

Class XII Session 2024-25
Subject - Mathematics
Sample Question Paper - 10

Time Allowed: 3 hours

Maximum Marks: 80

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If $A = [a_{ij}]$ is a scalar matrix of order $n \times n$ such that $a_{ij} = k$ for all i , then trace of A is equal to [1]
a) $\frac{n}{k}$ b) $n - k$
c) nk d) $n + k$
2. Let A and B be two invertible matrices of order 3×3 . If $\det(ABA') = 8$ and $\det(AB^{-1}) = 8$, then $\det(BA^{-1}B')$ is [1]
equal to
a) 16 b) $\frac{1}{4}$
c) 1 d) $\frac{1}{16}$
3. If A is a 3×3 matrix and $|A| = -2$, then value of $|A(\text{adj } A)|$ is [1]
a) -2 b) 8
c) 2 d) -8
4. If $y = \log(\sin e^x)$, then $\frac{dy}{dx}$ is: [1]
a) $\cot e^x$ b) $e^x \cot e^x$
c) $\text{cosec } e^x$ d) $e^x \text{ cosec } e^x$
5. The direction ratios of a line parallel to z -axis are: [1]
a) $\langle 0, 0, 0 \rangle$ b) $\langle 1, 1, 0 \rangle$
c) $\langle 1, 1, 1 \rangle$ d) $\langle 0, 0, 1 \rangle$
6. What is integrating factor of $\frac{dy}{dx} + y \sec x = \tan x$? [1]

a) $|\overrightarrow{AB} \times \overrightarrow{AC}|$

$$\text{b) } \frac{1}{8} |\overrightarrow{AC} \times \overrightarrow{AB}|$$

c) $\frac{1}{4} |\overrightarrow{AC} \times \overrightarrow{AB}|$

d) $\frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{AC}|$

17. If $f(x) = \begin{cases} kx + 5, & \text{when } x \leq 2 \\ x + 1, & \text{when } x > 2 \end{cases}$ is continuous at $x = 2$ then $k = ?$ **[1]**

- [illegible]

18. The lines l_1 and l_2 intersect. The shortest distance between them is **[1]**

- a) infinity b) negative
c) positive d) zero

19. **Assertion (A):** The rate of change of area of a circle with respect to its radius r when $r = 6$ cm is 12π cm²/cm. [1]

Reason (R): Rate of change of area of a circle with respect to its radius r is $\frac{dA}{dr}$, where A is the area of the circle.

- a) Both A and R are true and R is the correct explanation of A.
- b) Both A and R are true but R is not the correct explanation of A.
- c) A is true but R is false.
- d) A is false but R is true.

20. **Assertion (A):** A relation $R = \{(a, b) : |a - b| < 3\}$ defined on the set $A = \{1, 2, 3, 4\}$ is reflexive. [1]

Reason (R): A relation R on the set A is said to be reflexive if for $(a, b) \in R$ and $(b, c) \in R$, we have $(a, c) \in R$.

- a) Both A and R are true and R is the correct explanation of A.
- b) Both A and R are true but R is not the correct explanation of A.
- c) A is true but R is false.
- d) A is false but R is true.

Section B

21. Find the value of $\cos^{-1} \frac{1}{2} + 2\sin^{-1} \frac{1}{2}$. [2]

OR

Write the interval for the principal value of function and draw its graph: $\cos^{-1} x$.

22. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which y-coordinates is changing 2 times as fast as x - coordinates. [2]

23. Show that $f(x) = \cos(2x + \frac{\pi}{4})$ is an increasing function on $(\frac{3\pi}{8}, \frac{7\pi}{8})$ [2]

OR

Show that the function given by $f(x) = \sin x$ is neither increasing nor decreasing in $(0, \pi)$

24. Integrate the function: $\frac{1}{9x^2+6x+5}$ [2]

25. Find the matrix X for which: $\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} X \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$ [2]

Section C

26. Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}$ [3]

27. A doctor claims that 60% of the patients he examines are allergic to some type of weed. What is the probability that exactly 3 of his next 4 patients are allergic to weeds? [3]

28. Evaluate the integral: $\int \sec^{-1} \sqrt{x} dx$ [3]

OR

Prove that: $\int_0^\pi \frac{x}{1 - \cos \alpha \sin x} dx = \frac{\pi(\pi - \alpha)}{\sin \alpha}$

29. Solve the initial value problem: $\frac{dy}{dx} + y \tan x = 2x + x^2 \tan x$, $y(0) = 1$ [3]

OR

Solve differential equation: $\frac{dy}{dx} - y \tan x = e^x$

30. The two adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} - 5\hat{k}$ and $2\hat{i} + 2\hat{j} + 3\hat{k}$. Find the two unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram. [3]

OR

Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector \vec{d} which is perpendicular to both \vec{a} and \vec{b} , and $\vec{c} \cdot \vec{d} = 15$.

31. Find all points of discontinuity of f where f is defined as follows, $f(x) = \begin{cases} |x| + 3, & x \leq -3 \\ -2x, & -3 < x < 3 \\ 6x + 2, & x \geq 3 \end{cases}$ [3]

Section D

32. Find the area of the region $\{(x, y) : 0 \leq y \leq (x^2 + 1), 0 \leq y \leq (x + 1), 0 \leq x \leq 2\}$ [5]
33. Let $f : W \rightarrow W$ be defined as $f(n) = n - 1$, if n is odd and $f(n) = n + 1$, if n is even. Show that f is invertible. Find the inverse of f . Here, W is the set of all whole numbers. [5]

OR

Let $A = \{1, 2, 3\}$ and $R = \{(a, b) : a, b \in A \text{ and } |a^2 - b^2| \leq 5\}$. Write R as set of ordered pairs. Mention whether R is

- reflexive
- symmetric
- transitive

Give reason in each case.

34. Three shopkeepers A, B and C go to a store to buy stationery. A purchases 12 dozen notebooks, 5 dozen pens and 6 dozen pencils. B purchases 10 dozen notebooks, 6 dozen pens and 7 dozen pencils. C purchases 11 dozen notebooks, 13 dozen pens and 8 dozen pencils. A notebook costs 40 paise, a pen costs ₹1.25 and a pencil costs 35 paise. Use matrix multiplication to calculate each individual's bill. [5]
35. Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base. [5]

OR

Show that the semi-vertical angle of a cone of maximum volume and given slant height is $\tan^{-1}\sqrt{2}$ or $\cos^{-1}\frac{1}{\sqrt{3}}$.

Section E

36. Read the following text carefully and answer the questions that follow: [4]

A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is 0.65. The probability that many are not present and still the work gets completed on time is 0.35. The probability that work will be completed on time when all workers are present is 0.80.

Let: E_1 : represent the event when many workers were not present for the job;

E_2 : represent the event when all workers were present; and

E : represent completing the construction work on time.

- What is the probability that all the workers are present for the job? (1)
- What is the probability that construction will be completed on time? (1)
- What is the probability that many workers are not present given that the construction work is completed on time? (2)

OR

What is the probability that all workers were present given that the construction job was completed on time?

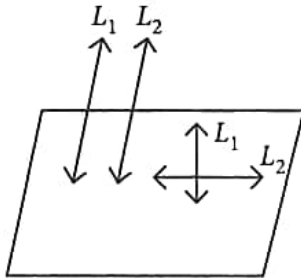
(2)

37. **Read the following text carefully and answer the questions that follow:**

[4]

If a_1, b_1, c_1 and a_2, b_2, c_2 are direction ratios of two lines say L_1 and L_2 respectively. Then $L_1 \parallel L_2$ iff

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ and $L_1 \perp L_2$ iff $a_1a_2 + b_1b_2 + c_1c_2 = 0$.



i. Find the coordinates of the foot of the perpendicular drawn from the point $A(1, 2, 1)$ to the line joining $B(1, 4, 6)$ and $C(5, 4, 4)$. (1)

ii. Find the direction ratios of the line which is perpendicular to the lines with direction ratios proportional to $(1, -2, -2)$ and $(0, 2, 1)$. (1)

iii. What is the relation between lines $\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-2}{0}$ and $\frac{x-1}{1} = \frac{y+\frac{3}{2}}{\frac{3}{2}} = \frac{z+5}{2}$. (2)

OR

If l_1, m_1, n_1 and l_2, m_2, n_2 are direction cosines of L_1 and L_2 respectively, then what is the condition for L_1 parallel to L_2 . (2)

38. **Read the following text carefully and answer the questions that follow:**

[4]

Sheetal rides her car at 25 km/hr. She has to spend ₹2 per km on diesel and if she rides it at a faster speed of 40 km/hr, the diesel cost increases to ₹ 5 per km. She has ₹ 100 to spend on diesel.



i. Formulate above information mathematically. (1)

ii. Represent the given information graphically. (1)

iii. Find the maximum distance covered by her in hour? (2)

OR

If $Z = 6x - 9y$ be the objective function, then find maximum value of Z . (2)

Solution

Section A

1.

(c) nk

Explanation: $\because A = [a_{ij}]_{n \times n}$

Trace of A, i.e., $\text{tr}(A) = \sum a_{ii}^n = 1 = a_{11} + a_{22} + \dots + a_{nn}$

$= k + k + k + k + k + \dots (n \text{ times})$

$= k(n)$

$= nk$

2.

(d) $\frac{1}{16}$

Explanation: $\frac{1}{16}$

3.

(d) -8

Explanation: -8

4.

(b) $e^x \cot e^x$

Explanation: $e^x \cot e^x$

$y = \log(\sin e^x)$

$\frac{dy}{dx} = \frac{d}{dx} \log(\sin e^x)$

$= \frac{1}{\sin e^x} \frac{d}{dx} \sin e^x$

$= \frac{1}{\sin e^x} \cos e^x \frac{d}{dx} e^x$

$= \cot e^x (e^x)$

$= e^x \cot e^x$

5.

(d) $< 0, 0, 1 >$

Explanation: $< 0, 0, 1 >$

6.

(c) $\sec x + \tan x$

Explanation: We have,

$\frac{dy}{dx} + y \sec x = \tan x$

Comparing with $\frac{dy}{dx} + Py = Q$

$P = \sec x, Q = \tan x$

I. $F \cdot = e^{\int \sec x dx} = e^{\log(\sec x + \tan x)} = \sec x + \tan x$

7.

(a) $q = 3p$

Explanation: Since Z occurs maximum at (15, 15) and (0, 20), therefore, $15p + 15q = 0p + 20q \Rightarrow q = 3p$.

8.

(b) $\left[\frac{-\pi}{2}, \frac{\pi}{2} \right] - \{0\}$

Explanation: We know that the principal value branch of $\text{cosec}^{-1} x$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2} \right] - \{0\}$

9.

(a) $(e - 1)$

Explanation: $I = \int_0^{\frac{\pi}{4}} e^{\tan x} \sec^2 x dx$

Let, $\tan x = t$,

Differentiating both side with respect to t

$$\sec^2 x \frac{dx}{dt} = 1$$

$$\Rightarrow \sec^2 x dx = dt$$

$$\text{At } x = 0, t = 0$$

$$\text{At } x = \frac{\pi}{4}, t = 1$$

$$I = \int_0^1 e^t dt$$

$$= e^t \Big|_0^1$$

$$= e^1 - e^0$$

$$= e - 1$$

10.

(b) 3×3

Explanation: $\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix}_{3 \times 2} \left\{ \begin{bmatrix} -1 & 0 & 2 \\ 2 & 0 & 1 \end{bmatrix}_{2 \times 3} - \begin{bmatrix} 0 & 1 & 23 \\ 1 & 0 & 21 \end{bmatrix}_{2 \times 3} \right\}$

$$= \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix}_{3 \times 2} \begin{bmatrix} -1 & -1 & -21 \\ 1 & 0 & -20 \end{bmatrix}_{2 \times 3}$$

$$= \begin{bmatrix} -2 & -1 & -1 \\ 2 & 0 & -40 \\ 1 & -2 & -102 \end{bmatrix}_{3 \times 3}$$

11.

(b) an open half-plane containing the origin.

Explanation: The strict inequality represents an open half plane and it contains the origin as (0, 0) satisfies it.

12.

(d) None of these

Explanation: Given vectors $4\vec{i} + 11\vec{j} + m\vec{k}$, $7\vec{i} + 2\vec{j} + 6\vec{k}$ and $\vec{i} + 5\vec{j} + 4\vec{k}$ are coplanar then

$$\begin{vmatrix} 4 & 11 & m \\ 7 & 2 & 6 \\ 1 & 5 & 4 \end{vmatrix} = 0$$

$$\Rightarrow 4(8-30) - 11(28-6) + m(35-2) = 0$$

$$-88 - 242 + 33m = 0$$

$$-330 + 33m = 0$$

$$m = 10$$

13.

(d) $(a^2 + b^2 + c^2 + d^2)$

Explanation: $\Delta = (a + ib)(a - ib) + (c - id)(c + id) = (a^2 + b^2 + c^2 + d^2)$

14.

(d) $\frac{7}{12}$

Explanation: Here, $P(A) = \frac{3}{10}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$

$$P(B/A) + P(A/B) = \frac{P(B \cap A)}{P(A)} + \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(A) + P(B) - P(A \cup B)}{P(A)} + \frac{P(A) + P(B) - P(A \cup B)}{P(B)}$$

$$= \frac{\frac{3}{10} + \frac{2}{5} - \frac{3}{5}}{\frac{3}{10}} + \frac{\frac{3}{10} + \frac{2}{5} - \frac{3}{5}}{\frac{2}{5}}$$

$$= \frac{\frac{1}{10}}{\frac{3}{10}} + \frac{\frac{1}{10}}{\frac{2}{5}} = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$$

15.

(d) $e^y - e^x = \frac{x^3}{3} + C$

Explanation: We have, $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

$$\Rightarrow e^y dy = (e^x + x^2) dx$$

$$\Rightarrow \int e^y dy = \int (e^x + x^2) dx$$

$$\Rightarrow e^y = e^x + \frac{x^3}{3} + c$$

$$\Rightarrow e^y - e^x = \frac{x^3}{3} + c$$

16.

(d) $\frac{1}{2} |\vec{AB} \times \vec{AC}|$

Explanation: The area of the parallelogram with adjacent sides AB and AC = $|\vec{AB} \times \vec{AC}|$. Hence, the area of the triangle with vertices A, B, C = $\frac{1}{2} |\vec{AB} \times \vec{AC}|$.

17.

(b) -1

Explanation: For continuity left hand limit must be equal to right hand limit and value at the point.

Continuous at $x = 2$.

$$\text{L.H.L} = \lim_{x \rightarrow 2^-} (kx + 5)$$

$$\Rightarrow \lim_{h \rightarrow 0} (k(2 - h) + 5)$$

$$\Rightarrow k(2 - 0) + 5 = 2k + 5$$

$$\text{R.H.L} = \lim_{x \rightarrow 2^+} (x + 1)$$

$$\Rightarrow \lim_{h \rightarrow 0} (2 + h + 1)$$

$$\Rightarrow 2 + 0 + 1$$

$$= 3$$

As $f(x)$ is continuous, we get

$$\therefore 2k + 5 = 3$$

$$k = -1.$$

18.

(d) zero

Explanation: Since the lines intersect. Hence they have a common point in them. Hence the distance will be zero.

19. (a) Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

20.

(c) A is true but R is false.

Explanation: Assertion is true because for each element $a \in A$, $|a - a| = 0 < 3$, so $(1, 1) \in R$, $(2, 2) \in R$, $(3, 3) \in R$, $(4, 4) \in R$ therefore R is reflexive.

Reason is false because a relation R on the set A is said to be transitive if for $(a, b) \in R$ and $(b, c) \in R$, we have $(a, c) \in R$

Section B

21. Let $\cos^{-1}\left(\frac{1}{2}\right) = x$. Then, $\cos x = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$.

$$\therefore \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

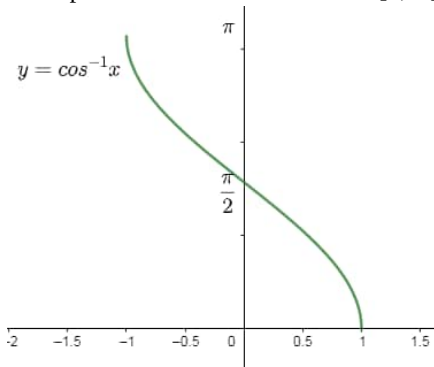
Let $\sin^{-1}\left(\frac{1}{2}\right) = y$. Then, $\sin y = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$.

$$\therefore \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

$$\therefore \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + \frac{2\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$$

OR

Principal value branch of $\cos^{-1} x$ is $[0, \pi]$ and its graph is shown here,



22. Given curve is,

$$6y = x^3 + 2$$

$$\Rightarrow 6 \frac{dy}{dt} = 3x^2 \cdot \frac{dx}{dt} \dots (i)$$

$$\text{Given: } \frac{dy}{dt} = 2 \cdot \frac{dx}{dt} \dots (ii)$$

$$\text{from (i) and (ii), } 2 \left(2 \frac{dx}{dt} \right) = x^2 \cdot \frac{dx}{dt}$$

$$\Rightarrow x = \pm 2$$

$$\text{when } x = 2, y = \frac{5}{3}; \text{ when } x = -2, y = -1$$

$$\text{Therefore, Points are } \left(2, \frac{5}{3} \right) \text{ and } (-2, -1)$$

23. Given: $f(x) = \cos\left(2x + \frac{\pi}{4}\right)$

$$f'(x) = -2 \sin\left(2x + \frac{\pi}{4}\right)$$

Now,

$$x \in \left(\frac{3\pi}{8}, \frac{7\pi}{8} \right)$$

$$\Rightarrow \frac{3\pi}{8} < x < \frac{7\pi}{8}$$

$$\Rightarrow \frac{3\pi}{4} < 2x < \frac{7\pi}{4}$$

$$\Rightarrow \frac{\pi}{4} + \frac{3\pi}{4} < 2x + \frac{\pi}{4} < \frac{7\pi}{4} + \frac{\pi}{4}$$

$$\Rightarrow \pi < 2x + \frac{\pi}{4} < 2\pi$$

$$\Rightarrow \sin\left(2x + \frac{\pi}{4}\right) < 0$$

$$\Rightarrow -2 \sin\left(2x + \frac{\pi}{4}\right) > 0$$

$$\Rightarrow f'(x) > 0$$

$$\text{Hence, } f(x) \text{ is increasing on } \left(\frac{3\pi}{8}, \frac{7\pi}{8} \right)$$

OR

The function is $f(x) = \sin x$

$$\text{Then, } f'(x) = \cos x$$

$$\text{Since for each } x \in \left(0, \frac{\pi}{2} \right), \cos x > 0, \text{ we have } f'(x) > 0$$

$$\text{Therefore, } f \text{ is strictly increasing in } \left(0, \frac{\pi}{2} \right) \dots (1)$$

Now, The function is $f(x) = \sin x$

$$\text{Then, } f'(x) = \cos x$$

$$\text{Since, for each } x \in \left(\frac{\pi}{2}, \pi \right), \cos x < 0, \text{ we have } f'(x) < 0$$

$$\text{Therefore, } f \text{ is strictly decreasing in } \left(\frac{\pi}{2}, \pi \right) \dots (2)$$

From (1) and (2),

It is clear that f is neither increasing nor decreasing in $(0, \pi)$.

24. Clearly, $9x^2 + 6x + 5 = (3x + 1)^2 + (2)^2$

$$\Rightarrow \int \frac{1}{9x^2 + 6x + 5} dx = \int \frac{1}{(3x+1)^2 + (2)^2} dx$$

$$\text{Let } 3x + 1 = t$$

$$\Rightarrow 3dx = dt$$

$$\therefore \int \frac{1}{(3x+1)^2 + (2)^2} dx = \frac{1}{3} \int \frac{1}{t^2 + 2^2} dt$$

$$= \frac{1}{3} \left[\frac{1}{2} \tan^{-1} \left(\frac{t}{2} \right) \right] + C$$

$$= \frac{1}{6} \tan^{-1} \left(\frac{3x+1}{2} \right) + C$$

25. Let $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix}$ $C = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$

Then The given equation becomes as

$$AXB = C$$

$$\Rightarrow X = A^{-1}CB^{-1}$$

$$\text{now } |A| = 35 - 14 = 21$$

$$\text{and } |B| = -1 + 2 = 1$$

$$\therefore A^{-1} = \frac{\text{adj}(A)}{|A|} = \frac{1}{21} \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

$$\text{and } B^{-1} = \frac{\text{adj}(B)}{|B|} = \frac{1}{1} \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$$

$$\Rightarrow X = A^{-1}CB^{-1} = \frac{1}{21} \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$$

$$= \frac{1}{21} \begin{bmatrix} 10 + 0 & -5 - 8 \\ -14 + 0 & 7 + 12 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$$

$$= \frac{1}{21} \begin{bmatrix} 10 & -13 \\ -14 & 19 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$$

$$= \frac{1}{21} \begin{bmatrix} 10 - 26 & -10 + 13 \\ -14 + 38 & 14 - 19 \end{bmatrix}$$

$$\text{Hence, } x = \frac{1}{21} \begin{bmatrix} -16 & 3 \\ 24 & -5 \end{bmatrix}$$

Section C

26. $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$

$$I = \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\frac{\sin x}{\cos x}}} = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \dots (1)$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos\left(\frac{\pi}{3} + \frac{\pi}{6} - x\right)}}{\sqrt{\cos\left(\frac{\pi}{3} + \frac{\pi}{6} - x\right)} + \sqrt{\sin\left(\frac{\pi}{3} + \frac{\pi}{6} - x\right)}} dx$$

$$\left[\because \int_a^b f(x) dx = \int_a^b f(a + b - x) dx \right]$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos\left(\frac{\pi}{2} - x\right)}}{\sqrt{\cos\left(\frac{\pi}{2} - x\right)} + \sqrt{\sin\left(\frac{\pi}{2} - x\right)}} dx$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \dots (2)$$

Adding (1) and (2), we get

$$2I = \int_{\pi/6}^{\pi/3} 1 dx$$

$$= [x]_{\pi/6}^{\pi/3}$$

$$= \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$$

$$I = \frac{\pi}{12}$$

27. Let us define the following events:

A = First patient is allergic to weeds, B = Second patient is allergic to weeds C = Third patient is allergic to weeds, D = Fourth patient is allergic to weeds

Clearly, A, B, C, D are independent events such that

$$P(A) = P(B) = P(C) = P(D) = \frac{60}{100} = \frac{3}{5}$$

Therefore, required probability is given by,

$$= P[(A \cap B \cap C \cap \bar{D}) \cup (A \cap B \cap \bar{C} \cap D) \cup (A \cap \bar{B} \cap C \cap D) \cup (\bar{A} \cap B \cap C \cap D)]$$

$$= P(A \cap B \cap C \cap \bar{D}) + P(A \cap B \cap \bar{C} \cap D) + P(A \cap \bar{B} \cap C \cap D) + P(\bar{A} \cap B \cap C \cap D)$$

$$= P(A) P(B) P(C) P(\bar{D}) + P(A) P(B) P(\bar{C}) P(D) + P(A) P(\bar{B}) P(C) P(D) + P(\bar{A}) P(B) P(C) P(D)$$

$$= \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{2}{5} + \frac{3}{5} \times \frac{3}{5} \times \frac{2}{5} \times \frac{3}{5} + \frac{3}{5} \times \frac{2}{5} \times \frac{3}{5} \times \frac{3}{5} + \frac{2}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} = \frac{216}{625}$$

28. Let the given integral be ,

$$I = \int \sec^{-1} \sqrt{x} dx$$

$$\text{Putting } \sqrt{x} = \sec \theta$$

$$\Rightarrow x = \sec^2 \theta$$

$$\Rightarrow dx = 2 \sec \theta \sec \theta \tan \theta d\theta$$

$$= 2 \sec^2 \theta \tan \theta d\theta$$

$$\therefore I = 2 \int \theta \sec^2 \theta \tan \theta d\theta$$

$$= 2 \int \theta \tan \theta \sec^2 \theta d\theta$$

Considering θ as first function and $\tan \theta \sec^2 \theta$ as second function

$$I = 2 \left[\theta \frac{\tan^2 \theta}{2} - \int 1 \frac{\tan^2 \theta}{2} d\theta \right] \left(\because \int \tan \theta \sec^2 \theta d\theta = \frac{\tan^2 \theta}{2} \right)$$

$$= \theta \tan^2 \theta - \int (\sec^2 \theta - 1) d\theta$$

$$= \theta \tan^2 \theta - \tan \theta + \theta + C$$

$$= \theta (1 + \tan^2 \theta) - \tan \theta + C$$

$$= \theta \sec^2 \theta - \sqrt{\sec^2 \theta - 1} + C$$

$$= \sec^{-1} \sqrt{x} - \sqrt{x-1} + C$$

$$= x \sec^{-1} \sqrt{x} - \sqrt{x-1} + C$$

OR

$$\text{Let } I = \int_0^\pi \frac{x}{1 - \cos \alpha \sin x} dx. \text{ Then,}$$

$$I = \int_0^\pi \frac{(\pi-x)}{1 - \cos \alpha \sin(\pi-x)} dx$$

By using property of definite integrals,

$$\Rightarrow I = \int_0^\pi \frac{\pi}{1 - \cos \alpha \sin x} dx - \int_0^\pi \frac{x}{1 - \cos \alpha \sin x} dx$$

$$\Rightarrow I = \pi \int_0^\pi \frac{1}{1 - \cos \alpha \sin x} dx - I$$

$$\Rightarrow 2I = \pi \int_0^\pi \frac{1}{1 - \cos \alpha \sin x} dx$$

$$\Rightarrow 2I = \pi \int_0^\pi \frac{1 + \tan^2 x/2}{(1 + \tan^2 x/2) - 2 \cos \alpha \tan x/2} dx$$

$$\Rightarrow 2I = \pi \int_0^\pi \frac{\sec^2 x/2}{\tan^2 x/2 - 2 \cos \alpha \tan x/2 + 1} dx$$

$$\Rightarrow I = \frac{\pi}{2} \int_0^\pi \frac{\sec^2 x/2}{\tan^2 x/2 - 2 \cos \alpha \tan x/2 + 1} dx$$

$$\text{Let } \tan \frac{x}{2} = t. \text{ Then, } d\left(\tan \frac{x}{2}\right) = dt \Rightarrow \sec^2 \frac{x}{2} dx = 2dt$$

$$\text{Also, } x = 0 \Rightarrow t = \tan 0 = 0 \text{ and } x = \pi \Rightarrow t = \tan \frac{\pi}{2} = \infty$$

$$\therefore I = \frac{\pi}{2} \int_0^\infty \frac{2dt}{t^2 - 2t \cos \alpha + 1}$$

$$\Rightarrow I = \pi \int_0^\infty \frac{1}{(t - \cos \alpha)^2 + (1 - \cos^2 \alpha)} dt$$

$$\Rightarrow I = \pi \int_0^\infty \frac{1}{\sin^2 \alpha + (t - \cos \alpha)^2} dt$$

$$\Rightarrow I = \frac{\pi}{\sin \alpha} \left[\tan^{-1} \left(\frac{t - \cos \alpha}{\sin \alpha} \right) \right]_0^\infty$$

$$\Rightarrow I = \frac{\pi}{\sin \alpha} \left[\tan^{-1} \alpha - \tan^{-1}(-\cot \alpha) \right]$$

$$\Rightarrow I = \frac{\pi}{\sin \alpha} \left[\frac{\pi}{2} + \tan^{-1}(\cot \alpha) \right]$$

$$\Rightarrow I = \frac{\pi}{\sin \alpha} \left[\frac{\pi}{2} + \tan^{-1} \left\{ \tan \left(\frac{\pi}{2} - \alpha \right) \right\} \right] = \frac{\pi}{\sin \alpha} \left[\frac{\pi}{2} + \frac{\pi}{2} - \alpha \right] = \frac{\pi(\pi - \alpha)}{\sin \alpha}$$

29. The given differential equation is,

$$\frac{dy}{dx} + y \tan x = 2x + x^2 \tan x, y(0) = 1$$

It is a linear differential equation. Comparing it with,

$$\frac{dy}{dx} + Py = Q$$

$$P = \tan x, Q = 2x + x^2 \tan x$$

$$\text{I.F.} = e^{\int P dx}$$

$$= e^{\int \tan x dx}$$

$$= e^{\log |\sec x|}$$

Solution of the equation is given by,

$$y \times (\text{I.F.}) = \int Q \times (\text{I.F.}) dx + c$$

$$y \sec x = \int (2x + x^2 \tan x) \sec x dx + c$$

$$= \int 2x \sec x dx + \int x^2 \sec \tan x dx + c$$

$$= \int 2x \times \sec x dx + [x^2 \times \int \sec x \tan x dx - \int (2x \int \sec \tan x dx) dx] + c$$

$$y \sec x = \int 2x \sec x dx + x^2 \sec x - \int 2x \sec x dx + c$$

$$y \sec x = x^2 \sec x + c \dots (i)$$

$$\text{Put } x = 0, y = 1$$

$$1 = 0 + c$$

$$c = 1$$

Put $c = 1$ in equation (i),

$$y \sec x = x^2 \sec x + 1$$

$$y = x^2 + \frac{1}{\sec x}$$

$$y = x^2 + \cos x$$

OR

The given differential equation is,

$$\frac{dy}{dx} - y \tan x = e^x$$

It is a linear differential equation.

Comparing it with $\frac{dy}{dx} + py = Q$

$$P = -\tan x, Q = e^x$$

$$\text{I.F.} = e^{\int p dx}$$

$$= e^{-\int \tan x dx}$$

$$= e^{-\log \sec x}$$

$$= \cos x$$

Solution of the given equation is given by,

$$y (\text{I.F.}) = \int Q \times (\text{I.F.}) dx + c_1$$

$$y \cos x = \int e^x \cos x dx + c_1$$

$$y \cos x = I + c_1 \dots (1)$$

$$I = \int e^x \cos x dx$$

Using equation by parts

$$I = e^x \int \cos x dx - \int (e^x \int \cos x dx) dx + c_2$$

$$= e^x \sin x - \int e^x \sin x dx + c_2$$

$$= e^x \sin x - [e^x (\int \sin x dx) - \int (e^x (\int \sin x dx) dx)] + c_2$$

$$I = e^x \sin x + e^x \cos x - \int e^x \cos x dx + c_2$$

$$I = e^x (\sin x + \cos x) - I + c_2$$

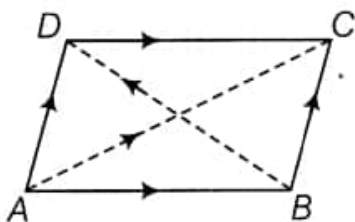
$$2I = e^x (\sin x + \cos x) + c_2$$

$$I = \frac{1}{2} e^x (\sin x + \cos x) + c_3$$

Using equation (i)

$$y \cos x = \frac{1}{2} e^x (\sin x + \cos x) + c$$

30. Let ABCD be the parallelogram.



According to the question,

$$\vec{AB} = 2\hat{i} - 4\hat{j} - 5\hat{k} \text{ and } \vec{AD} = 2\hat{i} + 2\hat{j} + 3\hat{k}.$$

By using parallelogram law of addition, we get

$$\text{The diagonal } \vec{AC} \text{ is given by } \vec{AB} + \vec{AD} = 4\hat{i} - 2\hat{j} - 2\hat{k}$$

$$\text{The diagonal } \vec{BD} \text{ is given by } \vec{BC} + \vec{BA} = \vec{AD} - \vec{AB} = 6\hat{j} + 8\hat{k}$$

Now, the unit vector along \vec{AC} is given by

$$\frac{\vec{AC}}{|\vec{AC}|} = \frac{4\hat{i} - 2\hat{j} - 2\hat{k}}{\sqrt{16+4+4}}$$

$$= \frac{4\hat{i} - 2\hat{j} - 2\hat{k}}{\sqrt{24}}$$

$$= \frac{4\hat{i} - 2\hat{j} - 2\hat{k}}{2\sqrt{6}}$$

$$= \frac{1}{\sqrt{6}}(2\hat{i} - \hat{j} - \hat{k})$$

the unit vector along \overrightarrow{BD} is given by

$$\begin{aligned}\frac{\overrightarrow{BD}}{|\overrightarrow{BD}|} &= \frac{6\hat{j}+8\hat{k}}{\sqrt{36+64}} \\ &= \frac{6\hat{j}+8\hat{k}}{10} \\ &= \frac{1}{5}(3\hat{j} + 4\hat{k})\end{aligned}$$

Now, area of parallelogram ABCD

$$= \frac{1}{2} |\overrightarrow{AC} \times \overrightarrow{BD}|$$

$$\text{Here, } \overrightarrow{AC} \times \overrightarrow{BD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -2 & -2 \\ 0 & 6 & 8 \end{vmatrix}$$

$$\begin{aligned}&= \hat{i}(-16 + 12) - \hat{j}(32 - 0) + \hat{k}(24 - 0) \\ &= -4\hat{i} - 32\hat{j} + 24\hat{k}\end{aligned}$$

$$\begin{aligned}|\overrightarrow{AC} \times \overrightarrow{BD}| &= \sqrt{(-4)^2 + (-32)^2 + (24)^2} \\ &= \sqrt{4^2 (1 + 8^2 + 6^2)} \\ &= 4\sqrt{1 + 64 + 36} \\ &= 4\sqrt{101}\end{aligned}$$

$$\begin{aligned}\therefore \text{Area of parallelogram ABCD} &= \frac{1}{2} \times 4\sqrt{101} \\ &= 2\sqrt{101} \text{ sq units.}\end{aligned}$$

OR

Given: Vectors $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$

We know that the cross-product of two vectors, $\vec{a} \times \vec{b}$ is a vector perpendicular to both \vec{a} and \vec{b}

Hence, vector \vec{d} which is also perpendicular to both \vec{a} and \vec{b} is $\vec{d} = \lambda (\vec{a} \times \vec{b})$ where $\lambda = 1$ or some other scalar.

$$\text{Therefore, } \vec{d} = \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 2 \\ 3 & -2 & 7 \end{vmatrix}$$

$$= \lambda [\hat{i}(28 + 4) - \hat{j}(7 - 6) + \hat{k}(-2 - 12)]$$

$$\Rightarrow \vec{d} = 32\lambda\hat{i} - \lambda\hat{j} - 14\lambda\hat{k} \dots (i)$$

Now given $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{c} \cdot \vec{d} = 15$

$$\vec{c} \cdot \vec{d} = 15$$

$$= 2(32\lambda) + (-1)(-\lambda) + 4(-14\lambda) = 15$$

$$\Rightarrow 64\lambda + \lambda - 56\lambda = 15$$

$$\Rightarrow 9\lambda = 15$$

$$\Rightarrow \lambda = \frac{15}{9}$$

$$\Rightarrow \lambda = \frac{5}{3}$$

Putting $\lambda = \frac{5}{3}$ in eq. (i), we get

$$\vec{d} = \frac{5}{3} [32\hat{i} - \hat{j} - 14\hat{k}]$$

$$\Rightarrow \vec{d} = \frac{1}{3} [160\hat{i} - 5\hat{j} - 70\hat{k}]$$

31. Given function is

$$f(x) = \begin{cases} |x| + 3, & x \leq -3 \\ -2x, & -3 < x < 3 \\ 6x + 2, & x \geq 3 \end{cases} = \begin{cases} -x + 3, & x \leq -3 \\ -2x, & -3 < x < 3 \\ 6x + 2, & x \geq 3 \end{cases}$$

First, we verify continuity at $x = -3$ and then at $x = 3$

Continuity at $x = -3$

$$\text{LHL} = \lim_{x \rightarrow (-3)^-} f(x) = \lim_{x \rightarrow (-3)^-} (-x + 3)$$

$$\Rightarrow \text{LHL} = \lim_{h \rightarrow 0} [-(-3 - h) + 3]$$

$$= \lim_{h \rightarrow 0} (3 + h + 3)$$

$$= 3 + 3 = 6$$

$$\text{and RHL} = \lim_{x \rightarrow (-3)^+} f(x) = \lim_{x \rightarrow (-3)^+} (-2x)$$

$$\Rightarrow \text{RHL} = \lim_{h \rightarrow 0} [-2(-3 + h)]$$

$$= \lim_{h \rightarrow 0} (6 - 2h)$$

$$\Rightarrow \text{RHL} = 6$$

Also, $f(-3)$ = value of $f(x)$ at $x = -3$

$$= -(-3) + 3$$

$$= 3 + 3 = 6$$

$$\therefore \text{LHL} = \text{RHL} = f(-3)$$

$\therefore f(x)$ is continuous at $x = -3$ So, $x = -3$ is the point of continuity.

Continuity at $x = 3$

$$\text{LHL} = \lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} [-(2x)]$$

$$\Rightarrow \text{LHL} = \lim_{h \rightarrow 0} [-2(3 - h)]$$

$$= \lim_{h \rightarrow 0} (-6 + 2h)$$

$$\Rightarrow \text{LHL} = -6$$

$$\text{and RHL} = \lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} (6x + 2)$$

$$\Rightarrow \text{RHL} = \lim_{h \rightarrow 0} [6(3 + h) + 2]$$

$$\Rightarrow \text{RHL} = 20$$

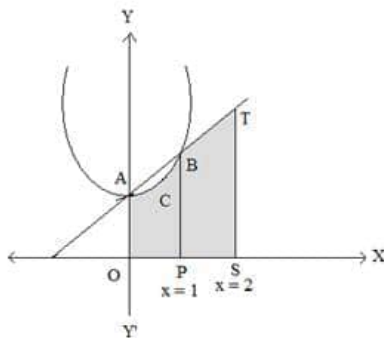
$$\therefore \text{LHL} \neq \text{RHL}$$

$\therefore f$ is discontinuous at $x = 3$ Now, as $f(x)$ is a polynomial function for $x < -3$, $-3 < x < 3$ and $x > 3$ so it is continuous in these intervals.

Hence, only $x = 3$ is the point of discontinuity of $f(x)$.

Section D

32.



There are three curves respectively,

$$C_1 = \{(x, y): 0 \leq y \leq x^2 + 1\}$$

$$C_2 = \{(x, y): 0 \leq y \leq x + 1\}$$

$$C_3 = \{(x, y): 0 \leq x \leq 2\}$$

The points of intersection of, $y = x^2$ and $y = x + 1$ are $A(0,1)$ and $B(1,2)$.

$$\text{The required area of shaded region} = \int_0^1 y_1 dx + \int_1^2 y_2 dx$$

$$\text{where } y_1 = x^2 + 1 \text{ and } y_2 = x + 1$$

$$\therefore A = \int_0^1 (x^2 + 1) dx + \int_1^2 (x + 1) dx$$

$$= \left[\frac{x^3}{3} + x \right]_0^1 + \left[\frac{x^2}{2} + x \right]_1^2$$

$$= \left(\frac{4}{3} - 0 \right) + \left(4 - \frac{3}{2} \right) = \frac{8+24-9}{6} = \frac{23}{6} \text{ sq. units}$$

33. Given: $f: W \rightarrow W$ defined as $f(n) = \begin{cases} n - 1, & \text{if } n \text{ is odd} \\ n + 1, & \text{if } n \text{ is even} \end{cases}$

Injectivity: Let n, m be any two odd real numbers, then $f(n) = f(m)$

$$\Rightarrow n - 1 = m - 1$$

$$\Rightarrow n = m$$

Again, let n, m be any two even whole numbers, then $f(n) = f(m)$

$$\Rightarrow n + 1 = m + 1$$

$$\Rightarrow n = m$$

If n is even and m is odd, then $n \neq m$.

Now n is even implies $f(n) = n+1$ and $f(m) = m-1$.

Therefore, $f(n) \neq f(m)$

Similarly n is odd and m is even gives $f(n) \neq f(m)$

Therefore in all cases f is one-one.

Surjectivity: Let n be an arbitrary whole number.

If n is an odd number, then there exists an even whole number $(n + 1)$ such that

$$f(n + 1) = n + 1 - 1 = n$$

If n is an even number, then there exists an odd whole number $(n - 1)$ such that

$$f(n - 1) = n - 1 + 1 = n$$

Therefore, every $n \in W$ has its pre-image in W .

So, $f : W \rightarrow W$ is a surjective. Thus f is invertible and f^{-1} exists.

$$\text{For } f^{-1} : y = n - 1$$

$$\Rightarrow n = y + 1 \text{ and } y = n + 1 \Rightarrow n = y - 1$$

$$\therefore f^{-1}(n) = \begin{cases} n - 1, & \text{if } n \text{ is odd} \\ n + 1, & \text{if } n \text{ is even} \end{cases}$$

$$\text{Hence, } f^{-1}(y) = y$$

OR

Given that

Let $A = \{1, 2, 3\}$ and $R = \{(a, b) : a, b \in A \text{ and } |a^2 - b^2| \leq 5\}$

Put $a = 1, b = 1$ $|1^2 - 1^2| \leq 5$, $(1, 1)$ is an ordered pair.

Put $a = 1, b = 2$ $|1^2 - 2^2| \leq 5$, $(1, 2)$ is an ordered pair.

Put $a = 1, b = 3$ $|1^2 - 3^2| > 5$, $(1, 3)$ is not an ordered pair.

Put $a = 2, b = 1$ $|2^2 - 1^2| \leq 5$, $(2, 1)$ is an ordered pair.

Put $a = 2, b = 2$ $|2^2 - 2^2| \leq 5$, $(2, 2)$ is an ordered pair.

Put $a = 2, b = 3$ $|2^2 - 3^2| \leq 5$, $(2, 3)$ is an ordered pair.

Put $a = 3, b = 1$ $|3^2 - 1^2| > 5$, $(3, 1)$ is not an ordered pair.

Put $a = 3, b = 2$ $|3^2 - 2^2| \leq 5$, $(3, 2)$ is an ordered pair.

Put $a = 3, b = 3$ $|3^2 - 3^2| \leq 5$, $(3, 3)$ is an ordered pair.

$R = \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 2), (3, 3)\}$

i. For $(a, a) \in R$

$$|a^2 - a^2| = 0 \leq 5. \text{ Thus, it is reflexive.}$$

ii. Let $(a, b) \in R$

$$(a, b) \in R, |a^2 - b^2| \leq 5$$

$$|b^2 - a^2| \leq 5$$

$$(b, a) \in R$$

Hence, it is symmetric

iii. Put $a = 1, b = 2, c = 3$

$$|1^2 - 2^2| \leq 5$$

$$|2^2 - 3^2| \leq 5$$

$$\text{But } |1^2 - 3^2| > 5$$

Thus, it is not transitive

34. Given the purchase details of three shopkeeper A, B, and C.

A: 12 dozen notebooks, 5 dozen pens, and 6 dozen pencils

B: 10 dozen notebooks, 6 dozen pens, and 7 dozen pencils

C: 11 dozen notebooks, 13 dozen pens, and 8 dozen pencils

Hence, the items purchased by A, B, and C can be represented in matrix X of order 3×3 where the rows denoting the each shopkeeper and columns denoting the number of dozens of items as –

$$X = \begin{bmatrix} 12 & 5 & 6 \\ 10 & 6 & 7 \\ 11 & 13 & 8 \end{bmatrix}$$

The price of each of the items is also given.

Cost of one notebook = 40 paise

\Rightarrow Cost of one dozen notebooks = 12×40 paise

\Rightarrow Cost of one dozen notebooks = 480 paise

\therefore Cost of one dozen notebooks = ₹ 4.80

Cost of one pen = ₹ 1.25

\Rightarrow Cost of one dozen pens = $12 \times ₹ 1.25$

\therefore Cost of one dozen pens = ₹ 15

Cost of one pencil = 35 paise

\Rightarrow Cost of one dozen notebooks = 12×35 paise

\Rightarrow Cost of one dozen notebooks = 420 paise

\therefore Cost of one dozen notebooks = ₹ 4.20

Hence, the cost of purchasing one dozen of the items can be represented in matrix form with each row corresponding to an item as

$$Y = \begin{bmatrix} 4.80 \\ 15 \\ 4.20 \end{bmatrix}$$

Now, the individual bill for each shopkeeper can be found by taking the product of the matrices X and Y.

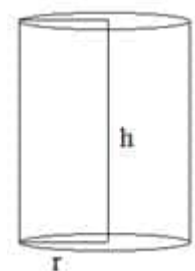
The product is feasible because the order of X is 3×3 and that of Y is 3×1 , which makes the number of columns of X equal to the number of the columns of Y. The product is given as follows.

$$\begin{aligned} XY &= \begin{bmatrix} 12 & 5 & 6 \\ 10 & 6 & 7 \\ 11 & 13 & 8 \end{bmatrix} \begin{bmatrix} 4.80 \\ 15 \\ 4.20 \end{bmatrix} \\ \Rightarrow XY &= \begin{bmatrix} 12 \times 4.80 + 5 \times 15 + 6 \times 4.20 \\ 10 \times 4.80 + 6 \times 15 + 7 \times 4.20 \\ 11 \times 4.80 + 13 \times 15 + 8 \times 4.20 \end{bmatrix} \\ \Rightarrow &\begin{bmatrix} 57.60 + 75 + 25.20 \\ 48 + 90 + 29.40 \\ 52.80 + 195 + 33.60 \end{bmatrix} \\ \therefore XY &= \begin{bmatrix} 157.80 \\ 167.40 \\ 281.40 \end{bmatrix} \end{aligned}$$

Thus, the bills of shopkeepers A, B and C are ₹ 157.80, ₹ 167.40 and ₹ 281.40 respectively.

35. Let radius of the cylinder = r

Height of the cylinder = h



$$s = 2\pi rh + 2\pi r^2 \quad (1)$$

$$\Rightarrow \frac{s - 2\pi r^2}{2\pi r} = h$$

Now volume of cylinder is, $v = \pi r^2 h$

$$\Rightarrow v = \pi \cdot r^2 \left(\frac{s - 2\pi r^2}{2\pi r} \right)$$

$$\Rightarrow v = \frac{1}{2} [sr - 2\pi r^3]$$

$$\text{Now, } \frac{dv}{dr} = \frac{1}{2} [s - 6\pi r^2]$$

$$\Rightarrow \frac{d^2v}{dr^2} = \frac{1}{2} [0 - 12\pi r]$$

For maximum/minimum

$$\frac{dv}{dr} = 0$$

$$\Rightarrow s = 6\pi r^2$$

From equ (1)

$$\Rightarrow 2\pi rh + 2\pi r^2 = 6\pi r^2$$

$$\Rightarrow r = \frac{h}{2}$$

$$\left[\frac{d^2v}{dr^2} \right]_{r=\frac{h}{2}} = \frac{1}{2} \left[0 - 12\pi \times \frac{h}{2} \right]$$

$$= -3\pi h < 0$$

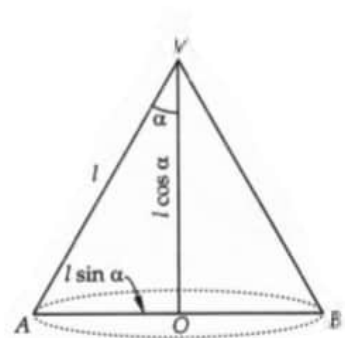
$$\Rightarrow s \text{ is maximum at } r = \frac{h}{2}$$

Hence $h = 2r$

OR

Let α be the semi-vertical angle of a cone VAB of given slant height l .

In $\triangle AOV$,



$$\cos \alpha = \frac{VO}{VA} \text{ and } \sin \alpha = \frac{OA}{VA}$$

$$\Rightarrow \cos \alpha = \frac{VO}{l} \text{ and } \sin \alpha = \frac{OA}{l}$$

$$\Rightarrow VO = l \cos \alpha, OA = l \sin \alpha$$

Let V be the volume of the cone, Then,

$$V = \frac{1}{3} \pi (OA)^2 (VO)$$

$$\Rightarrow V = \frac{1}{3} \pi (l \sin \alpha)^2 (l \cos \alpha)$$

$$\Rightarrow V = \frac{1}{3} \pi l^3 \sin^2 \alpha \cos \alpha$$

$$\Rightarrow \frac{dV}{d\alpha} = \frac{\pi l^3}{3} (-\sin^3 \alpha + 2 \sin \alpha \cos^2 \alpha)$$

$$\Rightarrow \frac{dV}{d\alpha} = \frac{\pi l^3}{3} \sin \alpha (-\sin^2 \alpha + 2 \cos^2 \alpha) \dots\dots\dots(i)$$

The critical points of V are given by $\frac{dV}{d\alpha} = 0$.

$$\therefore \frac{dV}{d\alpha} = 0$$

$$\Rightarrow \frac{\pi l^3}{3} \sin \alpha (-\sin^2 \alpha + 2 \cos^2 \alpha) = 0$$

$$\Rightarrow 2 \cos^2 \alpha = \sin^2 \alpha$$

$$\Rightarrow \tan^2 \alpha = 2 \Rightarrow \tan \alpha = \sqrt{2} \text{ [} \because \alpha \text{ is acute } \therefore \sin \alpha \neq 0 \text{]}$$

$$\therefore \cos \alpha = \frac{1}{\sqrt{1+\tan^2 \alpha}} = \frac{1}{\sqrt{3}} \text{ [} \because \tan \alpha = \sqrt{2} \text{]}$$

Differentiating (i) with respect to α , we get

$$\frac{d^2V}{d\alpha^2} = \frac{\pi l^3}{3} (-3 \sin^2 \alpha \cos \alpha + 2 \cos^3 \alpha - 4 \sin^2 \alpha \cos \alpha) = \frac{\pi l^3}{3} \cos^3 \alpha (2 - 7 \tan^2 \alpha)$$

$$\therefore \left(\frac{d^2V}{d\alpha^2} \right)_{\tan \alpha = \sqrt{2}} = \frac{1}{3} \pi l^3 \left(\frac{1}{\sqrt{3}} \right)^3 (2 - 7 \times 2) = \frac{-4\pi l^3}{3\sqrt{3}} < 0.$$

Thus, V is maximum, when $\tan \alpha = \sqrt{2}$ or $\alpha = \tan^{-1} \sqrt{2}$ i.e. when the semi-vertical angle of the cone is $\tan^{-1} \sqrt{2}$.

Section E

36. i. $P(E_2) = 1 - P(E_1) = 1 - 0.65 = 0.35$

ii. $P(E) = P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right)$
 $= 0.65 \times 0.35 + 0.35 \times 0.8$
 $= 0.35 \times 1.45$
 $= 0.51$

$$\text{iii. } P\left(\frac{E_1}{E}\right) = \frac{P(E_1) \cdot P\left(\frac{E}{E_1}\right)}{P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right)} = \frac{0.65 \times 0.35}{0.51} = 0.45$$

OR

$$P\left(\frac{E_2}{E}\right) = \frac{P(E_2) \cdot P\left(\frac{E}{E_2}\right)}{P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right)} = \frac{0.35 \times 0.8}{0.51} = 0.55$$

37. i. Equation of line joining B and C is

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

$$\frac{x-1}{4} = \frac{y-4}{0} = \frac{z-6}{-2} = \lambda$$

Let coordinates of foot of perpendicular be $D(4\lambda + 1, 4, -2\lambda + 6) \dots(i)$

\therefore D.R.'s of AD are $(4\lambda, 2, -2\lambda + 5)$.

Now, $4(4\lambda) + 0(2) + (-2)(-2\lambda + 5) = 0$

$$\lambda = \frac{1}{2}$$

Putting in eqn (i)

Coordinates of D are $(3, 4, 5)$

\therefore Required coordinates are $(3, 4, 5)$.

- ii. Let a, b, c be the direction ratios of the required line. Since it is perpendicular to the lines whose direction ratios are $(1, -2, -2)$ and $(0, 2, 1)$ respectively.

$$\therefore a - 2b - 2c = 0 \dots(i)$$

$$a + 2b + c = 0 \dots(ii)$$

On solving (i) and (ii) by cross-multiplication, we get

$$\frac{a}{-2+4} = \frac{b}{0-1} = \frac{c}{2} \Rightarrow \frac{a}{2} = \frac{b}{-1} = \frac{c}{2}$$

Thus, the direction ratios of the required line are $(2, -1, 2)$

- iii. Direction ratio of given lines are $(3, -2, 0)$ and $(1, \frac{3}{2}, 2)$

Now,

$$\text{as } 3 \cdot 1 + (-2) \cdot \left(\frac{3}{2}\right) + 0 \cdot 2 = 3 - 3 + 0 = 0$$

\therefore Given lines are perpendicular to each other.

OR

Since, direction ratio's are proportional to direction cosine's, therefore L_1 will be parallel to L_2 , iff

$$\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$$

38. i. Let she travels x kms with speed 25 km/hr and y kms with speed 40 km/hr.

$$x, y \geq 0$$

$$\frac{x}{25} + \frac{y}{40} \leq 1$$

$$2x + 5y \leq 100$$

$$Z = x + y$$

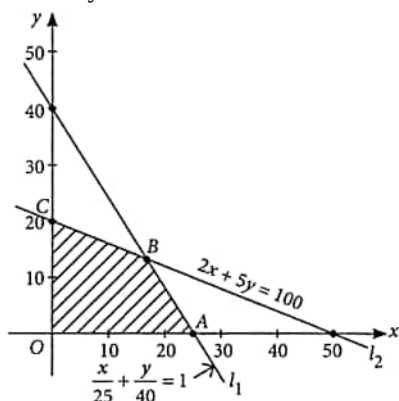
- ii. Let she travels x kms with speed 25 km/hr and y kms with speed 40 km/hr.

$$x, y \geq 0$$

$$\frac{x}{25} + \frac{y}{40} \leq 1$$

$$2x + 5y \leq 100$$

$$Z = x + y$$



- iii. Here $Z = x + y$

Corner Points	Value of $Z = x + y$
(0, 0)	0
(25, 0)	25
$\left(\frac{50}{3}, \frac{40}{3}\right)$	30 ← Maximum
(0, 20)	20

Thus, max $Z = 30$ occurs at point $\left(\frac{50}{3}, \frac{40}{3}\right)$.

Hence maximum distance covered by Sheetal in 1 hour = 30 km

OR

Corner Points	Value of $Z = 6x - 9y$
(0, 0)	0
(25, 0)	150 ← Maximum
$\left(\frac{50}{3}, \frac{40}{3}\right)$	-20
(0, 20)	-180

Maximum value is 150.