Class XII Session 2024-25
Subject - Mathematics
Sample Question Paper - 5

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If |A| = |kA|, where A is a square matrix of order 2, then sum of all possible values of k is **[1]**

a) 2 b) 1

c) -1 d) 0

2. The value of the determinant $\begin{vmatrix} 6 & 0 & -1 \\ 2 & 1 & 4 \\ 1 & 1 & 3 \end{vmatrix}$ is **[1]**

a) 10 b) -7

c) 7 d) 8

3. Let A be a 3 $\times$ 3 matrix such that |adj A| = 64. Then |A| is equal to: **[1]**

a) -8 only b) 64

c) 8 only d) 8 or -8

4. If A = $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$, then A^2 is: **[1]**

a) 27A b) 3A

c) 2A d) I

5. The Cartesian equations of a line are $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-5}{-1}$. Its vector equation is **[1]**

a) $\vec{r} = (\hat{i} - 2\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$ b) $\vec{r} = (2\hat{i} + 3\hat{j} - \hat{k}) + \lambda(\hat{i} - 2\hat{j} + 5\hat{k})$

c) $\vec{r} = (\hat{i} - 2\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 3\hat{j} - 4\hat{k})$ d) $\vec{r} = (\hat{i} - 2\hat{j} + 5\hat{k}) + \lambda(2\hat{i} - 3\hat{j} - 4\hat{k})$

6. The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$ is: **[1]**

a) e^x

b) $\frac{x}{e^x}$

c) $\frac{e^x}{x}$

d) xe^x

7. A Linear Programming Problem is as follows: **[1]**

Maximize/Minimize objective function Z = 2x - y + 5

Subject to the constraints

3x + 4y $\leq$ 60

x + 3y $\leq$ 30

x $\leq$ 0, y $\geq$ 0

In the corner points of the feasible region are A(0, 10), B(12, 6), C(20, 0) and O(0,0), then which of the following is true?

a) Minimum value of Z is -5

b) At two corner points, value of Z are equal

c) Maximum value of Z is 40

d) Difference of maximum and minimum values of Z is 35

8. If the angle between the vectors $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$ and $|\vec{a} \times \vec{b}|$ = 1, then $\vec{a} \cdot \vec{b}$ is equal to **[1]**

a) -1

b) $\frac{1}{\sqrt{2}}$

c) $\sqrt{2}$

d) 1

9. $\int \frac{1+\tan x}{1-\tan x} dx$ is equal to: **[1]**

a) $\sec^2\left(\frac{\pi}{4} - x\right) + C$

b) $\sec^2\left(\frac{\pi}{4} + x\right) + C$

c) $\log\left|\sec\left(\frac{\pi}{4} - x\right)\right| + C$

d) $\log\left|\sec\left(\frac{\pi}{4} + x\right)\right| + C$

10. Let A be a skew-symmetric matrix of order 3. If |A| = x, then $(2023)^x$ is equal to: **[1]**

a) 2023

b) $(2023)^2$

c) $\frac{1}{2023}$

d) 1

11. The corner points of the feasible region determined by the following system of linear inequalities: **[1]**

2x + y $\leq$ 10, x + 3y $\leq$ 15, x, y $\geq$ 0 are (0, 0), (5, 0), (3, 4) and (0, 5). Let Z = px + qy, where p, q $\geq$ 0. Condition on p and q so that the maximum of Z occurs at both (3, 4) and (0, 5) is

a) p = 3q

b) q = 3p

c) p = q

d) p = 2q

12. A unit vector $\hat{a}$ makes equal but acute angles on the co-ordinate axes. The projection of the vector $\hat{a}$ on the vector $\vec{b} = 5\hat{i} + 7\hat{j} - \hat{k}$ is **[1]**

a) $\frac{3}{5\sqrt{3}}$

b) $\frac{11}{15}$

c) $\frac{4}{5}$

d) $\frac{11}{5\sqrt{3}}$

13. Let X = $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, A = $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$. If AX = B, then X is equal to **[1]**

a)

b)

$\begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$ $\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

c) $\begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix}$ d) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

14. If for two events A and B, P(A - B) = $\frac{1}{5}$ and P(A) = $\frac{3}{5}$, then $\mathrm{P}\left(\frac{\mathrm{B}}{\mathrm{A}}\right)$ is equal to **[1]**

a) $\frac{1}{2}$ b) $\frac{2}{5}$

c) $\frac{2}{3}$ d) $\frac{3}{5}$

15. Find the particular solution for $2xy + y^2 - 2x^2\frac{dy}{dx} = 0$; $y = 2$ when x = 1 **[1]**

a) $y = \frac{2x}{1-\log|x|}$ $(x \neq 0, x \neq e)$ b) $y = \frac{3x}{1-\log|x|}$ $(x \neq 0, x \neq e)$

c) $y = \frac{2x}{1+\log|x|}$ $(x \neq 0, x \neq e)$ d) $y = \frac{5x}{1+\log|x|}$ $(x \neq 0, x \neq e)$

16. If $\vec{a}, \vec{b}$ and $(\vec{a}+\vec{b})$ are all unit vectors and θ is the angle between $\vec{a}$ and $\vec{b}$, then the value of θ is: **[1]**

a) $\frac{2\pi}{3}$ b) $\frac{\pi}{3}$

c) $\frac{5\pi}{6}$ d) $\frac{\pi}{6}$

17. If $y = \left(1 + \frac{1}{x}\right)^x$, then $\frac{dy}{dx} =$ **[1]**

a) $\left(1 + \frac{1}{x}\right)^x \log\left(1 + \frac{1}{x}\right)$ b) $\left(x + \frac{1}{x}\right)^x \left\{\log\left(1 + \frac{1}{x}\right) + \frac{1}{x+1}\right\}$

c) $\left(1 + \frac{1}{x}\right)^x \left\{\log\left(1 + \frac{1}{x}\right) - \frac{1}{x+1}\right\}$ d) $\left(x + \frac{1}{x}\right)^x \left\{\log(x+1) - \frac{x}{x+1}\right\}$

18. If the line $\frac{x-2}{2k} = \frac{y-3}{3} = \frac{z+2}{-1}$ and $\frac{x-2}{8} = \frac{y-3}{6} = \frac{z+2}{-2}$ are parallel, value of k is **[1]**

a) -2 b) 2

c) $\frac{1}{2}$ d) 4

19. **Assertion (A):** If 3 ≤ x ≤ 10 and 5 ≤ y ≤ 15, then minimum value of $\left(\frac{x}{y}\right)$ is 2. **[1]**

Reason (R): If 3 ≤ x ≤ 10 and 5 ≤ y ≤ 15, then minimum value of $\left(\frac{x}{y}\right)$ is $\frac{1}{5}$.

a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

20. If R is the relation in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a - b| is even}, **[1]**

Assertion (A): R is an equivalence relation.

Reason (R): All elements of {1, 3, 5} are related to all elements of {2, 4}.

a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

Section B

21. Using the principal values, write the value of $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$. **[2]**

OR

Write the interval for the principal value of function and draw its graph: $\sin^{-1}x$....

22. The volume of a cube is increasing at the rate of 7 cm^3 /sec. How fast is its surface area increasing at the instant **[2]**

when the length of an edge of the cube is 12 cm?

23. Find the maximum and minimum values of $2x^3 - 24x + 107$ on the interval [-3, 3]. **[2]**

OR

Show that $f(x) = \cos(2x + \frac{\pi}{4})$ is an increasing function on $(\frac{3\pi}{8}, \frac{7\pi}{8})$

24. Evaluate: $\int \frac{(3x+5)}{(x^3-x^2-x+1)} dx$ **[2]**

25. Show that the function $f(x) = \frac{x}{3} + \frac{3}{x}$ decreases in the intervals (-3, 0) ∪ (0, 3). **[2]**

Section C

26. Evaluate: $\int \frac{x+2}{\sqrt{x^2+2x+3}}$ **[3]**

27. Bag I contains 3 white and 4 black balls, while Bag II contains 5 white and 3 black balls. One ball is transferred at random from Bag I to Bag II and then a ball is drawn at random from Bag II. The ball so drawn is found to be white. Find the probability that the transferred ball is also white. **[3]**

28. Evaluate: $\int_0^{1/\sqrt{2}} \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx$ **[3]**

OR

Find $\int \frac{\sqrt{x}}{\sqrt{a^3-x^3}} dx.$

29. Find the particular solution of the differential equation $(1 + y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$, given that y = 0 when x = 1. **[3]**

OR

Solve: $\frac{dy}{dx} + \frac{y}{x} = \cos x + \frac{\sin x}{x}$

30. Show that the solution set of the linear constraints is empty: $x - 2y \geq 0, 2x - y \leq -2, x \geq 0$ and $y \geq 0$ **[3]**

OR

Solve the following LPP by graphical method:

Minimize Z = 20x + 10y

Subject to

$x + 2y \leq 40$

$3x + y \geq 30$

$4x + 3y \geq 60$

and $x, y \geq 0$

31. If x = a sin 2t (1 + cos 2t) and y = b cos 2t (1 – cos 2t), then find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$. **[3]**

Section D

32. Using integration, find the area of the region in the first quadrant enclosed by the Y-axis, the line y = x and the circle $x^2 + y^2 = 32$. **[5]**

33. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a – b| is divisible by 2} is an equivalence relation. Write all the equivalence classes of R. **[5]**

OR

Show that the function $f : R \to \{x \in R : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}$, x ∈ R is one-one and onto function.

34. Two schools P and Q want to award their selected students on the values of Tolerance, Kindness, and Leadership. The school P wants to award Rs x each, Rs y each and Rs z each for the three respective values to 3, 2 and 1 students respectively with total award money of Rs2200. **[5]**

School Q wants to spend Rs 3100 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as school P). If the total amount of award for one prize on each value is Rs1200, using matrices, find the award money for each value.

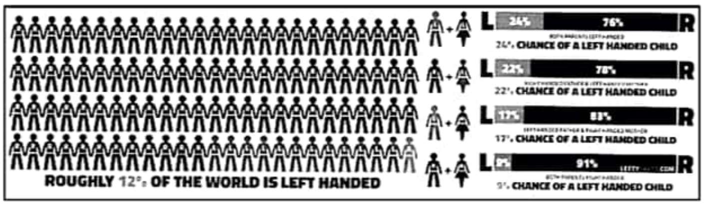
35. Find the length shortest distance between the lines: $\frac{x-3}{3} = \frac{y-8}{-1} = z - 3$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ **[5]**

OR

Find the image of the point (0, 2, 3) in the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$

Section E

36. Recent studies suggest that roughly 12% of the world population is left handed. **[4]**



Depending upon the parents, the chances of having a left handed child are as follows:

A. When both father and mother are left handed:
Chances of left handed child is 24%.

B. When father is right handed and mother is left handed:
Chances of left handed child is 22%.

C. When father is left handed and mother is right handed:
Chances of left handed child is 17%.

D. When both father and mother are right handed:
Chances of left handed child is 9%.

Assuming that P(A) = P(B) = P(C) = P(D) = $\frac{1}{4}$ and L denotes the event that child is left handed.

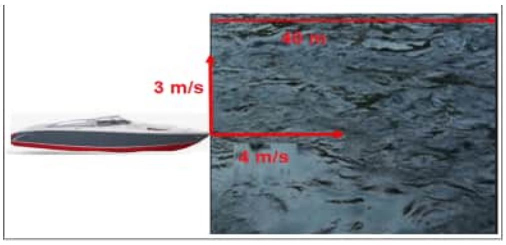
i. Find $P\left(\frac{L}{C}\right)$. (1)
ii. Find $P\left(\frac{\bar{L}}{A}\right)$. (1)
iii. Find $P\left(\frac{A}{L}\right)$. (2)

OR

Find the probability that a randomly selected child is left handed given that exactly one of the parents is left handed. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Once Ramesh was going to his native place at a village near Agra. From Delhi and Agra he went by flight, In the way, there was a river. Ramesh reached the river by taxi. Then Ramesh used a boat for crossing the river. The boat heads directly across the river 40 m wide at 4 m/s. The current was flowing downstream at 3 m/s.



i. What is the resultant velocity of the boat? (1)
ii. How much time does it take the boat to cross the river? (1)
iii. How far downstream is the boat when it reaches the other side? (2)

OR

If speeds of boat and current were 1.5 m/s and 2.0 m/s then what will be resultant velocity? (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Naina is creative she wants to prepare a sweet box for Diwali at home. She took a square piece of cardboard of side 18 cm which is to be made into an open box, by cutting a square from each corner and folding up the flaps to form the box. She wants to cover the top of the box with some decorative paper. Naina is interested in maximizing the volume of the box.

i. Find the volume of the open box formed by folding up the cutting each corner with x cm. (1)
ii. Naina is interested in maximizing the volume of the box. So, what should be the side of the square to be cut off so that the volume of the box is maximum? (1)
iii. Verify that volume of the box is maximum at x = 3 cm by second derivative test? (2)

OR

Find the maximum volume of the box. (2)

Solution

Section A

1.

(d) 0

Explanation: 0

2.

(b) -7

Explanation: -7

$\begin{vmatrix} 6 & 0 & -1 \\ 2 & 1 & 4 \\ 1 & 1 & 3 \end{vmatrix}$

=6(3-4) -0(6-4)+(-1)(2-1)

= 6 (-1) +0+ (-1)

= -6 -1

=-7

3.

(d) 8 or -8

Explanation: 8 or -8

Explanation

We know that |Adj A| = $|A|^{n-1}$, n is the order of the matrix.

$\because$ 64 = $|A|^{3-1}$

$|A|^2$ = 64

|A| = ± 8

4.

(b) 3A

Explanation: $\because$ A^2 = A $\times$ A

= $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} \cdot \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$ = $\begin{vmatrix} 1+1+1 & 1+1+1 & 1+1+1 \\ 1+1+1 & 1+1+1 & 1+1+1 \\ 1+1+1 & 1+1+1 & 1+1+1 \end{vmatrix}$

= $\begin{vmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{vmatrix} = 3 \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$

= 3A

5. **(a)** $\vec{r} = (\hat{i} - 2\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$

Explanation: Fixed point (1, -2, 5) and the parallel vector is $2\hat{\imath} + 3\hat{\jmath} - \hat{k}$

6.

(c) $\frac{e^x}{x}$

Explanation: We have, $\frac{dy}{dx} + y = \frac{1+y}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{x} + \frac{y(1-x)}{x}$

$\Rightarrow \frac{dy}{dx} - \left(\frac{1-x}{x}\right) y = \frac{1}{x}$

This is a linear differential equation.

On comparing it with $\frac{dy}{dx} + Py = Q$, we get

$P = \frac{-(1-x)}{x}, Q = \frac{1}{x}$

I.F. = $e^{\int Pdx} = e^{-\int \frac{1-x}{x} dx}$

$= e^{x-\log x}$ or $\frac{e^x}{x}$

7. **(a)** Minimum value of Z is -5

Explanation:

Corner points	Value of Z = 2x - y + 5
A(0, 10)	Z = 2(0) - 10 + 5 = -5 (Minimum)
B(12, 6)	Z = 2(12) - 6 + 5 = 23
C(20, 0)	Z = 2(20) - 0 + 5 = 45 (Maximum)
O(0, 0)	Z = 0(0) - 0 + 5 = 5

So the minimum value of Z is -5.

8.

(d) 1

Explanation: 1

9.

(d) $\log\left|\sec\left(\frac{\pi}{4} + x\right)\right| + C$

Explanation: $\log\left|\sec\left(\frac{\pi}{4} + x\right)\right| + C$

10.

(d) 1

Explanation: 1

11.

(b) q = 3p

Explanation: The maximum value of Z is unique.

It is given that the maximum value of Z occurs at two points (3,4) and (0,5)

∴ Value of Z at (3, 4) = Value of Z at (0, 5)

⇒ p(3) + q(4) = p(0) + q(5)

⇒ 3p + 4q = 5q

⇒ q = 3p

12.

(b) $\frac{11}{15}$

Explanation: $\frac{11}{15}$

13.

(b) $\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

Explanation: Given that

X = $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, A = $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$

Also AX = B and we have to find the value of X ,

Pre-multiplying A^{-1} both sides we get,

$A^{-1}AX = A^{-1}B$

$IX = A^{-1}B$ (∵ $A^{-1}A = I$)

$X = A^{-1}B$ (∵ IX = X) …(i)

Now,

$|A| = \begin{vmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{vmatrix}$ = 1(0 - 2) + 1(2 - 3) +2(4 - 0) = -2 - 1 + 8 = 5

And adjA = $\begin{bmatrix} -2 & 5 & -1 \\ 1 & -5 & 3 \\ 4 & -5 & 2 \end{bmatrix}$

$\therefore \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

On comparing both sides we get,

x_1 = -1, x_2 = 2 and x_3 = 3.

14.

(c) $\frac{2}{3}$

Explanation: $\frac{2}{3}$

15. **(a)** $y = \frac{2x}{1-\log|x|}$ $(x \neq 0, x \neq e)$

Explanation: Let y = vx

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

Question becomes $v + x\frac{dv}{dx} = \frac{2v+v^2}{2}$

$x\frac{dv}{dx} = \frac{2v+v^2}{2} - v$

$x\frac{dv}{dx} = \frac{2v+v^2-2v}{2}$

$2\int\frac{dv}{v^2} = \int\frac{dx}{x}$

$\frac{-2}{v} = logx + c$

When x=1 y=2 we get

$\frac{-2x}{y} = logx + c$

$\frac{-2}{2} = log1 + c \implies c = -1$

$\frac{-2x}{y} = logx - 1$

$y = \frac{2x}{1-log|x|}$

16. **(a)** $\frac{2\pi}{3}$

Explanation: $\frac{2\pi}{3}$

17.

(c) $\left(1+\frac{1}{x}\right)^x\left\{\log\left(1+\frac{1}{x}\right) - \frac{1}{x+1}\right\}$

Explanation: $y = \left(1+\frac{1}{x}\right)^x$

Taking log on both sides,

$\log y = \log\left(1+\frac{1}{x}\right)^x$

$\log y = x\log\left(1+\frac{1}{x}\right)$

Differentiate with respect to x,

$\frac{1}{y}\frac{dy}{dx} = x \times \frac{1}{1+\frac{1}{x}} \times \frac{-1}{x^2} + \log\left(1+\frac{1}{x}\right)$

$\frac{1}{y}\frac{dy}{dx} = \frac{x^2}{x+1} \times \frac{-1}{x^2} + \log\left(1+\frac{1}{x}\right)$

$\frac{dy}{dx} = y\left(\frac{-1}{x+1} + \log\left(1+\frac{1}{x}\right)\right)$

$\frac{dy}{dx} = \left(1+\frac{1}{x}\right)^x\left(\frac{-1}{x-1} + \log\left(1+\frac{1}{x}\right)\right)$

$\frac{dy}{dx} = \left(1+\frac{1}{x}\right)^x\left(\log\left(1+\frac{1}{x}\right) - \frac{1}{x+1}\right)$

18.

(b) 2

Explanation: Given lines are $\frac{x-2}{2k} = \frac{y-3}{3} = \frac{2+2}{-1}$ and $\frac{x-2}{8} = \frac{y-3}{6} = \frac{z+2}{-2}$

The direction ratio of the first line is (2k, 3, -1) and the direction ratio of second line is (8, 6, -2).

Lines are parallel;

So,

$\frac{2k}{8} = \frac{3}{6} = \frac{-1}{-2}$

$\Rightarrow \frac{k}{4} = \frac{1}{2} = \frac{1}{2}$

$\therefore$ k = 2

19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: For maximum value of $\frac{x}{y}$, we take maximum value of x and minimum value of y,

i.e., Maximum value of $\left(\frac{x}{y}\right) = \frac{10}{5} = 2$

Similarly, minimum value of $\left(\frac{x}{y}\right) = \frac{3}{15} = \frac{1}{5}$

20.

(c) A is true but R is false.

Explanation: Assertion: Given that, A = {1, 2, 3, 4, 5},

R = {(a, b) : |a - b| is even}

Let $a \in A \Rightarrow$ |a - a| = 0 (which is even), $\forall a$

So, R is reflexive.

Let (a, b) $\in R \Rightarrow$ |a - b| is even.

$\Rightarrow$ |a - b| = |-(b - a)| = |b - a|, therefore |b - a| is also even.

$\Rightarrow (b, a) \in R$. So, R is symmetric.

Now, let $(a, b) \in R$ and $(b, c) \in R$.

$\Rightarrow |a - b|$ is even and |b - c| is even.

$\Rightarrow (a - b)$ is even and (b - c) is even.

$\Rightarrow$ (a - c) = (a - b) + (b - c) is even

[$\because$ sum of two even integers is even]

$\Rightarrow |a - c|$ is even $\Rightarrow (a, c) \in R$.

So, R is transitive.

Hence, R is an equivalence relation.

Reason: Also, no element of the {1, 3, 5} can be related to any element of {2, 4}, as all elements of {1, 3, 5} are odd and all elements of {2, 4} are even.

So, the modulus of the difference between the two elements (from each of these two subsets) will not be even.

Hence Reason is not correct.

Section B

21. We have, $\cos^{-1}\left(\frac{1}{2}\right) = \cos^{-1}\left(\cos\frac{\pi}{3}\right)$

$= \frac{\pi}{3} \left[\because \frac{\pi}{3} \in [0, \pi]\right]$

Also $\sin^{-1}\left(-\frac{1}{2}\right) = \sin^{-1}\left(-\sin\frac{\pi}{6}\right)$

$= \sin^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right)$

$= -\frac{\pi}{6} \left[\because -\frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$

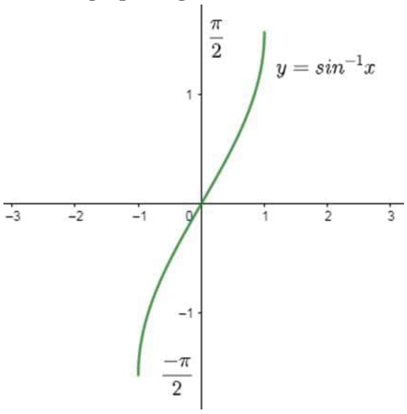
$\therefore \cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{3} - 2\left(-\frac{\pi}{6}\right)$

$= \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$

OR

Principal value branch of $\sin^{1}$ x is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

And its graph is given here



22. At any instant t, let the length of each edge of the cube be x, V be its volume and S be its surface area. Then,

$\frac{dV}{dt}$ = 7 cm^3 / sec ... (given) ... (i)

Now, V = $x^3 \Rightarrow \frac{dV}{dt} = \frac{dV}{dx} \cdot \frac{dx}{dt}$

$\Rightarrow 7 = \frac{d}{dx}\left(x^3\right) \cdot \frac{dx}{dt}$ .... [$\because$ V = x^3]

$\Rightarrow 3x^2 \cdot \frac{dx}{dt} = 7$

$\Rightarrow \frac{dx}{dt} = \frac{7}{3x^2}$

$\therefore$ S = $6x^2 \Rightarrow \frac{dS}{dt} = \frac{dS}{dx} \cdot \frac{dx}{dt}$

$= \frac{d}{dx}\left(6x^2\right) \cdot \frac{7}{3x^2}$

$= \left(12x \times \frac{7}{3x^2}\right) = \frac{28}{x}$

$\Rightarrow \left[\frac{dS}{dt}\right]_{x=12} = \left(\frac{28}{12}\right)$ cm^2 /sec = $2\frac{1}{3}\ cm^2/sec$

Hence, the surface area of the cube is increasing at the rate of $2\frac{1}{3}\ cm^2/sec$ at the instant when its edge is 12 cm.

23. We have maximum value is 139 at x = -2 and minimum value is 89 at x = 3

Also $F'(x) = 6x^2 - 24 = 0$

$6(x^2 - 4) = 0$

$6(x^2 - 2^2) = 0$

6(x - 2)(x + 2) = 0

x = 2, -2

Now, we shall evaluate the value of f at these points and the end points

$F(2) = 2(2)^3 - 24(2) + 107 = 75$

$F(-2) = 2(-2)^3 - 24(-2) + 107 = 139$

$F(-3) = 2(-3)^3 - 24(-3) + 107 = 125$

$F(3) = 2(3)^3 - 24(3) + 107 = 89$

OR

Given: $f(x) = \cos(2x + \frac{\pi}{4})$

$f'(x) = -2\sin(2x + \frac{\pi}{4})$

Now,

$x \in (\frac{3\pi}{8}, \frac{7\pi}{8})$

$\Rightarrow \quad \frac{3\pi}{8} < x < \frac{7\pi}{8}$

$\Rightarrow \quad \frac{3\pi}{4} < 2x < \frac{7\pi}{4}$

$\Rightarrow \quad \frac{\pi}{4} + \frac{3\pi}{4} < 2x + \frac{\pi}{4} < \frac{7\pi}{4} + \frac{\pi}{4}$

$\Rightarrow \quad \pi < 2x + \frac{\pi}{4} < 2\pi$

$\Rightarrow \quad \sin(2x + \frac{\pi}{4}) < 0$

$\Rightarrow \quad -2\sin(2x + \frac{\pi}{4}) > 0$

$\Rightarrow \quad f'(x) > 0$

Hence, f(x) is increasing on $(\frac{3\pi}{8}, \frac{7\pi}{8})$

24. Let I = $\int \frac{(3x+5)}{(x^3-x^2-x+1)} dx$

where $(x^3 - x^2 - x + 1) = x^2(x - 1) - (x - 1) = (x - 1)(x^2 - 1) = (x - 1)^2(x + 1)$

Now let $\frac{3x+5}{(x^3-x^2-x+1)} = \frac{3x+5}{(x-1)^2(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+1)}$

$\Rightarrow (3x + 5) = A(x - 1)(x + 1) + B(x + 1) + C(x - 1)^2$

Putting x = 1 on both sides of (i), we get B = 4

Putting x = -1 on both sides of (i), we get C $= \frac{1}{2}$

Comparing the coefficient of x^2 on both sides of (i), we get

A + C = 0 $\Rightarrow$ A = -C $= \frac{-1}{2}$

$\therefore \frac{(3x+5)}{(x^3-x^2-x+1)} = \frac{-1}{2(x-1)} + \frac{4}{(x-1)^2} + \frac{1}{2(x+1)}$

$\Rightarrow I=\int \frac{(3x+5)}{\left(x^{3}-x^{2}-x+1\right)} dx=-\frac{1}{2} \int \frac{dx}{(x-1)}+4 \int \frac{dx}{(x-1)^{2}}+\frac{1}{2} \int \frac{dx}{(x+1)}$

$=-\frac{1}{2} \log |x-1|-\frac{4}{(x-1)}+\frac{1}{2} \log |x+1|+C$

25. $f(x)=\frac{1}{3}-\frac{3}{x^{2}}$

for decreasing $f'(x)<0 \Rightarrow \frac{1}{3}-\frac{3}{x^{2}}<0$

$\Rightarrow x^2 < 9 \Rightarrow -3 < x < 3$

since, f(x) is not defined at x = 0

so f(x) decreasing in (-3, 0) ∪ (0, 3)

Section C

26. Let I = $\int \frac{x+2}{\sqrt{x^{2}+2x+3}}$

x + 2 = $A \frac{d}{dx}$ [x^2 + 2x + 3] + B

⇒ x + 2 = 2Ax + 2A + B

Comparing the coefficients, we have, 2A = 1 and 2A + B = 2

⇒ A = $\frac{1}{2}$

Substituting the value of A in 2 A + B = 2, we have, $2 \times \frac{1}{2}$ + B = 2

⇒ 1 + B = 2

⇒ B = 2 - 1

⇒ B = 1

Thus we have, x + 2 = $\frac{1}{2}[2x+2]+1$

Hence, using values of A , and B, we have

$I=\int \frac{x+2}{\sqrt{x^{2}+2x+3}} dx$

$=\int \frac{\left[\frac{1}{2}[2x+2]+1\right]}{\sqrt{x^{2}+2x+3}}\Bigg] dx$

$=\int \frac{\left[\frac{1}{2}[2x+2]\right]}{\sqrt{x^{2}+2x+3}} dx+\int \frac{dx}{\sqrt{x^{2}+2x+3}}$

$=\frac{1}{2} \int \frac{[2x+2]}{\sqrt{x^{2}+2x+3}} dx+\int \frac{dx}{\sqrt{x^{2}+2x+3}}$

Substituting t = x^2 + 2x + 3 and dt = 2 x + 2

in the first integrand, we have, I = $\frac{1}{2} \int \frac{dt}{\sqrt{t}}+\int \frac{dx}{\sqrt{x^{2}+2x+3}}$

= $\frac{1}{2} \times 2\sqrt{t}+\int \frac{dx}{\sqrt{x^{2}+2x+1+2}}+C$

= $\sqrt{t}+\int \frac{dx}{\sqrt{(x+1)^{2}+(\sqrt{2})^{2}}}+c$

I = $\sqrt{x^{2}+2x+3}$ + log [|x + 1| + $\sqrt{(x+1)^{2}+(\sqrt{2})^{2}}]+C$

⇒ I = $\sqrt{x^{2}+2x+3}$ + log [|x + 1| $\sqrt{x^{2}+2x+3}$] + c

27. Let E_1: Transferred ball is white

E_2: Transferred ball is black

A: white ball is found

Here, $\mathrm{P}\left(\mathrm{E}_{1}\right)=\frac{3}{7}, \mathrm{P}\left(\mathrm{E}_{2}\right)=\frac{4}{7}$

$\mathrm{P}\left(\mathrm{A}/\mathrm{E}_{1}\right)=\frac{6}{9}, \mathrm{P}\left(\mathrm{A}/\mathrm{E}_{2}\right)=\frac{5}{9}$

Using Baye's theorem

$P\left(E_{1}/A\right)=\frac{P(E_{1}) \cdot P(A/E_{1})}{P(E_{1}) \cdot P(A/E_{1})+P(E_{2}) \cdot P(A/E_{2})}$

$=\frac{\frac{3}{7} \times \frac{6}{9}}{\frac{3}{7} \times \frac{6}{9}+\frac{4}{7} \times \frac{5}{9}}$

$=\frac{18}{18+20}=\frac{9}{19}$

28. Let $\sin^{-1} x=\theta$ or, $x=\sin \theta$. Then, $dx=d(\sin \theta)=\cos \theta d\theta$

Now, $x=0 \Rightarrow \sin \theta=0 \Rightarrow \theta=0$ and $x=\frac{1}{\sqrt{2}} \Rightarrow \sin \theta=\frac{1}{\sqrt{2}} \Rightarrow \theta=\frac{\pi}{4}$

$\therefore \quad I=\int_{0}^{1/\sqrt{2}} \frac{\sin^{-1} x}{\left(1-x^{2}\right)^{3/2}} dx$

$\Rightarrow \quad I=\int_{0}^{\pi/4} \frac{\theta}{\cos^{3} \theta} \cos \theta d\theta=\int_{0}^{\pi/4} \theta \sec^{2} \theta d\theta$

Now using integratio by parts.

$\Rightarrow \quad I=[\theta \quad \tan\theta]_0^{\pi/4}+[\log\cos\theta]_0^{\pi/4} = \frac{\pi}{4}+\left\{\log\left(\frac{1}{\sqrt{2}}\right)-\log 1\right\}=\frac{\pi}{4}-\frac{1}{2}\log 2$

OR

Given $I=\int \frac{\sqrt{x}}{\sqrt{a^3-x^3}}dx=\int \frac{\sqrt{x}}{\sqrt{\left(a^{3/2}\right)^2-\left(x^{3/2}\right)^2}}dx$

$Let,\ \ x^{3/2}=a^{3/2}t$

$\Rightarrow \frac{3}{2}x^{1/2}dx=a^{3/2}dt$

$\Rightarrow \sqrt{x}dx=\frac{2}{3}a^{3/2}dt$

$\therefore \quad I=\int \frac{\frac{2}{3}a^{3/2}}{\sqrt{\left(a^{3/2}\right)^2-\left(a^{3/2}t\right)^2}}dt$

$=\frac{2}{3}a^{3/2}\int \frac{dt}{a^{3/2}\sqrt{1-t^2}}$

$=\frac{2}{3}\int \frac{dt}{\sqrt{1-t^2}}=\frac{2}{3}\sin^{-1}\left(\frac{t}{1}\right)+C$

$\left[\because \int \frac{dx}{a^2-x^2}=\sin^{-1}\left(\frac{x}{a}\right)+c\right]$

$=\frac{2}{3}\sin^{-1}\left(\frac{x^{3/2}}{a^{3/2}}\right)+C \quad \left[\text{put}\ t=\frac{x^{3/2}}{a^{3/2}}\right]$

$=\frac{2}{3}\sin^{-1}\left(\sqrt{\frac{x^3}{a^3}}\right)+C$

29. Given differential equation can be written as

$\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{e^{\tan^{-1}y}}{1+y^2}$

L.F. = $e^{\int \frac{dy}{1+y^2}} = e^{\tan^{-1}y}$

Solution is given by

x$e^{\tan^{-1}y} = \int \frac{e^{\tan^{-1}y}}{1+y^2}\times e^{\tan^{-1}y}$ dy = $\int \frac{e^{2\tan^{-1}y}}{1+y^2}$ dy

or x$e^{\tan^{-1}y} = \frac{e^{2\tan^{-1}y}}{2}$ + c

when x = 1, y = 0 or c = $\frac{1}{2}$

$\therefore$ Solution is given by x$e^{\tan^{-1}y} = \frac{1}{2}e^{2\tan^{-1}y}+\frac{1}{2}$

or x = $\frac{1}{2}\left(e^{\tan -1}y+e^{-\tan^{-1}y}\right)$

OR

We have,

$\frac{dy}{dx}+\left(\frac{1}{x}\right)$y = cos x + $\frac{\sin x}{x}$ ...(i)

This is a linear differential equation of the form

$\frac{dy}{dx}$ + Py = Q, where P = $\frac{1}{x}$ and Q = cos x + $\frac{\sin x}{x}$

$\therefore$ I. $F=e^{\int Pdx}=e^{\int \frac{1}{x}dx}=e^{\log x}=x$

Multiplying both sides of (i) by I.F. = x, we get

$x\frac{dy}{dx}$ + y = x cos x + sin x

Integrating both sides with respect to x, we get

yx = $\int$ (x cos x + sin x) dx + C [Using: y = (I.F.) = $\int$ Q (I.F.) dx + C]

$\Rightarrow$xy = $\int \underset{I}{x}\ \underset{II}{\cos}$ x dx + $\int$ sin x dx + C

$\Rightarrow$ xy = x sin x - $\int$ sin x dx + $\int$ sin x dx + C [Integrating 1st integral by parts]

$\Rightarrow$ xy = x sin x + C

$\Rightarrow$ y = sin x + $\frac{C}{x}$.

30. Here, it is given that the equations

x - 2y $\geq$ 0...... (i)

2x -y $\leq$ -2 ...... (ii)

x $\geq$ 0 .... (iii)

y $\geq$ 0...... (iv)

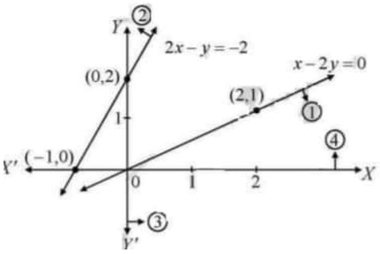
The line corresponding to (i) is x - 2y = 0 .... (v)

on the line (v) put x = 0

$\Rightarrow$ 0 - 2y = 0

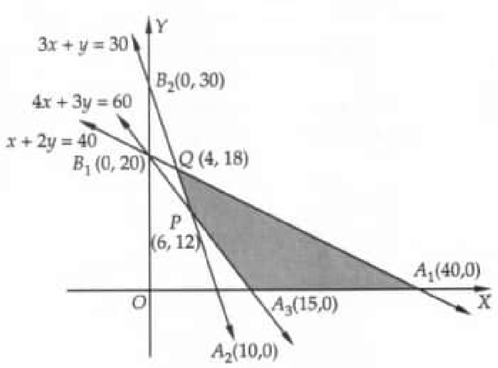
$\Rightarrow$ y =0

and x = 2 ⇒ 2 - 2 y = 0 = y = 1
(0,0) and (2,1) are on the line (v)
(0,1) is not on this line and it lies in the half
plane of (i) which is not true
The line corresponding to (ii) is
$2x - y = -2$ ......(vi)
on the line (vi) put $x = 0 \Rightarrow 0 - y = -2 \Rightarrow y = 2$
and y = 0 ⇒ $2x - 0 = -2 \Rightarrow x = -1$
The inequation x > 0 represents the closed half-plane on the right of the y-axis.
The inequation y > 0 represent the closed half-plane above the x-axis
The graph of the given system is the intersection of half-planes of the inequation
The intersection of half-planes is empty
The solution set of the given inequation is empty. This is the required solution



OR

Converting the given inequations into equations, we obtain the following equations:
x + 2y = 40, 3x + y = 30, 4x + 3y = 60, x = 0 and y = 0
Region represented by x + 2y $\leq$ 40:
The line x + 2y = 40 meets the coordinate axes at A_1 (40, 0) and B_1 (0, 20) respectively. Join these points to obtain the line x + 2y = 40.
Clearly, (0, 0) satisfies the inequation x + 2y $\leq$ 40. So, the region in xy-plane that contains the origin represents the solution set of the given inequation.
Region represented by 3x + y $\geq$ 30:
The line 3x + y = 30 meets x and y axes at A_2 (10, 0) and B_2 (0, 30) respectively. Join these points to obtain this line.
We find that the point O (0, 0) does not satisfy the inequation 3x + y $\geq$ 30.
So, that region in xy-plane which does not contain the origin is the solution set of this inequation.
Region represented by 4x + 3y $\geq$ 60:
The line 4x + 3y = 60 meets x and y axes at A_3 (15, 0) and B_1 (0, 20) respectively.
Join these points to obtain the line 4x + 3y = 60. We observe that the point O (0, 0) does not satisfy the inequation 4x + 3y $\geq$ 60.
So, the region not containing the origin in xy-plane represents the solution set of the given inequation.
Region represented by x $\geq$ 0, y $\geq$ 0:
Clearly, the region represented by the non-negativity restrictions x $\geq$ 0 and y $\geq$ 0 is the first quadrant in xy-plane.
The shaded region A_3A_1QP in a figure represents the common region of the regions represented by the above inequations.
This region represents the feasible region of the given LPP.



The coordinates of the corner points of the shaded feasible region are A_3 (15, 0), A_1 (40, 0), Q (4,18) and P (6,12). These points have been obtained by solving the equations of the corresponding intersecting lines, simultaneously.
The values of the objective function at these points are given in the following table:

Point (x, y)	Value of the objective function Z = 20x + 10y
A_3 (15, 0)	Z = 20×15 + 10×0 = 300
A_1 (40, 0)	Z = 20×40 + 10×0 = 800
Q(4,18)	Z = 20×4 + 0×18 = 260
P(6,12)	Z = 20×6 + 10×12 = 240

Out of these values of Z, the minimum value is 240 which is attained at point P (6,12). Hence, x = 6, y = 12 is the optimal solution of the given LPP and the optimal value of Z is 240.

31. We know that, x = a sin 2t (1 + cos 2t) and y = b cos 2t (1 – cos 2t)

$\Rightarrow \frac{dy}{dt}$ = a[-2sin 2t sin 2t + 2cos 2t (1 + cos 2t)]....(I)

and $\frac{dx}{dt}$ = b[2sin 2t cos 2t - 2sin 2t (1 - cos 2t)]...(ii)

$\therefore \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{b[2\sin 2t\cos 2t - 2\sin 2t(1-\cos 2t)]}{a\left[-2\sin^2 2t + 2\cos 2t(1+\cos 2t)\right]}$. [Using (i) and (ii)]

$\Rightarrow \left.\frac{dy}{dx}\right]_{at\ t=\pi/4} = \frac{b\left[2\sin\frac{\pi}{2}\cos\frac{\pi}{2} - 2\sin\frac{\pi}{2}\left(1-\cos\frac{\pi}{2}\right)\right]}{a\left[-2\sin^2\left(\frac{\pi}{2}\right) + 2\cos\frac{\pi}{2}\left(1+\cos\frac{\pi}{2}\right)\right]} = \frac{b}{a} \cdot \frac{(0-1)}{(-1-0)} = \frac{b}{a}$

Section D

32. According to the question ,

Given, equation of circle is $x^2 + y^2 = 32$ ......(i)

Given ,equation of line is y = x ....(ii)

Consider $x^2 + y^2 = 32$,

$\Rightarrow x^2 + y^2 = (4\sqrt{2})^2$

Given circle has centre at (0, 0) and

radius of circle is = $4\sqrt{2}$

To find the point of intersection ,

On substituting y = x in Eq. (i), we get

$2x^2 = 32 \Rightarrow x^2 = 16 \Rightarrow x = \pm 4$

When x = 4 , then y = 4

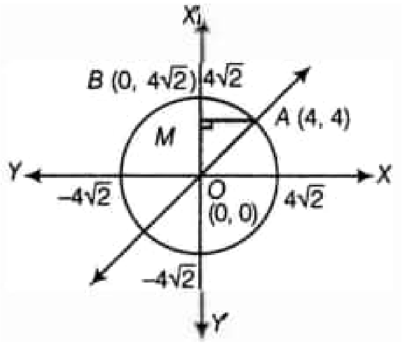
When x = -4 , then y = -4

Thus, the points of intersection are (4, 4) and (-4, - 4)

So, given line and the circle intersect in the first quadrant at point A(4, 4) and

The circle cut the Y-axis at point B $(0, 4\sqrt{2})$.

Now, let us sketch the graph of given curves, we get



Let us draw AM perpendicular to Y-axis.

Required area = Area of shaded region OABO

$= \int_0^4 x_{(\text{line})} dy + \int_4^{4\sqrt{2}} x_{(\text{circle})} dy$

$\because x^2 + y^2 = 32 \Rightarrow x = \pm\sqrt{32 - y^2}$,but we need area of region enclosed in the first quadrant only,so x = $\sqrt{32 - y^2}$

$= \int_0^4 y dy + \int_4^{4\sqrt{2}} \sqrt{32 - y^{-2}} dy$

$= \left[\frac{y^2}{2}\right]_0^4 + \int_4^{4\sqrt{2}} \sqrt{(4\sqrt{2})^2 - y^2} dy$

$= \frac{1}{2}(16 - 0) + \left[\frac{y}{2}\sqrt{32 - y^2} + \frac{32}{2}\sin^{-1}\left(\frac{y}{4\sqrt{2}}\right)\right]_4^{4\sqrt{2}}$

= 8 + $\left[16\sin^{-1}(1) - \left\{2 \times 4 + 16\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right\}\right]$

= 8 + $\left[16 \cdot \frac{\pi}{2} - 8 - 16 \cdot \frac{\pi}{4}\right]$
$= 16\left(\frac{\pi}{2} - \frac{\pi}{4}\right)$
$= 16 \cdot \frac{\pi}{4}$
$= 4\pi$ sq units

33. R = {(a,b) = |a.b| is divisible by 2.
where $a, b \in A = \{1, 2, 3, 4, 5\}$
reflexivty
For any a∈A,|a−a|=0 Which is divisible by 2.
∴ (a, a) ∈ r for all a ∈ A
So ,R is Reflexive
Symmetric :
Let $(a, b) \in R$ for all a, b ∈ R
|a−b| is divisible by 2
|b−a| is divisible by 2
$(a, b) \in r \Rightarrow (b, a) \in R$
So, R is symmetirc .
Transitive :
Let $(a, b) \in R$ and $(b, c) \in R$ then
$(a, b) \in R$ and $(b, c) \in R$
|a−b| is divisible by 2
|b−c| is divisible by 2
Two cases :
Case 1:
When b is even
$(a, b) \in R$ and $(b, c) \in R$
|a−c| is divisible by 2
|b−c| is divisible by 2
|a−c| is divisible by 2
∴ (a, c) ∈ R
Case 2:
When b is odd
$(a, b) \in R$ and $(b, c) \in R$
|a−c| is divisible by 2
|b−c| is divisible by 2
|a−c| is divisible by 2
Thus, $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$
So R is transitive.
Hence , R is an equivalence relation

OR

f is one-one: For any x, y ∈ R - {+1}, we have $f(x) = f(y)$
$\Rightarrow \frac{x}{1+|x|} = \frac{y}{|y|+1}$
$\Rightarrow$ xy + x = xy + y
$\Rightarrow$ x = y
Therefore, f is one-one function.
If f is one-one, let y = R - {1}, then f(x) = y
$\Rightarrow \frac{x}{x+1} = y$
$\Rightarrow x = \frac{y}{1-y}$
It is cleat that x ∈ R for all y = R - {1}, also x = $\neq$ -1
Because x = -1
$\Rightarrow \frac{y}{1-y} = -1$
$\Rightarrow$ y = -1 + y
which is not possible.

Thus for each R - {1} there exists $x = \frac{y}{1-y} \in$ R - {1} such that

$f(x) = \frac{x}{x+1} = \frac{\frac{y}{1-y}}{\frac{y}{1-y}+1} = y$

Therefore f is onto function.

34. Three equations are formed from the given statements:

3x + 2y + z = 2200

4x + y + 3z = 3100 and

x + y + z = 1200

Converting the system of equations in matrix form we get,

$\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$

i.e. AX = B

where A = $\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$

$\Rightarrow$ X = A^{-1} B

$A^{-1} = \frac{1}{|A|}$ (Adj A)

|A| = 3(1 − 3) −2(4 − 3) + 1(4 − 1) = −6 −2 + 3 = −5

Adj A = $\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{-5} \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$

$= \frac{1}{5} \begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{bmatrix}$

$\Rightarrow X = \frac{1}{5} \begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{bmatrix} \begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$

$= \frac{1}{5} \begin{bmatrix} 4400 + 3100 - 6000 \\ 2200 - 6200 + 6000 \\ -6600 + 3100 + 6000 \end{bmatrix}$

$= \begin{bmatrix} 300 \\ 400 \\ 500 \end{bmatrix}$

$\Rightarrow$ x = 300, y = 400 and z = 500

i.e. The award money for each value are Rs.300 for Tolerance, Rs.400 for Kindness and Rs.500 for Leadership.

35. Here,it is given that the equation of lines

$\mathrm{L1}: \frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$

$\mathrm{L2} = \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$

Direction ratios of L_1 and L_2 are (3, -1, 1) and (-3, 2, 4) respectively.

Suppose general point on line L_1 is P = (x_1, y_1, z_1)

$x_1 = 3s + 3$, $y_1 = -s + 8$, $z_1 = s + 3$

and suppose general point on line L_2 is Q = (x_2, y_2, z_2)

$x_2 = -3t - 3$, $y_2 = 2t - 7$, $Z_2 = 4t + 6$

$\therefore \overrightarrow{PQ} = (x_2 - x_1)\,\hat{i} + (y_2 - y_1)\,\hat{j} \; + (z_2 - z_1)\,\hat{k}$

$= (-3t - 3 - 3s - 3)\hat{i} + (2t - 7 + s - 8)j + (4t + 6 - s - 3)\hat{k}$

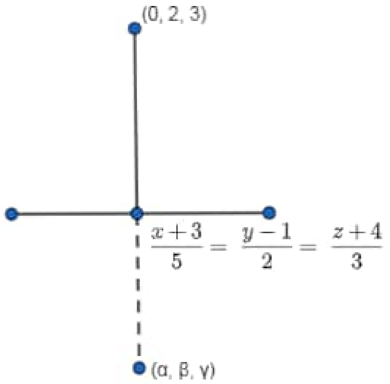
$\therefore \overrightarrow{PQ} = (-3t - 3s - 6)\hat{i} + (2t + s - 15)j + (4t - s + 3)\hat{k}$

Direction ratios of $\overrightarrow{PQ}$ are ((-3t - 3s - 6, (2t + s - 15), (4t - s + 3))

PQ will be the shortest distance if it perpendicular to both the given lines
Thus, by the condition of perpendicularity,
$\implies$ 3(-3t - 3s - 6) - 1(2t + s - 15) + 1(4t - s + 3) = 0 and
$\implies$ -3(-3t - 3s - 6) + 2(2t + s - 15) + 4(4t - s + 3) = 0
⇒ -9t - 9s - 18 -2t - s + 15 + 4t - s + 3 = 0 and
9t + 9s + 18 + 4t + 2s - 30 + 16t - 4s + 12 = 0
⇒ -7t - 11s = 0 and
29t + 7s = 0
Solving above two equations, we get,
t = 0 and s = 0
therefore,
P = (3, 8, 3) and Q = (-3, -7, 6)
Now distance between points P and Q is
$d=\sqrt{(3+3)^2+(8+7)^2+(3-6)^2}$
$=\sqrt{(6)^2+(15)^2+(-3)^2}$
$=\sqrt{36+225+9}$
$=\sqrt{270}$
$=3\sqrt{30}$
Thus, the shortest distance between two given lines is
$d=3\sqrt{30}$ units
Now, equation of line passing through points P and Q is,
$\frac{x-x_1}{x_1-x_2}=\frac{y-y_1}{y_1-y_2}=\frac{z-z_1}{z_1-z_2}$
$\therefore \frac{x-3}{3+3}=\frac{y-8}{8+7}=\frac{z-3}{3-6}$
$\therefore \frac{x-3}{6}=\frac{y-8}{15}=\frac{z-3}{-3}$
$\therefore \frac{x-3}{2}=\frac{y-8}{5}=\frac{z-3}{-1}$
Thus, equation of line of shortest distance between two given lines is
$\frac{x-3}{2}=\frac{y-8}{5}=\frac{z-3}{-1}$

OR

We have,
$\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}=\lambda$
Therefore, the foot of the perpendicular is $(5\lambda-3, 2\lambda+1, 3\lambda-4)$
The direction ratios of the perpendicular is
$(5\lambda-3-0):(2\lambda+1-2):(3\lambda-4-3)$
$\Rightarrow (5\lambda-3):(2\lambda-1):(3\lambda-7)$
Direction ratio of the line is 5 : 2 : 3



From the direction ratio of the line and direction ratio of its perpendicular, we have
$5(5\lambda-3)+2(2\lambda-1)+3(3\lambda-7)=0$
$\Rightarrow 25\lambda-15+4\lambda-2+9\lambda-21=0$
$\Rightarrow 38\lambda=38$
$\Rightarrow \lambda=1$
Therefore, the foot of the perpendicular is (2, 3, -1)
The foot of the perpendicular is the mid-point of the line joining (0, 2, 3) and (a, β, y)

Therefore, we have

$\frac{\alpha+0}{2} = 2 \Rightarrow \alpha = 4$

$\frac{\beta+2}{2} = 3 \Rightarrow \beta = 4$

$\frac{\gamma+3}{2} = -1 \Rightarrow \gamma = -5$

Thus, the imgae is (4, 4, -5)_

Section E

36. i. $P\left(\frac{L}{C}\right) = \frac{17}{100}$

ii. $P\left(\frac{\bar{L}}{A}\right) = 1 - P\left(\frac{L}{A}\right) = 1 - \frac{24}{100} = \frac{76}{100}$ or $\frac{19}{25}$

iii. $P\left(\frac{A}{L}\right) = \frac{\frac{1}{4}\times\frac{24}{100}}{\frac{1}{4}\times\frac{24}{100}+\frac{1}{4}\times\frac{22}{100}+\frac{1}{4}\times\frac{17}{100}+\frac{1}{4}\times\frac{9}{100}} = \frac{24}{72} = \frac{1}{3}$

Probability that a randomly selected child is left-handed given that exactly one of the parents is left-handed.

$= P\left(\frac{L}{B\cup C}\right) = \frac{22}{100} + \frac{17}{100} = \frac{39}{100}$

37. i. Resultant velocity of boat

$= \sqrt{4^2+3^2}$

$= \sqrt{25}$

= 5 m/s

ii. Time taken by boat to cross the river $= \frac{\text{Width of river}}{\text{Resultant velocity of boat}}$

$= \frac{40}{5}$

= 8 sec

iii. Downstream distance travelled by boat = downstream speed $\times$ time taken by boat to cross the river

= 3 $\times$ 8

= 24 m

OR

Resultant velocity of boat $= \sqrt{(1.5)^2+(2)^2}$

$= \sqrt{2.25+4}$

$= \sqrt{6.25}$

= 2.5 m/sec

38. i. Let the side of square to be cut off be 'x' cm. then, the length and the breadth of the box will be (18 - 2x) cm each and the height of the box is 'x' cm.

The volume V(x) of the box is given by $V(x) = x(18 - x)^2$

ii. $V(x) = x(18 - 2x)^2$

$\frac{dV(x)}{dx} = (18 - 2x)^2 - 4x(18 - 2x)$

For maxima or minima = $\frac{dV(x)}{dx} = 0$

$\Rightarrow$ (18 - 2x)[18 - 2x - 4x] = 0

$\Rightarrow$ x = 9 or x = 3

$\Rightarrow$ x = not possible

$\Rightarrow$ x = 3 cm

The side of the square to be cut off so that the volume of the box is maximum is x = 3 cm

iii. $\frac{dV(x)}{dx} = (18 - 2x)(18 - 6x)$

$\frac{d^2V(x)}{dx^2} = (18 - 6x)(-2) + (18 - 2x)(-6)$

$\Rightarrow \frac{d^2V(x)}{dx^2} = -12[3 - x + 9 - x] = -24(6 - x)$

$\Rightarrow \left.\frac{d^2V(x)}{dx^2}\right]_{x=3} = -72 < 0$

$\Rightarrow$ volume is maximum at x = 3

OR

$V(x) = x(18 - 2x)^2$

When x = 3

$V(3) = 3(18 - 2 \times 3)^2$

$\Rightarrow$ Volume = $3 \times 12 \times 12 = 432\ cm^3$