$=\sec^2\theta-\cos^2\theta$

$=(1+\tan^2\theta)-(1-\sin^2\theta)\ \begin{bmatrix}\because \sec^2\theta=1+\tan^2\theta \\ \cos^2\theta=1-\sin^2\theta\end{bmatrix}$

$=1+\tan^2\theta-1+\sin^2\theta$

$=\tan^2\theta+\sin^2\theta$

= RHS

Hence proved

OR

L.H.S = $\sin^2 A.\cos^2 B$ - $\cos^2 A.\sin^2 B$

$= \sin^2 A(1 - \sin^2 B) - \{(1 - \sin^2 A)\sin^2 B\}$ $\left[\because \cos^2\theta = 1 - \sin^2\theta\right]$

$= \sin^2 A - \sin^2 A \sin^2 B - \{\sin^2 B - \sin^2 A \sin^2 B\}$

$= \sin^2 A - \sin^2 A \sin^2 B - \sin^2 B + \sin^2 A \sin^2 B$

$= \sin^2 A - \sin^2 B$

= R.H.S

Hence proved.

25. Here, r = 45 cm and $\theta = \frac{360^\circ}{8} = 45^\circ$

Area between two consecutive ribs of the umbrella = $\frac{\theta}{360^\circ} \times \pi r^2$

= $\frac{45^\circ}{360^\circ} \times \frac{22}{7} \times 45 \times 45 \quad = \frac{22275}{28}$ cm ^{2}

OR

Length of arc of circle = 44 cm

Radius of circle = 17.5 cm

Area of sector $= \frac{1}{2}$r $\times arc$

$= \left(\frac{1}{2} \times 44 \times 17.5\right) \text{cm}^2$

$= (22 \times 17.5)\text{cm}^2$

= 385 cm^2

Section C

26. We have to take LCM of 2, 4,3

2 = 2×1

4 = 2×2

3 = 3×1

LCM = 12

Thus they will meet at a gap of 12 days.

7 May - 19 May - 31 May - 12 June - 24 June - 6 July - 18 July - 30 July- 11 August - 23 August - 4 Sept.

Therefore the last day before 4th Sept. will be 23rd of August.

27. x^2 - 6

Let p(x) = x^2 - 6

For zeroes of p(x), p(x) = 0

$\Rightarrow x^2 - 6 = 0 \Rightarrow (x)^2 - (\sqrt{6})^2 = 0$

$\Rightarrow (x - \sqrt{6})(x + \sqrt{6}) = 0$

Using the identity $a^2 - b^2 = (a - b)(a + b)$

$\Rightarrow x - \sqrt{6} = 0$ or $x + \sqrt{6} = 0$

$\Rightarrow x = \sqrt{6}$ or $x = -\sqrt{6} \Rightarrow x = \sqrt{6}, -\sqrt{6}$

So, the zeroes of x^2 - 6 are $\sqrt{6}$ and $-\sqrt{6}$

Sum of zeroes

$= (\sqrt{6}) + (-\sqrt{6}) = 0 = \frac{-0}{1} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of zeroes

$= (\sqrt{6}) \times (-\sqrt{6}) = -6 = \frac{-6}{1} = \frac{\text{Constant term}}{\text{Coefficient of x}^2}$

Hence the relation between zeroes and coefficient is verified.

28. Let the one's digit be 'a' and ten's digit be 'b'.

Given, two digit number is 4 times the sum of its digits and twice the product of the digits.

$\Rightarrow$ 10b + a = 4(a + b)

$\Rightarrow$ a = 2b

Also, 10b + a = 2ab

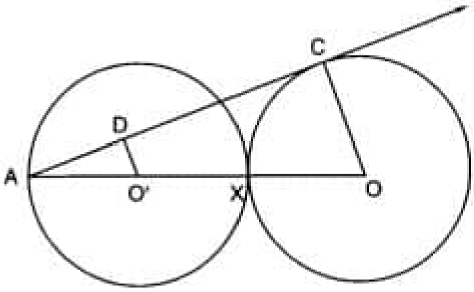
Substituting value of a.

$\Rightarrow$ 10b + 2b = 2 $\times$ 2b $\times$ b

$\Rightarrow$ b = 3
Thus, a = 6
Thus, the number is 36.

OR

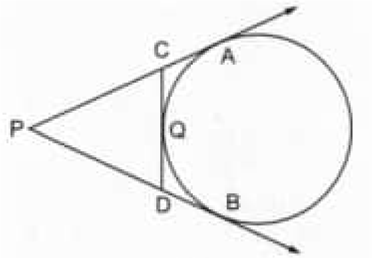
Let the number of 20 paisa coins be x and that of 25 paisa coins be y. Then,
x + y = 50 .... (i)
Total value of 20 paisa coins = 20 x paisa
Total value of 25 paisa coins = 25 y paisa
$\therefore$20 x+25 y = 1125 ...(because Rs11.25 = 1125 paisa)
$\Rightarrow$ 4x + 5y = 225 ... (ii)
Thus, we get the following system of linear equations
x + y - 50 = 0
4x + 5y - 225 = 0
By using cross-multiplication, we have
$\frac{x}{-225+250} = \frac{-y}{-225+200} = \frac{1}{5-4}$
$\Rightarrow \quad \frac{x}{25} = \frac{y}{25} = \frac{1}{1} \Rightarrow$ x = 25 and y = 25
Hence, there are 25 coins of each kind.

29. 

Two equal circles
$\therefore$ O'X = OX
and O'A = O'X
Now, in ΔAO'D and ΔAOC
$\angle A = \angle A$ [Common]
$\angle ADO' = \angle ACO = 90^\circ$
So, by AA similarly, we have
ΔAO'D ~ ΔAOC
$\therefore \frac{AO}{AO'} = \frac{CO}{DO'}$
$\Rightarrow \frac{AO'+AO'+AO'}{AO'} = \frac{CO}{DO'}$
$\Rightarrow \frac{3AO'}{AO'} = \frac{CO}{DO'}$
$\Rightarrow 3 = \frac{CO}{DO'}$
$\Rightarrow \frac{DO'}{CO} = \frac{1}{3}$

OR

Here, we have to find the perimeter of triangle PCD.



Perimeter is nothing but sum of all sides of the triangle. Therefore we have,
Perimeter of $\triangle PCD = PC + CD + PD$
In the given figure we can see that,
CD = CQ + QD
Therefore,

Perimeter of $\triangle PCD = PC + CQ + QD + PD$

We know that the two tangents drawn to a circle from a common external point will be equal in length. From this property we have,

CQ = CA

QD = DB

Now let us replace CQ and QD with CA and DA. We get,

Perimeter of $\triangle PCD = PC + CA + DB + PD$

Also from the figure we can see that,

DB + PD = PB

PC + CA = PA

Now, let us replace these in the equation for perimeter of $\triangle PCD$. We have,

Perimeter of $\triangle PCD$ = PB +PA

Also, from the property of tangents we know that, two tangents drawn to a circle from the same external point will be equal in length. Therefore,

PB = PA

Let us replace PA with PB in the above equation. We get,

Perimeter of $\triangle PCD$ = 2PB

It is given in the question that PB = 10 cm. Therefore,

Perimeter of $\triangle PCD$ = 2×10

Perimeter of $\triangle PCD$ = 20 cm

Hence, the perimeter of $\triangle PCD$ is 20 cm

30. We have,

$5 \tan\theta = 4 \Rightarrow \tan\theta = \frac{4}{5}$

Now, L.H.S = $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta} = \frac{\frac{5\sin\theta - 3\cos\theta}{\cos\theta}}{\frac{5\sin\theta + 2\cos\theta}{\cos\theta}}$ [Dividing Numbers and Denominator by $\cos\theta$]

$= \frac{\frac{5\sin\theta}{\cos\theta} - \frac{3\cos\theta}{\cos\theta}}{\frac{5\sin\theta}{\cos\theta} + \frac{2\cos\theta}{\cos\theta}} = \frac{5\tan\theta - 3}{5\tan\theta + 2} = \frac{5\times\frac{4}{5} - 3}{5\times\frac{4}{5} + 2} \quad \left[\because \tan\theta = \frac{4}{5}\right]$

$= \frac{4-3}{4+2} = \frac{1}{6}$ = R.H.S

Hence proved.

31. Let the frequency of the class 30 - 40 be f_1 and that of the class 50 - 60 be f_2. The total frequency is 229.

12 + 30 + f_1 + 65 + f_2 + 25 + 18 = 229

$\Rightarrow f_1 + f_2 = 79$

It is given that the median is 46

Clearly, 46 lies in the class 40 - 50. So, 40 - 50 is the median class.

$\therefore$ l = 40, h = 10, f = 65 and

F = 12 + 30 + f_1

= 42 + f_1

N = 229

Median = l + $\frac{\frac{N}{2} - F}{f} \times$ h

46 = 40 + $\frac{\frac{229}{2} - (42 + f_1)}{65} \times$ 10

46 = 40 + $\frac{145 - 2f_1}{13}$

6 = $\frac{145 - 2f_1}{13} \Rightarrow 2f_1 = 67 \Rightarrow f_1$ = 33.5 or 34 (say)

Since $f_1 + f_2 = 79$,

f_2 = 79 - 34

= 45

Hence, f_1 = 34 and f_2 = 45

Section D

32. Here roots are equal,

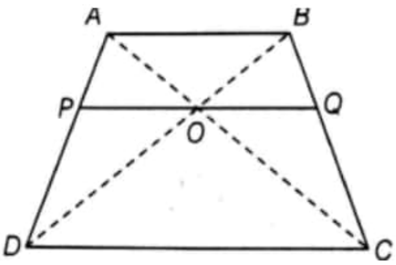
$\therefore D = B^2 - 4AC = 0$

Here, $A=1+m^2, B=2mc, C=\left(c^2-a^2\right)$
$\therefore (2mc)^2-4\left(1+m^2\right)\left(c^2-a^2\right)=0$
or, $4m^2c^2-4\left(1+m^2\right)\left(c^2-a^2\right)=0$
or, $m^2c^2-\left(c^2-a^2+m^2c^2-m^2n^2\right)=0$
or, $m^2c^2-c^2+a^2-m^2c^2+m^2a^2=0$
or, $-c^2+a^2+m^2a^2=0$
or, $c^2=a^2\left(1+m^2\right)$
Hence Proved.

OR

Let the average speed of truck be x km/h.
$\frac{150}{x}+\frac{200}{x+20}=5$
or, 150x + 3000 + 200x = 5x(x + 20)
or, $x^2-50x-600=0$
or, $x^2-60x+10x-600=0$
or, $x(x-60)+10(x-60)=0$
or, (x-60)(x + 10) = 0
or, x = 60 ; or x = -10
as, speed cannot be negative
Therefore, x=60 km/h
Hence, first speed of the truck = 60 km/h

33. Given:
ABCD is a trapezium,
Diagonals AC and BD are intersect at 0.
To prove: PQ || AB || DC.
PO = QO
Concepts Used:
AAA Similarity Criterion: If all three angles of a triangle equals to angles of another triangle, then both the triangles are similar.Basic Proportionality theorem: If a line is parallel to a side of a triangle which intersects the other sides into two distinct points, then the line divides those sides in proportion
Proof:



In △ABD and △POD,
PO || AB ...[∵ PQ || AB]
∠D = ∠D ...[common angle]
∠ABD = ∠POD ...[corresponding angles]
∴ △ABD ~ △POD ...[by AAA similarity criterion]
Then,
OP/AB = PD/AD …(i) [by basic proportionality theorem]
In △ABC and △OQC,
OQ || AB ...[∵ OQ || AB]
∠C = ∠C ...[common angle]
∠BAC = ∠QOC ...[corresponding angle]
∴ △ABC ~ △OQC ...[by AAA similarity criterion]
Then,
OQ/AB = QC/BC …(ii) ...[by basic proportionality theorem]
Now, in △ADC,
OP || DC

∴ AP/PD = OA/OC ...[by basic proportionality theorem] …(iii)

In △ABC, OQ ∥ AB

∴ BQ/QC = OA/OC ...[by basic proportionality theorem] …(iv)

From Equation (iii) and (iv),

AP/PD = BQ/QC

Adding 1 on both sides, we get,

= AP/PD + 1 = BQ/QC + 1

= ((AP + PD))/PD = (BQ + QC)/QC

= AD/PD = BC/QC

= PD/AD = QC/BC

= OP/AB = OQ/BC ...[from Equation (i) and (ii)]

⇒ OP/AB = OQ/AB ...[from Equation (iii)]

⇒ OP = OQ

Hence proved.

34. According to question

Diameterdiameter of the well = 7m

Radius of the well (r) = $\frac{7}{2}$ m = 3.5m and, height of the well (h) = 22.5 m

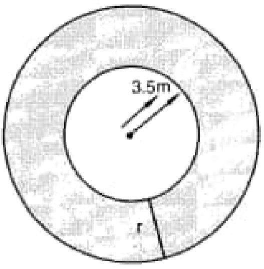
∴ Volume of the earth dugout = $\pi \times (3.5)^2 \times 22.5\ m^3 = \pi \times \frac{7}{2} \times \frac{7}{2} \times \frac{45}{2}\ m^3$

Let the width of the embankment be r metres. Clearly, embankment forms a cylindrical shell whose inner and outer radii are 3.5 m and (r + 3.5) m respectively and height 1.5 m.

∴ Volume of the embankment = Area of ring at top $\times$ height of the embankment

$= \pi\{(r + 3.5)^2 - (3.5)^2\} \times 1.5\ m^3 = \pi(r + 7)\ r \times \frac{3}{2}\ m^3$

But, Volume of the embankment = Volume of the well



$\Rightarrow \pi r(r+7) \times \frac{3}{2} = \pi \times \frac{7}{2} \times \frac{7}{2} \times \frac{45}{2}$

$\Rightarrow r(r + 7) = \frac{49}{4} \times 15$

$\Rightarrow 4r^2 + 28r = 735$

$\Rightarrow 4r^2 + 28r - 735 = 0$

$4r^2 + 70x - 42x - 735 = 0$

$\Rightarrow 2r(2r + 35) - 21(2r + 35) = 0$

$\Rightarrow (2r + 35)(2r - 21) = 0$

$\Rightarrow 2r + 35 = 0$ or $2r - 21 = 0$

$\Rightarrow r = \frac{-35}{2}$ or $x = \frac{21}{2}$

$\frac{-35}{2}$ is negative, hence neglect this value

$\Rightarrow x = \frac{21}{2} = 10.5$ m

Hence, the width of the embankment is 10.5 m

OR

Given side of a cube = 21 cm

Diameter of the hemisphere is equal to the side of the cubical piece (d) = 21 cm

⇒ Radius of the hemisphere = 10.5 cm

Volume of cube = $Side^3$

$= (21)^3$

$= 9261\ cm^3$

Surface area of cubical piece of wood = $6a^2$

= 6 × 21 × 21 cm^2

= 2646 cm^2

Volume of the hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times 10.5 \times 10.5 \times 10.5$

= 44 × 0.5 × 10.5 × 10.5

= 2425.5 cm^3

Surface area of hemisphere = $2\pi r^2$

= 2 × π × 10.5 × 10.5 cm

= 693 cm

Volume of remaining solid = Volume of cubical piece of wood – Volume of hemisphere

⇒ Volume of the remaining solid = 9261 - 2425.5

= 6835.5 cm^3

Surface area remaining piece of solid = surface area of cubical piece of wood – Area of circular base of hemisphere + Curved Surface area of hemisphere

= $6a^2 - \pi r^2 + 2\pi r^2$

= (2646 – π × 10.5^2 + 693) cm^2

= 2992.5 cm^2.

35. Calculation of median:

Class interval	Frequency(f_i)	Cumulative frequency
0 - 15	5	5
15 - 30	20	25
30 - 45	40	65
45 - 60	50	115
60 - 75	25	140

N = 140⇒ $\frac{N}{2}$= 70.

The median class is 45 -60.

$\therefore l = 45, h = 15, f = 50, c.f. = 65$

Median, M =$l + \left\{h \times \frac{\left(\frac{N}{2} - cf\right)}{f}\right\}$

$= 45 + \left\{15 \times \frac{(70-65)}{50}\right\}$

$= 45 + \left\{15 \times \frac{5}{50}\right\}$

$= 45 + 1.5 = 46.5$

Hence, the median age of diabetic patients is 46.5 years.

Section E

36. **Read the text carefully and answer the questions:**

Students of a school thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of class I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII. There are three sections of each class.

(i) Each class has 3 section

class 1 plants = 3 trees

class 2 plants = 6 trees

class 3 plants = 9 trees

$\therefore$ 3, 6, 9, ...

The no of trees planted by each class is in AP.

$S_n = \frac{n}{2}\{2a + (n - 1)d\}$

$S_5 = \frac{5}{2}\{2 \times 3 + (5 - 1)3\}$

$S_5 = \frac{5}{2}\{6 + 12\}$

$S_5 = \frac{5}{2} \times 18$

$S_5 = 45$

$\therefore$ class 1 to 5 students plant 45 trees.

(ii) $S_n = \frac{n}{2}\{2a + (n - 1)d\}$

$S_{12} = \frac{12}{2}\{2 \times 3 + (12 - 1)3\}$

$S_{12} = 6\{6 + 33\}$

$S_{12} = 6 \times 39$

$S_{12} = 234$

$\therefore$ total no of trees planted by school = 234

OR

$\because$ Class 12^{th} has 3 sections and each section plants 12 trees.

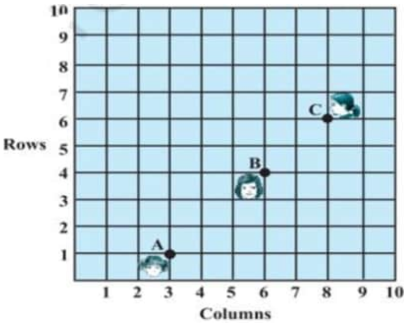
$\therefore$ total no of trees = 12×3

= 36 trees.

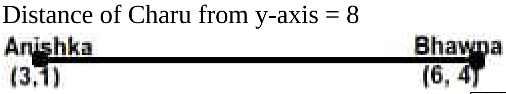
(iii) 30

37. **Read the text carefully and answer the questions:**

There is a function in the school. Anishka, Bhawna and Charu are standing in a rectangular ground at points A, B and C respectively as shown in the figure. They are ready to perform an aerobic dance.



(i) Distance of Charu from y-axis = 8

(ii)


Distance between Anishka and Bhawna = $\sqrt{(6-3)^2 + (4-1)^2}$

$= \sqrt{3^2 + 3^2}$

$= 3\sqrt{2}$

OR

Yes, because AB + BC = AC

(iii) AB = $3\sqrt{2}$

BC = $\sqrt{(8-6)^2 + (6-4)^2}$

$= \sqrt{2^2 + 2^2}$

$= 2\sqrt{2}$

AC = $\sqrt{(8-3)^2 + (6-1)^2}$

$= \sqrt{25 + 25}$

$= 5\sqrt{2}$

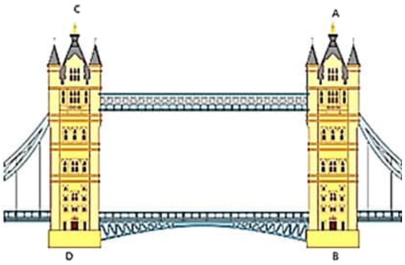
AC = $5\sqrt{2}$

AB + BC = AC

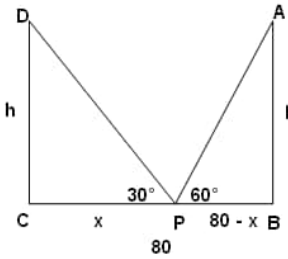
38. **Read the text carefully and answer the questions:**

Tower Bridge is a Grade I listed combined bascule and suspension bridge in London, built between 1886 and 1894, designed by Horace Jones and engineered by John Wolfe Barry. The bridge is 800 feet (240 m) in length and consists of two bridge towers connected at the upper level by two horizontal walkways, and a central pair of bascules that can open to allow shipping.

In this bridge, two towers of equal heights are standing opposite each other on either side of the road, which is 80 m wide. During summer holidays, Neeta visited the tower bridge. She stood at some point on the road between these towers. From that point between the towers on the road, the angles of elevation of the top of the towers was 60^{o} and 30^{o} respectively.



(i)



Suppose AB and CD are the two towers of equal height h m. BC be the 80 m wide road. P is any point on the road. Let CP be x m, therefore BP = (80 – x).

Also, $\angle$APB = 60^{o} and $\angle$DPC = 30^{o}

In right angled triangle DCP,

tan 30^{o} = $\frac{CD}{CP}$

$\Rightarrow \frac{h}{x} = \frac{1}{\sqrt{3}}$

$\Rightarrow h = \frac{x}{\sqrt{3}}$ ....(i)

In right angled triangle ABP,

tan 60^{o} = $\frac{AB}{AP}$

$\Rightarrow \frac{h}{80-x} = \sqrt{3}$

$\Rightarrow h = \sqrt{3}(80 - x)$

$\Rightarrow \frac{x}{\sqrt{3}} = \sqrt{3}(80 - x)$

$\Rightarrow$ x = 3(80 – x)

$\Rightarrow$ x = 240 – 3x

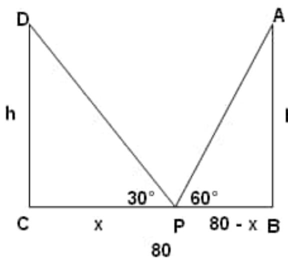
$\Rightarrow$ x + 3x = 240

$\Rightarrow$ 4x = 240

$\Rightarrow$ x = 60

Thus, the position of the point P is 60 m from C.

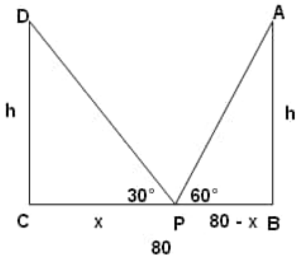
(ii)



Height of the tower, h = $\frac{x}{\sqrt{3}} = \frac{60}{\sqrt{3}}$ = $20\sqrt{3}$

The height of each tower is $20\sqrt{3}$ m.

OR



The distance between Neeta and top of tower CD.

In $\triangle$CDP

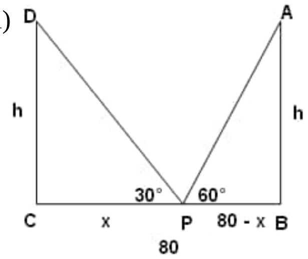
sin $30^o = \frac{CD}{PD}$

$\Rightarrow PD = \frac{CD}{\sin 30^0}$

$\Rightarrow PD = \frac{20\sqrt{3}}{\frac{1}{2}} = 40\sqrt{3}$

$\Rightarrow PD = 40\sqrt{3}$

(iii)



The distance between Neeta and top of tower AB.

In $\triangle$ABP

sin $60^o = \frac{AB}{AP}$

$\Rightarrow AP = \frac{AB}{\sin 60^0}$

$\Rightarrow AP = \frac{20\sqrt{3}}{\frac{\sqrt{3}}{2}}$

$\Rightarrow$ AP = 40 m