Class XII Session 2023-24
Subject - Mathematics
Sample Question Paper - 3

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If A = $\begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and kA = $\begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then the values of k, a, b, are respectively **[1]**

   a) - 6, - 12, - 18 b) -6, 12, 18

   c) - 6, 4, 9 d) - 6, - 4, - 9

2. The system of equations, x + y + z = 1, 3x + 6y + z = 8, αx + 2y + 3z = 1 has a unique solution for **[1]**

   a) all real α b) α not equal to 0

   c) all integral α d) all rational α

3. If A and B are square matrices of same order and A' denotes the transpose of A, then **[1]**

   a) AB = O $\Rightarrow$|A| = 0 and |B| = 0 b) (AB)' = A'B'

   c) (AB)' = B'A' d) AB = O $\Rightarrow$ A = 0 or B = 0

4. For the curve $\sqrt{x} + \sqrt{y} = 1, \frac{dy}{dx}$ at (1/4, 1/4) is **[1]**

   a) 2 b) -1

   c) 1/2 d) 1

5. If the line lies $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$ in the plane 2x - 4 y + z = 7, then the value of k is **[1]**

   a) 7 b) -4

   c) -7 d) 4

6. General solution of $\left(1 + x^2\right) dy + 2xy\, dx = \cot x\, dx\ (x \neq 0)$ is **[1]**

   a) $y(1 + x^2) = log|sinx| + c$ b) $y = (1 + x)^{-1} \log|\sin x| - C(1 + x^2)^{-1}$

c) $y=(1+x)^{-1}\log|\sin x|+C(1-x^2)^{-1}$ d) $y=(1+x)^{-1}\log|\sin x|-C(1-x^2)^{-1}$

7. Maximize Z = 100x + 120y , subject to constraints 2x + 3y ≤ 30, 3x + y ≤ 17, x ≥ 0, y ≥ 0. **[1]**

a) 1260 b) 1280

c) 1300 d) 1200

8. The two adjacent side of a triangle are represented by the vectors $\vec{a}=3\hat{i}+4\hat{j}$ and $\vec{b}=-5\hat{i}+7\hat{j}$ The area of the triangle is **[1]**

a) 41 sq units b) none of these

c) 37 sq units d) $\frac{41}{2}$ sq units

9. $\int \frac{dx}{\sin^2 x \cos^2 x}$ equals **[1]**

a) tan x + cot x + C b) tan x - cot x + C

c) $(\tan x - \cot x)^2 + C$ d) $(\tan x + \cot x)^2 + C$

10. If A is 3×4 matrix and B is a matrix such that $A^T B$ and BA^T are both defined. Then, B is of the type **[1]**

a) 4×4 b) 4×3

c) 3×3 d) 3×4

11. Objective function of an LPP is **[1]**

a) a function to be optimized b) None of these

c) a constraint d) a relation between the variables

12. The magnitude of the vector $6\hat{i}+2\hat{j}+3\hat{k}$ is **[1]**

a) 7 b) 5

c) 12 d) 1

13. Let $A=\begin{bmatrix}1 & 2\\ 3 & -5\end{bmatrix}$ and $B=\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}$ and X be a matrix such that A = BX, then X is equal to **[1]**

a) $\frac{1}{2}\begin{bmatrix}-2 & 4\\ 3 & 5\end{bmatrix}$ b) $\begin{bmatrix}2 & 4\\ 3 & -5\end{bmatrix}$

c) $\frac{1}{2}\begin{bmatrix}2 & 4\\ 3 & -5\end{bmatrix}$ d) None of these

14. Number X is randomly selected from the set of odd numbers and Y is randomly selected from the set of even numbers of the set {1, 2, 3, 4, 5, 6, 7}. Let Z = (X + Y). What is P (Z = 5) equal to? **[1]**

a) $\frac{1}{6}$ b) $\frac{1}{4}$

c) $\frac{1}{3}$ d) $\frac{1}{2}$

15. What is the degree of the differential equation y = $x\frac{dy}{dx}+\left(\frac{dy}{dx}\right)^{-1}$? **[1]**

a) -1 b) 1

c) Does not exist d) 2

16. The vector component of $\vec{b}$ perpendicular to $\vec{a}$ is **[1]**

a) $\frac{\vec{a}\times(\vec{b}\times\vec{a})}{|\vec{a}|^2}$ b) $(\vec{b},\vec{c})\vec{a}$

c) $\vec{a} \times (\vec{b} \times \vec{a})$ d) none of these

17. If $y = \tan^{-1}$x and $z = \cot^{-1} x$ then $\frac{dy}{dz}$ is equal to **[1]**

a) 1 b) $\frac{\pi}{2}$

c) – 1 d) None of these

18. A line makes equal angles with co-ordinate axis. Direction cosines of this line are **[1]**

a) $\pm\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ b) $\pm\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

c) $\pm(1,1,1)$ d) $\pm\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$

19. **Assertion (A):** $y = \frac{e^x+e^{-x}}{2}$ is an increasing function on [0, ∞). **[1]**

Reason (R): $y = \frac{e^x-e^{-x}}{2}$ is an increasing function on (- ∞, ∞).

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** A function f: Z → Z defined as f(x) = x^3 is injective. **[1]**

Reason (R): A function f: A → B is said to be injective if every element of B has a pre-Image in A.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$ **[2]**

OR

Find the value of $\cos^{-1}\frac{1}{2} + 2\sin^{-1}\frac{1}{2}$.

22. A balloon which always remains spherical, has a variable diameter $\frac{3}{2}(2x+1)$ Find the rate of change of its volume with respect to x. **[2]**

23. Let I be any interval disjoint from [-1, 1]. Prove that the function f given by $f(x) = x + \frac{1}{x}$ is increasing on I. **[2]**

OR

The total revenue in Rs received from the sale of x units of the product is given by $R(x) = 13x^2 + 26x + 15$. find Marginal Revenue when 17 unit are produce.

24. Evaluate: $\int_0^{\sqrt{2}} \sqrt{2-x^2}\,dx$ **[2]**

25. A spherical soap bubble is expanding so that its radius is increasing at the rate of 0.02 cm/sec. At what rate is the surface area increasing when its radius is 5 cm? (Take π = 3.14) **[2]**

Section C

26. Evaluate: $\int \frac{(x+2)}{\sqrt{2x^2+2x-3}}dx$ **[3]**

27. The contents of three bags I, II and III are as follows: **[3]**

Bag 1 : 1 white, 2 black and 3 red balls,

Bag II : 2 white, 1 black and 1 red ball;

Bag III : 4 white, 5 black and 3 red balls.

A bag is chosen at random and two balls are drawn. What is the probability that the balls are white and red?

28. Show that: $\int_0^{\pi/2} f(\sin 2x)\sin x\,dx = \sqrt{2}\int_0^{\pi/4} f(\cos 2x)\cos x\,dx$ **[3]**

OR

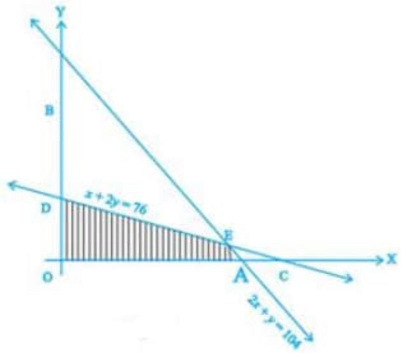
Evaluate: $\int(e^{\log x} + \sin x) \cos x \, dx$

29. Solve: $\frac{dy}{dx}$ = y sin 2x, it being given that y(0) = 1 **[3]**

OR

Find the general solution of the differential equation $\left(x^3+x^2+x+1\right)\frac{dy}{dx}=2x^2+x$

30. Determine the maximum value of Z = 3x + 4y if the feasible region (shaded) for a LPP is shown in Fig. **[3]**



OR

Minimize Z = 2x + 3y subject to the constraints $x \geq 0$, $y \geq 0$, $x + 2y \geq 1$ and $x+2y \leq 10$

31. If y = e^x cosx, prove that $\frac{d^2y}{dx^2}=2e^x\cos\left(x+\frac{\pi}{2}\right)$ **[3]**

Section D

32. Find the area of the region $\left\{(x,y):0\leqslant y\leqslant\left(x^2+1\right),0\leqslant y\leqslant(x+1),0\leqslant x\leqslant 2\right\}$ **[5]**

33. Show that the function f : R $\rightarrow$ R defined by $f(x)=\frac{x}{x^2+1}$, $\forall x\in R$, is neither one-one nor onto. **[5]**

OR

Let A = R – {3} and B = R- {1}. Consider the function of f: A $\rightarrow$ B defined by f(x) = $\frac{x-2}{x-3}$ is one – one and onto.

34. Given $A=\begin{bmatrix}2&2&-4\\-4&2&-4\\2&-1&5\end{bmatrix}$, $B=\begin{bmatrix}1&-1&0\\2&3&4\\0&1&2\end{bmatrix}$, find BA and use this to solve the system of equations y + 2z = 7, x - y = 3, 2x + 3y + 4z = 17. **[5]**

35. Find the length shortest distance between the lines: $\frac{x-6}{3}=\frac{y-7}{-1}=\frac{z-4}{1}$ and $\frac{x}{-3}=\frac{y+9}{2}=\frac{z-2}{4}$ **[5]**

OR

Find the vector equation of the line passing through the point (1, 2, 3) and parallel to the planes $\vec{r}\cdot(\hat{i}-\hat{j}+2\hat{k})=5$ and $\vec{r}\cdot(3\hat{i}+\hat{j}+\hat{k})=6$.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

A shopkeeper sells three types of flower seeds A_1, A_2, A_3. They are sold in the form of a mixture, where the proportions of these seeds are 4 : 4 : 2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.

Based on the above information:

(i) Calculate the probability that a randomly chosen seed will germinate.

(ii) Calculate the probability that the seed is of type A2, given that a randomly chosen seed germinates.

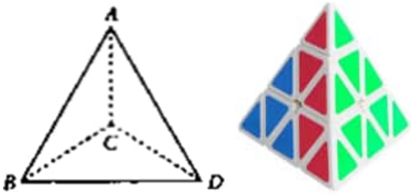
(iii) A die is throw and a card is selected at random from a deck of 52 playing cards. Then find the probability of getting an even number on the die and a spade card.

OR

If A and B are any two events such that P(A) + P(B) - P(A and B) = P(A), then find P(A|B).

37. **Read the text carefully and answer the questions:** **[4]**

A building is to be constructed in the form of a triangular pyramid, ABCD as shown in the figure.



Let its angular points are A(0, 1, 2), B(3, 0, 1), C(4, 3, 6) and D(2, 3, 2) and G be the point of intersection of the medians of $\triangle$BCD.

(i) Find the coordinates of point G

(ii) Find the length of vector $\overrightarrow{AG}$.

(iii) Find the area of $\triangle$ABC (in sq. units).

OR

Find the length of the perpendicular from the vertex D on the opposite face.

38. **Read the text carefully and answer the questions:** **[4]**

In a street two lamp posts are 600 feet apart. The light intensity at a distance d from the first (stronger) lamp post is $\frac{1000}{d^2}$, the light intensity at distance d from the second (weaker) lamp post is $\frac{125}{d^2}$ (in both cases the light intensity is inversely proportional to the square of the distance to the light source). The combined light intensity is the sum of the two light intensities coming from both lamp posts.

(i) If l(x) denotes the combined light intensity, then find the value of x so that I(x) is minimum.

(ii) Find the darkest spot between the two lights.

Solution

Section A

1.

(d) - 6, - 4, - 9

Explanation: A = $\begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and kA = $\begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix} = \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix}$

Comparing the equations,

-4k = 24

k = -6

3k = 2b

3(-6) = 2b

2b = -18

b = -9

3a = 2k

3a = 2(-6)

3a = -12

a = -4

Values are

k = -6, a = -4 & b = -9.

Which is the required solution.

2.

(c) all integral α

Explanation: The given system of equations has unique solution, if:

$\begin{vmatrix} 1 & 1 & 1 \\ 3 & 6 & 1 \\ \alpha & 2 & 3 \end{vmatrix} \neq 0$

$\Rightarrow$ 1(18 - 2) - 1(9 - α) + 1(6 - 6α) $\neq$ 0

$\Rightarrow$ 13 - 5α $\neq$ 0

$\Rightarrow \alpha \neq \frac{13}{5}$. (Since α is not integral value)

Thus, unique solution exists for all integral values of α.

3.

(c) (AB)’ = B’A’

Explanation: By the property of transpose of a matrix, (AB)’ = B’A’.

4.

(b) -1

Explanation: $\sqrt{x} + \sqrt{y} = 1$

Differentiating with respect to x,

$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\sqrt{\frac{y}{x}}$

$\frac{dy}{dx}\left(\frac{1}{4^{-1}\frac{1}{4}}\right) = -\sqrt{\frac{\frac{1}{4}}{\frac{1}{4}}} = -1$

5. **(a)** 7

Explanation: Clearly, the given line passes through the point (4, 2, k).

Since, the given line lies in the plane 2x - 4y + z = 7, so the above point lies in this plane.

(2 × 4) - (4 × 2) + k = 7

$\Rightarrow$ k = 7 .

6. **(a)** $y(1+x^2)=log|sinx|+c$

Explanation: $(1+x^2)dy=(cotx-2xy)dx$

$\frac{dy}{dx}=\frac{cotx-2xy}{1+x^2}$

$\frac{dy}{dx}+\frac{2x}{1+x^2}y=\frac{cotx}{1+x^2}$

It is a linear differential equation in y.

Therefore, Solution is

$ye^{\int\frac{2xdx}{1+x^2}}=\int\frac{cotx}{1+x^2}e^{\int\frac{2xdx}{1+x^2}}dx+c$

$y(1+x^2)=\int\frac{cotx}{1+x^2}(1+x^2)dx+c$

$y(1+x^2)=\int cotxdx+c$

$y(1+x^2)=log|sinx|+c$

7. **(a)** 1260

Explanation: We have , Maximize Z = 100x + 120y , subject to constraints 2x + 3y ≤ 30, 3x + y ≤ 17, x ≥ 0, y ≥ 0.

Corner points	Z = 100x +120 y
P(0 , 0)	0
Q(3 , 8)	1260…………………(Max.)
R(0, 10)	1200
S(17/3 , 0)	1700/3

Hence the maximum value is 1260

8.

(d) $\frac{41}{2}$ sq units

Explanation: $\vec{a}=3\hat{\imath}+4\hat{\jmath}$

$\vec{b}=-5\hat{\imath}+7\hat{\jmath}$

For area of triangle we require $\frac{1}{2}|\vec{a}\times\vec{b}|$

$\vec{a}\times\vec{b}=41\hat{k}$

$\frac{1}{2}|\vec{a}\times\vec{b}|=\frac{1}{2}\sqrt{41^2}=\frac{41}{2}$

9.

(b) tan x - cot x + C

Explanation: Let I = $\int\frac{dx}{\sin^2x\cos^2x}=\int\frac{1}{\sin^2x\cos^2x}dx$

$=\int\frac{\sin^2x+\cos^2x}{\sin^2x\cos^2x}dx$

$=\int\frac{\sin^2x}{\sin^2x\cos^2x}dx+\int\frac{\cos^2x}{\sin^2x\cos^2x}dx$

$=\int\sec^2xdx+\int cosec^2xdx$

= tanx - cotx + C

10.

(d) 3×4

Explanation: We have to find: Order of A = 3×4

Order of A’ = 4×3

As A^TB and BA^T are both defined, so the number of columns in B should be equal to the number of rows in A’ for BA’ and also the number of columns in A’ should be equal to the number of rows in A’ for BA’.

Therefore, the order of matrix B is 3×4.

11. **(a)** a function to be optimized

Explanation: a function to be optimized

The objective function of a linear programming problem is either to be maximized or minimized i.e. objective function is to be optimized.

12. **(a)** 7

Explanation: 7

13.

(c) $\frac{1}{2}\begin{bmatrix}2 & 4\\3 & -5\end{bmatrix}$

Explanation: A = BX

$B^{-1}A = B^{-1}BX$

$X = B^{-1}A$

Using Adjoint method of inverse

$B^{-1} = \frac{1}{2}\begin{bmatrix}2 & 0\\0 & 1\end{bmatrix}$

$X = B^{-1}A$

$X = \frac{1}{2}\begin{bmatrix}2 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\3 & -5\end{bmatrix}$

$X = \frac{1}{2}\begin{bmatrix}2 & 4\\3 & -5\end{bmatrix}$

14. **(a)** $\frac{1}{6}$

Explanation: Given that, X = Set of odd numbers from the set A.

Y = Set of even numbers from the set A.

Let set A = {1, 2, 3, 4, 5, 6, 7} and Z = X + Y

Now, Z = 5 is only possible when X = 1, 3 and Y = 4, 2

Sample space = {(1, 2,), (1, 4), (1, 6), (3, 2), (3, 4), (3, 6), (5, 2), (5, 4), (5, 6), (7, 2), (7, 4), (7, 6)}

$\therefore$ n(S) = 12 and favourable space = {(1, 4), (3, 2)}

$\therefore$ n(E) = 2

So, P(Z = 5) = $\frac{n(E)}{n(S)} = \frac{2}{12} = \frac{1}{6}$

15.

(d) 2

Explanation: Given differential equation is

$y = x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^{-1} \Rightarrow y = x\frac{dy}{dx} + \frac{1}{(dy/dx)}$

$\Rightarrow y\left(\frac{dp}{dx}\right) = x\left(\frac{dy}{dx}\right)^2 + 1$

$\therefore$ Degree = Power of highest derivative = 2

16. **(a)** $\frac{\vec{a}\times(\vec{b}\times\vec{a})}{|\vec{a}|^2}$

Explanation: we know that the component of $\vec{b}$ on $\vec{a}$ is $\frac{(\vec{b}.\vec{a}).\vec{a}}{|\vec{a}|^2}$

$\Longrightarrow$ the components of $\vec{b}$ perpendicular to $\vec{a}$ is $\vec{b}$ - $\frac{(\vec{b}.\vec{a}).\vec{a}}{|\vec{a}|^2} \Longrightarrow \frac{\vec{b}(\vec{a}.\vec{a})-(\vec{b}.\vec{a})(\vec{a})}{|\vec{a}|^2}$

$\Longrightarrow \frac{\vec{a}\times(\vec{b}\times\vec{a})}{|\vec{a}|^2}$

17.

(c) – 1

Explanation: $\frac{dy}{dz} = \frac{\frac{d}{dx}(\tan^{-1}x)}{\frac{d}{dx}(\cot^{-1}x)} = \frac{\frac{1}{1+x^2}}{-\frac{1}{1+x^2}} = -1$

18. **(a)** $\pm\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Explanation: Let the line makes angle α with each of the axis. Then, its direction cosines are cos α, cos α, cos α.

Since $\cos^2 \alpha + \cos^2 \alpha + \cos^2 \alpha = 1$

$\Rightarrow 3\cos^2 \alpha = 1$

$\Rightarrow \cos\alpha = \pm\frac{1}{\sqrt{3}}$

Therefore, direction cosines are $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$

19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: Assertion Let f(x) = $\frac{e^x+e^{-x}}{2}$

$\Rightarrow$ f'(x) = $\frac{e^x-e^{-x}}{2} = \frac{1}{2}\left(e^x - \frac{1}{e^x}\right)$

= $\frac{1}{2}\left(\frac{e^{2x}-1}{e^x}\right)$ ...(i)

Now, for x $\geq$ 0, we have

2x $\geq$ 0 $\Rightarrow$ $e^{2x} \geq e^0$ [$\because$ e^x is an increasing function]

$\Rightarrow$ $e^{2x} \geq 1$

Also, for x $\geq$ 0

$\Rightarrow$ $e^x \geq 1$

$\therefore$ From Eq. (i), we have

f'(x) = $\frac{1}{2}\left(\frac{e^{2x}-1}{e^x}\right) \geq 0$

So, f(x) is an increasing function on [0, ∞).

Reason: Let g(x) = $\frac{e^x-e^{-x}}{2}$

$\Rightarrow$ g'(x) = $\frac{e^x+e^{-x}}{2} > 0$ [$\because$.e^x and e^{-x} both are greater than zero in (-∞, ∞)]

So, g(x) is an increasing function on (-∞, ∞).

Hence, both Assertion and Reason are true.

20.

(c) A is true but R is false.

Explanation: Assertion is true because distinct elements in Z (domain) has distinct images in Z (codomain).

Reason is false because of: A $\rightarrow$ B is said to be surjective if every element of B has at least one pre-Image in A.

Section B

21. Let $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = y$

$\Rightarrow \cos y = -\frac{1}{\sqrt{2}}$

$\Rightarrow \cos y = -\cos\frac{\pi}{4}$

$\Rightarrow \cos y = \cos\left(\pi - \frac{\pi}{4}\right) = \cos\frac{3\pi}{4}$

Since, the principal value branch of $\cos^{-1}$ is $[0, \pi]$.

Therefore, Principal value of $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$ is $\frac{3\pi}{4}$.

OR

Let $\cos^{-1}\left(\frac{1}{2}\right) = x$. Then, $\cos x = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$.

$\therefore \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

Let $\sin^{-1}\left(\frac{1}{2}\right) = y$. Then, $\sin y = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$.

$\therefore \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\therefore \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + \frac{2\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$

22. Given: Diameter of the balloon $= \frac{3}{2}(2x+1)$

$\therefore$ Radius of the balloon $= \frac{3}{4}(2x+1)$

$\therefore$ Volume of the balloon $= \frac{4}{3}\pi\left(\frac{3}{4}(2x+1)\right)^3$

$= \frac{9\pi}{16}(2x+1)^3$ cube units

$\therefore$ Rate of change of volume w.r.t. $x = \frac{dV}{dx}$

$= \frac{9\pi}{16}.3(2x+1)^2.\frac{d}{dx}(2x+1)$

$= \frac{27\pi}{16}(2x+1)^2.2$

$= \frac{27\pi}{8}(2x+1)^2$

23. Given: $f(x) = x + \frac{1}{x} = x + x^{-1}$

$\Rightarrow f'(x) = 1 + (-1)x^{-2} = 1 - \frac{1}{x^2} = \frac{x^2-1}{x^2}$

$\Rightarrow f'(x) = \frac{(x-1)(x+1)}{x^2}$ ...(i)

Here for every x either $x < -1$ or x > 1

$\therefore$ for x< -1, x = -2 (say)

$f'(x)\frac{(-)(-)}{(+)} = (+) > 0$

And for x > 1, x = 2 (say),

$f'(x)\frac{(+)(+)}{(+)}=(+)>0$

$\therefore f'(x)>0$ for all $x \in I$, hence f(x) is strictly increasing on I.

OR

Given $R(x)=13x^2+26x+15$

Now, $MR=\frac{d}{dx}(R(x))=26x+26$

$MR]_{x=17}=25\times 17+26$

= 425 + 26

= 451

24. Let I = $\int_0^{\sqrt{2}}\sqrt{2-x^2}dx$. Then, we have

$I=\int_0^{\sqrt{2}}\sqrt{(\sqrt{2})^2-x^2}dx$

$\Rightarrow I=\left[\frac{1}{2}x\sqrt{2-x^2}+\frac{1}{2}(\sqrt{2})^2\sin^{-1}\frac{x}{\sqrt{2}}\right]_0^{\sqrt{2}}$

$=\left\{0+\sin^{-1}(1)\right\}-\left\{0+\sin^{-1}0\right\}=\frac{\pi}{2}$

25. A soap bubble is in the form of a sphere. At an instant t, let its radius be r and surface area S. Then,

$\frac{dr}{dt}$ = 0.02 cm /sec ... (given) ... (i)

Now, S = $4\pi r^2$

$\Rightarrow \frac{dS}{dt}$ = $8\pi r.\frac{dr}{dt}$= (8 × 3.14 × r × 0.02) cm^2/sec

$\Rightarrow \left[\frac{dS}{dt}\right]_{r=5}$ = (8 × 3.14 × 5 × 0.02)cm^2/sec = 2.512 cm^2/sec

Hence, the surface area of the bubble is increasing at the rate of 2.512 cm^2/sec at the instance when its radius is 5 cm.

Section C

26. Formula to be used $\int\frac{dx}{\sqrt{x^2\pm a^2}}=\log(x+\sqrt{x^2\pm a^2})+c$ where is the integrating constant

$\therefore\int\frac{(x+2)}{\sqrt{2x^2+2x-3}}dx$

$=\int\frac{\frac{1}{4}\times(4x+2)+\frac{3}{2}}{\sqrt{2x^2+2x-3}}dx=I_1+I_2$

$Now, I_1=\frac{1}{4}\int\frac{(4x+2)}{\sqrt{2x^2+2x-3}}dx=\frac{1}{4}\int\frac{(4x+2)}{\sqrt{2x^2+2x-3}}dx$

Tip - Assuming $2x^2 + 2x - 3 = a^2$, (4x + 2)dx = 2ada

$\therefore\frac{1}{4}\int\frac{(4x+2)}{\sqrt{2x^2+2x-3}}dx$

$=\frac{1}{4}\int\frac{2ada}{a}$

$=\frac{1}{4}\int\frac{2ada}{a}$

$=\frac{a}{2}+c_1$

$=\frac{\sqrt{2x^2+2x-3}}{2}+c_1$

$Now, I_2=\frac{3}{2}\int\frac{1}{\sqrt{2x^2+2x-3}}dx$

$=\frac{3}{2}\int\frac{dx}{\sqrt{2\left(x+\frac{1}{2}\right)^2-(\sqrt{\frac{7}{2}})^2}}$

$=\frac{3}{2\sqrt{2}}\int\frac{dx}{\sqrt{\left(x+\frac{1}{2}\right)^2-\left(\frac{\sqrt{7}}{2}\right)^2}}$

$=\frac{3}{2\sqrt{7}}\log\left|\left(x+\frac{1}{2}\right)\right|$

$=\frac{3}{2\sqrt{7}}\log\left|\left(x+\frac{1}{2}\right)\right|$

$\therefore\frac{1}{4}\int\frac{(4x+2)}{\sqrt{2x^2+2x-3}}dx+\frac{3}{2}\int\frac{1}{\sqrt{2x^2+2x-3}}dx$

$=\frac{\sqrt{2x^2+2x-3}}{2}+\frac{3}{2\sqrt{7}}\log\left|\left(x+\frac{1}{2}\right)\right|$ + C,

Where C is the integrating constant.

27. A white ball and a red ball can be drawn in three mutually exclusive ways:

i. Selecting bag I and then drawing a white and a red ball from it
ii. Selecting bag II and then drawing a white and a red ball from it
iii. Selecting bag III and then drawing a white and a red ball from it

Consider the following events:
E_1 = Selecting bag I
E_2 = Selecting bag II
E_3 = Selecting bag II
A = Drawing a white and a red ball
It is given that one of the bags is selected randomly.
$\therefore P\left(E_1\right)=\frac{1}{3}$
P(E_2) = $\frac{1}{3}$
P(E_3) = $\frac{1}{3}$
Now,
$P\left(\frac{A}{E_1}\right)=\frac{{}^1C_1\times{}^3C_1}{{}^6C_2}=\frac{3}{15}$
$P\left(\frac{A}{E_2}\right)=\frac{{}^2C_1\times{}^1C_1}{{}^4C_2}=\frac{2}{6}$
$P\left(\frac{A}{E_3}\right)=\frac{{}^4C_1\times{}^3C_1}{{}^{12}C_2}=\frac{12}{66}$
Using the law of total probability, we get
Required probability = P(A) = P(E_1)P($\frac{A}{E_1}$) + P(E_2)P($\frac{A}{E_2}$) + P(E_3)P($\frac{A}{E_3}$)
$=\frac{1}{3}\times\frac{3}{15}+\frac{1}{3}\times\frac{2}{6}+\frac{1}{3}\times\frac{12}{66}$
$=\frac{1}{15}+\frac{1}{9}+\frac{2}{33}$
$=\frac{33+55+30}{495}=\frac{118}{495}$

28. Let $I=\int_0^{\pi/2}f(\sin 2x)\sin x dx$ ...(i)
Then, $I=\int_0^{\pi/2}f\left\{\sin 2\left(\frac{\pi}{2}-x\right)\right\}\sin\left(\frac{\pi}{2}-x\right)dx$ $\left[\because\int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$
$\Rightarrow\quad I=\int_0^{\pi/2}f\{\sin(\pi-2x)\}\cos x dx$
$\Rightarrow\quad I=\int_0^{\pi/2}f(\sin 2x)\cos x dx$ ...(ii)
Adding (i) and (ii), we get
$2I=\int_0^{\pi/2}f(\sin 2x)(\sin x+\cos x)dx$
$\Rightarrow\quad 2I=2\int_0^{\pi/4}f(\sin 2x)(\sin x+\cos x)dx$
$\Rightarrow\quad 2I=2\sqrt{2}\int_0^{\pi/4}f(\sin 2x)\left(\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x\right)dx$
$\Rightarrow\quad 2I=2\sqrt{2}\iint_0^{\pi/4}f(\sin 2x)\sin\left(x+\frac{\pi}{4}\right)dx$
$\Rightarrow\quad 2I=2\sqrt{2}\int_0^{\pi/4}f\left\{\sin 2\left(\frac{\pi}{4}-x\right)\right\}\sin\left(\frac{\pi}{4}-x+\frac{\pi}{4}\right)dx$ By using the property of defnite integrals
$\left[\because\int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$
$\Rightarrow\quad 2I=2\sqrt{2}\int_0^{\pi/4}f\left\{\sin\left(\frac{\pi}{2}-2x\right)\right\}\sin\left(\frac{\pi}{2}-x\right)dx$
$\Rightarrow\quad 2I=2\sqrt{2}\int_0^{\pi/4}f(\cos 2x)\cos x dx$
$\Rightarrow\quad I=\sqrt{2}\int_0^{\pi/4}f(\cos 2x)\cos x dx$
Hence, $\int_0^{\pi/2}f(\sin 2x)\sin x dx=\sqrt{2}\int_0^{\pi/4}f(\cos 2x)\cos x dx$

OR

Let the given integral be ,
I = $\int$($e^{\log x}$ + sin x) cos x dx
= $\int$(x + sin x) cos x dx
= $\int$ x cos x dx + $\int$ sin x cos x dx
= [x $\int$cos x dx - $\int$(1 $\int$cos x dx) dx] + $\frac{1}{2}\int$sin 2x dx
= [x sin x - $\int$sin x dx] + $\frac{1}{2}\left(-\frac{\cos 2x}{2}\right)$ + C
I = x som x + cos x - $\frac{1}{4}$cos 2x + c
I = x sin x + cos x - $\frac{1}{4}$[1 - 2 $\sin^2$ x] + c
I = x sin x + cos x - $\frac{1}{4}+\frac{1}{2}\sin^2$ x + c
I = x sin x + cos x - $\frac{1}{2}\sin^2 x+c-\frac{1}{4}$
I = x sin x + cos x - $\frac{1}{2}\sin^2 2$ x + k, where k = c - $-\frac{1}{4}$

29. The given differential equation is,

$\frac{dy}{dx}$ = y sin 2x

$\Rightarrow \frac{1}{y}$ dy = sin 2x dx

$\Rightarrow \int \frac{1}{y} dy = \int$ sin 2x dx [Integrating both sides]

$\Rightarrow$ log |y| = $-\frac{1}{2}$ cos 2x + C ...(i)

It is given that y (0) = 1 i.e. y = 1 when x = 0. Putting x = 0 and y = 1 in (i), we get

$0 = -\frac{1}{2} + C \Rightarrow C = \frac{1}{2}$

Putting C = $\frac{1}{2}$ in (i), we get,

log |y| = $-\frac{1}{2}$ cos 2x + $\frac{1}{2}$

$\Rightarrow$ log |y| = $\frac{1}{2}$ (1 - cos 2x)

$\Rightarrow$ log |y| = $\sin^2$ x

$\Rightarrow |y| = e^{\sin^2 x}$

$\Rightarrow y = \pm e^{\sin^2 x}$

$\Rightarrow y = e^{\sin^2 x}$ or, $y = -e^{\sin^2 x}$

But, $y = -e^{\sin^2 x}$ is not satisfied by y(0) = 1

Hence, $y = e^{\sin^2 x}$ is the required solution.

OR

The given differential equation can be rewritten as

$\frac{dy}{dx} = \frac{2x^2+x}{(x^3+x^2+x+1)} = \frac{(2x^2+x)}{x^2(x+1)+(x+1)} = \frac{(2x^2+x)}{(x+1)(x^2+1)}$

$\Rightarrow dy = \left\{\frac{(2x^2+x)}{(x+1)(x^2+1)}\right\} dx$

$\Rightarrow \int dy = \int \frac{(2x^2+x)}{(x+1)(x^2+1)} dx$ ... (i) [integrating both sides]

$\Rightarrow y = \int \frac{(2x^2+x)}{(x+1)(x^2+1)} dx + C$ where C is an arbitrary constant.

Let $\frac{2x^2+x}{(x+1)(x^2+1)} = \frac{A}{(x+1)} + \frac{Bx+C}{(x^2+1)}$

Therefore, 2 x^2 + x = A (x^2+1) + (Bx+C)(x+1) ...(ii)

Put x = -1 in (ii), we have, 2A = 1 $\Rightarrow A = \frac{1}{2}$

Put x = 0 in (ii), we have, A + C = 0 $\Rightarrow C = -A = \frac{-1}{2}$

Put x = 1 in (ii), we have, 2A + 2B + 2C = 3.

$\therefore \quad A + B + C = \frac{3}{2} \Rightarrow \frac{1}{2} + B - \frac{1}{2} = \frac{3}{2} \Rightarrow B = \frac{3}{2}$

$\therefore \quad A = \frac{1}{2}, B = \frac{3}{2}$ and $C = \frac{-1}{2}$

$\therefore \quad y = \frac{1}{2} \int \frac{dx}{(x+1)} + \int \frac{\left(\frac{3}{2}x - \frac{1}{2}\right)}{(x^2+1)} dx + C$

$\Rightarrow y = \frac{1}{2}\log|x+1| + \frac{3}{4} \int \frac{2x}{(x^2+1)} dx - \frac{1}{2}\tan^{-1} x + C$

$\Rightarrow y = \frac{1}{2}\log|x+1| + \frac{3}{4} \int \frac{2x}{(x^2+1)} dx - \frac{1}{2}\tan^{-1} x + C$

$\Rightarrow \quad y = \frac{1}{2}\log|x+1| + \frac{3}{4}\log\left|x^2+1\right| - \frac{1}{2}\tan^{-1} x + C$

Therefore, y = $\frac{1}{2}\log|x+1| + \frac{3}{4}\log\left|x^2+1\right| - \frac{1}{2}\tan^{-1} x + C$ is the required solution.

30. As it is clear from the given graph, the coordinates of the corner points O, A, E and Dare given as (0, 0), (52, 0), (144, 16) and (0, 38), respectively. Also given region is bounded.

Given that the objective function, Z = 3x + 4y to be maximised.

$\because$ Also converting the given inequalities into their equations to find their points of intersection ,

2x + y = 104.............(i) and

2x + 4y = 152.............(ii)

solving above equations (i) and (ii), we get

$\Rightarrow$ -3y = -48

$\Rightarrow$ y = 16 and x = 44.

Hence the point of intersection of the lines (i) and (ii) is (16,44)

The table below gives the values of the objective function Z, at the corner points of the feasible region.

Corner Points	**Corresponding value of Z**
(0, 0)	Z = 0

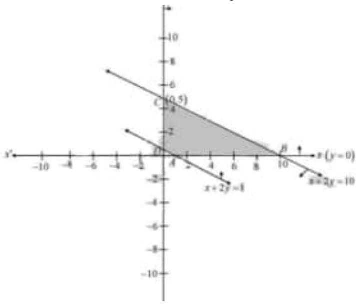
(52, 0)	Z = 156
(44, 16)	Z = 196 (maximum)
(0, 38)	Z = 152

Hence, Z attains its maximum at the point (44,16) and its maximum value is 196.

OR

We have, Z = 2x + 3y subject to the constraints $x \geq 0$, $y \geq 0$, $x + 2y \geq 1$ and $x+2y \leq 10$

Now draw the line x + 2y = 1 and x + 2y = 10



and shaded region satisfied by above inequalities Here the feasible region is bounded.

The corner points are given as A(1, 0), B(10, 0), C(0, 5), and $D\left(0, \frac{1}{2}\right)$

The value of Z at A (1,0) = 2 ,B (10,0) = 20, at C (0,5) = 15 and $D\left(0, \frac{1}{2}\right) = \frac{3}{2}$

At corner points, the minimum value of Z is $\frac{3}{2}$ this is the required solution which occurs at $D\left(0, \frac{1}{2}\right)$

31. Given,

$y = e^x \cos x$

To prove:

$\frac{d^2y}{dx^2} = 2e^x \cos\left(x + \frac{\pi}{2}\right)$

We have,

$\frac{dy}{dx} = \frac{d}{dx}(e^x \cos x)$

Using product rule, we get;

$\frac{dy}{dx} = e^x \frac{d}{dx}(\cos x) + \cos x \frac{dy}{dx} e^x$

$\frac{dy}{dx} = -e^x \sin x + e^x \cos x \quad \left[\because \frac{d}{dx}(\cos x) = -\sin x \& \frac{d}{dx} e^x = e^x\right]$

Again differentiating w.r.t x:

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(-e^x \sin x + e^x \cos x)$

$= \frac{d}{dx}(-e^x \sin x) + \frac{d}{dx}(e^x \cos x)$

Again using the product rule:

$\frac{d^2y}{dx^2} = -e^x \frac{d}{dx}(\sin x) - \sin x \frac{d}{dx} e^x + e^x \frac{d}{dx}(\cos x) + \cos x \frac{d}{dx}(e^x)$

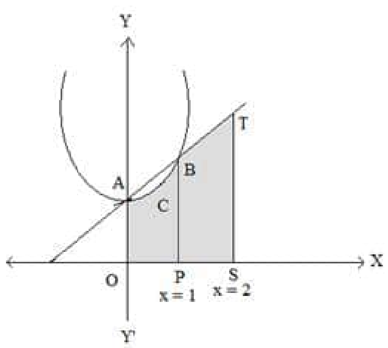
$\frac{d^2y}{dx^2}$ = $-e^x \cos x - e^x \sin x - e^x \sin x + e^x \cos x$

$\frac{d^2y}{dx^2}$ = $-2e^x \sin x$ [$\because -\sin x = \cos(x + \pi/2)$]

$\frac{d^2y}{dx^2} = -2e^x \cos\left(x + \frac{\pi}{2}\right)$

Hence proved

Section D

32. 

There are three curves respectively,

$C_1 = \{(x, y): 0 \leq y \leq x^2 + 1\}$

$C_2 = \{(x, y): 0 \leq y \leq x + 1\}$

$C_3 = \{(x, y): 0 \leq x \leq 2\}$

The points of intersection of, $y = x^2$ and y = x + 1y are A(0,1) and B(1,2).

The required area of shaded region = $\int_0^1 y_1 dx + \int_1^2 y_2 dx$

where $y_1 = x^2 + 1$ and $y_2 = x + 1$

$\therefore A = \int_0^1 (x^2 + 1)dx + \int_1^2 (x + 1)dx$

$= [\frac{x^3}{3} + x]_0^1 + [\frac{x^2}{2} + x]_1^2$

$= (\frac{4}{3} - 0) + (4 - \frac{3}{2}) = \frac{8+24-9}{6} = \frac{23}{6}\ sq. units$

33. For $x_1, x_2 \in R$, consider

$f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1}{x_1^2+1} = \frac{x_2}{x_2^2+1}$

$\Rightarrow x_1 x_2^2 + x_1 = x_2 x_1^2 + x_2$

$\Rightarrow x_1 x_2(x_2 - x_1) = x_2 - x_1$

$\Rightarrow x_1 = x_2$ or $x_1 x_2 = 1$

We note that there are point, x_1 and x_2 with $x_1 \neq x_2$ and $f(x_1) = f(x_2)$ for instance, if we take $x_1 = 2$ and $x_2 = \frac{1}{2}$, then we have $f(x_1)\frac{2}{5}$ and $f(x_2) = \frac{2}{5}$ but $2 \neq \frac{1}{2}$. Hence f is not one-one. Also, f is not onto for if so then for $1 \in R \exists x \in R$ such that f(x) = 1 which gives $\frac{x}{x^2+1} = 1$. But there is no such x in the domain R, since the equation $x^2 - x + 1 = 0$ does not give any real value of x.

OR

For all $x_1\ x_2 \in A$

if $f(x_1) = f(x_2)$ implies $x_1 = x_2$ then f is one one

Now $f(x_1) = f(x_2)$

$\frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$

Cross multiplying and solving, we get

$x_1 = x_2$

f is one – one

$y = \frac{(x-2)}{(x-3)}$

$(x-3)y = x - 2$

$xy - 3y = x - 2$

$xy - x = 3y - 2$

$x = \frac{(3y-2)}{(y-1)}$

$f\left(\frac{3y-2}{y-1}\right) = y$

Hence f is onto.

34. We have, A = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, B = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$

$\therefore$ BA = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$ = 6I

$\therefore B^{-1} = \frac{A}{6} = \frac{1}{6}$ A = $\frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$

Also, x - y = 3, 2x + 3y + 4z = 17 and y + 2z = 7

$\Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$= \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$ [using Eq. (i)]

$= \frac{1}{6} \begin{bmatrix} 6+34-28 \\ -12+34-28 \\ 6-17+35 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$

$\therefore$ x = 2, y = -1 and z = 4

35. Here,it is given equations of lines:

$L_1 : \frac{\mathrm{x}-6}{3} = \frac{\mathrm{y}-7}{-1} = \frac{\mathrm{z}-4}{1}$

$L_2 : \frac{x}{-3} = \frac{y+9}{2} = \frac{z-2}{4}$

Direction ratios of L_1 and L_2 are (3, -1, 1) and (-3, 2, 4) respectively.

Suppose general point on line L_1 is P = (x_1, y_1, z_1)

$x_1 = 3s + 6$, $y_1 = -s + 7$, $z_1 = s + 4$

and suppose general point on line L_2 is Q = (x_2, y_2, z_2)

$x_2 = -3t$, $y_2 = 2t - 9$, $z_2 = 4t + 2$

$\therefore \overrightarrow{\mathrm{PQ}} = (\mathrm{x}_2 - \mathrm{x}_1)\,\hat{\mathrm{i}} + (\mathrm{y}_2 - \mathrm{y}_1)\,\hat{\mathrm{j}} + (z_2 - z_1)\,\hat{k}$

$= (-3t - 3s - 6)i + (2t - 9 + s - 7)\hat{j} + (4t + 2 - s - 4)\hat{k}$

$\therefore \overrightarrow{PQ} = (-3t - 3s - 6)\,\hat{\imath} + (2t + s - 16)\} + (4t - s - 2)\hat{k}$

Direction ratios of $\overrightarrow{\mathrm{PQ}}$ are ((-3t - 3s - 6), (2t + s - 16), (4t - s - 2))

PQ will be the shortest distance if it is perpendicular to both the given lines

Thus, by the condition of perpendicularity,

$\Longrightarrow$ 3(-3t - 3s - 60) - 1(2t + s - 16) + 1(4t - s - 2) = 0 and

$\Longrightarrow$ -3(-3t - 3s - 6) + 2(2t + s - 16) + 4(4t - s - 2) = 0

$\Rightarrow$ -9t - 9s - 18 - 2t - s + 16 + 4t - s - 2 = 0 and

9t + 9s + 18 + 4t + 2s + - 32 + 16t - 4s - 8 = 0

$\Rightarrow$ -7t - 11s = 4 and

29t + 7s = -22

Solving above two equations,the we obtain

t = 1 and s = -1

therefore

P = (3, 8, 3) and Q = (-3, -7, 6)

Now, distance between points P and Q is

$d = \sqrt{(3+3)^2 + (8+7)^2 + (3-6)^2}$

$= \sqrt{(6)^2 + (15)^2 + (-3)^2}$

$= \sqrt{36 + 225 + 9}$

$= \sqrt{270}$

$= 3\sqrt{30}$

Thus, the shortest distance between two given lines is

$d = 3\sqrt{30}$ units

Now, the equation of the line passing through points P and Q is

$\frac{x-x_1}{x_1-x_2} = \frac{y-y_1}{y_1-y_2} = \frac{z-z_1}{z_1-z_2}$

$\therefore \frac{x-3}{3+3} = \frac{y-8}{8+7} = \frac{z-3}{3-6}$

$\therefore \frac{x-3}{6} = \frac{y-8}{15} = \frac{z-3}{-3}$

$\therefore \frac{x-3}{2} = \frac{y-8}{5} = \frac{z-3}{-1}$ thus, the equation of the line of the shortest distance between two given lines is

$\frac{x-3}{2} = \frac{y-8}{5} = \frac{z-3}{-1}$

OR

Suppose the required line is parallel to vector $\vec{b}$

Which is given by $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

We know that the position vector of the point (1, 2, 3) is given by

$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

The equation of line passing through (1, 2, 3) and parallel to $\vec{b}$ is given by

$\vec{r} = \vec{a} + \lambda\vec{b}$

$\Rightarrow \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right)$ ...(i)

The equation of the given planes are

$\vec{r}.(\hat{i} - \hat{j} + 2\hat{k}) = 5$ ...(ii)

and $\vec{r}.(3\hat{i} + \hat{j} + \hat{k}) = 6$ ...(iii)

The line in Equation (i) and plane in Eq. (ii) are parallel.

Therefore, the normal to the plane of Eq. (ii) is perpendicular to the given line

$\therefore (\hat{i} - \hat{j} + 2\hat{k}) \cdot \left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right) = 0$

$\Rightarrow (b_1 - b_2 + 2b_3) = 0$ ....(iv)

Similarly, from Eqs. (i) and (iii), we get

$(3\hat{i} + \hat{j} + \hat{k}).(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) = 0$

$\Rightarrow (3b_1 + b_2 + b_3) = 0$ ......(v)

On solving Equations . (iv) and (v) by cross-multiplication, we get

$\frac{b_1}{(-1)\times1-1\times2} = \frac{b_2}{2\times3-1\times1} = \frac{b_3}{1\times1-3(-1)}$

$\Rightarrow \frac{b_1}{-3} = \frac{b_2}{5} = \frac{b_3}{4}$

Therefore, the direction ratios of $\vec{b}$ are (-3, 5, 4).

$\vec{b} = -3\hat{i} + 5\hat{j} + 4\hat{k}\left[\because \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right]$

On substituting the value of b in Equation (i), we get

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-3\hat{i} + 5\hat{j} + 4\hat{k})$

which is the equation of the required line.

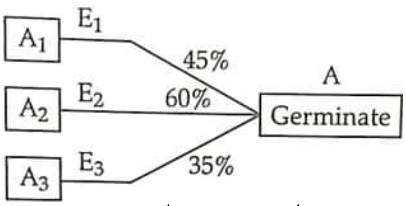
Section E

36. **Read the text carefully and answer the questions:**

A shopkeeper sells three types of flower seeds A_1, A_2, A_3. They are sold in the form of a mixture, where the proportions of these seeds are 4 : 4 : 2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.

Based on the above information:

(i)



Here, $P(E_1) = \frac{4}{10}$, $P(E_2) = \frac{4}{10}$, $P(E_3) = \frac{2}{10}$

$P\left(\frac{A}{E_1}\right) = \frac{45}{100}, P\left(\frac{A}{E_2}\right) = \frac{60}{100}, P\left(\frac{A}{E_3}\right) = \frac{35}{100}$

$\therefore P(A) = P(E_1) \cdot P\left(\frac{A}{E_1}\right) + P(E_2) \cdot P\left(\frac{A}{E_2}\right) + P(E_3) \cdot P\left(\frac{A}{E_3}\right)$

$= \frac{4}{10} \times \frac{45}{100} + \frac{4}{10} \times \frac{60}{100} + \frac{2}{10} \times \frac{35}{100}$

$= \frac{180}{1000} + \frac{240}{1000} + \frac{70}{100}$

$= \frac{490}{1000} = 4.9$

(ii) Required probability = $P\left(\frac{E_2}{A}\right)$

= $\frac{P(E_2)\cdot P\left(\frac{A}{E_2}\right)}{P(A)}$

= $\frac{\frac{4}{10}\times\frac{60}{100}}{\frac{490}{1000}}$

= $\frac{240}{490}=\frac{24}{49}$

(iii) Let,

E_1 = Event for getting an even number on die and

E_2 = Event that a spade card is selected

$\therefore$ P(E_1) = $\frac{3}{6}$

= $\frac{1}{2}$

and P(E_2) = $\frac{13}{52}=\frac{1}{4}$

Then, P($E_1 \cap E_2$) = P(E_1) $\cdot$ P(E_2)

= $\frac{1}{2}, \frac{1}{4}=\frac{1}{8}$

OR

P(A) + P(B) - P(A and B) = P(A)

$\Rightarrow$ P(A) + P(B) - P(A $\cap$ B) = P(A)

$\Rightarrow$ P(B) - P(A $\cap$ B) = 0

$\Rightarrow$ P(A $\cap$ B) = P(B)

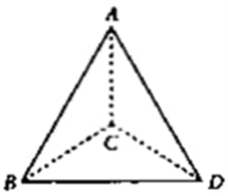
$\therefore$ P(A|B) = $\frac{P(A\cap B)}{P(B)}$

= $\frac{P(B)}{P(B)}$

= 1

37. **Read the text carefully and answer the questions:**

A building is to be constructed in the form of a triangular pyramid, ABCD as shown in the figure.



Let its angular points are A(0, 1, 2), B(3, 0, 1), C(4, 3, 6) and D(2, 3, 2) and G be the point of intersection of the medians of $\triangle$BCD.

(i) Clearly, G be the centroid of $\triangle$BCD, therefore coordinates of G are $\left(\frac{3+4+2}{3}, \frac{0+3+3}{3}, \frac{1+6+2}{3}\right)$ = (3, 2, 3)

(ii) Since, A $\equiv$ (0, 1, 2) and G = (3, 2, 3)

$\therefore \overrightarrow{AG}$ = (3 - 0)$\hat{i}$ + (2 - 1)$\hat{j}$ + (3 - 2)$\hat{k}$ = 3$\hat{i}$ + $\hat{j}$ + $\hat{k}$

$\Rightarrow |\overrightarrow{AG}|^2 = 3^2 + 1^2 + 1^2 = 9 + 1 + 1 = 11$

$\Rightarrow |\overrightarrow{AG}| = \sqrt{11}$

(iii) Clearly, area of $\triangle$ABC = $\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|$

Here, $\overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3-0 & 0-1 & 1-2 \\ 4-0 & 3-1 & 6-2 \end{vmatrix}$

= $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & -1 \\ 4 & 2 & 4 \end{vmatrix}$

= $\hat{i}$(-4 + 2) - $\hat{j}$(12 + 4) + $\hat{k}$(6 + 4) = -2$\hat{i}$ - 16$\hat{j}$ + 10$\hat{k}$

$\therefore |\overrightarrow{AB}\times\overrightarrow{AC}| = \sqrt{(-2)^2+(-16)^2+10^2}$

= $\sqrt{4+256+100}=\sqrt{360}=6\sqrt{10}$

Hence, area of $\triangle$ABC = $\frac{1}{2}\times 6\sqrt{10}=3\sqrt{10}$ sq. units

OR

The length of the perpendicular from the vertex D on the opposite face

= |Projection of $\overrightarrow{AD}$ on $\overrightarrow{AB} \times \overrightarrow{AC}$|

= $\left|\frac{(2\hat{i}+2\hat{j})\cdot(-2\hat{i}-16\hat{j}+10\hat{k})}{\sqrt{(-2)^2+(-16)^2+10^2}}\right|$

= $\left|\frac{-4-32}{\sqrt{360}}\right| = \frac{36}{6\sqrt{10}} = \frac{6}{\sqrt{10}}$ units

38. **Read the text carefully and answer the questions:**

In a street two lamp posts are 600 feet apart. The light intensity at a distance d from the first (stronger) lamp post is $\frac{1000}{d^2}$, the light intensity at distance d from the second (weaker) lamp post is $\frac{125}{d^2}$ (in both cases the light intensity is inversely proportional to the square of the distance to the light source). The combined light intensity is the sum of the two light intensities coming from both lamp posts.

(i) We have, I(x) = $\frac{1000}{x^2} + \frac{125}{(600-x)^2}$

$\Rightarrow$ I'(x) = $\frac{-2000}{x^3} + \frac{250}{(600-x)^3}$ and

$\Rightarrow$ I''(x) = $\frac{6000}{x^4} + \frac{750}{(600-x)^4}$

For maxima/minima, I'(x) - 0

$\Rightarrow \frac{2000}{x^3} = \frac{250}{(600-x)^3} \Rightarrow 8(600 - x)^3 = x^3$

Taking cube root on both sides, we get

2(600 - x) = x $\Rightarrow$ 1200 = 3x $\Rightarrow$ x = 400

Thus, I(x) is minimum when you are at 400 feet from the strong intensity lamp post.

(ii) At a distance of 200 feet from the weaker lamp post.

Since I(x) is minimum when x = 400 feet, therefore the darkest spot between the two light is at a distance of 400 feet from a stronger lamp post, i.e., at a distance of 600 - 400 = 200 feet from the weaker lamp post.