Class XII Session 2024-25
Subject - Mathematics
Sample Question Paper - 2

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. A matrix $A = [a_{ij}]_{3\times3}$ is defined by $a_{ij} = \begin{cases} 2i+3j & , \quad i<j \\ 5 & , \quad i=j \\ 3i-2j & , \quad i>j \end{cases}$ **[1]**

The number of elements in A which are more than 5, is 4:

a) 5 b) 6

c) 4 d) 3

2. If $A = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$, then A^{-1} = ? **[1]**

a) $\begin{bmatrix} \frac{3}{7} & \frac{1}{7} \\ \frac{-1}{7} & \frac{2}{7} \end{bmatrix}$ b) $\begin{bmatrix} \frac{3}{7} & \frac{-1}{7} \\ \frac{1}{7} & \frac{2}{7} \end{bmatrix}$

c) $\begin{bmatrix} \frac{1}{5} & \frac{2}{7} \\ \frac{1}{7} & \frac{3}{7} \end{bmatrix}$ d) $\begin{bmatrix} \frac{1}{3} & \frac{1}{7} \\ \frac{1}{7} & \frac{2}{7} \end{bmatrix}$

3. For any 2-rowed square matrix A, if A · (adj A) = $\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$ then the value of |A| is **[1]**

a) 8 b) 4

c) 0 d) 64

4. If x = a sec θ, y = b tan θ then $\frac{dy}{dx}$ = ? **[1]**

a) $\frac{b}{a}\sec\theta$ b) $\frac{b}{a}\tan\theta$

c) $\frac{b}{a}cosec\theta$ d) $\frac{b}{a}\cot\theta$

5. The equation of a line passing through point (2, -1, 0) and parallel to the line $\frac{x}{1} = \frac{y-1}{2} = \frac{2-z}{2}$ is: **[1]**

a) $\frac{x+2}{1}=\frac{y-1}{2}=\frac{z}{2}$

b) $\frac{x+2}{1}=\frac{y-1}{2}=\frac{z}{-2}$

c) $\frac{x-2}{1}=\frac{y+1}{2}=\frac{z}{-2}$

d) $\frac{x-2}{1}=\frac{y-1}{2}=\frac{z}{2}$

6. What are the order and degree respectively of the differential equation whose solution is y = cx + c^2 - $3c^{3/2}$ + 2, where c is a parameter? **[1]**

a) 1, 3

b) 1, 4

c) 2, 2

d) 1, 2

7. Which of the following is a convex set? **[1]**

a) $\left\{(x,y) : y^2 \geq x\right\}$

b) $\{(x,y) : x^2+y^2 \geq 1\}$

c) $\{(x,y) : x \geq 2, y \leq 4\}$

d) $\{x,y) : 3x^2+4y^2 \geq 5\}$

8. If the vectors $3\hat{i}+\lambda\hat{j}+\hat{k}$ and $2\hat{i}-\hat{j}+8\hat{k}$ are perpendicular, then λ is equal to **[1]**

a) 7

b) -14

c) 1/7

d) 14

9. $\int_0^\pi \frac{1}{1+\sin x}$ dx equals **[1]**

a) 2

b) $\frac{3}{2}$

c) $\frac{1}{2}$

d) 0

10. If I is a unit matrix, then 3I will be **[1]**

a) A null matrix

b) A unit matrix

c) A triangular matrix

d) A scalar matrix

11. A point out of following points lie in plane represented by 2x + 3y $\leq$ 12 is **[1]**

a) (4, 3)

b) (0, 3)

c) (0, 5)

d) (3, 3)

12. If $\vec{a}+\vec{b}=\hat{i}$ and $\vec{a}$ = $2\hat{i}-2\hat{j}+2\hat{k}$, then $|\vec{b}|$ equals: **[1]**

a) $\sqrt{14}$

b) $\sqrt{17}$

c) $\sqrt{12}$

d) 3

13. The area of the triangle, whose vertices are (3, 8), (-4, 2) and (5, 1), is **[1]**

a) 60 sq. units

b) $\frac{61}{2}$ sq. units

c) 61 sq. units

d) 30 sq. units

14. If A and B are independent events, then $P(\bar{A}/\bar{B})$ = ? **[1]**

a) 1 - P(A / $\overline{B}$)

b) 1 - P(A)

c) 1 - P(B)

d) -P($\overline{A}$ / B)

15. The general solution of the DE $x^2\frac{dy}{dx}$ = x^2 + xy + y^2 is **[1]**

a) $\tan^{-1}\frac{y}{x}=\log x+C$

b) $\tan^{-1}\frac{y}{x}=\log y+C$

c) $\tan^{-1}\frac{x}{y}=\log x+C$

d) $\tan^{-1}\frac{y}{x}=\log x+C$

16. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$ and $|\vec{a}\times\vec{b}|$ = $3\sqrt{3}$, then the value of $\vec{a}\cdot\vec{b}$ is **[1]**

a) $\frac{1}{3}$

b) $\frac{1}{9}$

c) 9

d) 3

17. Let f(x) = |sin x|; $0 \leq x \leq 2\pi$ then **[1]**

a) f(x) is discontinuous at 3 points

b) f(x) is differentiable function at infinite number of points

c) f(x) is non-differentiable at 3 points and continuous everywhere

d) f(x) is discontinuous everywhere

18. The straight line $\frac{x-2}{3} = \frac{y-3}{1} = \frac{z+1}{0}$ is **[1]**

a) parallel to the y-axis

b) perpendicular to the z-axis

c) parallel to the x-axis

d) parallel to the z-axis

19. **Assertion (A):** The absolute maximum value of the function $2x^3$ - 24x in the interval [1, 3] is 89. **[1]**

Reason (R): The absolute maximum value of the function can be obtained from the value of the function at critical points and at boundary points.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The function f(x) = x^2 + bx + c, where b and c are real constants, describes onto mapping. **[1]**

Reason (R): Let A = {1 2, 3,..., n} and B = {a, b}. Then, the number of surjections from A into B is 2^n - 2.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. $\sin^{-1}\left(\frac{-1}{2}\right)$ **[2]**

OR

Find the domain of f(x) = $\sin^{-1}(-x^2)$.

22. The volume of a spherical balloon is increasing at the rate of 25 cm^3/sec. Find the rate of change of its surface area at the instant when radius is 5 cm. **[2]**

23. Determine whether f(x) = - $\frac{\pi}{2}$ + sin x is increasing or decreasing on (- $\frac{\pi}{3}$, $\frac{\pi}{3}$). **[2]**

OR

Find the intervals in which the function f given by $f(x) = 4x^3 - 6x^2 - 72x + 30$ is

i. increasing
ii. decreasing.

24. Integrate the function e^x (sin x + cos x) **[2]**

25. Prove that the function f given by f(x) = x^2 - x + 1 is neither strictly increasing nor strictly decreasing on (-1, 1). **[2]**

Section C

26. Evaluate: $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x+\cos x} dx$ **[3]**

27. There are two boxes, namely box-I and box-II. Box-I contains 3 red and 6 black balls. Box-II contains 5 red and 5 black balls. One of the two boxes, is selected at random and a ball is drawn at random. The ball drawn is found **[3]**

to be red. Find the probability that this red ball comes out from box-II.

28. Evaluate: $\int \frac{1}{\cos x-\sin x} dx$ **[3]**

OR

Evaluate: $\int \frac{x^3+x^2+2x+1}{x^2-x+1} dx$

29. Solve: 2y $e^{x/y}$ dx + (y - 2x $e^{x/y}$) dy = 0 **[3]**

OR

Solve the differential equation: $(x^3 + x^2 + x + 1) \frac{dy}{dx} = 2x^2 + x$

30. Minimise Z = 3x + 5y subject to the constraints: **[3]**

$x+2y \geqslant 10$

$x+y \geqslant 6$

$3x+y \geqslant 8$

$x, y \geqslant 0$

OR

Solve graphically the following linear programming problem:

Maximise z = 6x + 3y,

Subject to the constraints:

$4x + y \geq 80$,

$3x + 2y \leq 150$,

$x + 5y \geq 115$,

$x > 0, y \geq 0$.

31. If $x\sqrt{1+y}+y\sqrt{1+x}=0$ and $x \neq y$, prove that $\frac{dy}{dx}=-\frac{1}{(x+1)^2}$. **[3]**

Section D

32. Find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$. **[5]**

33. Show that the function f : R $\rightarrow$ R defined by $f(x)=\frac{x}{x^2+1}, \forall x \in R$, is neither one-one nor onto. **[5]**

OR

Let R be a relation on $N \times N$, defined by (a, b) R (c, d) $\Leftrightarrow$ a + d = b + c for all (a, b), (c, d) $\in N \times N$. Show that R is an equivalence relation.

34. Solve the system of the following equations: (Using matrices): **[5]**

$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4; \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1; \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2;$

35. Find the perpendicular distance of the point (1, 0, 0) from the line $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$. Also, find the coordinates of the foot of the perpendicular and the equation of the perpendicular. **[5]**

OR

Find the shortest distance between the lines $\vec{r}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(\hat{i}-3\hat{j}+2\hat{k}\right)$ and $\vec{r}=\left(4\hat{i}+5\hat{j}+6\hat{k}\right)+\mu\left(2\hat{i}+3\hat{j}+\hat{k}\right)$

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

There are two antiaircraft guns, named as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.

i. What is the probability that the shell fired from exactly one of them hit the plane? (1)

ii. If it is known that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B? (1)

iii. What is the probability that the shell was fired from A? (2)

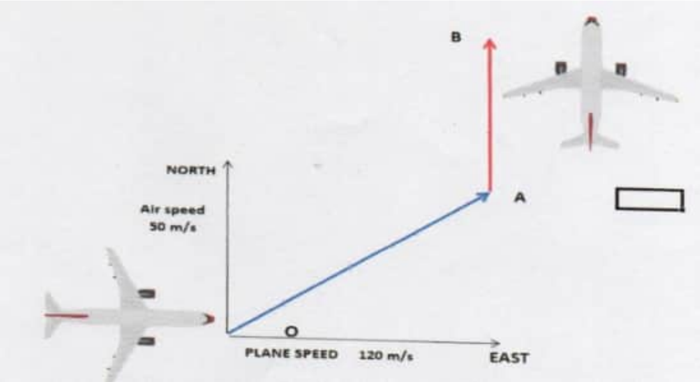
OR

How many hypotheses are possible before the trial, with the guns operating independently? Write the conditions of these hypotheses. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

A plane started from airport O with a velocity of 120 m/s towards east. Air is blowing at a velocity of 50 m/s towards the north As shown in the figure.

The plane travelled 1 hr in OA direction with the resultant velocity. From A and B travelled 1 hr with keeping velocity of 120 m/s and finally landed at B.



i. What is the resultant velocity from O to A? (1)

ii. What is the direction of travel of plane O to A with east? (1)

iii. What is the total displacement from O to A? (2)

OR

What is the resultant velocity from A to B? (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

In a street two lamp posts are 600 feet apart. The light intensity at a distance d from the first (stronger) lamp post is $\frac{1000}{d^2}$, the light intensity at distance d from the second (weaker) lamp post is $\frac{125}{d^2}$ (in both cases the light intensity is inversely proportional to the square of the distance to the light source). The combined light intensity is the sum of the two light intensities coming from both lamp posts.

i. If l(x) denotes the combined light intensity, then find the value of x so that I(x) is minimum. (1)

ii. Find the darkest spot between the two lights. (1)

iii. If you are in between the lamp posts, at distance x feet from the stronger light, then write the combined light intensity coming from both lamp posts as function of x. (2)

OR

Find the minimum combined light intensity? (2)

Solution

Section A

1.

(c) 4

Explanation: Here, A = $\begin{bmatrix} 5 & 8 & 11 \\ 4 & 5 & 13 \\ 7 & 5 & 5 \end{bmatrix}$

Thus, number of elements more than 5, is 4.

2. **(a)** $\begin{bmatrix} \frac{3}{7} & \frac{1}{7} \\ \frac{-1}{7} & \frac{2}{7} \end{bmatrix}$

Explanation: $A^{-1} = \frac{1}{|\mathbf{A}|} \operatorname{adj} A$ ...(i)

|A| = 3 $\times$ 2 - (1) $\times$ (-1)

= 7

C_{11} = 3, C_{12} = -1

C_{21} = 1, C_{22} = 2

Co-factor matrix $A = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$

$\operatorname{Adj} A = \begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix}'$

$= \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$

Putting in 1

$A^{-1} = \frac{1}{|7|} \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$

$= \begin{pmatrix} \frac{3}{7} & \frac{1}{7} \\ \frac{-1}{7} & \frac{2}{7} \end{pmatrix}$

3. **(a)** 8

Explanation: $(\operatorname{adj} A) = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$

$= 8 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

= |A| I

|A| = 8.

4.

(c) $\frac{b}{a} cosec\theta$

Explanation: x = a secθ ,we get

$\therefore \frac{dx}{d\theta} = \operatorname{asec} \theta \cdot \tan \theta$

$\therefore \frac{d\theta}{dx} = \frac{1}{\operatorname{asec} \theta \cdot \tan \theta}$

y = b tan θ ,we get

$\therefore \frac{dy}{d\theta} = b \cdot \sec^2 \theta$

$\Rightarrow \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$

$\Rightarrow \frac{dy}{dx} = b \cdot \sec^2 \theta \times \frac{1}{\operatorname{asec} \theta \cdot \tan \theta}$

$\Rightarrow \frac{dy}{dx} = \frac{b \sec \theta}{\operatorname{atan} \theta}$

$\Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\mathrm{b} \cdot \frac{1}{\cos \theta}}{\mathrm{a} \cdot \frac{\sin \theta}{\cos \theta}}$

$\Rightarrow \frac{dy}{dx} = \frac{b}{a} cosec\theta$

5.

(c) $\frac{x-2}{1}=\frac{y+1}{2}=\frac{z}{-2}$

Explanation: $\frac{x-2}{1}=\frac{y+1}{2}=\frac{z}{-2}$

6.

(b) 1, 4

Explanation: Given, y = cx + c^2 -3$c^{3/2}$ + 2 ...(i)

On differentiating both sides w.r.t. x, we get

$\frac{dy}{dx}$ = C ...(ii)

From Eqs. (i) and (ii), we have

y = $\frac{dy}{dx}$ $\times$ x + $\left(\frac{dy}{dx}\right)^2$ - 3$\left(\frac{dy}{dx}\right)^{3/2}$ + 2

$\Rightarrow$ y - x$\frac{dy}{dx}$ $-$ $\left(\frac{dy}{dx}\right)^2$ - 2 = -3$\left(\frac{dy}{dx}\right)^{3/2}$

$\Rightarrow$ $\left[y-x\left(\frac{dy}{dx}\right)-\left(\frac{dy}{dx}\right)^2-2\right]^2$ = 9$\left(\frac{dy}{dx}\right)^3$

Hence, order is 1 and degree is 4.

7.

(c) $\{(x,y): x \geq 2, y \leq 4\}$

Explanation: $\{(x,y): x \geq 2, y \leq 4\}$ is the region between two parallel lines, so any line segment joining any two points in it lies in it. Hence, it is a convex set.

8.

(d) 14

Explanation: given vectors $3\hat{i}+\lambda\hat{j}+\hat{k} and 2\hat{i}-\hat{j}+8\hat{k}$ are perpendicular to each other

$\implies (3\hat{i}+\lambda\hat{j}+\hat{k}).(2\hat{i}-\hat{j}+8\hat{k})=0$

$\implies$ 6-$\lambda+8=0 \implies \lambda=14$

9. **(a)** 2

Explanation: $\int_0^\pi \frac{1}{1+\sin x}dx$

$=\int_0^\pi \frac{1}{1+\sin x}\times\frac{1-\sin x}{1-\sin x}dx$

$=\int_0^\pi \frac{1-\sin x}{1-\sin^2 x}dx$

$=\int_0^\pi \frac{1-\sin x}{\cos^2 x}dx$

$=\int_0^\pi \left(\sec^2 x-\sec x\tan x\right)dx$

$=[\tan x-\sec x]_0^\pi$

= 0 + 1 - 0 + 1

= 2

10.

(d) A scalar matrix

Explanation: A scalar matrix

11.

(b) (0, 3)

Explanation: (0, 3) satisfy the equation 2x + 3y $\leq$ 12

2 $\times$ 0 + 3 $\times$ 3 $\leq$12

9 $\leq$ 12

But (3, 3), (4, 3), (0, 5) does not satisfy 2x + 3y $\leq$ 12.

12.

(d) 3

Explanation: 3

13.

(b) $\frac{61}{2}$ sq. units

Explanation: The area of triangle is given by

$\Delta=\frac{1}{2}\begin{vmatrix}3 & 8 & 1\\ -4 & 2 & 1\\ 5 & 1 & 1\end{vmatrix}$

$=\frac{1}{2}$ [3(2 - 1)-8(-4 - 5)+1(-4 - 10)]

$=\frac{1}{2}(3+72-14)=\frac{61}{2}$ sq. units

14.

(b) 1 - P(A)

Explanation: $P(\overline{\mathrm{A}}/\overline{\mathrm{B}})$

$=\frac{P(\bar{A}\cap\bar{B})}{P(\bar{B})}$

$=\frac{P(\bar{A})P(\bar{B})}{1-P(B)}$

= 1 - P(A)

15. **(a)** $\tan^{-1}\frac{y}{x}=\log x+C$

Explanation: We have, $x^2\frac{dy}{dx}=x^2+xy+y^2$

$\frac{dy}{dx}=1+\frac{y}{x}+\frac{y^2}{x^2}$

Let y = vx

$\frac{dy}{dx}=v+x\frac{dv}{dx}$

$1+v+v^2=v+x\frac{dv}{dx}$

$1+v^2=x\frac{dv}{dx}$

$\frac{dx}{x}=\frac{dv}{v^2+1}$

On integrating on both sides, we obtain

log x = $\tan^{-1}$v + C

$\tan^{-1}\frac{y}{x}=\log x+c$

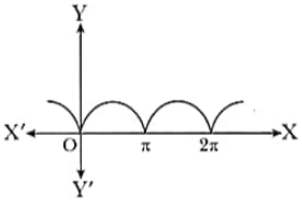
16.

(d) 3

Explanation: 3

17.

(c) f(x) is non-differentiable at 3 points and continuous everywhere

Explanation: 

It is clear from graph that f(x) is continuous everywhere in $0\leq x\leq 2\pi$. And has sharp edge at x = 0, π, 2π so it is not differentiable at x = 0, π, 2π.

Because it has no unique tangents.

18.

(b) perpendicular to the z-axis

Explanation: It is perpendicular to z-axis.

Given, direction ratios of the line : a_1 =3 ,a_2=1, a_3=0 & direction ratios of z-axis is b_1=0, b_2=0,b_3=1.

Now, $a_1a_2+b_1b_2+c_1c_2$= 3.0+1.0+0.1=0 which implies that line is perpendicular to z-axis.

19.

(d) A is false but R is true.

Explanation: Let f(x) = $2x^3$ - 24x

$\Rightarrow$ f'(x) = $6x^2$ - 24 = 6(x^2 - 4)

= 6(x + 2) (x - 2)

For maxima or minima put f'(x) = 0.

$\Rightarrow$ 6(x + 2)(x - 2) = 0

$\Rightarrow$ x = 2, -2

We first consider the interval [1, 3].

So, we have to evaluate the value of f at the critical point x = 2 $\in$ [1, 3] and at the end points of [1, 3].

At x = 1, f(1) = 2 $\times$ 1^3 - 24 $\times$ 1 = -22

At x = 2, f(2) = 2 $\times$ 2^3 - 24 $\times$ 2 = -32

At x = 3, f(3) = 2 $\times$ 3^3 - 24 $\times$ 3 = -18

$\therefore$ The absolute maximum value of f(x) in the interval [1, 3] is -18 occurring at x = 3.

Hence, Assertion is false and Reason is true.

20.

(d) A is false but R is true.

Explanation: Assertion: Given function is f(x) = x^2 + bx + c.

It is a quadratic equation in x.

So, we will get a parabola either downward or upward.

Hence, it is a many-one mapping and not onto mapping.

Hence, it is neither one-one nor onto mapping.

Reason: Total number of functions = $(n(B))^{n(A)} = 2^n$.

Clearly, a function will not be onto if all elements of A map to either a or b.

Section B

21. Let $\sin^{-1}\left(\frac{-1}{2}\right) = y$

$\Rightarrow \sin y = -\frac{1}{2}$

$\Rightarrow \sin y = -\sin\frac{\pi}{6}$

$\Rightarrow \sin y = \sin\left(-\frac{\pi}{6}\right)$

Since, the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Therefore, principal value of $\sin^{-1}\left(\frac{-1}{2}\right)$ is $-\frac{\pi}{6}$.

OR

The domain of $\sin^{-1}$ x is [-1,1]. Therefore, f(x) = $\sin^{-1}(-x^2)$ is defined for all x satisfying $-1 \leq -x^2 \leq 1$

$\Rightarrow \quad 1 \geq x^2 \geq -1$

$\Rightarrow 0 \leq x^2 \leq 1$

$\Rightarrow x^2 \leq 1$

$\Rightarrow x^2 - 1 \leq 0$

$\Rightarrow$ (x - 1)(x + 1) $\leq$ 0

$\Rightarrow -1 \leq x \leq 1$

Hence, the domain of f(x) = $\sin^{-1}(-x^2)$ is [-1, 1].

22. Let r be the radius and V be the volume of the sphere at any t then,

$V = \frac{4}{3}\pi r^3$

Differentiating both sides with respect to t,

$\Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\Rightarrow \frac{dr}{dt} = \frac{1}{4\pi r^2}\frac{dV}{dt}$

$\Rightarrow \frac{dr}{dt} = \frac{25}{4\pi(5)^2}$

$\Rightarrow \frac{dr}{dt} = \frac{1}{4\pi}\text{cm/sec}$

Now, let S be the surface area of the sphere area at any t then

$S = 4\pi r^2$

Differentiating both sides with respect to t,

$\Rightarrow \frac{dS}{dt} = 8\pi r\frac{dr}{dt}$

$\Rightarrow \frac{dS}{dt} = 8\pi(5) \times \frac{1}{4\pi}$

$\Rightarrow \frac{dS}{dt} = 10\text{cm}^2/\text{sec}$

23. Given:

$f(x) = -\frac{x}{2} + \sin x$

$\Rightarrow \mathrm{f}'(\mathrm{x}) = -\frac{1}{2} + \cos \mathrm{x}$

Now

$x \in \left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$

$\Rightarrow -\frac{\pi}{3} < x < \frac{\pi}{3}$

$\Rightarrow \cos\left(-\frac{\pi}{3}\right) < \cos x < \cos\frac{\pi}{3}$

$\Rightarrow \cos\left(\frac{\pi}{3}\right) < \cos x < \cos\frac{\pi}{3}$

$\Rightarrow \frac{1}{2} < \cos x < \frac{1}{2}$

$\Rightarrow -\frac{1}{2} + \cos x > 0$

$\Rightarrow \mathrm{f}'(\mathrm{x}) > 0$

Hence, f(x) is an increasing function on (- $\frac{\pi}{3}$, $\frac{\pi}{3}$).

OR

$f(x) = 4x^3 - 6x^2 - 72x + 30$

$f'(x) = 12x^2 - 12x - 72$

$= 12(x^2 - x - 6)$

$= 12(x^2 - 3x + 2x - 6)$

$= 12\left[x(x-3) + 2(x-3)\right]$

$= 12(x-3)(x+2)$

Put f ' (x) = 0

$x = -2, 3$

int	Sign of f'(x)	Result
$(-\infty, -2)$	+ tive	Increase
$(-2, 3)$	+ tive	Decrease
$(3, \infty)$	+ tive	increase

Hence function is

i. increasing in $(-\infty, -2)$
ii. $(3, \infty)$ decreasing in $(-2, 3)$

24. $I = \int e^x(\sin x + \cos x)dx$

Now,

Let sin x = f(x) ⇒ f'(x) = cos x

We know that,

$\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + c$

Thus,

$\int e^x(\sin x + \cos x)dx = e^x \sin x + C$

25. Given: $f(x) = x^2 - x + 1$ f(x) = x^2 - x + 1

⇒ f'(x) = 2x - 1

f(x) is strictly increasing if f'(x) < 0

⇒2x - 1 > 0

$\Rightarrow x > \frac{1}{2}$

i.e., increasing on the interval $\left(\frac{1}{2}, 1\right)$

f(x) is strictly decreasing if f'(x) < 0

⇒ 2x - 1 < 0

$\Rightarrow x < \frac{1}{2}$

i.e., decreasing on the interval $\left(-1, \frac{1}{2}\right)$

hence, f(x) is neither strictly increasing nor decreasing on the interval (-1, 1).

Section C

26. Let the given integral be, $I = \int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx.$ ...(i)

Then,

$\Rightarrow \quad I=\int_0^{\pi/2} \frac{\sin^2\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)} dx \quad \left[\text{ Using } : \int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$

$\Rightarrow \quad I=\int_0^{\pi/2} \frac{\cos^2 x}{\cos x+\sin x} dx$ ...(ii)

Adding (i) and (ii), we get

$2I=\int_0^{\pi/2} \frac{\sin^2 x}{\sin x+\cos x}+\frac{\cos^2 x}{\sin x+\cos x} dx=\int_0^{\pi/2} \frac{1}{\sin x+\cos x} dx$

$\Rightarrow \quad 2I=\int_0^{\pi/2} \frac{1}{\frac{2\tan x/2}{1+\tan^2 x/2}+\frac{1-\tan^2 x/2}{1+\tan^2 x/2}} dx$

$\Rightarrow \quad 2I=\int_0^{\pi/2} \frac{1+\tan^2 \frac{x}{2}}{2\tan\frac{x}{2}+1-\tan^2\frac{x}{2}} dx=\int_0^{\pi/2} \frac{\sec^2 \frac{x}{2}}{2\tan\frac{x}{2}+1-\tan^2\frac{x}{2}} dx$

Let $\tan\frac{x}{2}=t$. Then, $d\left(\tan\frac{x}{2}\right)=dt \Rightarrow \left(\sec^2\frac{x}{2}\right)\frac{1}{2}dx=dt \Rightarrow \sec^2\frac{x}{2}dx=2dt$

Also, $x=0 \Rightarrow t=\tan 0=0$ and $x=\frac{\pi}{2} \Rightarrow t=\tan\frac{\pi}{4}=1$

$\therefore \quad 2I=\int_0^1 \frac{2dt}{2t+1-t^2}=2\int_0^1 \frac{1}{(\sqrt{2})^2-(t-1)^2} dt$

$\Rightarrow \quad 2I=2\times\frac{1}{2\sqrt{2}}\left[\log\left|\frac{\sqrt{2}+t-1}{\sqrt{2}-t+1}\right|\right]_0^1$

$\Rightarrow \quad 2I=\frac{1}{\sqrt{2}}\left\{\log\left(\frac{\sqrt{2}}{\sqrt{2}}\right)-\log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right\}=\frac{1}{\sqrt{2}}\left\{0-\log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right\}$

$\Rightarrow \quad 2I=-\frac{1}{\sqrt{2}}\log\left\{\frac{(\sqrt{2}-1)^2}{(\sqrt{2}+1)(\sqrt{2}-1)}\right\}=-\frac{1}{\sqrt{2}}\log(\sqrt{2}-1)^2=-\frac{2}{\sqrt{2}}\log(\sqrt{2}-1)$

$\Rightarrow \quad I=-\frac{1}{\sqrt{2}}\log(\sqrt{2}-1)$

27. Let E_1 be the event when box I is selected

E_2 be the event when box II is selected

A be the event of getting red ball in any box

$P(E_1)=\frac{1}{2},\ P(E_2)=\frac{1}{2}$

$P\left(\frac{A}{E_1}\right)=\frac{3}{9}=\frac{1}{2}$

$P\left(\frac{A}{E_2}\right)=\frac{5}{10}=\frac{1}{2}$

Now, required probability =

$P\left(\frac{E_2}{A}\right)=\frac{P\left(\frac{A}{E_2}\right)P(E_2)}{P\left(\frac{A}{E_1}\right)P(E_1)+P\left(\frac{A}{E_2}\right)P(E_2)}$

$P\left(\frac{E_2}{A}\right)=\frac{\frac{1}{2}\times\frac{1}{2}}{\frac{1}{2}\times\frac{1}{3}+\frac{1}{2}\times\frac{1}{2}}=\frac{\frac{1}{4}}{\frac{5}{12}}$

$P\left(\frac{E_2}{A}\right)=\frac{3}{5}$

28. Let the given integral be,

I = $\int \frac{1}{\cos x-\sin x} dx$

Putting cos x = $\frac{1-\tan^2\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$ and sin x = $\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$\Rightarrow I=\int \frac{dx}{\frac{1-\tan^2\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}-\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}}$

$=\int \frac{\sec^2\left(\frac{x}{2}\right)dx}{1-\tan^2\left(\frac{x}{2}\right)-2\tan\left(\frac{x}{2}\right)}$

Let tan $\left(\frac{x}{2}\right)$ = t

$\Rightarrow \frac{1}{2}$ sec^2 $\left(\frac{x}{2}\right)$ dx = dt

sec^2 $\left(\frac{x}{2}\right)$ dx = 2dt

$\therefore I=\int \frac{2dt}{1-t^2-2t}$

$=\int \frac{-2dt}{t^2+2t-1}$

$=\int \frac{-2dt}{t^2+2t+1-2}$

$=-\int \frac{2dt}{(t+1)^2-(\sqrt{2})^2}$

$=\int \frac{2dt}{(\sqrt{2})^2-(t-1)^2}$

$= \frac{2}{2\sqrt{2}} \ln\left|\frac{\sqrt{2}+t+1}{\sqrt{2}-t-1}\right| + C$

$= \frac{1}{\sqrt{2}} \ln\left|\frac{\sqrt{2}+\tan\frac{x}{2}+1}{\sqrt{2}-\tan\frac{x}{2}-1}\right| + C$

OR

Let I = $\int \frac{x^3+x^2+2x+1}{x^2-x+1} dx$

Here we have,

$\frac{x^3+x^2+2x+1}{x^2-x+1}$ = x + 2 + $\frac{3x-1}{x^2-x+1}$ ...(i)

Let 3x - 1 = $A\frac{d}{dx}$ (x - x + 1) + B

$\Rightarrow$ 3x - 1 = A (2x + 1) + B

$\Rightarrow$ 3x - 1 = (2A) x + B - A

Equating Coefficients of like terms, we get

2A = 3 $\Rightarrow A = \frac{3}{2}$

Also, B - A = -1

$\Rightarrow B - \frac{3}{2}$ = -1

$\Rightarrow B = \frac{1}{2}$

So, $\int \left(\frac{x^3+x^2+2x+1}{x^2-x+1}\right) dx$ = $\int$ (x + 2) dx + $\int \left(\frac{\frac{3}{2}(2x-1)+\frac{1}{2}}{x^2-x+1}\right)$

= $\int$ (x + 2) dx + $\frac{3}{2} \int \left(\frac{2x-1}{x^2-x+1}\right) dx + \frac{1}{2} \int \frac{dx}{x^2-x+1}$

= $\int$ (x + 2) dx + $\frac{3}{2} \int \frac{(2x-1)dx}{x^2-x+1} + \frac{1}{2} \int \frac{dx}{x^2-x+\frac{1}{4}-\frac{1}{4}+1}$

= $\int$ (x + 2) dx + $\frac{3}{2} \int \frac{(2x-1)dx}{x^2-x+1} + \frac{1}{2} \int \frac{dx}{\left(x-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$

= $\frac{x^2}{2}$ + 2x + $\frac{3}{2}$ log |x^2 - x + 1| + $\frac{1}{2} \times \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + C$

= $\frac{x^2}{2}$ + 2x + $\frac{3}{2}$ log |x^2 - x + 1| + $\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + C$

29. We have, 2y $e^{x/y}$ dx + (y - 2x $e^{x/y}$) dy = 0

$\Rightarrow \frac{dx}{dy} = \frac{2xe^{x/y}-y}{2ye^{x/y}}$ ...(i)

Clearly, the given differential equation is a homogeneous differential equation.

As the right hand side of (i) is expressible as a function of $\frac{x}{y}$.

So, we put x = vy and $\frac{dx}{dy} = v + y\frac{dv}{dy}$ to get

$v + y\frac{dv}{dy} = \frac{2ve^v-1}{2e^v}$

$\Rightarrow y\frac{dv}{dy} = \frac{2ve^v-1}{2e^v} - v$

$\Rightarrow y\frac{dv}{dy} = -\frac{1}{2e^v}$

$\Rightarrow$ 2ye^v dv = -dy

$\Rightarrow$ 2e^v dv = $-\frac{1}{y}$ dy

Integrating both sides,

$\Rightarrow 2\int e^v dv = -\int \frac{1}{y} dy$

$\Rightarrow$ 2e^v = -log |y| + log C

$\Rightarrow$ 2e^v = log $\left|\frac{c}{y}\right|$

$\Rightarrow$ 2$e^{x/y}$ = log $\left|\frac{c}{y}\right|$

Hence, 2$e^{x/y}$ = log $\left|\frac{c}{y}\right|$ gives the general solution of the given differential equation.

OR

The given differential equation is,

(x^3 + x^2 + x + 1) $\frac{dy}{dx}$ = 2x^2 + x

$\Rightarrow \frac{dy}{dx} = \frac{2x^2+x}{x^3+x^2+x+1}$

$\Rightarrow dy = \frac{2x^2+x}{(x+1)(x^2+1)} dx$

Integrating both sides, we get

$\int dy = \int \left\{ \frac{2x^2+x}{(x+1)(x^2+1)} \right\} dx$

$\Rightarrow y = \int \left\{ \frac{2x^2+x}{(x+1)(x^2+1)} \right\} dx$

Let $\frac{2x^2+x}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$

$\Rightarrow 2x^2 + x = Ax^2 + A + Bx^2 + Bx + Cx + C$

$\Rightarrow 2x^2 + x = (A + B)\, x^2 + (B + C)\, x + (A + C)$

Comparing the coefficients on both sides, we get

A + B = 2 ...(i)

B + C = 1 ...(ii)

A + C = 0 ...(iii)

Solving (i), (ii) and (iii), we get

A = $\frac{1}{2}$, B = $\frac{3}{2}$, C = $-\frac{1}{2}$

$\therefore y = \frac{1}{2} \int \frac{1}{(x+1)} dx + \int \frac{\frac{3}{2}x - \frac{1}{2}}{x^2+1} dx$

$= \frac{1}{2} \int \frac{1}{(x+1)} dx + \frac{1}{2} \int \frac{3x}{x^2+1} dx - \frac{1}{2} \int \frac{1}{x^2+1} dx$

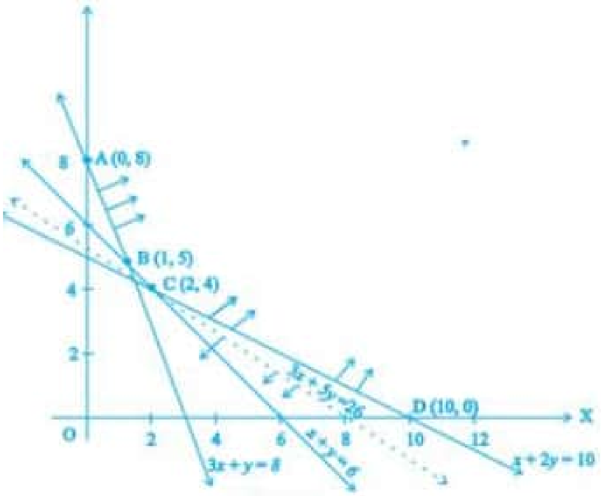
$= \frac{1}{2} \int \frac{1}{(x+1)} dx + \frac{3}{4} \int \frac{2x}{x^2+1} dx - \frac{1}{2} \int \frac{1}{x^2+1} dx$

$= \frac{1}{2} \log|x+1| + \frac{3}{4} \log\left|x^2+1\right| - \frac{1}{2} \tan^{-1} x + C$

Hence, y = $\frac{1}{2}$ log |x + 1| + $\frac{3}{4}$ log |x^2 + 1| $-\frac{1}{2}$ $\tan^{-1}$ x + C is the solution to the given differential equation.

30. We first draw the graphs of x + 2y = 10, x + y = 6, 3x + y = 8. The shaded region ABCD is the feasible region (R) determined by the above constraints. The feasible region is unbounded. Therefore, minimum of Z may or may not occur. If it occurs, it will be on the corner point.

Corner Point	**Value of Z**
A(0, 8)	40
B(1, 5)	28
C(2, 4)	26 (smallest)
D(10, 0)	30



Let us draw the graph of 3x + 5y < 26 as shown in Fig by dotted line.

We see that the open half plane determined by 3x + 5y < 26 and R do not have a point in common. Thus, 26 is the minimum value of Z.

OR

Subject to the constraints are

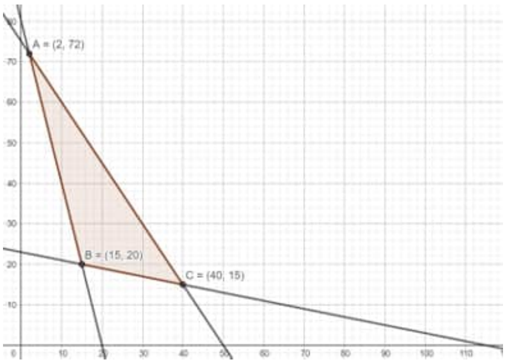
4x + y $\geq$ 80

x + 5y $\geq$ 115

3x + 2y $\leq$ 150

and the non negative constraint x,y $\geq$ 0

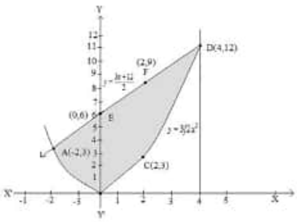
Converting the given inequations into equations, we get 4x + y = 80, x + 5y = 115, 3x + 2y = 150, x = 0 and y = 0
These lines are drawn on the graph and the shaded region ABC represents the feasible
region of the given LPP.



It can be observed that the feasible region is bounded. The coordinates of the corner
points of the feasible region are A(2, 72), B(15, 20) and C(40, 15) The values of the objective function, Z at these corner points are given in the following table:
Corner Point Value of the Objective Function Z = 6x + 3y
A(2, 72) : $Z = 6 \times 2 + 3 \times 72 = 228$
B(15, 20) : $Z = 6 \times 15 + 3 \times 20 = 150$
C(40, 15) : $Z = 6 \times 40 + 3 \times 15 = 285$
From the table, Z is minimum at x = 15 and y = 20 and the minimum value of Z is 150. Thus, the minimum value of Z is 150.

31. According to the question,we have to prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$ if $x\sqrt{1+y} + y\sqrt{1+x} = 0$
where x$\neq$y.
we shall first write y in terms of x explicitly i.e y=f(x)
Clearly, $x\sqrt{1+y} = -y\sqrt{1+x}$
Squaring both sides,we get,
$x^2(1 + y) = y^2 (1 + x)$
$\Rightarrow x^2 + x^2y = y^2(1 + x)$
$\Rightarrow x^2 - y^2 = y^2x - x^2y$
$\Rightarrow$(x - y)(x + y) = -xy(x - y)
$\Rightarrow$(x - y)(x + y) + xy(x - y) = 0
$\Rightarrow$(x - y)(x + y + xy) = 0
$\therefore$Either, x - y = 0 or x + y + xy = 0
Now, x - y = 0 $\Rightarrow$ x = y
But, it is given that x $\neq$ y
So, it is a contradiction
Therefore, x - y = 0 is rejected.
Now, consider y + xy + x = 0
$\Rightarrow$ y(1 + x) = -x $\Rightarrow$y = $\frac{-x}{1+x}$ ....(i)
Therefore,on differentiating both sides w.r.t x, we get,
$\frac{dy}{dx} = \frac{(1+x)\times\frac{d}{dx}(-x)-(-x)\times\frac{d}{dx}(1+x)}{(1+x)^2}$ [By using quotient rule of derivative]
$\Rightarrow \frac{dy}{dx} = \frac{(1+x)(-1)+x(1)}{(1+x)^2}$
$\Rightarrow \frac{dy}{dx} = \frac{-1-x+x}{(1+x)^2}$
$\therefore \frac{dy}{dx} = \frac{-1}{(1+x)^2}$

Section D

32. 

$4y = 3x^2$ ......(1)

2y = 3x + 12......(2)

From (2), $y = \frac{3x+12}{2}$

Using this value of y in (1), we get,

$x^2 - 6x - 8 = 0$

$\Rightarrow$ (x + 2) (x - 4) = 0

$\Rightarrow$ x = -2, 4

From (2),

When, x = -2, y = 3

When, x = 4, y = 12

Thus, points of intersection are, (-2, 3) and (4, 12).

Area $= \int_{-2}^{4} \frac{3x+12}{2} dx - \int_{-2}^{4} \frac{3}{4} x^2 dx$

$= \frac{1}{2}[\frac{3x^2}{2} + 12x]_{-2}^{4} - \frac{3}{4}[\frac{x^3}{3}]_{-2}^{4}$

$\frac{1}{2}[(24+48) - (6-24)] - \frac{1}{4}[64 - (-8)]$

= 45 - 18 = 27 sq units.

33. For $x_1, x_2 \in R$, consider

$f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1}{x_1^2+1} = \frac{x_2}{x_2^2+1}$

$\Rightarrow x_1 x_2^2 + x_1 = x_2 x_1^2 + x_2$

$\Rightarrow x_1 x_2 (x_2 - x_1) = x_2 - x_1$

$\Rightarrow x_1 = x_2$ or $x_1 x_2 = 1$

We note that there are point, x_1 and x_2 with $x_1 \neq x_2$ and $f(x_1) = f(x_2)$ for instance, if we take $x_1 = 2$ and $x_2 = \frac{1}{2}$, then we have $f(x_1)\frac{2}{5}$ and $f(x_2) = \frac{2}{5}$ but $2 \neq \frac{1}{2}$. Hence f is not one-one. Also, f is not onto for if so then for $1 \in R \; \exists \; x \in R$ such that f(x) = 1 which gives $\frac{x}{x^2+1} = 1$. But there is no such x in the domain R, since the equation $x^2 - x + 1 = 0$ does not give any real value of x.

OR

Here R is a relation on $N \times N$, defined by (a, b) R (c, d) $\Leftrightarrow$ a + d = b + c for all (a, b), (c, d) $\in N \times N$

We shall show that R satisfies the following properties

i. Reflexivity:
   We know that a + b = b + a for all a, b $\in$ N.
   $\therefore$ (a, b) R (a, b) for all (a, b) $\in (N \times N)$
   So, R is reflexive.
ii. Symmetry:
   Let (a, b) R (c, d). Then,
   (a, b) R (c, d) $\Rightarrow$ a + d = b + c
   $\Rightarrow$ c + b = d + a
   $\Rightarrow$ (c, d) R (a, b).
   $\therefore$ (a, b) R (c, d) $\Rightarrow$ (c, d) R (a, b) for all (a, b), (c, d) $\in N \times N$
   This shows that R is symmetric.
iii. Transitivity:
   Let (a, b) R (c, d) and (c, d) R (e, f). Then,
   (a, b) R (c, d) and (c, d) R (e, f)
   $\Rightarrow$ a + d = b + c and c + f = d + e
   $\Rightarrow$ a + d + c + f = b + c + d + e
   $\Rightarrow$ a + f = b + e
   $\Rightarrow$ (a, b) R (e, f).

Thus, (a, b) R (c, d) and (c, d) R (e, f) ⇒ (a, b) R (e, f)

This shows that R is transitive.

∴ R is reflexive, symmetric and transitive

Hence, R is an equivalence relation on $N \times N$

34. Put $\frac{1}{x} = u, \frac{1}{y} = v$ and $\frac{1}{z} = w$ in the given equations,

$2u + 3v + 10w = 4;\ 4u - 6v + 5w = 1;\ 6u + 9v - 20w = 2$

∴ The matrix form of given equations is $\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix} \begin{bmatrix} x \\ v \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$ [AX= B]

Here, $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$, $X \begin{bmatrix} x \\ v \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix}$

$= 2(120 - 45) - 3(-80 - 30) + 10(36 + 36)$

$= 150 + 330 + 750 = 1200 \neq 0$

$\therefore A^{-1}$ exists and unique solution is $X = A^{-1}B$….(i)

Now $A_{11} = 75, A_{12} = 110, A_{13} = 72$ and $A_{21} = 150, A_{22} = -100, A_{23} = 0$ and $A_{31} = 75, A_{32} = 30, A_{33} = -24$

$\therefore adj.\ A = \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}' = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$

And $A^{-1} = \frac{adj.A}{|A|} = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$

∴ From eq. (i),

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{1200} \begin{bmatrix} 300 + 150 + 150 \\ 440 - 100 + 60 \\ 288 + 0 - 48 \end{bmatrix}$

$= \frac{1}{1200} \begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$

$\therefore u = \frac{1}{2}, v = \frac{1}{3}, w = \frac{1}{5}$

$\Rightarrow x = \frac{1}{u} = 2, y = \frac{1}{v} = 3, z = \frac{1}{w} = 5$

35. Suppose the point (1, 0, 0) be P and the point through which the line passes be Q(1,-1,-10). The line is parallel to the vector $\vec{b} = 2\hat{i} - 3\hat{j} + 8\hat{k}$

Now,

$\overrightarrow{PQ} = 0\hat{i} - \hat{j} - 10\hat{k}$

$\therefore \vec{b} \times \overrightarrow{PQ}$ = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 8 \\ 0 & -1 & -10 \end{vmatrix}$

= $38\hat{i} + 20\hat{j} - 2\hat{k}$

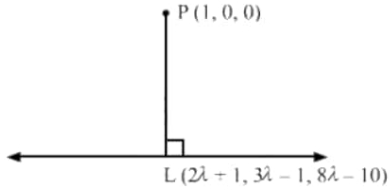
$\Rightarrow |\vec{b} \times \overrightarrow{PQ}| = \sqrt{38^2 + 20^2 + 2^2}$

= $\sqrt{1444 + 400 + 4}$

= $\sqrt{1848}$

d = $\frac{|\vec{b} \times \overrightarrow{PQ}|}{|\vec{b}|}$

$= \frac{\sqrt{1848}}{\sqrt{77}}$
$= \sqrt{24}$
$= 2\sqrt{6}$
Suppose L be the foot of the perpendicular drawn from the point P(1,0,0) to the given line-



The coordinates of a general point on the line
$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ are given by
$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} = \lambda$
$\Rightarrow x = 2\lambda + 1$
$y = -3\lambda - 1$
$z = 8\lambda - 10$
Suppose the coordinates of L be
$(2\lambda + 1, -3\lambda - 1, 8\lambda - 10)$
Since,The direction ratios of PL are proportional to,
$2\lambda + 1 - 1, -3\lambda - 1 - 0, 8\lambda - 10 - 0$, i.e., $2\lambda, -3\lambda - 1, 8\lambda - 10$
Since,The direction ratios of the given line are proportional to 2, -3, 8, but PL is perpendicular to the given line.
$\therefore 2(2\lambda) - 3(-3\lambda - 1) + 8(8\lambda - 10) = 0$
$\Rightarrow \lambda = 1$ Substituting $\lambda = 1$ in $(2\lambda + 1, -3\lambda - 1, 8\lambda - 10)$ we get the coordinates of L as (3, -4, -2). Equation of the line PL is given by
$\frac{x-1}{3-1} = \frac{y-0}{-4-0} = \frac{z-0}{-2-0}$
$= \frac{x-1}{1} = \frac{y}{-2} = \frac{z}{-1}$
$\Rightarrow \vec{r} = \hat{i} + \lambda(\hat{i} - 2\hat{j} - \hat{k})$

OR

$\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$
$\vec{b}_1 = \hat{i} - 3\hat{j} + 2\hat{k}$
$\vec{a}_2 = 4\hat{i} + 5\hat{j} + 6\hat{k}$
$\vec{b}_2 = 2\hat{i} + 3\hat{j} + \hat{k}$
$\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j} + 3\hat{k}$
$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$
$= -9\hat{i} + 3\hat{j} + 9\hat{k}$
$\therefore |\vec{b}_1 \times \vec{b}_2| = \sqrt{(-9)^2 + 3^2 + 9^2}$
$= \sqrt{3^2}\sqrt{3^2 + 1 + 3^2} = 3\sqrt{19}$
Required shortest distance
$= \left|\frac{(a_2 - a_1).(b_2 - b_1)}{|b_1 \times b_2|}\right| = \left|\frac{-9\times3+3\times3+9\times3}{3\sqrt{19}}\right| = \left|\frac{9}{3\sqrt{19}}\right| = \frac{3}{\sqrt{19}} units$

Section E

36. i. Let P be the event that the shell fired from A hits the plane and Q be the event that the shell fired from B hits the plane. The following four hypotheses are possible before the trial, with the guns operating independently:
$E_1 = PQ$, $E_2 = \bar{P}\bar{Q}$, $E_3 = \bar{P}Q$, $E_4 = P\bar{Q}$

Let E = The shell fired from exactly one of them hits the plane.
$P(E_1) = 0.3 \times 0.2 = 0.06$, $P(E_2) = 0.7 \times 0.8 = 0.56$, $P(E_3) = 0.7 \times 0.2 = 0.14$, $P(E_4) = 0.3 \times 0.8 = 0.24$
$P\left(\frac{E}{E_1}\right) = 0$, $P\left(\frac{E}{E_2}\right) = 0$, $P\left(\frac{E}{E_3}\right) = 1$, $P\left(\frac{E}{E_4}\right) = 1$
$P(E) = P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right) + P(E_3) \cdot P\left(\frac{E}{E_3}\right) + P(E_4) \cdot P\left(\frac{E}{E_4}\right)$
= 0.14 + 0.24 + = 0.38

ii. By Bayes' Theorem, $P\left(\frac{E_3}{E}\right) = \frac{P(E_3)\cdot P\left(\frac{E}{E_3}\right)}{P(E_1)\cdot P\left(\frac{E}{E_1}\right)+P(E_2)\cdot P\left(\frac{E}{E_2}\right)+P(E_3)\cdot P\left(\frac{E}{E_3}\right)+P(E_4)\cdot P\left(\frac{E}{E_4}\right)}$

$= \frac{0.14}{0.38} = \frac{7}{19}$

NOTE: The four hypotheses form the partition of the sample space and it can be seen that the sum of their probabilities is 1. The hypotheses E_1 and E_2 are actually eliminated as $P\left(\frac{E}{E_1}\right) = P\left(\frac{E}{E_2}\right) = 0$

iii. By Bayes' Theorem, $P\left(\frac{E_4}{E}\right) = \frac{P(E_4)\cdot P\left(\frac{E}{E_4}\right)}{P(E_1)\cdot P\left(\frac{E}{E_1}\right)+P(E_2)\cdot P\left(\frac{E}{E_2}\right)+P(E_3)\cdot P\left(\frac{E}{E_3}\right)+P(E_4)\cdot P\left(\frac{E}{E_4}\right)}$

$= \frac{0.24}{0.38} = \frac{12}{19}$

OR

Let P be the event that the shell fired from A hits the plane and Q be the event that the shell fired from B hits the plane. The following four hypotheses are possible before the trial, with the guns operating independently:

E_1 = PQ, $E_2 = \bar{P}\bar{Q}$, $E_3 = \bar{P}Q$, $E_4 = P\bar{Q}$

Let E = The shell fired from exactly one of them hits the plane.

$P(E_1) = 0.3 \times 0.2 = 0.06$, $P(E_2) = 0.7 \times 0.8 = 0.56$, $P(E_3) = 0.7 \times 0.2 = 0.14$, $P(E_4) = 0.3 \times 0.8 = 0.24$

37. i. Resultant velocity from O to A

$= \sqrt{(V_{\text{Plane}})^2 + (V_{\text{wind}})^2}$

$= \sqrt{(120)^2 + (50)^2}$

$= \sqrt{14400 + 2500}$

$= \sqrt{16900}$

= 130 m/s

ii. $\tan\theta = \frac{V_{\text{wind}}}{V_{\text{aeroplane}}}$

$\tan\theta = \frac{50}{120}$

$\tan\theta = \frac{5}{12}$

$\theta = \tan^{-1}\left(\frac{5}{12}\right)$

iii. Displacement from O to A = Resultant velocity $\times$ time

$|\overrightarrow{OA}| = |\vec{V}| \times t$

$= 130 \times \frac{18}{5} \times 1$

= 468 km

OR

Since, from A to B both Aeroplane and wind have velocity in North direction.

So,

$\vec{V}_{plane,AtoB}$ = 120 + 50

= 170 m/s

38. i. We have, I(x) = $\frac{1000}{x^2} + \frac{125}{(600-x)^2}$

$\Rightarrow$ I'(x) = $\frac{-2000}{x^3} + \frac{250}{(600-x)^3}$ and

$\Rightarrow$ I''(x) = $\frac{6000}{x^4} + \frac{750}{(600-x)^4}$

For maxima/minima, I'(x) - 0

$\Rightarrow \frac{2000}{x^3} = \frac{250}{(600-x)^3} \Rightarrow 8(600 - x)^3 = x^3$

Taking cube root on both sides, we get

2(600 - x) = x $\Rightarrow$ 1200 = 3x $\Rightarrow$ x = 400

Thus, I(x) is minimum when you are at 400 feet from the strong intensity lamp post.

ii. At a distance of 200 feet from the weaker lamp post.

Since I(x) is minimum when x = 400 feet, therefore the darkest spot between the two light is at a distance of 400 feet from a stronger lamp post, i.e., at a distance of 600 - 400 = 200 feet from the weaker lamp post.

iii. $\frac{1000}{x^2} + \frac{125}{(600-x)^2}$

Since, the distance is x feet from the stronger light, therefore the distance from the weaker light will be 600 - x.

So, the combined light intensity from both lamp posts is given by $\frac{1000}{x^2} + \frac{125}{(600-x)^2}$.

OR

We know that l(x) = $\frac{1000}{x^2} + \frac{125}{(600-x)^2}$

When x = 400

l(x) = $\frac{1000}{160000} + \frac{125}{(600-400)^2}$

= $\frac{1}{160} + \frac{125}{40000} = \frac{1}{160} + \frac{1}{320} = \frac{3}{320}$ units