Class XII Session 2024-25
Subject - Mathematics
Sample Question Paper - 7

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If matrices A and B anticommute then **[1]**

a) $(AB) = (BA)^{-1}$ b) AB = BA

c) $(AB)^{-1} = (BA)$ d) AB = -BA

2. If A is skew symmetric matrix of order 3, then the value of |A| is: **[1]**

a) 9 b) 3

c) 0 d) 27

3. The adjoint of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is **[1]**

a) $\begin{bmatrix} 4 & -2 \\ -3 & -1 \end{bmatrix}$ b) $\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 4 & -2 \\ 1 & -3 \end{bmatrix}$

4. Let g(x) = $\begin{cases} e^{2x}, & x < 0 \\ e^{-2x}, & x \geq 0 \end{cases}$ then g(x) does not satisfy the condition **[1]**

a) differentiable at x = 0 b) continuous $\forall$ x $\in$ R

c) continuous $\forall$ x $\in$ R and non differentiable at x = ± 1 d) not differentiable at x = 0

5. The straight line $\frac{x-3}{3} = \frac{y-2}{1} = \frac{z-1}{0}$ is **[1]**

a) perpendicular to z-axis b) parallel to z-axis

c) parallel to y-axis d) parallel to x-axis

6. The solution of the differential equation $\frac{dy}{dx}+\frac{2xy}{1+x^2}=\frac{1}{(1+x^2)^2}$ is: **[1]**

a) $\frac{y}{1+x^2}$ = c + $\tan^{-1}$ x

b) y (1 + x^2) = c + $\sin^{-1}$ x

c) y(1 + x^2) = c + $\tan^{-1}$ x

d) y log (1 + x^2) = c + $\tan^{-1}$ x

7. The feasible region for an LPP is always a **[1]**

a) convex polygon

b) Straight line

c) concave polygon

d) type of polygon

8. Let θ = $\sin^{-1}$ (sin (– 600°)), then value of θ is **[1]**

a) $\frac{-2\pi}{3}$

b) $\frac{2\pi}{3}$

c) $\frac{\pi}{2}$

d) $\frac{\pi}{3}$

9. $\int \frac{1}{x\sqrt{x^4-1}}dx=?$ **[1]**

a) cosec^{-1} x^2 + C

b) $\frac{1}{2}\sec^{-1}x^2+C$

c) $\sec^{-1}$ x^2 + C

d) 2cosec^{-1} x^2 + C

10. The number of all possible matrices of order 2 $\times$ 3 with each entry 1 or 2 is **[1]**

a) 24

b) 64

c) 6

d) 16

11. The point which does not lie in the half plane 2x + 3y - 12 $\leq$ 0 is **[1]**

a) (2,1)

b) (-3, 2)

c) (1, 2)

d) (2, 3)

12. The vector with initial point P (2, -3, 5) and terminal point Q(3, -4, 7) is **[1]**

a) $-\hat{i}+\hat{j}-2\hat{k}$

b) $\hat{i}-\hat{j}+2\hat{k}$

c) $5\hat{i}-7\hat{j}+12\hat{k}$

d) $5\hat{i}-7\hat{j}-12\hat{k}$

13. If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix}+3=0$, then the value of x is **[1]**

a) 1

b) 0

c) -1

d) 3

14. The probabilities of A, B and C of solving a problem are $\frac{1}{6}, \frac{1}{5}$ and $\frac{1}{3}$ respectively. What is the probability that the problem is solved? **[1]**

a) $\frac{5}{9}$

b) $\frac{4}{9}$

c) $\frac{1}{3}$

d) $\frac{1}{7}$

15. The integrating factor of differential equation $\cos x\frac{dy}{dx}$ + y sin x = 1 is **[1]**

a) sin x

b) sec x

c) tan x

d) cos x

16. Find $\lambda \; and \; \mu \; if \; \left(2\hat{i}+6\hat{j}+27\hat{k}\right)\times\left(\hat{i}+\lambda\hat{j}+\mu\hat{k}\right)=\vec{0}$ **[1]**

a) 5, $\frac{27}{2}$

b) 3, $\frac{27}{2}$

c) 3, $\frac{27}{5}$

d) 4, $\frac{27}{2}$

17. The derivative of $\sin^2 x$ w.r.t. $e^{\cos x}$ is **[1]**

a) $\frac{2}{e^{\cos x}}$

b) $\frac{2\cos x}{e^{\cos x}}$

c) $-\frac{2\cos x}{e^{\cos x}}$

d) $\frac{e^{\cos x}}{-2}$

18. If a vector makes an angle of $\frac{\pi}{4}$ with the positive directions of both x-axis and y-axis, then the angle which it makes with positive z-axis is: **[1]**

a) 0

b) $\frac{\pi}{4}$

c) $\frac{3\pi}{4}$

d) $\frac{\pi}{2}$

19. **Assertion (A):** Minimum value of (x - 5)(x -7) is -1. **[1]**

Reason (R): Minimum value of $ax^2 + bx + c$ is $\frac{4ac-b^2}{4a}$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** A function f: N $\rightarrow$ N be defined by $f(n) = \begin{cases} \frac{n}{2} & \text{if } n \text{ is even} \\ \frac{(n+1)}{2} & \text{if } n \text{ is odd} \end{cases}$ for all n $\in$ N; is one-one. **[1]**

Reason (R): A function f: A $\rightarrow$ B is said to be injective if a $\neq$ b then f(a) $\neq$ f(b).

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$. **[2]**

OR

$\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

22. Find the least value of a such that the function f given by f(x) = x^2 + ax + 1 is strictly increasing on (1, 2). **[2]**

23. Find the absolute maximum value and the absolute minimum value of the function: **[2]**

$f(x) = 4x - \frac{1}{2}x^2, x \in \left[-2, \frac{9}{2}\right]$

OR

Find the point on the curve y^2 = 8x + 3 for which the y-coordinate change 8 times more than coordinate of x.

24. Evaluate: $\int \sec^{\frac{4}{3}} x \csc^{\frac{8}{3}} x dx$ **[2]**

25. Evaluate the determinant $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$ **[2]**

Section C

26. Find $\int \frac{\cos\theta}{(4+\sin^2\theta)(5-4\cos^2\theta)} d\theta$. **[3]**

27. Three groups of children contain 3 girls and 1 boy; 2 girls and 2 boys; 1 girl and 3 boys respectively. One child is selected at random from each group. Find the chance that the three selected comprise one girl and 2 boys. **[3]**

28. Evaluate: $\int_0^{\pi} \frac{1}{5+4\cos x} dx$ **[3]**

OR

Find $\int \frac{x^2-3x+1}{\sqrt{1-x^2}} dx$

29. Find the general solution of the differential equation x ($y^3 + x^3$) dy = ($2y^4 + 5x^3y$) dx. **[3]**

OR

Find a particular solution of x$\frac{dy}{dx}$ - y = log x, given that y = 0 when x = 1.

30. If $\vec{a} = \hat{i} + j + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, then find a vector $\vec{c}$, such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$. **[3]**

OR

If $\vec{a} = 3\hat{i} - \hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$, then express $\vec{b}$ in the form $\vec{b} = \vec{b}_1 + \vec{b}_2$, where $\vec{b}_1 \| \vec{a}$ and $\vec{b}_2 \perp \vec{a}$.

31. If x= a(cos t + t sin t) and y = a (sin t - t cos t), then find $\frac{d^2x}{dt^2}$, $\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$. **[3]**

Section D

32. Prove that the curves y^2 = 4x and x^2 = 4y divide the area of the square bounded by sides x = 0, x = 4, y = 4 and y = 0 into three equal parts. **[5]**

33. Let R be a relation on $N \times N$, defined by (a, b) R (c, d) ⇔ a + d = b + c for all (a, b), (c, d) $\in N \times N$. Show that R is an equivalence relation. **[5]**

OR

Let A = [-1, 1]. Then, discuss whether the following functions defined on A are one-one, onto or bijective:

i. $f(x) = \frac{x}{2}$

ii. g(x) = |x|

iii. h(x) = x|x|

iv. k(x) = x^2

34. If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$, Prove I + A = (I - A)$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ **[5]**

35. The sum of the surface areas of a cuboid with sides x, 2x and $\frac{x}{3}$ and a sphere is given to be constant. Prove that the sum of their volumes is minimum, if x is equal of three times the radius of sphere. Also, find the minimum value of the sum of their volumes. **[5]**

OR

Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$ Also find the maximum volume.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

Akash and Prakash appeared for first round of an interview for two vacancies. The probability of Nisha’s selection is $\frac{1}{3}$ and that of Ayushi's selection is $\frac{1}{2}$.

i. Find the probability that both of them are selected. (1)

ii. The probability that none of them is selected. (1)

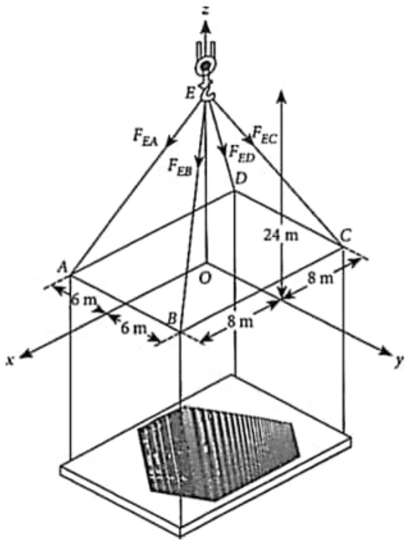
iii. Find the probability that only one of them is selected.(2)

OR

Find the probability that atleast one of them is selected. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Consider the following diagram, where the forces in the cable are given.



i. What is the cartesian equation of line along EA? (1)

ii. The vector $\overrightarrow{ED}$ is (1)

iii. The length of the cable EB is (2)

OR

What is the result of adding up all the vectors along the cables? (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Dinesh is having a jewelry shop at Green Park, normally he does not sit on the shop as he remains busy in political meetings. The manager Lisa takes care of jewelry shop where she sells earrings and necklaces. She gains profit of ₹30 on pair of earrings & ₹40 on neckless. It takes 30 minutes to make a pair of earrings and 1 hour to make a necklace, and there are 10 hours a week to make jewelry. In addition, there are only enough materials to make 15 total of jewelry items per week.

Solution

i. Formulate the above information mathematically. (1)

ii. Graphically represent the given data. (1)

iii. To obtain maximum profit how many pair of earing and neckleses should be sold? (2)

OR

What would be the profit if 5 pairs of earrings and 5 necklaces are made? (2)

Soluion

Section A

1.

(d) AB = -BA

Explanation: If A and B anticommute then AB = -BA

2.

(c) 0

Explanation: Determinant value of skew-symmetric matrix is always '0'.

3.

(b) $\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

Explanation: Let A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then |A| = $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$.

Now, cofactors of elements of |A| are

$C_{11} = (-1)^{1+1} 4 = 4$,

$C_{12} = (-1)^{1+2} (3) = -3$,

$C_{21} = (-1)^{2+1} (2) = -2$

and $C_{22} = (-1)^{2+2} (1) = 1$

Now, adj (A) = $\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

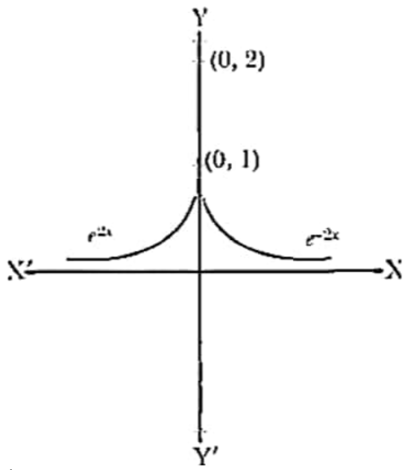
$= \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}^T$

$= \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

4.

(c) continuous $\forall$ x $\in$ R and non differentiable at x = ± 1

Explanation:



Given g(x) = $\begin{cases} e^{2x}, & x < 0 \\ e^{-2x}, & x \geq 0 \end{cases}$

g'(x) = $\begin{cases} 2e^{2x}, & x < 0 \\ -2e^{-2x}, & x \geq 0 \end{cases}$

$\therefore$ LHD at x = 0, g'(0) = $2e^{2 \times 0} = 2e^0 = 2$

RHD at x = 0, g'(0) = $-2e^0 = -2 \times 1 = -2$

As LHD $\neq$ RHD at x = 0

$\therefore$ g(x) is not differentiable at x = 0

Again RHL = $\lim_{x\to 0^+}$ g(x) = $\lim_{x\to 0^+}$ $e^{-2x} = e^0 = 1$

LHL = $\lim_{x\to 0^-}$ g(x) = $\lim_{x\to 0^-}$ $e^{2x} = e^0 = 1$

$g(0) = e^0 = 1$

As LHL = RHL = f(0)

∴ g(x) is continuous ∀ x ∈ R

5. **(a)** perpendicular to z-axis

Explanation: We have,

$\frac{x-3}{3} = \frac{y-2}{1} = \frac{z-1}{0}$

Also, the given line is parallel to the vector $\vec{b} = 3\hat{i} + \hat{j} + 0\hat{k}$

Let $x\hat{i} + y\hat{j} + z\hat{k}$ be perpendicular to the given line.

Now,

3x + 4y + 0z = 0

It is satisfied by the coordinates of z-axis, i.e. (0, 0, 1)

Hence, the given line is perpendicular to z-axis.

6.

(c) $y(1 + x^2) = c + \tan^{-1} x$

Explanation: We have, $\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{\left(1+x^2\right)^2}$

Which is linear differential equation.

Here, P = $\frac{2x}{1+x^2}$ and Q = $\frac{1}{\left(1+x^2\right)^2}$

∴ I.F. = $e^{\int \frac{2x}{1+x^2}dx} = e^{\log\left(1+x^2\right)} = 1 + x^2$

∴ the general solution is

$y\left(1+x^2\right) = \int \left(1+x^2\right) \frac{1}{\left(1+x^2\right)^2} + C$

$\Rightarrow y\left(1+x^2\right) = \int \frac{1}{1+x^2}dx + C$

$\Rightarrow y(1 + x^2) = \tan^{-1} x + C$

7. **(a)** convex polygon

Explanation: Feasible region for an LPP is always a convex polygon.

8.

(d) $\frac{\pi}{3}$

Explanation: $\sin^{-1} \sin\left(-600 \times \frac{\pi}{180}\right) = \sin^{-1} \sin\left(\frac{-10\pi}{3}\right)$

= $\sin^{-1}\left[-\sin\left(4\pi - \frac{2\pi}{3}\right)\right] = \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

= $\sin^{-1}\left(\sin\left(\pi - \frac{\pi}{3}\right)\right) = \sin^{-1}\left(\sin\frac{\pi}{3}\right) = \frac{\pi}{3}$

9.

(b) $\frac{1}{2}\sec^{-1} x^2 + C$

Explanation: Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c; \int \frac{1}{t\sqrt{t^2-1}}dt = \sec^{-1} t + c$

Therefore,

Put $x^2 = t$

$\Rightarrow$ 2 x dx = dt

$= \int \frac{1}{x\sqrt{t^2-1}} \times \frac{dt}{2x} \Rightarrow \frac{1}{2}\int \frac{1}{t\sqrt{t^2-1}}dt$

$= \frac{1}{2}\sec^{-1}t + c$

= $\frac{1}{2}\sec^{-1}x^2 + c$

10.

(b) 64

Explanation: The order of the matrix = 2 × 3

The number of elements = 2 × 3 = 6

Each place can have either 1 or 2. So, each place can be filled in 2 ways.

Thus, the number of possible matrices = 2^6 = 64

11.

(d) (2, 3)

Explanation: Since (2, 3) does not satisfy 2x + 3y - 12 $\leq$ 0 as 2 $\times$ 2 + 3 $\times$ 3 - 12 = 4 + 9 - 12 = 1 $\neq$ 0

12.

(b) $\hat{i}$-$\hat{j}$+2$\hat{k}$

Explanation: To find the vector we need to find the $\overrightarrow{PQ}$

=$3\hat{i} - 4\hat{j} + 7\hat{k} - (2\hat{i} + 3\hat{j} - 5\hat{k})$.

Hence, the vector formed by above points is with the following (1,-1,2).

13.

(c) -1

Explanation: -1

14. **(a)** $\frac{5}{9}$

Explanation: The probability that the problem is solved = $P(A \cup B \cup C)$ = P(A) + P(B) + P(C) - P(A∩B) - P(B∩C) - P(C ∩A) + 3P(A∩B∩C)

Considering independent events, P(A∩B) =P(A).P(B),

P(BC) = P(B).P(C), P(C∩A) = P(C).P(A),

P(A∩B∩C) =P(A).P(B).P(C),

Thus, P(A∪B∪C) is,

$\Rightarrow \frac{1}{6} + \frac{1}{5} + \frac{1}{3} - \frac{1}{30} - \frac{1}{15} - \frac{1}{18} + 3\left(\frac{1}{90}\right) = \frac{5}{9}$

15.

(b) sec x

Explanation: Given that,

cos x$\frac{dy}{dx}$ + y sin x = 1

$\Rightarrow \frac{dy}{dx}$ + y tan x = sec x

Here, P = tan x and Q = sec x

IF = $e^{\int Pdx}$

= $e^{\int \tan x dx}$

= $e^{\ln \sec x}$

$\therefore$ IF = sec x

16.

(b) 3, $\frac{27}{2}$

Explanation: It is given that:

$\left(2\hat{i} + 6\hat{j} + 27\hat{k}\right) X \left(\hat{i} + \lambda\hat{j} + \mu\hat{k}\right) = \overrightarrow{0}$

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{vmatrix} = \hat{i}\,(6\mu - 27\lambda) - \hat{j}\,(2\mu - 27) + \hat{k}\,(2\lambda - 6) = \overrightarrow{0}$, equating the coefficients of $\hat{i}, \hat{j}, \hat{k}$on both sides,we get

$(6\mu - 27\lambda) = 0, (2\mu - 27) = 0, (2\lambda - 6) = 0.$

solving,we get $\lambda = 3, \mu = \frac{27}{2}$

17.

(c) $-\frac{2\cos x}{e^{\cos x}}$

Explanation: Let u(x) = \sin^2 x and v(x) = $e^{\cos x}$.

We want to find $\frac{du}{dv}$ = $\frac{\frac{du}{dx}}{\frac{dv}{dx}}$.

Clearly, $\frac{du}{dx}$ = 2sin x cos x and $\frac{dv}{dx}$ = $e^{\cos x}$(-sin x) = -(sin x) $e^{\cos x}$

$\frac{du}{dv} = \frac{2\sin x \cos x}{-\sin x e^{\cos x}} = -\frac{2\cos x}{e^{\cos x}}$

18.

(d) $\frac{\pi}{2}$

Explanation: $\frac{\pi}{2}$

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: We have, (x - 5)(x - 7)

$\Rightarrow x^2 - 12x + 35$

We know that, $ax^2 + bx + c$ has minimum value $\frac{4ac-b^2}{4a}$.

Here, a = 1, b = -12 and c = 35

$\therefore$ Minimum value of (x - 5)(x - 7) $= \frac{4.1 \cdot 35-(-12)^2}{4.1}$

$= \frac{140-144}{4}$

$= -\frac{4}{4} = -1$

20.

(d) A is false but R is true.

Explanation: Assertion is false because distinct elements in N has equal images.

for example f(1) = $\frac{(1+1)}{2}$ = 1

f(2) = $\frac{2}{2}$ = 1

Reason is true because for injective function if elements are not equal then their images should be unequal.

Section B

21. We have, $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$.

$= \tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cot^{-1}\left(\cot\frac{\pi}{3}\right) + \tan^{-1}(-1)$.

$= \tan^{-1}\left[\tan\left(\pi - \frac{\pi}{6}\right)\right] + \cot^{-1}\left[\cot\left(\frac{\pi}{3}\right)\right] + tan^{-1}\left[\tan\left(\pi - \frac{\pi}{4}\right)\right]$

$= \tan^{-1}\left(-\tan\frac{\pi}{6}\right) + \cot^{-1}\left(\cot\frac{\pi}{3}\right) + \tan^{-1}\left(-\tan\frac{\pi}{4}\right) \begin{bmatrix} \because \tan^{-1}(\tan x) = x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \\ \cot^{-1}(\cot x) = x, x \in (0,\ \pi) \\ and \tan^{-1}(-x) = -\tan^{-1}x \end{bmatrix}$

$= -\frac{\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4} = \frac{-2\pi+4\pi-3\pi}{12}$

$= \frac{-5\pi+4\pi}{12} = \frac{-\pi}{12}$

OR

Let $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = y$

$\Rightarrow \cos y = -\frac{1}{\sqrt{2}}$

$\Rightarrow \cos y = -\cos\frac{\pi}{4}$

$\Rightarrow \cos y = \cos\left(\pi - \frac{\pi}{4}\right) = \cos\frac{3\pi}{4}$

Since, the principal value branch of $\cos^{-1}$ is $[0, \pi]$.

Therefore, Principal value of $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$ is $\frac{3\pi}{4}$.

22. $f(x) = x^2 + ax + 1$

$\Rightarrow$ f'(x) = 2x + a

Since f(x) is strictly increasing on (1, 2), therefore f'(x) = 2x + a > 0 for all x in (1, 2)

$\therefore$ On (1, 2) 1 < x < 2

$\Rightarrow$ 2 < 2x < 4

$\Rightarrow$ 2 + a < 2x + a < 4 + a

$\therefore$ Minimum value of f' (x) is 2 + a and maximum value is 4 + a.

Since f'(x) > 0 for all x in (1, 2)

$\therefore$ 2 + a > 0 and 4 + a > 0

$\Rightarrow$ a > -2 and a > -4

Therefore least value of a is - 2.

Which is the required solution.

23. Given that $f(x) = 4x - \frac{1}{2}x^2, x \in \left[-2, \frac{9}{2}\right]$

$\Rightarrow f'(x) = 4 - \frac{1}{2}(2x) = 4 - x$

Now, f'(x) = 0

⇒ x = 4

Now, we evaluate the value of f at critical point x = 0 and at end points of the interval $\left[-2, \frac{9}{2}\right]$

$f(4) = 16 - \frac{1}{2}(16) = 16 - 8 = 8$

$f(-2) = -8 - \frac{1}{2}(4) = -8 - 2 = -10$

f $\left(\frac{9}{2}\right) = 4\left(\frac{9}{2}\right) - \frac{1}{2}\left(\frac{9}{2}\right)^2 = 18 - \frac{81}{8} = 18 - 10.125 = 7.875$

Therefore, the absolute maximum value of f on $\left[-2, \frac{9}{2}\right]$ is 8 occurring at x = 4

And, the absolute minimum value of f on $\left[-2, \frac{9}{2}\right]$ is -10 occurring at x = -2

OR

y^2 = 8x + 3 ...(i) (given)

$\therefore 2y\frac{dy}{dt} = 8\frac{dx}{dt}$

$\frac{dy}{dt} = 8\frac{dx}{dt}$ ...(ii) (given)

$\therefore 2y \cdot 8\frac{dx}{dt} = 8\frac{dx}{dt}$

$\Rightarrow y = \frac{8}{16} = \frac{1}{2}$

For $y = \frac{1}{2}$

From eq (i), $\left(\frac{1}{2}\right)^2 = 8x + 3$

or, $\frac{1}{4} - 3 = 8x$

or, $x = -\frac{11}{32}$

Hence, required point is $\left(-\frac{11}{32}, \frac{1}{2}\right)$.

24. Let I = $\int \sec^{\frac{4}{3}} x \csc^{\frac{8}{3}} x dx$. Then, we have

I = $\int \frac{1}{\cos^{4/3} x \sin^{8/3} x} dx$ = $\int \cos^{\frac{-4}{3}} x \sin^{\frac{-8}{3}} x dx$

since - $\left(\frac{4}{3} + \frac{8}{3}\right)$ = - 4, which is an even integer. So, we divide both numerator and denominator by $\cos^4$x.

$\therefore \quad I = \int \frac{\sec^4 x}{\tan^{8/3} x} dx = \int \frac{(1+\tan^2 x)}{\tan^{8/3} x} \sec^2 x dx$

Put tan x = t and $\sec^2$ = dt, we get

I = $\int \frac{1+t^2}{t^{\frac{8}{3}}} dt = \int \left(t^{\frac{-8}{3}} + t^{\frac{-2}{3}}\right) dt = -\frac{3}{5} t^{\frac{-5}{3}} + 3t^{\frac{1}{3}} + c$

$\Rightarrow \quad I = -\frac{3}{5} \tan^{\frac{-5}{3}} x + 3\tan^{\frac{1}{3}} x + C$

25. Let A = $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$

⇒ |A| = (a + ib)(a - ib) - (c + id)(-c + id)

= (a + ib)(a - ib) + (c + id)(c - id)

= $a^2 - i^2 b^2 + c^2 - i^2 d^2$

= $a^2 - (-1)b^2 + c^2 - (-1)d^2$

= $a^2 + b^2 + c^2 + d^2$

Thus, |A| = $a^2 + b^2 + c^2 + d^2$

Section C

26. According to the question, $I = \int \frac{\cos\theta}{(4+\sin^2\theta)(5-4\cos^2\theta)]} d\theta$

$= \int \frac{\cos\theta}{(4+\sin^2\theta)[5-4(1-\sin^2\theta)]} d\theta \; [\therefore cos^2\theta = 1 - sin^2\theta]$

$= \int \frac{\cos\theta}{(4+\sin^2\theta)(5-4+4\sin^2\theta)} d\theta$

$= \int \frac{\cos\theta}{(4+\sin^2\theta)(1+4\sin^2\theta)} d\theta$

Let $\sin\theta = t \Rightarrow \cos\theta d\theta = dt$

Then, $I = \int \frac{dt}{(4+t^2)(1+4t^2)}$

let , $\frac{1}{(4+t^2)(1+4t^2)} = \frac{A}{4+t^2} + \frac{B}{1+4t^2}$

using partial fractions

At t = 0, $\frac{A}{4} + \frac{B}{1} = \frac{1}{4\times 1} \Rightarrow \quad A + 4B = 1$ ...(i)

At $t=1, \frac{A}{5}+\frac{B}{5}=\frac{1}{5\times5} \Rightarrow 5A+5B=1$ ...(ii)

On solving Equations (i) and (ii), we get

$A=\frac{-1}{15}\ and\ B=\frac{4}{15}$

$\frac{1}{(4+t^2)(1+4t^2)}=\frac{-\frac{1}{15}}{4+t^2}+\frac{\frac{4}{15}}{1+4t^2}$

$\Rightarrow \quad \frac{1}{(4+t^2)(+4t^2)}=\frac{-1}{15(4+t^2)}+\frac{4}{15(1+4t^2)}$

Integrating both sides w.r.t. t,

$\Rightarrow \int \frac{1}{(4+t^2)(1+4t^2)}dt=\frac{-1}{15}\int \frac{1}{4+t^2}dt+\frac{4}{15}\int \frac{1}{1+4t^2}dt$

$=\frac{-1}{15}\int \frac{1}{2^2+t^2}+\frac{4}{15\times4}\int \frac{1}{\left(\frac{1}{2}\right)^2+t^2}dt$

$=\frac{-1}{15}\cdot\frac{1}{2}\tan^{-1}\frac{t}{2}+\frac{1}{15}\cdot\frac{1}{1/2}\tan^{-1}\frac{t}{1/2}+C\ \left[\because \int \frac{dx}{x^2+a^2}=\frac{1}{a}\tan^{-1}\frac{x}{a}+c\right]$

put $t=\sin\theta$

$=\frac{-1}{30}\tan^{-1}\frac{\sin\theta}{2}+\frac{2}{15}\tan^{-1}2\sin\theta+C$

27. One girl and 2 boys can be selected in the following mutually exclusive ways:

	Group 1	**Group 2**	**Group 3**
(I)	Girl	Boy	Boy
(II)	Boy	Girl	Boy
(III)	Boy	Boy	Girl

Therefore, if we define G_1, G_2, G_3 as the events of selecting a girl from first, second and third group respectively and B_1, B_2, B_3 as the events of selecting a boy from first, second and third group respectively. Then B_1, B_2, B_3, G_1, G_2, G_3 are independent events such that

$P(G_1)=\frac{3}{4}, P(G_2)=\frac{2}{4}, P(G_3)=\frac{1}{4}$

$P(B_1)=\frac{1}{4}, P(B_2)=\frac{2}{4}, P(B_3)=\frac{3}{4}$

Therefore, required probability is given by,

P(Selecting 1 girl and 2 boys)

= (I or II or III)

= $P(\text{I}\cup\text{II}\cup\text{III})$

= $P[(G_1\cap B_2\cap B_3)\cup(B_1\cap G_2\cap B_3)\cup(B_1\cap B_2\cap G_3)]$

= $P(G_1\cap B_2\cap B_3)+P(B_1\cap G_2\cap B_3)+P(B_1\cap B_2\cap G_3)$

= $P(G_1)\ P(B_2)\ P(B_3) + P(B_1)\ P(B_2)\ P(G_3)$

= $\frac{3}{4}\times\frac{2}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{2}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{2}{4}\times\frac{1}{4}$ = $\frac{9}{32}+\frac{3}{32}+\frac{1}{32}=\frac{13}{32}$ $P(B_3) + P(B_1)P(G_2)$

28. Let $I=\int_0^\pi \frac{1}{5+4\cos x}dx$. Then

$I=\int_0^\pi \frac{1}{5+4\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}dx=\int_0^\pi \frac{1+\tan^2\frac{x}{2}}{5\left(1+\tan^2\frac{x}{2}\right)+4\left(1-\tan^2\frac{x}{2}\right)}dx$

$\Rightarrow I=\int_0^\pi \frac{1+\tan^2\frac{x}{2}}{9+\tan^2\frac{x}{2}}dx=\int_0^\pi \frac{\sec^2\frac{x}{2}}{9+\tan^2\frac{x}{2}}dx$

By using substitution

Let $\tan\frac{x}{2}=t.$ Then, $d\left(\tan\frac{x}{2}\right)=dt \Rightarrow \frac{1}{2}\sec^2\frac{x}{2}dx=dt \Rightarrow dx=\frac{2dt}{\sec^2\frac{x}{2}}$

Also, $x=0 \Rightarrow t=\tan 0=0$ and $x=\pi \Rightarrow t=\tan\frac{\pi}{2}=\infty$

$\therefore I=\int_0^\infty \frac{\sec^2\frac{x}{2}}{9+t^2}\times\frac{2dt}{\sec^2\frac{x}{2}}$

$\Rightarrow I=2\int_0^\infty \frac{dt}{3^2+t^2}=\frac{2}{3}\left[\tan^{-1}\frac{t}{3}\right]_0^\infty=\frac{2}{3}\left(\tan^{-1}\infty-\tan^{-1}0\right)=\frac{2}{3}\left(\frac{\pi}{2}-0\right)=\frac{\pi}{3}$

OR

Let $I=\int \frac{x^2-3x+1}{\sqrt{1-x^2}}$

$=(-1)\int \frac{-x^2+3x-1}{\sqrt{1-x^2}}dx$

$= (-1)\int \frac{-x^2+3x-1+1-1}{\sqrt{1-x^2}}dx$
$= (-1)\int \frac{1-x^2+3x-2}{\sqrt{1-x^2}}dx$
$= (-1)\int \left[\frac{1-x^2}{\sqrt{1-x^2}} + \frac{3x-2}{\sqrt{1-x^2}}\right]dx$
$= (-1)\int \left[\sqrt{1-x^2} + \int \frac{3x-2}{\sqrt{1-x^2}}\right]dx$
$= (-1)\left[\int \sqrt{1-x^2}dx + \int \frac{3x-2}{\sqrt{1-x^2}}dx\right]$
$= (-1)(I_1 + I_2)$ ....(i)
consider, $I_1 = \int \sqrt{1-x^2}dx$
$= \frac{1}{2}\left[x\sqrt{1-x^2} + \sin^{-1}(x)\right] + C_1$ ...(ii) $\left[\because \int \sqrt{a^2-x^2}dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\sin^{-1}\left(\frac{x}{a}\right)\right] + C\right]$
consider $I_2 = \int \frac{3x-2}{\sqrt{1-x^2}}dx$
$= \int \frac{3x}{\sqrt{1-x^2}}dx - 2\int \frac{dx}{\sqrt{1-x^2}}$
$= -\frac{3}{2}\int \frac{-2x}{\sqrt{1-x^2}}dx - 2\int \frac{dx}{\sqrt{1-x^2}}$
$= -\frac{3}{2} \times 2\sqrt{1-x^2} - 2\sin^{-1}(x) + C_2 \left[\because \int \frac{f'(x)}{\sqrt{f(x)}}dx = 2\sqrt{f(x)} + C\right]$
$= -3\sqrt{1-x^2} - 2\sin^{-1}(x) + C_2$ ...(iii)
From Equations (i), (ii) and (iii), we get
$I = (-1)\left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}(x) + C_1 - 2\sin^{-1}(x) - 3\sqrt{1-x^2} + C_2\right]$
$I = \frac{3}{2}\sin^{-1}(x) - \frac{x}{2}\sqrt{1-x^2} + 3\sqrt{1-x^2} + C$ $[\ where\ C = -C_1 - C_2\ ]$

29. $x(y^3 + x^3)\ dy = (2y^4 + 5x^3y)\ dx$
$\frac{dy}{dx} = \frac{2y^4+5x^3y}{xy^3+x^4}$
It is a homogeneous differential equation
So put y = vx, $\frac{dy}{dx} = v + x\frac{dv}{dx}$
$v + x\frac{dv}{dx} = \frac{2v^4+5v}{v^3+1}$
$x\frac{dv}{dx} = \frac{2v^4+5v}{v^3+1} - v$
$x\frac{dv}{dx} = \frac{2v^4+5v-v^4-v}{v^3+1}$
$\frac{v^3+1}{v^4+4v}dv = \frac{dx}{x}$
$\int \frac{4v^3+4}{v^4+4v}dv = 4\int \frac{dx}{x}$
$\log|v^4 + 4v| = 4\log x + \log C$
$\log|v^4 + 4v| = \log(x)^4 + \log C$
$\log|v^4 + 4v| = \log Cx^4$
$v^4 + 4v = Cx^4$
Put $v = \frac{y}{x}$
$\frac{y^4}{x^4} + 4\frac{y}{x} = Cx^4$
$y^4 + 4yx^3 = Cx^8$

OR

The given differential equation is,
$x\frac{dy}{dx} - y = \log x$
$\Rightarrow \frac{dy}{dx} + \left(-\frac{1}{x}\right)y = \frac{1}{x}\log x$
This is of the form $\frac{dy}{dx} + Py = Q$
Where, $P = -\frac{1}{x}, Q = \frac{1}{x}\cdot\log x$
Here $I.F = e^{\int\left(-\frac{1}{x}\right)dx} = e^{-\log x} = \frac{1}{x}$
$\therefore y\cdot(IF) = \int(IF)Q\ dx + C$
$\Rightarrow y\cdot\frac{1}{x} = \int \frac{1}{x}\cdot\frac{1}{x}\cdot\log x\ dx + C \ \Rightarrow \frac{y}{x} = \int \frac{1}{x^2}\cdot\log x\ dx + C$

$\Rightarrow \frac{y}{x} = \log x \int \frac{1}{x^2} dx - \int \left\{ \frac{d}{dx}(\log x) \int \frac{1}{x^2} dx \right\} dx + C$

$\Rightarrow \frac{y}{x} = -\frac{1}{x} \cdot \log x + \int \frac{1}{x} \cdot \frac{1}{x} dx + C$

$\Rightarrow \frac{y}{x} = -\frac{1}{x} \cdot \log x - \frac{1}{x} + C$ ...(i)

Putting x = 1 and y = 0,we get,

0 = -log 1 - 1 + C

C = 1

Putting C = 1 in equation (i) we have $\frac{y}{x} = -\frac{1}{x} \cdot \log x - \frac{1}{x} + 1$

⇒ y = x - 1 - log x

30. According to the question ,

Given vectors are , $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and

$\vec{b} = \hat{j} - \hat{k}$

Let $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$

Then ,$\vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix}$

$= \hat{i}(z-y) - \hat{j}(z-x) + \hat{k}(y-x)$

Given that $\vec{a} \times \vec{c} = \vec{b}$.

$\Rightarrow \hat{i}(z-y) + \hat{j}(x-z) + \hat{k}(y-x) = 0\hat{i} + 1\hat{j} + (-1)\hat{k}$ $[\because \vec{b} = \hat{j} - \hat{k}]$

On comparing the coefficients of i, j, and k. from both sides, we get

z - y = 0,

x - z = 1, and

y - x = -1

x - y = 1 ...(i)

Also given that , $\vec{a} \cdot \vec{c} = 3$

$\Rightarrow (\hat{i} + \hat{j} + \hat{k}) \cdot (x\hat{i} + y\hat{j} + z\hat{k}) = 3$

⇒x + y + z = 3

⇒ x + 2y = 3 [∵y = z] ...(ii)

On subtracting Eq. (i) from Eq. (ii), we get

$3y = 2$

$\Rightarrow y = \frac{2}{3} = z$ [∵ y = z]

From Eq. (i), $x = 1 + y = 1 + \frac{2}{3} = \frac{5}{3}$

$\therefore \vec{c} = \frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$

OR

According to the question,

$\vec{a} = 3\hat{i} - \hat{j}$ and

$\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$

Let $\overrightarrow{b_1} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ and

$\overrightarrow{b_2} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

$\overrightarrow{b_1} + \overrightarrow{b_2} = \vec{b}, \overrightarrow{b_1} \| \vec{a}$ and

$\overrightarrow{b_2} \perp \vec{a}$.

Consider, $\overrightarrow{b_1} + \overrightarrow{b_2} = \vec{b}$

$\Rightarrow \quad (x_1 + x_2)\hat{i} + (y_1 + y_2)\hat{j} + (z_1 + z_2)\hat{k} = 2\hat{i} + \hat{j} - 3\hat{k}$

On comparing the coefficient of $\hat{i}, \hat{j}$ and $\hat{k}$ both sides; we get

$\Rightarrow x_1 + x_2 = 2$ ...(i)

$y_1 + y_2 = 1$...(ii)

and $z_1 + z_2 = -3$ ...(iii)

Now, consider $\overrightarrow{b_1} \| \vec{a}$

$\Rightarrow \quad \frac{x_1}{3} = \frac{y_1}{-1} = \frac{z_1}{0} = \lambda(\text{say})$

$\Rightarrow \quad x_1 = 3\lambda, y_1 = -\lambda \text{ and } z_1 = 0$ ...(iv)

On substituting the values of x,y and z, form Eq. (iv) to Eq. (i), (ii) and (iii), respectively, we get

$x_2 = 2 - 3\lambda, y_2 = 1 + \lambda$ and $z_2 = -3$ ...(v)

Since, $\overrightarrow{b_2} \perp \vec{a}$, therefore $\vec{b}_2 \cdot \vec{a} = 0$

$\Rightarrow \quad 3x_2 - y_2 = 0$

$\Rightarrow \quad 3(2 - 3\lambda) - (1 + \lambda) = 0$

$\Rightarrow \quad 6 - 9\lambda - 1 - \lambda = 0$

$\Rightarrow \quad 5 - 10\lambda = 0 \Rightarrow \lambda = \frac{1}{2}$

On substituting $\lambda = \frac{1}{2}$ in Eqs. (iv) and Eqs. (iv) and (v), we get

$x_1 = \frac{3}{2}, y_1 = \frac{-1}{2}, z_1 = 0$

and $x_2 = \frac{1}{2}, y_2 = \frac{3}{2}$ and $z_2 = -3$

Hence, $\vec{b} = \vec{b}_1 + \vec{b}_2 = \left(\frac{3}{2}\hat{i} - \frac{1}{2}\hat{j}\right) + \left(\frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}\right)$

$= 2\hat{i} + \hat{j} - 3\hat{k}$

31. Given, x = a(cos t + t sin t)

On differentiating both sides w.r.t t, we get

$\frac{dx}{dt} = a\left[-\sin t + \frac{d}{dt}(t) \cdot \sin t + t\frac{d}{dt}(\sin t)\right]$ [by using product rule of derivative]

$\Rightarrow \frac{dx}{dt}$ = a(-sin t + 1. sin t + t cos t) = a t cos t.......(i)

Also, given, y = a(sin t - t cos t)

On differentiating both sides w.r.t t, we get

$\frac{dy}{dt} = a\left[\cos t - \frac{d}{dt}(t)\cos t - t\frac{d}{dt}(\cos t)\right]$ [by using product rule of derivative]

$\frac{dy}{dt}$ = a(cos t - cos t.1 + t sin t)

= a t sin t..............(ii)

Now, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{at\sin t}{at\cos t}$ = tan t [From Eqs.(i) and (ii)]

Again,differentiating both sides w.r.t x, we get

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}(\tan t)\frac{dt}{dx} = \sec^2 t\frac{1}{dx/dt}$

$= \frac{\sec^2 t}{at\cos t} = \frac{\sec^3 t}{at}$ [From Eq(i)]

Also, $\frac{d^2x}{dt^2} = \frac{d}{dt}(at\cos t)$

$= a\frac{d}{dt}(t\cos t)$

$= a\left[\frac{d}{dt}(t) \cdot \cos t + t\frac{d}{dt}(\cos t)\right]$ [by using product rule of derivative]

= a[cos t - sin t]

and $\frac{d^2y}{dt^2} = \frac{d}{dt}\left(\frac{dy}{dt}\right) = \frac{d}{dt}(at\sin t)$

= a(sin t + t cos t)

Section D

32. The given curves are $y^2 = 4x$ and $x^2 = 4y$

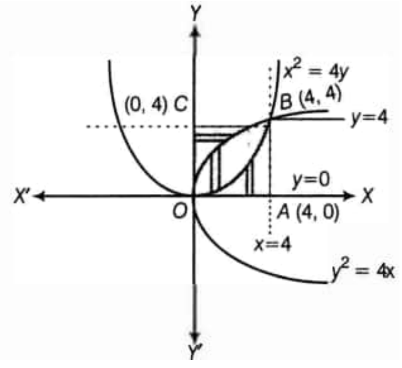
Let OABC be the square whose sides are represented by following equations

Equation of OA is y = 0

Equation of AB is x = 4

Equation of BC is y = 4

Equation of CO is x = 0



On solving equations $y^2 = 4x$ and $x^2 = 4y$, we get A(0, 0) and B(4, 4) as their points of intersection.

The Area bounded by these curves
$= \int_0^4 \left[y_{(\text{ parabola } y^2=4x)} - y_{(\text{ parabola } x^2=4y)}\right] dx$
$= \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right) dx$
$= \left[2 \cdot \frac{2}{3}x^{3/2} - \frac{x^3}{12}\right]_0^4$
$= \left[\frac{4}{3}x^{3/2} - \frac{x^3}{12}\right]_0^4$
$= \frac{4}{3} \cdot (4)^{3/2} - \frac{64}{12}$
$= \frac{4}{3} \cdot \left(2^2\right)^{3/2} - \frac{64}{12}$
$= \frac{4}{3} \cdot (2)^3 - \frac{64}{12}$
$= \frac{32}{3} - \frac{16}{3}$
$= \frac{16}{3}$ sq units

Hence, area bounded by curves $y^2 = 4x$ and $x = 4y$ is $\frac{16}{3}$sq units ......(i)

Area bounded by curve $x^2 = 4y$ and the lines x = 0, x = 4 and X-axis
$= \int_0^4 y_{(\text{ parabola } x^2=4y)} dx$
$= \int_0^4 \frac{x^2}{4} dx$
$= \left[\frac{x^3}{12}\right]_0^4$
$= \frac{64}{12}$
$=\frac{16}{3}$sq units ....(ii)

The area bounded by curve $y^2 = 4x$, the lies y = 0, y = 4 and Y-axis
$= \int_0^4 x_{(\text{ parabola } y^2=4x)} dy$
$= \int_0^4 \frac{y^2}{4} dy$
$= \left[\frac{y^3}{12}\right]_0^4$
$= \frac{64}{12}$
$= \frac{16}{3}$sq units ....(iii)

From Equations. (i), (ii) and (iii), area bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$ divides the area of square into three equal parts.

33. Here R is a relation on $N \times N$, defined by (a, b) R (c, d) ⇔ a + d = b + c for all (a, b), (c, d) ∈ $N \times N$

We shall show that R satisfies the following properties

i. Reflexivity:
We know that a + b = b + a for all a, b ∈ N.
∴ (a, b) R (a, b) for all (a, b) ∈ $(N \times N)$
So, R is reflexive.

ii. Symmetry:
Let (a, b) R (c, d). Then,
(a, b) R (c, d) ⇒ a + d = b + c
⇒ c + b = d + a
⇒ (c, d) R (a, b).
∴ (a, b) R (c, d) ⇒ (c, d) R (a, b) for all (a, b), (c, d) ∈ $N \times N$
This shows that R is symmetric.

iii. Transitivity:
Let (a, b) R (c, d) and (c, d) R (e, f). Then,
(a, b) R (c, d) and (c, d) R (e, f)
⇒ a + d = b + c and c + f = d + e
⇒ a + d + c + f = b + c + d + e
⇒ a + f = b + e
⇒ (a, b) R (e, f).
Thus, (a, b) R (c, d) and (c, d) R (e, f) ⇒ (a, b) R (e, f)
This shows that R is transitive.

$\therefore$ R is reflexive, symmetric and transitive

Hence, R is an equivalence relation on $N \times N$

OR

Given that A = [-1, 1]

i. $f(x) = \frac{x}{2}$

Let f(x_1) = f(x_2)

$\Rightarrow \frac{x_1}{2} = \frac{x_2}{2} \Rightarrow$ $x_1 = x_2$

So, f(x) is one-one.

Now, let $y = \frac{x}{2}$

$\Rightarrow x = 2y \notin A,\ \forall y \in A$

As for $y = 1 \in A,\ x = 2 \notin A$

So, f(x) is not onto.

Also, f(x) is not bijective as it is not onto.

ii. g(x) = |x|

Let g(x_1) = g(x_2)

$\Rightarrow |x_1| = |x_2| \Rightarrow x_1 = \pm x_2$

So, g(x) is not one-one.

Now, x=$\pm$y $\notin A$ for all y $\in$ R

So, g(x) is not onto, also, g(x) is not bijective.

iii. h(x) = x|x|

$\Rightarrow x_1|x_1| = x_2|x_2| \Rightarrow x_1 = x_2$

So, h(x) is one-one

Now, let y = x|x|

$\Rightarrow y = x^2 \in A, \forall x \in A$

So, h(x) is onto also, h(x) is a bijective.

iv. k(x) = x^2

Let k(x_1) = k(x_2)

$\Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$

Thus, k(x) is not one-one.

Now, let y = x^2

$\Rightarrow x\sqrt{y} \notin A, \forall y \in A\ x = \sqrt{y} \notin A, \forall y \in A$

As for y = -1, $x = \sqrt{-1} \notin A$

Hence, k(x) is neither one-one nor onto.

34. LHS = I + A

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix} = \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$$

$$\text{RHS} = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}\right)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha + \tan\frac{\alpha}{2}\sin\alpha & -\sin\alpha + \tan\frac{\alpha}{2}\cos\alpha \\ -\tan\frac{\alpha}{2}\cos\alpha + \sin\alpha & \tan\frac{\alpha}{2}\sin\alpha + \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha + \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}\sin\alpha & -\sin\alpha + \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}\cos\alpha \\ -\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}\cos\alpha + \sin\alpha & \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}\sin\alpha + \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha + \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}\sin\alpha & -\sin\alpha + \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}\cos\alpha \\ -\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}\cos\alpha + \sin\alpha & \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}\sin\alpha + \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\cos\alpha\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}\sin\alpha}{\cos\frac{\alpha}{2}} & \frac{-\sin\alpha\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}\cos\alpha}{\cos\frac{\alpha}{2}} \\ \frac{-\cos\alpha\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}\sin\alpha}{\cos\frac{\alpha}{2}} & \frac{\sin\alpha\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}\cos\alpha}{\cos\frac{\alpha}{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\cos\left(\alpha-\frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} & \frac{-\sin\left(\alpha-\frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} \\ \frac{\sin\left(\alpha-\frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} & \frac{\cos\left(\alpha-\frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} \end{bmatrix} = \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix} = LHS$$

35. Let r be the radius of the sphere and dimensions of cuboid are x, 2x and $\frac{x}{3}$.

$\therefore 4\pi r^2 + 2\left[\frac{x}{3}\times x + x\times 2x + 2x\times\frac{x}{3}\right]$ =k (constant) [given]

$\Rightarrow 4\pi r^2 + 6x^2 = k$

$\Rightarrow \quad r^2 = \frac{k-6x^2}{4\pi} \Rightarrow r = \sqrt{\frac{k-6x^2}{4\pi}}$ ......(i)

Sum of the volumes, $V = \frac{4}{3}\pi r^3 + \frac{x}{3}\times x\times 2x$

$= \frac{4\pi r^3}{3} + \frac{2}{3}x^3$ ....(ii)

$\Rightarrow \quad V = \frac{4}{3}\pi\left(\frac{k-6x^2}{4\pi}\right)^{\frac{3}{2}} + \frac{2}{3}x^3$

On differentiating both sides w.r.t. x, we get

$\frac{dV}{dx} = \frac{4}{3}\pi\times\frac{3}{2}\left(\frac{k-6x^2}{4\pi}\right)^{\frac{1}{2}}\left(\frac{-12x}{4\pi}\right) + \frac{2}{3}\times 3x^2$

$= 2\pi\sqrt{\frac{k-6x^2}{4\pi}}\left(\frac{-3x}{\pi}\right) + 2x^2$

$= (-6x)\sqrt{\frac{k-6x^2}{4\pi}} + 2x^2$

For maxima or minima, put $\frac{dV}{dx} = 0$

$\Rightarrow \quad (-6x)\sqrt{\frac{k-6x^2}{4\pi}} + 2x^2 = 0$

$\Rightarrow \quad 2x^2 = 6x\sqrt{\frac{k-6x^2}{4\pi}}$

$\Rightarrow \quad x = 3\sqrt{\frac{k-6x^2}{4\pi}}$

$\Rightarrow$ x=3r

[using Eq. (i)]

Again, on differentiating $\frac{dV}{dx}$w.r.t. x, we get

$\frac{d^2V}{dx^2} = -6\frac{d}{dx}\left(x\sqrt{\frac{k-6x^2}{4\pi}}\right) + 4x$

$= -6\left(\sqrt{\frac{k-6x^2}{4\pi}} + x\cdot\frac{1}{2} + \frac{1}{\sqrt{\frac{k-6x^2}{4}}}\left(\frac{-12x}{4\pi}\right)\right) + 4x$

$= -6\left(r - \frac{3x^2}{2\pi r}\right) + 4x$

$= -6r + \frac{9x^2}{\pi r} + 4x$

Now, $\left(\frac{d^2r}{dx^2}\right)_{x=3r} = -6r + \frac{9\times 9r^2}{\pi} + 12r = 6r + \frac{18r}{\pi} > 0$

Hence, V is minimum when x is equal to three times the radius of the sphere.

Hence proved.

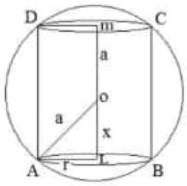
Now, on putting $r = \frac{x}{3}$ in Eq. (ii), we get

$V_{\min} = \frac{4\pi}{3}\left(\frac{x}{3}\right)^3 + \frac{2}{3}x^3 = \frac{4\pi}{81}x^3 + \frac{2}{3}x^3$

$= \frac{2}{3}x^2\left(\frac{2\pi}{27} + 1\right) = \frac{2}{3}x^3\left(\frac{44}{189} + 1\right)$

$= \frac{2}{3}x^3\left(\frac{233}{189}\right) = \frac{466}{567}x^3$

OR



$V = \pi r^2 .2x \; [\because OL = x, LM = 2x]$
$= \pi .\,(a^2 - x^2).2x$
$V = 2\pi(a^2 x - x^3)$
$\frac{dv}{dx} = 2\pi(a^2 - 3x^2)$
$\frac{d^2v}{dx^2} = 2\pi\,[0 - 6x]$
$= -12\pi x$
For maximum/minimum
$\frac{dv}{dx} = 0$
$2\pi[a^2 - 3x^2] = 0$
$a^2 = 3x^2 \Rightarrow \sqrt{\frac{a^2}{3}} = x$
$\Rightarrow x = \frac{a}{\sqrt{3}}$
$\left.\frac{d^2v}{dx^2}\right]_{x=\frac{a}{\sqrt{3}}} = -12\pi .\,\frac{a}{\sqrt{3}}$
= negative maximum
Volume is maximum at $x = \frac{a}{\sqrt{3}}$
Height of cylinder of maximum volume is
= 2x
$= 2 \times \frac{a}{\sqrt{3}}$
$= \frac{2a}{\sqrt{3}}$

Section E

36. i. P(A) = $\frac{1}{3}$, P(A') = $1 - \frac{1}{3} = \frac{2}{3}$
P(B) = $\frac{1}{2}$, P(b') = $1 - \frac{1}{3} = \frac{1}{2}$
P(Both are selected) = $P(A \cap B) = P(A) \cdot P(B)$ = $\frac{1}{3} . \frac{1}{2}$
P(Both are selected) = $\frac{1}{6}$
ii. P(A) = $\frac{1}{3}$, P(A') = $1 - \frac{1}{3} = \frac{2}{3}$
P(B) = $\frac{1}{2}$, P(b') = $1 - \frac{1}{3} = \frac{1}{2}$
P(none of them selected) = $P(A' \cap B') = P(A') \cdot P(B')$ = $\frac{2}{3} . \frac{1}{2}$
P(Both are selected) = $\frac{1}{3}$
iii. P(A) = $\frac{1}{3}$, P(A') = $1 - \frac{1}{3} = \frac{2}{3}$
P(B) = $\frac{1}{2}$, P(b') = $1 - \frac{1}{3} = \frac{1}{2}$
P(none of them selected) = $P(A') \cdot P(B) + P(A) \cdot P(B')$ = $\frac{2}{3} . \frac{1}{2} + \frac{1}{3} . \frac{1}{2}$
P(Both are selected) = $\frac{3}{6} = \frac{1}{2}$
OR
P(A) = $\frac{1}{3}$, P(A') = $1 - \frac{1}{3} = \frac{2}{3}$
P(B) = $\frac{1}{2}$, P(b') = $1 - \frac{1}{3} = \frac{1}{2}$
P(atleast one of them selected) = 1 - P(none selected) = $1 - \frac{1}{3}$
P(atleast one of them selected) = $\frac{2}{3}$

37. i. Clearly, the coordinates of A are (8, -6, 0) and that of E are (0, 0, 24).
Also, cartesian equation of line along EA is given by
$\frac{x-0}{8-0} = \frac{y-0}{-6-0} = \frac{z-24}{0-24}$
$\Rightarrow \frac{x}{8} = \frac{y}{-6} = \frac{z-24}{-24} \Rightarrow \frac{x}{-4} = \frac{y}{3} = \frac{z-24}{12}$
ii. Clearly, the coordinates of D are (-8, -6, 0) and that of E are (0, 0, 24)
$\therefore$ Vector $\overrightarrow{ED}$ is (-8 - 0)$\hat{i}$ + (-6 - 0)$\hat{j}$ + (0 - 24)$\hat{k}$, i.e., -8$\hat{i}$ - 6$\hat{j}$ - 24$\hat{k}$.

iii. Since, the coordinates of B are (8, 6, 0) and that of E are (0, 0, 24), therefore length of cable

EB = $\sqrt{(8-0)^2+(6-0)^2+(0-24)^2}$

$=\sqrt{64+36+576}=\sqrt{676}$ = 26 units

OR

Sum of all vectors along the cables

$=\overrightarrow{EA}+\overrightarrow{EB}+\overrightarrow{EC}+\overrightarrow{ED}$

= $(8\hat{i} - 6\hat{j} - 24\hat{k}) + (8\hat{i} + 6\hat{j} - 24\hat{k}) + (-8\hat{i} + 6\hat{j} - 24\hat{k}) + (-8\hat{i} - 6\hat{j} - 24\hat{k})$

= $-96\hat{k}$

38. i. Let number of pairs of earing = x and number of Necklaces = y

As per the given information

x, y $\geq$ 0

0.5x + y $\leq$ 10

x + y $\leq$ 15

Profit function = Z = 30x + 40y

ii. Let number of pairs of earing = x and number of Necklaces = y

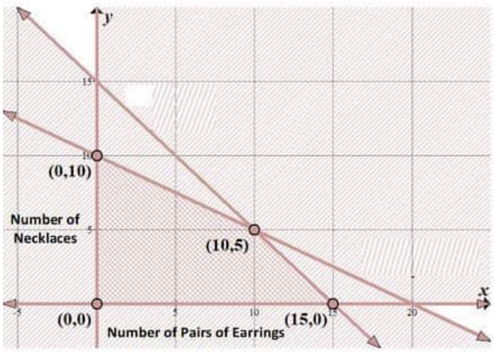
As per the given information

x, y $\geq$ 0

0.5x + y $\leq$ 10

x + y $\leq$ 15

Profit function = Z = 30x + 40y



iii. From graph corner points are (0, 0), (0, 10), (10, 5) and (15, 0).

corner points	maximum profit = Z = 30x + 40y
(0, 0)	Z = 0
(0, 10)	Z = ₹400
(10, 5)	**Z = ₹500**
(15, 0)	Z = ₹450

Hence profit is maximum when x = number of pair of Earings = 10 and y = Number of Neckleses

OR

When x = 5 and y = 5

Z = 30x + 40y = 150 + 200 = ₹350