Class X Session 2023-24
Subject - Mathematics (Standard)
Sample Question Paper - 7

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub- parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

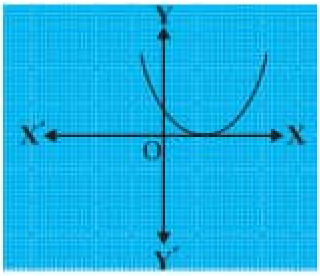
Section A

1. If two positive integers m and n are expressible in the form m pq^3 and n = p^3q^2, where p, q are prime numbers, then HCF (m, n) = **[1]**

   a) pq b) pq^2

   c) p^2q^3 d) p^3q^3

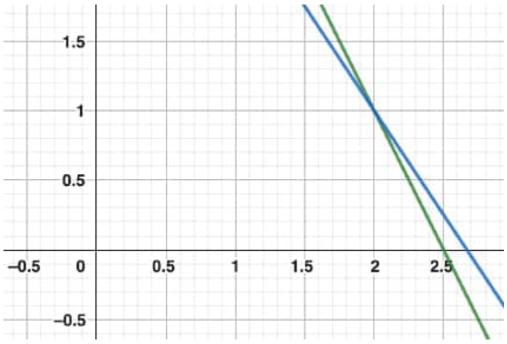
2. Find the number of zeroes of p(x) in the figure given below. **[1]**



   a) 3 b) 1

   c) 2 d) 0

3. The pair of equations 2x + y = 5, 3x + 2y = 8 has **[1]**



a) a unique solution

b) two solutions

c) infinitely many solutions

d) no solution

4. If one root of the equation $2x^2 + ax + 6 = 0$ is 2 then a = ? **[1]**

a) -7

b) $\frac{7}{2}$

c) $\frac{-7}{2}$

d) 7

5. Which of the following is an A.P.? **[1]**

a) 1, 3, 9, 27, ...

b) $1^2, 5^2, 7^2, 73, ...$

c) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, ...$

d) $1^2, 3^2, 5^2, 7^2, ...$

6. The perimeter of the triangle formed by the points (0, 0), (1, 0) and (0, 1) is **[1]**

a) $2 + \sqrt{2}$

b) 3

c) $\sqrt{2} + 1$

d) $1 \pm \sqrt{2}$

7. A line intersects the y-axis and x-axis at the points P and Q, respectively. If (2, –5) is the mid-point of PQ, then the coordinates of P and Q are, respectively **[1]**

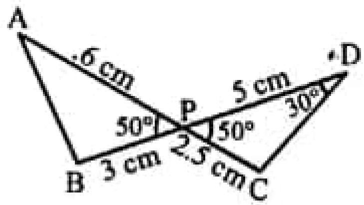
a) (0, – 5) and (2, 0)

b) (0, 4) and (– 10, 0)

c) (0, 10) and (– 4, 0)

d) (0, – 10) and (4, 0)

8. In the given figure, two line segments AC and BD intersect each other at the point P such that PA = 6 cm, PB = 3 cm, PC = 2.5 cm, PD = 5 cm, ∠APB = 50° and ∠CDP = 30°, ∠PBA = ? **[1]**



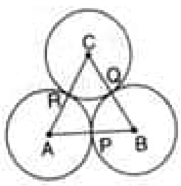
a) 60°

b) 100°

c) 50°

d) 30°

9. In the given figure, three circles with centres A, B, C respectively touch each other externally. If AB = 6 cm, BC = 8 cm and CA = 7 cm, then the radius of the circle with centre A is **[1]**



a) 2.5 cm

b) 1.5 cm

c) 2 cm d) 3 cm

10. PQ is a tangent drawn from a point P to a circle with centre O and QOR is a diameter of the circle such $\angle POR = 120^\circ$, then $\angle OPQ$ is **[1]**

a) 60° b) 35°

c) 30° d) 45°

11. $\sqrt{\frac{1+\cos A}{1-\cos A}} = ?$ **[1]**

a) cosec A - cot A b) None of these

c) cosec A + cot A d) cosec A cot A

12. $\left(\frac{2\tan 30^\circ}{1+\tan^2 30^\circ}\right)$ is equal to: **[1]**

a) cos 60° b) sin 60°

c) sin 30° d) tan 60°

13. A plane is observed to be approaching the airport. It is at a distance of 12 km from the point of observation and makes an angle of elevation of 30° there at. Its height above the ground is **[1]**

a) 10 km b) 12 km

c) 6 km d) none of these

14. The area of a sector whose perimeter is four times its radius r units, is **[1]**

a) $\frac{r^2}{2}$sq units b) $2r^2$ sq. units

c) r^2 sq. units d) $\frac{r^2}{4}$sq units

15. If the perimeter of a sector of a circle of radius 6.5 cm is 29 cm, then its area is **[1]**

a) 56 cm^2 b) 58 cm^2

c) 52 cm^2 d) 25 cm^2

16. The probability that a leap year selected at random will have 53 Fridays is **[1]**

a) $\frac{1}{7}$ b) $\frac{2}{7}$

c) $\frac{4}{7}$ d) $\frac{6}{7}$

17. A number x is chosen at random from the numbers -4, -3, -2, -1, 0, 1, 2, 3, 4, 5. The probability that $|x| < 3$ is **[1]**

a) 1 b) 0

c) $\frac{1}{2}$ d) $\frac{7}{10}$

18. The relation between mean, mode and median is **[1]**

a) mode = (3 $\times$ mean) - (2 $\times$ median) b) mode = (3 $\times$ median) - (2 $\times$ mean)

c) mean = (3 $\times$ median) - (2 $\times$mode) d) median = (3 $\times$mean) - (2 $\times$ mode)

19. **Assertion (A):** A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water is holds, a child finds its volume to be 345 cm^3. **[1]**

Reason (R): To calculate the volume of vessel the expression used here is v = $\pi r^2 h + \frac{4}{3}\pi r^3$.

a) Both A and R are true and R is the correct b) Both A and R are true but R is not the explanation of A. correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

20. **Assertion (A):** The 11th term of an AP is 7, 9, 11, 13 is 67. **[1]**

Reason (R): If s_n is the sum of first n terms of an AP then its nth term an is given by a n = $s_n + s_{n-1}$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

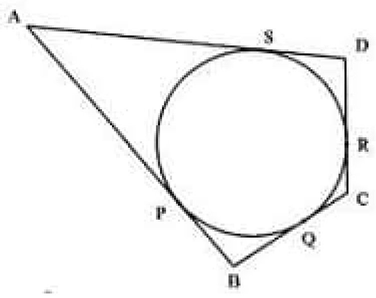
d) A is false but R is true.

Section B

21. Find the largest four-digit number which when divided by 4, 7 and 13 leaves a remainder 3 in each case. **[2]**

22. In ΔABC, P and Q are points on sides AB and AC respectively such that $PQ || BC$. If AP = 4 cm, PB = 6 cm and PQ = 3 cm, determine BC. **[2]**

23. A quadrilateral ABCD is drawn to the circumference of a circle. Prove that: AB + CD = AD + BC **[2]**



24. If sin X + $\sin^2$ X = 1, prove that $\cos^2$ X+ $\cos^4$ X = 1. **[2]**

OR

If a $\cos\theta$ + b $\sin\theta$ = m and a $\sin\theta$ - b $\cos\theta$ = n, prove that $a^2 + b^2 = m^2 + n^2$

25. The areas of two sectors of two different circles are equal. Is it necessary that their corresponding arc lengths are equal? Why? **[2]**

OR

Find the diameter of the circle whose area is equal to the sum of the areas of two circles having radii 4 cm and 3 cm.

Section C

26. In a morning walk, three persons step off together. Their steps measure 80 cm, 85 cm and 90 cm respectively. What is the minimum distance each should walk so that all can cover the same distance in complete steps? **[3]**

27. Find a quadratic polynomial, the sum and product of whose zeroes are $\frac{1}{4}$ and -1, respectively. **[3]**

28. Solve the system of the equation: **[3]**

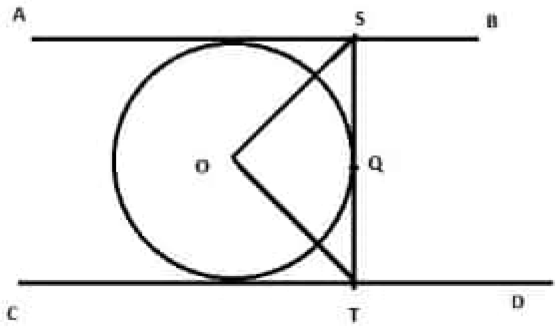
99x + 101y = 499

101x + 99y = 501

OR

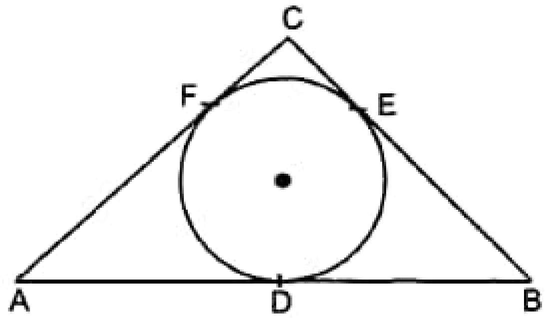
The monthly incomes of A and B are in the ratio of 9: 7 and their monthly expenditures are in the ratio of 4: 3. If each saves Rs 1,600 per month, find the monthly incomes of each.

29. In the adjoining figure, AB and CD are two parallel tangents to a circle with centre O. ST is the tangent segment between two parallel tangents touching the circle at Q. Show that $\angle SOT = 90^\circ$ **[3]**



OR

In figure, a circle inscribed in triangle ABC touches its sides AB, BC and AC at points D, E and F respectively. If AB = 12 cm, BC = 8 cm and AC = 10 cm, then find the lengths of AD, BE and CF.



30. In a $\triangle$ABC, $\angle$B = 90° and tan A = $\frac{1}{\sqrt{3}}$. Prove that: **[3]**

i. sin A cos C + cos A sin C = 1

ii. cos A cos C - sin A sin C = 0

31. If the mean of the following data is 20.6, find the missing frequency (x). **[3]**

x	**f**
10	3
15	10
20	x
25	7
35	5

Section D

32. Sum of the areas of two squares is 544 m^2. If the difference of their perimeters is 32 m, find the sides of the two squares. **[5]**

OR

A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/h more than the first speed. If it takes 3 hours to complete the total journey, what was its first average speed?

33. In trapezium ABCD, AB $\parallel$ DC and DC = 2AB. EF $\parallel$ AB, where E and F lie on BC and AD respectively, such that $\frac{BE}{EC} = \frac{4}{3}$. Diagonal DB intersects EF at G. Prove that 7EF = 11 AB. **[5]**

34. A solid is in the shape of a right-circular cone surmounted on a hemisphere, the radius of each of them is being 3.5 cm and the total height of solid is 9.5 cm. Find the volume of the solid. **[5]**

OR

A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

35. In the following data, find the values of p and q. Also, find the median class and modal class. **[5]**

Class	Frequency(f)	Cumulative frequency(cf)
100 - 200	11	11
200 - 300	12	p
300 - 400	10	33
400 - 500	q	46
500 - 600	20	66
600 - 700	14	80

Section E

36. **Read the text carefully and answer the questions:** **[4]**

In a school garden, Dinesh was given two types of plants viz. sunflower and rose flower as shown in the following figure.

The distance between two plants is to be 5m, a basket filled with plants is kept at point A which is 10 m from the first plant. Dinesh has to take one plant from the basket and then he will have to plant it in a row as shown in the figure and then he has to return to the basket to collect another plant. He continues in the same way until all the flower plants in the basket. Dinesh has to plant ten numbers of flower plants.

(i) Write the above information in the progression and find first term and common difference.

(ii) Find the distance covered by Dinesh to plant the first 5 plants and return to basket.

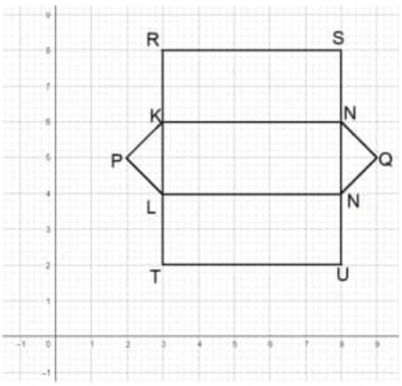
OR

If the speed of Dinesh is 10 m/min and he takes 15 minutes to plant a flower plant then find the total time taken by Dinesh to plant 10 plants.

(iii) Find the distance covered by Dinesh to plant all 10 plants and return to basket.

37. **Read the text carefully and answer the questions:** **[4]**

The camping alpine tent is usually made using high-quality canvas and it is waterproof. These alpine tents are mostly used in hilly areas, as the snow will not settle on the tent and make it damp. It is easy to layout and one need not use a manual to set it up. One alpine tent is shown in the figure given below, which has two triangular faces and three rectangular faces. Also, the image of canvas on graph paper is shown in the adjacent figure.



(i) What is the distance of point Q from y-axis?

(ii) What are the coordinates of U?

OR

What is the Perimeter of image of a rectangular face?

(iii) What is the distance between the points P and Q?

38. **Read the text carefully and answer the questions:** **[4]**

An observer on the top of a 40m tall light house (including height of the observer) observes a ship at an angle of depression 30^{o} coming towards the base of the light house along straight line joining the ship and the base of the light house. The angle of depression of ship changes to 45^{o} after 6 seconds.

(i) Find the distance of ship from the base of the light house after 6 seconds from the initial position when angle of depression is 45^{o}.

(ii) Find the distance between two positions of ship after 6 seconds?

OR

Find the distance of ship from the base of the light house when angle of depression is 30^{o}.

(iii) Find the speed of the ship?

Solution

Section A

1.

(b) pq^2

Explanation: Two positive integers are expressed as follows:

$m = pq^3$

$n = p^3q^2$

p and q are prime numbers.

Then, taking the smallest powers of p and q in the values for m and n we get

HCF (m, n) = pq^2

2.

(b) 1

Explanation: The number of zeroes is 1 as both the zeros are same here.

3. **(a)** a unique solution

Explanation: Given: 2x + y - 5 = 0 and 3x + 2y - 8 = 0

We know that the general form for a pair of linear equations in 2 variables x and y is $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$.

Comparing with above equations,

we have $a_1 = 2$, $b_1 = 1$, $c_1 = -5$; $a_2 = 3$, $b_2 = 2$, $c_2 = -8$

$\frac{a_1}{a_2} = \frac{2}{3}$

$\frac{b_1}{b_2} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{-5}{-8} = \frac{5}{8}$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

The lines are intersecting.

$\therefore$ The pair of equations has a unique solution.

4. **(a)** -7

Explanation: One root of the equation $2x^2 + ax + 6 = 0$ is 2 i.e. it satisfies the equation

$2(2)^2 + 2a + 6=0$

8 + 2 a + 6=0

2a = - 14

a = - 7

5.

(b) $1^2, 5^2, 7^2, 73, ...$

Explanation: In $1^2, 5^2, 7^2, 73, ... = 1, 25, 49, 73, ...$

$d = a_2 - a_1 = 25 - 1 = 24$

And $d = a_3 - a_2 = 49 - 25 = 24$

Also $d = a_4 - a_3 = 73 - 49 = 24$

Here, the common difference is the same for all terms, therefore, it is an AP.

6. **(a)** $2 + \sqrt{2}$

Explanation: Let the vertices of $\triangle ABC$ be A(0, 0), B(1, 0) and C(0, 1)

Now length of AB $= \sqrt{(1-0)^2 + (0-0)^2}$

$= \sqrt{(1)^2 + 0^2} = \sqrt{1^2} = 1$

Length of AC $= \sqrt{(0-0)^2 + (1-0)^2} = \sqrt{0^2 + (1)^2}$

$= \sqrt{1^2} = 1$

and length of BC $= \sqrt{(0-1)^2 + (1-0)^2}$
$= \sqrt{(1)^2 + (1)^2} = \sqrt{1+1} = \sqrt{2}$
Perimeter of $\triangle ABC$ = Sum of sides
$= 1 + 1 + \sqrt{2} = 2 + \sqrt{2}$

7.

(d) (0, – 10) and (4, 0)

Explanation:

Let the coordinates of P (0, y) and Q (x, 0).

So, the mid - point of P (0, y) and Q (x, 0) = M

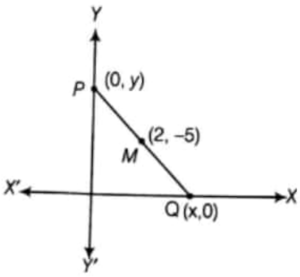
Coordinates of M = $\left(\frac{0+x}{2}, \frac{y+0}{2}\right)$

$\therefore$ Mid - point of a line segment having points (x_1, y_1) and (x_2, y_2)

= $\left(\frac{(x_1+x_2)}{2}, \frac{(y_1+y_2)}{2}\right)$

Given,

Mid - point of PQ is (2, - 5)



$\therefore 2 = \frac{x+0}{2}$ = 4 = x + 0

x = 4

$-5 = \frac{y+0}{2}$ = - 10 = y + 0

- 10 = y

So,

x = 4 and y = - 10

Thus, the coordinates of P and Q are (0, - 10) and (4, 0)

8.

(b) 100°

Explanation: In the given figure, two line segments AC and BD intersect each other at P such that PA = 6 cm, PB = 3 cm, PC = 2.5 cm, PD = 5 cm, $\angle$APB = 50° and $\angle$CDP = 30°

In $\triangle$ABP and $\triangle$CPD,

$\angle APB = \angle CPD$ (each = 50°) and vertically)

opposite angles.

$\frac{AP}{PB} = \frac{6}{5}, \frac{BP}{PC} = \frac{3}{2.5} = \frac{6}{5}$

$\therefore \frac{AP}{PB} = \frac{BP}{PC}$

$\therefore \Delta ABP \sim \Delta CPD$ (SAS axiom)

$\therefore \angle PAB = \angle CDP = 30^\circ$

and $\angle ABP = \angle DCP$

But $\angle ABP + \angle APB + \angle BAP = 180^\circ$ (sum of angles of a triangles)

$\Rightarrow \angle ABP + 50^\circ + 30^\circ = 180^\circ$

$\Rightarrow \angle ABP = 180^\circ - 50^\circ - 30^\circ = 100^\circ$

9. **(a)** 2.5 cm

Explanation: Let AP = x, then BP = 6 - x

and BP = BQ = 6 - x

Also AR = x [Radii] $\Rightarrow$ CR = 7 - x ……….(i)

And CQ = 8 - 6 + x = 2 + x ……….(ii)

But CR = CQ [Radii] $\therefore$7 - x = 2 + x

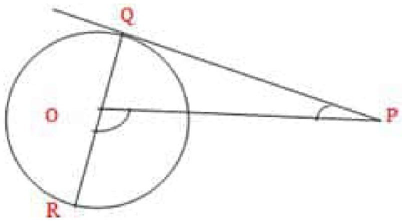
[From eq. (i) and (ii)]

$\Rightarrow$ 2x = 5

$\Rightarrow$ x = 2.5 cm

10.

(c) 30°

Explanation: 

Here $\angle PQO = 90^\circ$ Since, $\angle QOR = 180^\circ$

$\therefore \angle POQ + \angle POR = 180^\circ$

$\Rightarrow \angle POQ + 120^\circ = 180^\circ$

$\Rightarrow \angle POQ = 60^\circ$

Now, in triangle OPQ,

$\angle OPQ + \angle PQO + \angle QOP = 180^\circ$

$\Rightarrow \angle OPQ + 90^\circ + 60^\circ = 180^\circ$

$\Rightarrow \angle OPQ = 30^\circ$

11.

(c) cosec A + cot A

Explanation: $\sqrt{\frac{1+\cos A}{1-\cos A}} = \sqrt{\frac{(1+\cos A)}{(1-\cos A)} \times \frac{(1+\cos A)}{(1+\cos A)}} = \frac{(1+\cos A)}{\sqrt{1-\cos^2 A}} = \frac{(1+\cos A)}{\sqrt{\sin^2 A}}$

$= \frac{(1+\cos A)}{\sin A} = \left(\frac{1}{\sin A} + \frac{\cos A}{\sin A}\right) = (cosec\ A + \cot A)$

12.

(b) sin 60^o

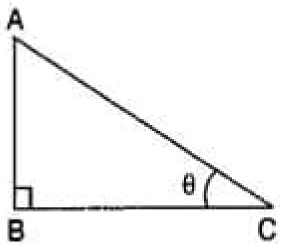
Explanation: $\frac{2tan30^o}{1+tan^2 30^o} = \frac{2\times\frac{1}{\sqrt{3}}}{1+(\frac{1}{\sqrt{3}})^2}$

$\frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{\sqrt{3}}{2}$

= $\sin 60^o$

13.

(c) 6 km

Explanation: 

Let the height of the flying plane be AB = h meters, distance from the poisnt of observation AC = 12 m and angle of elevation $\theta = 30^\circ$

$\therefore \sin 30^\circ = \frac{AB}{AC} \Rightarrow \frac{1}{2} = \frac{h}{12}$ $\Rightarrow$h = 6 meters

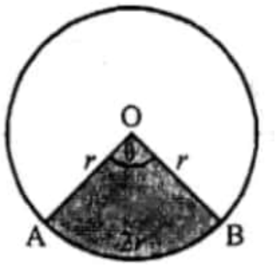
14.

(c) r^2 sq. units

Explanation: Radius of sector = r

Perimeter = 4r

and length of arc = 4r - 2r = 2r



$\therefore$ Let angle at the centre = θ

Then, $2\pi r = \frac{\theta}{360^\circ}$ and $2r = \frac{\theta}{360^\circ}$

$\Rightarrow \pi \times \frac{\theta}{360^\circ}$ = 1 ...(i)

Now area = $\pi r^2 \times \frac{\theta}{360^\circ} = r^2 \left(\pi \times \frac{\theta}{360^\circ}\right)$

= $r^2 \times 1$ [from (i)]

= r^2

15.

(c) 52 cm^2

Explanation: We know that perimeter of a sector of radius, $r = 2r + \frac{\theta}{360} \times 2\pi r \ldots(1)$

Therefore, substituting the corresponding values of perimeter and radius in equation (1), we get,

$29 = 2 \times 6.5 + \frac{\theta}{360} \times 2\pi \times 6.5 \ \ldots(2)$

$29 = 2 \times 6.5 \left(1 + \frac{\theta}{360} \times \pi\right)$

$\frac{29}{2\times 6.5} = \left(1 + \frac{\theta}{360} \times \pi\right)$

$\frac{29}{2\times 6.5} - 1 = \frac{\theta}{360} \times \pi \ \ldots\ldots\ldots(3)$

We know that area of the sector $= \frac{\theta}{360} \times \pi r^2$

From equation (3), we get

Area of the sector $= \left(\frac{29}{2\times 6.5} - 1\right) r^2$

Substituting r = 6.5 we get,

Area of the sector $= \left(\frac{29}{2\times 6.5} - 1\right) 6.5^2$

$= \left(\frac{29\times 6.5^2}{2\times 6.5} - 6.5^2\right)$

$= \left(\frac{29\times 6.5}{2} - 6.5^2\right)$

$= \left(\frac{29\times 6.5}{2} - 6.5^2\right)$

$= (94.25 - 42.25)$

= 52

Therefore, area of the sector is 52 cm^2.

16.

(b) $\frac{2}{7}$

Explanation: Leap year contains 366 days = 364 days + 2 days = $\frac{364}{7}$ weeks + 2 additional days = 52 weeks + 2 additional days

52 weeks contain 52 Fridays

We will get 53 Fridays if one of the remaining two additional days is a Friday

These additional days can be :

{(Sunday, Monday), (Monday, Tuesday), (Tuesday, Wednesday), (Wednesday, Thursday), (Thursday, **Friday**), (**Friday**, Saturday), (Saturday, Sunday)}

Number of total outcomes = 7

Number of possible outcomes = 2

$\therefore$ Required Probability of the event $= \frac{Number\,of\,possible\,outcomes}{Number\,of\,total\,outcomes} = \frac{2}{7}$

17.

(c) $\frac{1}{2}$

Explanation: Number of total outcomes = 10

Number of possible outcomes = {-2, -1, 0, 1, 2} = 5

$\therefore$ Required Probability = $\frac{5}{10} = \frac{1}{2}$

18.

(b) mode = (3 $\times$ median) - (2 $\times$ mean)

Explanation: mode = (3 $\times$ median) - (2 $\times$ mean)

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

20.

(c) A is true but R is false.

Explanation: A is true but R is false.

Section B

21. LCM of (4,7,13) = 364

Largest 4 digit number = 9999

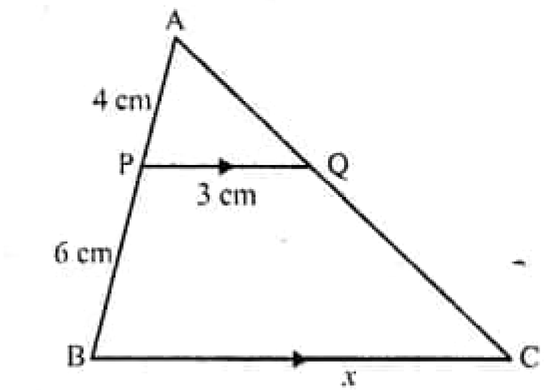
On dividing 9999 by 364 we get remainder as 171

Greatest number of 4 digits divisible by 4, 7 and 13 = (9999 – 171) = 9828

Hence, required number = (9828 + 3) = 9831

Therefore 9831 is the number.

22. Let BC = x cm



In Δ's APQ and ABC,we have,

$\angle A = \angle A$

$\angle APQ = \angle ABC$

Therefore,by AA criteria of similar Δ's ,we have,

$\because \text{PQ} \| \text{BC}$

$\therefore \Delta\text{APQ} \sim \Delta\text{ABC}$

$\therefore \frac{\text{AP}}{\text{AB}} = \frac{\text{AQ}}{\text{AC}} = \frac{\text{PQ}}{\text{BC}}$

$\Rightarrow \frac{\text{AP}}{\text{AP+PB}} = \frac{\text{PQ}}{\text{BC}}$

$\Rightarrow \frac{4}{4+6} = \frac{3}{x} \Rightarrow \frac{4}{10} = \frac{3}{x}$

$\Rightarrow x = \frac{10\times3}{4} = \frac{15}{2}$

$\therefore \text{BC} = \frac{15}{2}\text{cm} = 7.5 \text{ cm}$

23. Let the sides of the quadrilateral ABCD touch the circle at P, Q, R and S. Since, the lengths of the tangents from an external point to a given circle are equal.

$\therefore$ AP = AS

$\Rightarrow$ BP = BQ

CR = CQ

$\Rightarrow$ DR = DS

Adding, (AP + BP) + (CR + DR) = (BQ + CQ) + (AS + DS)

$\Rightarrow$ AB + CD = BC + AD.

Hence proved

24. Given sin X + $\sin^2$X = 1 .... (i)

$\Rightarrow$ sin X = 1 - $\sin^2$X = $\cos^2$ X ...... (ii)

Now we show that $\cos^2 X + \cos^4 X = 1$

L.H.S = $\cos^2 X + \cos^4 X$

= $1 - \sin^2 X + (1 - \sin^2 X)^2$ [Using (ii)]

= $\sin X + \sin^2 X$ [Using (ii)]

= 1 [Using (i)]

= R.H.S

OR

Given,

R.H.S = $m^2 + n^2$

= $(a \cos\theta + b \sin\theta)^2 + (a \sin\theta - b \cos\theta)^2$ $[\sin ce,\ m = a\ \cos\theta + b\ \sin\theta\ and\ n = a\ \sin\theta - b\ \cos\theta]$

= $(a^2\cos^2\theta + b^2\sin^2\theta + 2ab \cos\theta \sin\theta) + (a^2\sin^2\theta + b^2\cos^2\theta - 2ab \sin\theta \cos\theta)$ $\left[\because (a \pm b)^2 = a^2 + b^2 \pm 2ab\right]$

= $a^2 (\cos^2\theta + \sin^2\theta) + b^2 (\sin^2\theta + \cos^2\theta) = a^2 + b^2$ = L.H.S $\left[\because \sin^2\theta + \cos^2\theta = 1\right]$

therefore, $m^2 + n^2 = a^2 + b^2$

Hence proved.

25. Area of first sector = $\left(\frac{1}{2}\right)(r_1)^2\theta_1,$

where r1 is the radius,

θ_1 is the angle in radians subtended by the arc at the center of the circle.

Area of second sector = $\left(\frac{1}{2}\right)(r_2)^2\theta_2$

where r_2 is the radius,

θ_2 is the angle in radians subtended by the arc at the centrer of the circle.

Given that: $\left(\frac{1}{2}\right)(r_1)^2\theta_1 = \left(\frac{1}{2}\right)(r_2)^2\theta_2$

$\Rightarrow (r_1)^2\theta_1 = (r_2)^2\theta_2$

It depends on both radius and angle subtended at the center. But arc length only depends on radius of the circle. Therefore, it is not necessary that the corresponding arc lengths are equal. It is possible only if corresponding angles are equal (because then, the corresponding radii will be equal and hence the arc lengths will be equal).

OR

Let the radius of the large circle be R.

Then, we have

Area of large circle of radius R = Area of a circle of radius 4 cm + Area of circle of radius 3 cm

$\Rightarrow \pi R^2 = \left(\pi \times 4^2 + \pi \times 3^2\right)$

$\Rightarrow \pi \mathrm{R}^2 = (16\pi + 9\pi)$

$\Rightarrow \pi R^2 = 25\pi$

$\Rightarrow$ R^2 = 25

$\Rightarrow$ R = 5 cm

$\Rightarrow$ Diameter = 2R = 10 cm

Section C

26. GIVEN: Their steps measure 80 cm, 85 cm and 90 cm.

We have to find the L.C.M of the measures of their steps i.e. 80 cm, 85 cm, and 90 cm, to calculate the required distance each should walk.

L.C.M of 80 cm, 85 cm, and 90 cm.

80 = 2^4 x 5

85 = 17 x 5

90 = 2 x 3 x 3 x 5

L.C.M of 80, 85 and 90 = 2^4 x 3 x 3 x 5 x 17

L.C.M of 80, 85 and 90 = 12240 cm

Hence, the minimum distance each should walk so that all can cover the same distance in complete steps is 12240 cm.

27. Let the required polynomial be $ax^2 + bx + c$

and let its zeroes be α and β

Then, $\alpha + \beta = \frac{1}{4} = -\frac{b}{a}$ and $\alpha\beta = -1 = \frac{c}{a}$

If a = 4, then b = -1 and c = -4

So, one quadratic polynomial which satisfies the given conditions is $4x^2 - x - 4$

Or

If α and β zeroes of the polynomials then standard quadratic polynomial is given by

$x^2-(\alpha+\beta)x+\alpha\beta$, where $\alpha+\beta=\frac{1}{4}\ and\ \alpha\beta=-1$ [Given]

Now, we have,

$x^2-(\alpha+\beta)x+\alpha\beta$

$=x^2-\left(\frac{1}{4}\right)x+(-1)$

$=\frac{1}{4}\left(4x^2-x-4\right)$

Required polynomial is $4x^2 - x - 4$

28. Given, 99x + 101y = 499 ....(i)

101x + 99y = 501... (ii)

Adding eqn. (i) and (ii),

(99x + 101y) + (101x + 99y) = 499 + 501

99x + 101y + 101x + 99y = 1000

200x + 200y = 1000

x + y = 5 ...(iii)

Subtracting eqn. (ii) from eqn. (i), we get

(99x + 101y) - (101x + 99y) = 499 - 501

99x + 101y - 101x - 99y = -2

-2x + 2y = - 2

or, x - y= 1 .... (iv)

Adding equations (iii) and (iv)

x + y + x - y = 5 + 1

2x = 6

$\therefore$ x = 3

Substituting the value of x in eqn. (iii), we get

3 + y = 5

y = 2

Hence the value of x and y of given equation are 3 and 2 respectively.

OR

The monthly incomes of A and B are in the ratio of 9: 7.

So incomes of A and B be 9x and 7x.

And monthly expenditures of A and B are in the ratio of 4: 3.

So, monthly expenditures be 4y and 3y

then, as per given condition

9x-4y=1600 .... (i)

7x-3y=1600 ... (ii)

Multiplying eq (i) by 3 and eq (ii) by 4, we get

27x-12y=4800 ... (iii)

28x-12y=6400 ... (iv)

Subtracting eq (iii) from eq (iv), we get

x=1600

Put x = 1600 in (ii), we get

7x-3y=1600

7(1600)- 3y = 1600

3y = 11200 - 1600

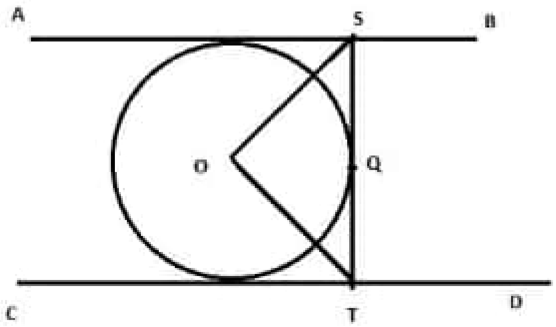
3y = 9600

y=3200

Hence the monthly income of A and B are 14400,11200.

And monthly expenditures of A and B are 12800, 9600.

29.



From the given figure we have, AB⊥ST, then ∠ASQ = 90° and CD⊥TS then ∠CTQ = 90°

$\angle ASO = \angle QSO = \frac{90}{2}$ = 45°

Similarly, $\angle OTQ$ = 45°

To find $\angle SOT$

Consider △SOT

$\angle OTS$ = 45° and $\angle OST$ = 45°

$\angle SOT + \angle OTS + \angle OST$ = 180° (by angle sum property of a triangle)

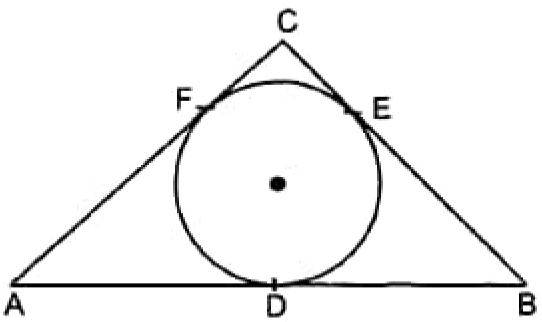
$\angle SOT$ = 180° - ($\angle OTS + \angle OST$) = 180° - (45° + 45°) = 180° - 90° = 90°

Therefore, $\angle SOT$ = 90°

Hence proved

OR

Given,



Let ,

$AD = x\ cm,$

$BD = AB - AD$

$= (12 - x)cm$

AD = AF [tangents from point A]

$AF = x\ cm$

Now, $CF = AC - AF = (10 - x)cm$

$CE = CF$

$CE = (10 - x)cm$

$BD = BE$

$BE = (12 - x)cm$

Now, $BC = CE + BE$

$\Rightarrow 8 = (10 - x) + (12 - x)$

$\Rightarrow 8 = 22 - 2x \Rightarrow 2x = 14$

⇒ x = 7 cm

⇒ AD = 7 cm.

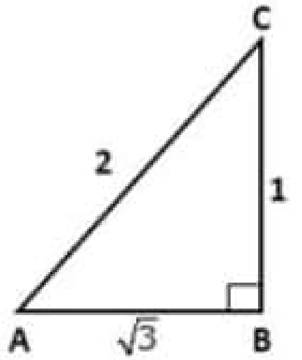
$BE = 12 - x$

$= 12 - 7 = 5cm.$

$\Rightarrow CF = 10 - x$

$= 10 - 7 \quad = 3cm$

30. Let us draw a triangle ABC in which ∠B = 90°.



Let ∠A = θ°.

$\tan A = \frac{1}{\sqrt{3}}$

Then, $\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB} = \frac{1}{\sqrt{3}}$

Let BC = 1 and AB = $\sqrt{3}$,

By Pythagoras' theorem, we have

$AC^2 = AB^2 + BC^2$

$=(\sqrt{3})^2 + 1^2$ = 3 + 1 = 4

$\Rightarrow \quad AC = 2$

Now,

$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{1}{2}$ and $\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$

$\sin C = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$ and $\cos C = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{1}{2}$

i. LHS = sin A cos C + cos A sin C

$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{1}{4} + \frac{3}{4}$

$= \frac{4}{4}$

= 1

= R.H.S

ii. cos A cos C - sin A sin C

= $\frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$

= 0

= R.H.S

31.

x	**f**	**$f_i x_i$**
10	3	30
15	10	150
20	x	20x
25	7	175
35	5	175

Let the missing frequency is 'x'.

$\Sigma f_i x_i = 530 + 20x$,

$\Sigma f_i = 25 + x$

Mean = $\frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{530+20x}{25+x} = 20.6$

$530 + 20x = 25 \times 20.6 + 20.6x$

$\Rightarrow 530 + 20x = 515 + 20.6x$

$\Rightarrow 15 = 0.6x$

$\Rightarrow \quad x = \frac{15}{0.6} = 25$

Section D

32. Let sides of two squares be a cm and b cm

Sum of areas of squares = $a^2 + b^2$

Sum of Perimater = 4a + 4b

A.T.Q $a^2 + b^2 = 544$

4a - 4b = 32

or, a - b = 8

a = b + 8

$\Rightarrow a^2 + b^2 = 544$

$(b + 8)^2 + b^2 = 544$

$\Rightarrow b^2 + 64 + 16b + b^2 = 544$

$\Rightarrow 2b^2 + 16b + 64 = 544$

$\Rightarrow b^2 + 8b + 32 = 272$

$\Rightarrow b^2 + 8b - 240 = 0$

$\Rightarrow b^2 + 20b - 12b - 240 = 0$

$\Rightarrow b(b + 20) - 12(b + 20) = 0$

b = 12 or b = -20

Sides cant be -ve

b = 12

a = 20

Therefore, sides of two squares are 20 cm and 12 cm respectively

OR

Let the speed of the train be x km/hr for first 54 km and for next 63 km, speed is (x + 6) km/hr.

According to the question

$\frac{54}{x} + \frac{63}{x+6} = 3$

$\frac{54(x+6)+63x}{x(x+6)} = 3$

or, 54x + 324 + 63x = 3x(x + 6)

or, $117x + 324 = 3x^2 + 18x$

or, $3x^2 - 99x - 324 = 0$

or, $x^2 - 33x - 108 = 0$

or, $x^2 - 36x + 3x - 108 = 0$

or, x(x - 36) + 3(x - 36) = 0

(x - 36)(x + 3) = 0

x = 36

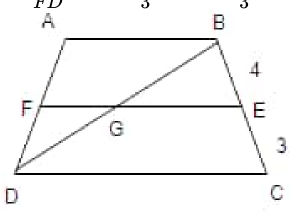
x = - 3 rejected.

(as speed is never negative)

Hence First speed of train = 36 km/h

33. $\frac{AF}{FD} = \frac{BE}{EC} = \frac{4}{3}$

$\Rightarrow \frac{AD}{FD} = \frac{AF+FD}{3} = \frac{4+3}{3} = \frac{7}{3}$



In $\triangle DFG$ and $\triangle DAB$

$\angle$FDG $\cong$ $\angle$ADB ($\because$ common angle)

$\angle$ DFG $\cong$ $\angle$DAB ($\because$ FG $\parallel$ AB)

So,$\triangle DFG \sim \triangle DAB$ (AA similarity)

Also, $\triangle BGE \sim \triangle BDC$

$\Rightarrow \frac{DF}{DA} = \frac{FG}{AB}$ and $\frac{GE}{DC} = \frac{BE}{BC}$

$\Rightarrow \frac{3}{7} = \frac{FG}{AB}$ and $\frac{GE}{2AB} = \frac{4}{7}$

Now, $EF = FG + GE = \frac{3}{7}AB + \frac{8}{7}AB$
$\Rightarrow EF = (\frac{3}{7} + \frac{8}{7})AB$
$\Rightarrow EF = (\frac{3+8}{7})AB$
$\Rightarrow EF = (\frac{11}{7})AB$
$\Rightarrow 7EF = 11AB$

34. 

From the given figure,
Height (AB) of the cone = AC - BC (Radius of the hemisphere)
Thus, height of the cone = Total height - Radius of the hemisphere
= 9.5 - 3.5
= 6 cm
Volume of the solid = Volume of the cone + Volume of the hemisphere
$= \left(\frac{1}{3}\pi r^2 h\right) + \left(\frac{2}{3}\pi r^3\right)$
$= \frac{1}{3}\pi r^2(h + 2r)$
$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5(6 + 2 \times 3.5)$
$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 13$
= 166.83 cm^3
Thus, total volume of the solid is 166.83 cm^3.

OR

According to the question,a hemispherical depression is cut from one face of the cubical block such that the diameter l of the hemisphere is equal to the edge of the cube.
Let the radius of hemisphere = r
$\therefore r = \frac{l}{2}$
Now, the required surface area = Surface area of cubical block - Area of base of hemisphere + Curved surface area of hemisphere.
$= 6(\text{ side })^2 - \pi r^2 + 2\pi r^2$
$= 6l^2 - \pi\left(\frac{l}{2}\right)^2 + 2\pi\left(\frac{l}{2}\right)^2$
$= 6l^2 - \frac{\pi l^2}{4} + \frac{\pi}{2}l^2$
$= 6l^2 + \frac{\pi l^2}{4}$
Surface area $= \frac{1}{4}(24 + \pi)l^2$ units.
$= \frac{1}{4}\left(24 + \frac{22}{7}\right)l^2$
$= \frac{1}{4} \times \frac{190}{7} \times l^2$
$= 184.18l^2 \text{ unit }^2$

35. p = Frequency of the class + cf of preceding class
$= 12 + 11 = 23$
q = cf of the class - cf of preceding class
$= 46 - 33 = 13$
Table:

Class Interval	Frequency	Cumulative Frequency
100 - 200	11	11
200 - 300	12	23
300 - 400	10	33

400 - 500	13	46
500 - 600	20	66
600 - 700	14	80

$N = 80 \Rightarrow \frac{N}{2} = 40$

The cumulative frequency just greater than 40 is 46.

Hence, median class is $400 - 500$.

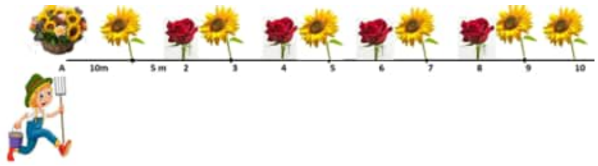
Here, maximum frequency $= 20$

Hence, modal class is $500 - 600$.

Section E

36. **Read the text carefully and answer the questions:**

In a school garden, Dinesh was given two types of plants viz. sunflower and rose flower as shown in the following figure.



The distance between two plants is to be 5m, a basket filled with plants is kept at point A which is 10 m from the first plant. Dinesh has to take one plant from the basket and then he will have to plant it in a row as shown in the figure and then he has to return to the basket to collect another plant. He continues in the same way until all the flower plants in the basket. Dinesh has to plant ten numbers of flower plants.

(i) The distance covered by Dinesh to pick up the first flower plant and the second flower plant,

= 2 $\times$ 10 + 2 $\times$ (10 + 5) = 20 + 30

therefore, the distance covered for planting the first 5 plants

= 20 + 30 + 40 + ... 5 terms

This is in AP where the first term a = 20

and common difference d = 30 - 20 = 10

(ii) We know that a = 20, d = 10 and number of terms = n = 5 so,

$S_n = \frac{n}{2}[2a + (n-1)d]$

So, the sum of 5 terms

$S_5 = \frac{5}{2}[2 \times 20 + 4 \times 10] = \frac{5}{2} \times$ 80 = 200 m

Hence, Dinesh will cover 200 m to plant the first 5 plants.

OR

Total distance covered by Ramesh 650 m

Time = $\frac{\text{distance}}{\text{speed}} = \frac{650}{10}$ = 65 minutes

Time taken to plant all 10 plants = 15 $\times$ 10 = 150 minutes

Total time = 65 + 150 = 215 minutes = 3 hrs 35 minutes

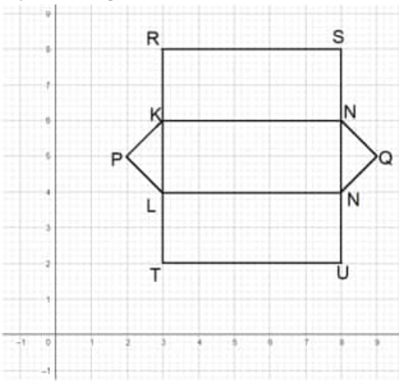
(iii) As a = 20, d = 10 and here n = 10

So, $S_{10} = \frac{10}{2}[2 \times 20 + 9 \times 10] = 5 \times 130$ = 650 m

So, hence Ramesh will cover 650 m to plant all 10 plants.

37. **Read the text carefully and answer the questions:**

The camping alpine tent is usually made using high-quality canvas and it is waterproof. These alpine tents are mostly used in hilly areas, as the snow will not settle on the tent and make it damp. It is easy to layout and one need not use a manual to set it up. One alpine tent is shown in the figure given below, which has two triangular faces and three rectangular faces. Also, the image of canvas on graph paper is shown in the adjacent figure.



(i) Coordinates of Q are (9, 5).

∴ Distance of point Q from y-axis = 9 units

(ii) Coordinates of point U are (8, 2).

OR

Length of TU = 5 units and of TL = 2 units

∴ Perimeter of image of a rectangular face = 2(5 + 2) = 14 units

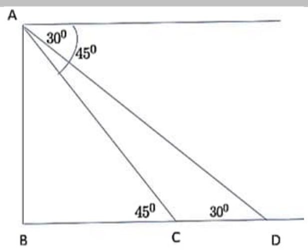
(iii)We have, P(2, 5) and Q (9, 5)

∴ PQ = $\sqrt{(2-9)^2+(5-5)^2}$ = $\sqrt{49+0}$ = 7 units

38. **Read the text carefully and answer the questions:**

An observer on the top of a 40m tall light house (including height of the observer) observes a ship at an angle of depression 30^o coming towards the base of the light house along straight line joining the ship and the base of the light house. The angle of depression of ship changes to 45^o after 6 seconds.

(i)



The distance of ship from the base of the light house after 6 seconds from the initial position when angle of depression is 45^o.

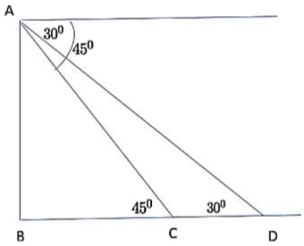
In △ABC

tan 45^o = $\frac{AB}{BC}$

$\Rightarrow 1 = \frac{40}{BC}$

⇒ BC = 40 m

(ii)



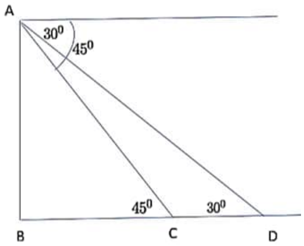
The distance between two positions of ship after 6 seconds

CD = BD - BC

$\Rightarrow$ CD = $40\sqrt{3}$ - 40 = $40(\sqrt{3}$ - 1)

$\Rightarrow$ CD = 29.28 m

OR



The distance of ship from the base of the light house when angle of depression is 30^o.

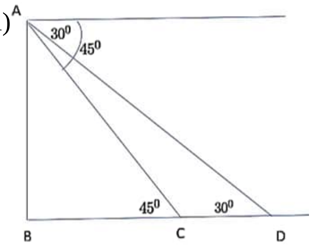
In $\triangle$ABD

tan 30^o = $\frac{AB}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{40}{BD}$

$\Rightarrow$ BD = $40\sqrt{3}$ m

(iii)



Speed of ship = $\frac{Distance}{Time} = \frac{29.28}{6}$ = 4.88 m/sec