Class XII Session 2023-24
Subject - Mathematics
Sample Question Paper - 4

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If A = $\begin{bmatrix} 2 & 0 & -3 \\ 4 & 3 & 1 \\ -5 & 7 & 2 \end{bmatrix}$ is expressed as the sum of a symmetric and skew-symmetric matrix, then the symmetric matrix is **[1]**

a) $\begin{bmatrix} 2 & 2 & -4 \\ 2 & 3 & 4 \\ -4 & 4 & 2 \end{bmatrix}$

b) $\begin{bmatrix} 2 & 4 & -5 \\ 0 & 3 & 7 \\ -3 & 1 & 2 \end{bmatrix}$

c) $\begin{bmatrix} 4 & 4 & -8 \\ 4 & 6 & 8 \\ -8 & 8 & 4 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2. If x, y, z are non-zero real numbers, then the inverse of matrix $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ is **[1]**

a) $\frac{1}{xyz}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

b) $\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

c) $\frac{1}{xyz}\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$

d) $\frac{5yz}{xyz}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3. If A is a 3 $\times$ 3 matrix such that |A| =8, then |3 A| equals. **[1]**

a) 8

b) 72

c) 216

d) 24

4. The function f(x) = $e^{-|x|}$ is **[1]**

a) continuous everywhere but not differentiable at x = 0

b) continuous and differentiable everywhere

c) none of these

d) not continuous at x = 0

5. The projections of a line segment on X, Y and Z axes are 12, 4 and 3 respectively. The length and direction cosines of the line segment are **[1]**

a) $11; \frac{12}{11}, \frac{14}{11}, \frac{3}{11}$

b) $13; \frac{12}{13}, \frac{4}{13}, \frac{3}{13}$

c) $19; \frac{12}{19}, \frac{4}{19}, \frac{3}{19}$

d) none of these

6. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 - \left(\frac{dy}{dx}\right) = y^3$, is **[1]**

a) 4

b) $\frac{1}{2}$

c) 2

d) 3

7. The region represented by the inequation system x, y $\geq$ 0, y $\leq$ 6, x + y $\leq$ 3 is **[1]**

a) unbounded in first and second quadrants

b) bounded in first quadrant

c) None of these

d) unbounded in first quadrant

8. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then which one of the following is correct? **[1]**

a) $\vec{a}$ is parallel to $\vec{b}$

b) $\vec{a}$ is a unit vector

c) $\vec{a}$ is perpendicular to $\vec{b}$

d) $\vec{a} = \lambda\vec{b}$ for some scalar λ

9. $\int e^x \left\{\sin^{-1} x + \frac{1}{\sqrt{1-x^2}}\right\}$ dx = ? **[1]**

a) None of these

b) $e^x \sin^{-1}$ + C

c) $\frac{-e^x}{\sin^{-1} x}$ + C

d) $e^x \cdot \frac{1}{\sqrt{1-x^2}}$ + C

10. The matrix $\begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$ is a **[1]**

a) symmetric matrix

b) scalar matrix

c) diagonal matrix

d) skew-symmetric matrix

11. The corner points of the feasible region determined by the system of linear inequalities are (0, 0), (4, 0), (2, 4), and (0, 5). If the maximum value of z = ax + by, where a, b > 0 occurs at both (2, 4) and (4, 0), then: **[1]**

a) 3a = b

b) 2a = b

c) a = 2b

d) a = b

12. The two lines x = ay + b, z = cy + d; and x = $a'y + b', z = c'y + d'$ are perpendicular to each other, if: **[1]**

a) $aa' + cc' = -1$

b) aa' + cc' = 1

c) $\frac{a}{a'} + \frac{c}{c'} = -1$

d) $\frac{a}{a'} + \frac{c}{c'} = 1$

13. If A is singular matrix, then adj A is **[1]**

a) symmetric

b) non-singular

c) not defined

d) singular

14. If $P(A \cap B) = \frac{7}{10}$ and P(B) = $\frac{17}{20}$, then $P(\frac{A}{B})$ equals **[1]**

a) $\frac{7}{8}$ b) $\frac{17}{20}$

c) $\frac{14}{17}$ d) $\frac{1}{8}$

15. Solution of the differential equation $\frac{dx}{x}+\frac{dy}{y}=0$ is **[1]**

a) log x . log y = c b) x + y = c

c) xy = c d) $\frac{1}{x}+\frac{1}{y}=c$

16. The vector $(\cos\alpha\cos\beta)\hat{i}+(\cos\alpha\sin\beta)\hat{j}+(\sin\alpha)\hat{k}$ is a **[1]**

a) none of these b) constant vector

c) null vector d) unit vector

17. If y = log $\left(\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}-x}\right)$ then $\frac{dy}{dx}=?$ **[1]**

a) $\frac{-2}{\sqrt{1+x^2}}$ b) $\frac{2\sqrt{1+x^2}}{x^2}$

c) none of these d) $\frac{2}{\sqrt{1+x^2}}$

18. The angle between two lines having direction ratios 1, 1, 2 and $(\sqrt{3}-1)$, $(-\sqrt{3}-1)$, 4 is **[1]**

a) $\frac{\pi}{4}$ b) $\frac{\pi}{3}$

c) $\frac{\pi}{6}$ d) $\frac{\pi}{2}$

19. **Assertion (A):** The function f(x) = x^2 - 4x + 6 is strictly increasing in the interval (2, ∞). **[1]**

Reason (R): The function f(x) = x^2 - 4x + 6 is strictly decreasing in the interval ($-\infty$, 2).

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** If A = {0, 1} and N be the set of natural numbers. Then, the mapping f : N $\rightarrow$ A defined by f (2n - 1) = 0, f (2n) = 1, $\forall$ n$\in$ N, is onto. **[1]**

Reason (R): Range = Codomain

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Write the interval for the principal value of function and draw its graph: $\cos^{-1}$ x. **[2]**

OR

Find the value of $\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$.

22. If f(x) = x^3 + ax^2 + bx + c has a maximum at x = -1 and minimum at x = 3. determine a, b and c. **[2]**

23. Find the intervals in which $f(x)=(x+2)e^{-x}$ is increasing or decreasing. **[2]**

OR

What are the values of 'a' for which f(x) = a^x is decreasing on R?

24. By using the properties of definite integrals, evaluate the integral $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\sin^7 x\,dx$ **[2]**

25. Find the interval in function $f(x)=\frac{x^4}{4}+\frac{2}{3}x^3-\frac{5}{2}x^2-6x+7$ is increasing or decreasing. **[2]**

Section C

26. Evaluate the definite integral: $\int_1^2 \frac{x+3}{x(x+2)}dx$ **[3]**

27. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six. **[3]**

28. Evaluate $\int_0^1 \log\left|\frac{1}{x}-1\right|dx.$ **[3]**

OR

Evaluate $\int_0^4[|x|+|x-2|+|x-4|]$ dx.

29. Solve the differential equation: $(1 + x^2)\frac{dy}{dx} - x = 2\tan^{-1} x$ **[3]**

OR

Solve the differential equation $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, given that y = 1 when x = 0.

30. Exhibit graphically the solution set of the system of linear inequations x + y > 1, 7 x + 9y $\leq 63, y\leq 5, x\leq 6, x\geq 0$ and $y\geq 0$ **[3]**

OR

Solve the Linear Programming Problem graphically:

Minimize Z = $3x_1 + 5x_2$

Subject to

$x_1 + 3x_2 \geq 3$

$x_1 + x_2 \geq 2$

$x_1, x_2 \geq 0$

31. Prove that the function $f(x)=\begin{cases}\frac{\sin x}{x}, & x<0\\ x+1, & x\geq 0\end{cases}$ is everywhere continuous. **[3]**

Section D

32. Find the area enclosed by the parabola $4y = 3x^2$ and the line 2y = 3x + 12. **[5]**

33. Let A = R - {3} and B = R- {1}. Consider the function of f: A $\rightarrow$ B defined by $f(x)=\frac{x-2}{x-3}$, is one - one and onto. **[5]**

OR

If A = {1, 2, 3, 4}, define relations on A which have properties of being:

a. reflexive, transitive but not symmetric
b. symmetric but neither reflexive nor transitive
c. reflexive, symmetric and transitive.

34. For the matrix $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{bmatrix}$, show that $A^3 - 6A^2 + 5A + 11I = 0$. Hence find A^{-1}. **[5]**

35. Prove that if a plane has the intercepts a,b,c is at a distance of p units from the origin then $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}$ **[5]**

OR

Find the distance of a point (2, 4, –1) from the line $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}$.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

To hire a marketing manager, it's important to find a way to properly assess candidates who can bring radical changes and has leadership experience.

Ajay, Ramesh and Ravi attend the interview for the post of a marketing manager. Ajay, Ramesh and Ravi chances of being selected as the manager of a firm are in the ratio 4 : 1 : 2 respectively. The respective probabilities for them to introduce a radical change in marketing strategy are 0.3, 0.8, and 0.5. If the change does take place.

(i) Find the probability that it is due to the appointment of Ajay (A).

(ii) Find the probability that it is due to the appointment of Ramesh (B).

(iii) Find the probability that it is due to the appointment of Ravi (C).

OR

Find the probability that it is due to the appointment of Ramesh or Ravi.

37. **Read the text carefully and answer the questions:** **[4]**

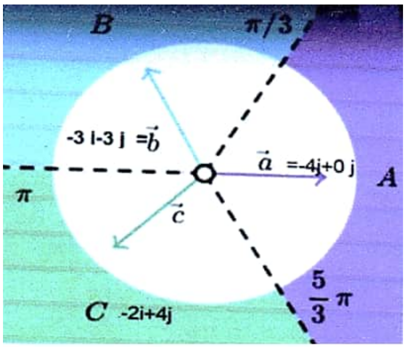
Three friends Ganesh, Dinesh and Ramesh went for playing a Tug of war game. Team A, B, and C belong to Ganesh, Dinesh and Ramesh respectively.

Teams A, B, C have attached a rope to a metal ring and is trying to pull the ring into their own area (team areas shown below).

Team A pulls with $F_1 = 4\hat{i} + 0\hat{j}$ KN

Team B $\rightarrow F_2 = -2\hat{i} + 4\hat{j}$ KN

Team C $\rightarrow F_3 = -3\hat{i} - 3\hat{j}$ KN



(i) Which team will win the game?

(ii) What is the magnitude of the teams combine Force?

(iii) What is the magnitude of the force of Team B?

OR

How many KN Force is applied by Team A?

38. **Read the text carefully and answer the questions:** **[4]**

A tin can manufacturer designs a cylindrical tin can for a company making sanitizer and disinfectors. The tin can is made to hold 3 litres of sanitizer or disinfector. The cost of material used to manufacture the tin can is

₹$100/m^2$.

(i) If r cm be the radius and h cm be the height of the cylindrical tin can, then express the surface area as a function of radius (r)

(ii) Find the radius of the can that will minimize the cost of tin used for making can?

Solution

Section A

1. **(a)** $\begin{bmatrix} 2 & 2 & -4 \\ 2 & 3 & 4 \\ -4 & 4 & 2 \end{bmatrix}$

Explanation: A = $\begin{bmatrix} 2 & 0 & -3 \\ 4 & 3 & 1 \\ -5 & 7 & 2 \end{bmatrix}$ and T' = $\begin{bmatrix} 2 & 4 & -5 \\ 0 & 3 & 7 \\ -3 & 1 & 2 \end{bmatrix}$

As, sum is expressed as

B = $\frac{1}{2}$ (A + A') {Newly formed Symmetric Matrix}

$\frac{1}{2}\left(\mathrm{A}+\mathrm{A}'\right)=\frac{1}{2}\left[\begin{bmatrix} 2 & 0 & -3 \\ 4 & 3 & 1 \\ -5 & 7 & 2 \end{bmatrix}+\begin{bmatrix} 2 & 4 & -5 \\ 0 & 3 & 7 \\ -3 & 1 & 2 \end{bmatrix}\right]$

= $\frac{1}{2}\begin{bmatrix} 4 & 4 & -8 \\ 4 & 6 & 8 \\ -8 & 8 & 4 \end{bmatrix}$

= $\begin{bmatrix} 2 & 2 & -4 \\ 2 & 3 & 4 \\ -4 & 4 & 2 \end{bmatrix}$

2.

(b) $\begin{bmatrix} \mathrm{x}^{-1} & 0 & 0 \\ 0 & \mathrm{y}^{-1} & 0 \\ 0 & 0 & \mathrm{z}^{-1} \end{bmatrix}$

Explanation: Here, $A=\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$

Clearly, we can see that

$adjA=\begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix}\ \ and\ \ |A|=xyz$

$\therefore\ A^{-1}=\frac{adjA}{|A|}=\frac{1}{xyz}\begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix}$

$=\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

3.

(c) 216

Explanation: Given A is a square matrix of order 3 and also |A| = 8

$|3A|=(3)^3\times|A|=27\times8$ = 216

4. **(a)** continuous everywhere but not differentiable at x = 0

Explanation: Given that f(x) = $e^{-|x|}$

$\Rightarrow \mathrm{f(x)}=\left\{\begin{array}{l} \mathrm{e^x, x<0} \\ \mathrm{1, x=0} \\ \mathrm{e^{-x}, x>0} \end{array}\right\}$

Checking continuity and differentiability at x = 0,

LHL:

$\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim e_{h \to 0}^{-h} = 1$

RHL:

$\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} e^{-h} = 1$

And f(0)= 1

$\therefore$ LHL = RHL = f(0)

f(x) is continuous at x = 0.

LHD at x =0,

$\lim_{x \to 0^-} \frac{f(x)-f(0)}{x-0} = \lim_{h \to 0} \frac{f(0-h)-f(0)}{0-h-0}$

$= \lim_{h \to 0} \frac{e^{-h}-(0)}{-h} = \infty$

$\therefore$LHD does not exist, so f(x) is not differentiable at x = 0

5.

(b) $13; \frac{12}{13}, \frac{4}{13}, \frac{3}{13}$

Explanation: $13; \frac{12}{13}, \frac{4}{13}, \frac{3}{13}$

If a line makes angles α, β and γ with the axis, then $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma$ = 1 ....(i)

Let r be the length of the line segment. Then,

$r \cos \alpha = 12, r \cos \beta = 4, r \cos \gamma = 3$ ....(ii)

$\Rightarrow (r \cos \alpha)^2 + (r \cos \beta)^2 + (r \cos \gamma)^2 = 12^2 + 4^2 + 3^2$

$\Rightarrow r^2 \left(\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma\right) = 169$

$\Rightarrow r^2$ (1) = 169 [From (i)]

$\Rightarrow r = \sqrt{169}$

$\Rightarrow$ r = ± 13

$\Rightarrow$ r = 13 (since length cannot be negative)

Substituting r = 13 in (ii)

We get,

$\cos \alpha = \frac{12}{13}, \cos \beta = \frac{4}{13}, \cos \gamma = \frac{1}{13}$

Thus, the direction cosines of the line are

$\frac{12}{13}, \frac{4}{13}, \frac{1}{13}$

6.

(c) 2

Explanation: We know that,

The degree is the power of the highest order derivative.

The highest order is 2 and its power is 2.

Hence, the degree of a differential equation is 2.

7.

(b) bounded in first quadrant

Explanation: Converting the given inequations into equations, we obtain

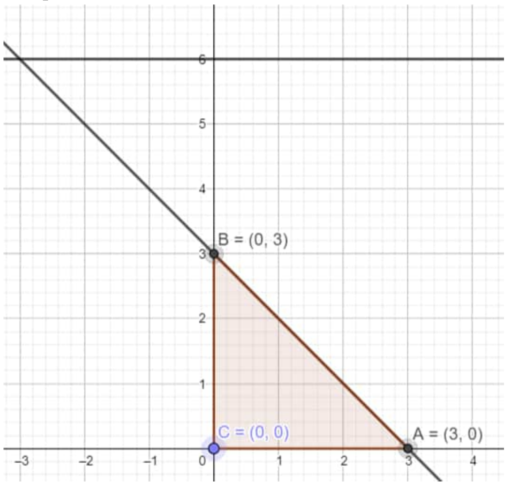
y = 6, x + y = 3, x = 0 and y = 0, y = 6 is the line passing through (0, 6) and parallel to the X axis.The

region below the line y = 6 will satisfy the given inequation.

The line x + y = 3 meets the coordinate axis at A(3, 0) and B(0, 3). Join these points to obtain the line x + y = 3 Clearly, (0, 0) satisfies the inequation x + y $\leq$3 . So, the region in x y -plane that contains the origin represents the solution set of the given equation.

The region represented by x $\geq$ 0 and y $\geq$ 0 :

Since every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations.



8.

(c) $\vec{a}$ is perpendicular to $\vec{b}$

Explanation: Since, $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$

$\Rightarrow [|\vec{a}+\vec{b}|]^2=[\vec{a}-\vec{b}]^2$

$\Rightarrow \vec{a}\cdot\vec{a}+\vec{b}\cdot\vec{b}$ + $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{a}$

= $\vec{a}\cdot\vec{a}+\vec{b}\cdot\vec{b}$ - $\vec{a}\cdot\vec{b}-\vec{b}\cdot\vec{a}$

$\Rightarrow \vec{a}\cdot\vec{b}$ = 0 [$\because \vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$]

Here $\vec{a}$ is perpendicular to $\vec{b}$.

9.

(b) $e^x \sin^{-1}$ + C

Explanation: I = $\int$ e^x {f(x) + f'{x)} dx, where f(x) = $\sin^{-1}$ x

= $e^x \sin^{-1}$ + C

10.

(d) skew-symmetric matrix

Explanation: We have A = $\begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$

$\therefore$ A' = $\begin{bmatrix} 0 & 5 & -8 \\ -5 & 0 & -12 \\ 8 & 12 & 0 \end{bmatrix}$ = - A

So, matrix A is skew-symmetric

11.

(c) a = 2b

Explanation: The maximum value of 'z' occurs at (2, 4) and (4, 0)

$\therefore$ Value of z at (2, 4) = value of z at (4, 0)

a(2) + b(4) = a(4) + b(0)

2a + 4b = 4a + 0

4b = 4a - 2a

4b = 2a

a = 2b

12. **(a)** $aa'+cc'=-1$

Explanation: x = ay + b, z = cy + d

L_1 : $\frac{x-b}{a}$ = y = $\frac{z-d}{c}$

x = $a'y+b', z=c'y+d'$

L_2 : $\frac{x-b'}{a'}$ = y = $\frac{z-d'}{c'}$

if two lines are perpendicular, angle between their direction ratio is $\frac{\pi}{2}$

cos $\frac{\pi}{2}$ = 0

$aa' + cc' \pm -1 = 0$
$aa' + cc' = -1$

13.

(d) singular

Explanation: If A is singular matrix then adjoint of A is also singular.This is true becaue , if A is a singular matrix, Then det(A) = 0 ,and hence the adjoint will also be zero.

14.

(c) $\frac{14}{17}$

Explanation: Here, $\mathrm{P(A \cap B)} = \frac{7}{10}$ and P(B) = $\frac{17}{20}$,
$\therefore P(\frac{A}{B})$ = $\frac{P(A \cap B)}{P(B)} = \frac{7/10}{17/20} = \frac{14}{17}$

15.

(c) xy = c

Explanation: From the given equation, we get log x + log y = log c giving xy = c.

16.

(d) unit vector

Explanation: Given vector $(cos\alpha cos\beta)\hat{i} + (cos\alpha sin\beta)\hat{j} + sin\alpha\hat{k}$
The magnitude of the vector = $\sqrt{cos^2\alpha cos^2\beta + cos^2\alpha sin^2\beta + sin^2\alpha}$
= $\sqrt{cos^2\alpha(cos^2\beta + sin^2\beta) + sin^2\alpha}$
= $\sqrt{cos^2\alpha + sin^2\alpha}$
= 1
$\therefore$ It is unit vector

17.

(d) $\frac{2}{\sqrt{1+x^2}}$

Explanation: Given that y $= \log_e\left(\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}-x}\right)$

Differentiating with respect to x, we obtain

$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{1+x^2}-x}{\sqrt{1+x^2}+x} \times \frac{(\sqrt{1+x^2}-x)\times\left(\frac{1}{2\sqrt{1+x^2}}\times 2x+1\right)-(\sqrt{1+x^2}+x)\times\left(\frac{1}{2\sqrt{1+x^2}}\times 2x-1\right)}{(\sqrt{1+x^2}-x)^2}$
Hence, $\frac{dy}{dx} = \frac{2}{\sqrt{1+x^2}}$

18.

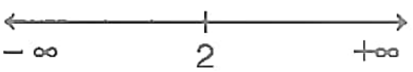
(d) $\frac{\pi}{2}$

Explanation: Let $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\overrightarrow{\mathrm{b}} = (\sqrt{3}-1)\hat{\mathrm{i}} + (-\sqrt{3}-1)\hat{\mathrm{j}} + 4\hat{\mathrm{k}}$
$|\overrightarrow{\mathrm{a}}| = \sqrt{6}$, $|\vec{b}| = \sqrt{(4-2\sqrt{3}) + (4+2\sqrt{3}) + 16} = 2\sqrt{6}$
$\cos\alpha = \frac{(\hat{i}+\hat{j}+2\hat{\mathrm{k}})\cdot((\sqrt{3}-1)\hat{\mathrm{i}}+(-\sqrt{3}-1)\hat{\mathrm{j}}+4\hat{\mathrm{k}})}{\sqrt{6}\times 2\sqrt{6}}$
$\cos\alpha = \frac{\sqrt{3}-1-\sqrt{3}-1+8}{12}$
$\cos\alpha = \frac{1}{2}$
$\alpha = 60^\circ$

19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: We have, f(x) = x^2 - 4x + 6
or f'(x) = 2x - 4 = 2 (x - 2)



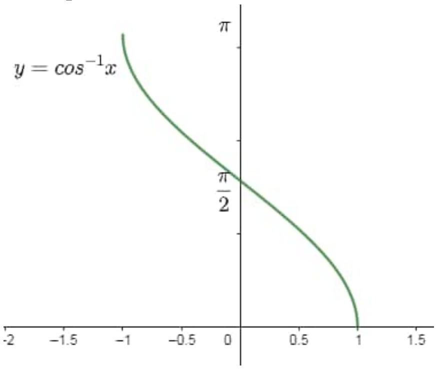
Therefore, f'(x) = 0 gives x = 2.
Now, the point x = 2 divides the real line into two disjoint intervals namely, $(-\infty, 2)$ and $(2, \infty)$.
In the interval $(-\infty, 2)$, f'(x) = 2x - 4 < 0.
Therefore, f is strictly decreasing in this interval.

Also, in the interval $(2, \infty)$, $f'(x) > 0$ and so the function f is strictly increasing in this interval.
Hence, both the statements are true but Reason is not the correct explanation of Assertion.

20. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation: Given A = {0, 1}
$f(2n - 1) = 0, f(2n) = 1 \ \forall \ n \in N$
$\Rightarrow$ every element in A has its preimage in N.
so A is true.
and we know range is subset or equal to codomain
so R is true.
and for onto function, Range = Codomain
so R is correct explanation of A.

Section B

21. Principal value branch of $\cos^{-1} x$ is $[0, \pi]$ and its graph is shown here,



OR

We have, $\tan^{-1}\left(\tan \frac{2\pi}{3}\right) = \tan^{-1} \tan\left(\pi - \frac{\pi}{3}\right)$
$= \tan^{-1}\left(-\tan \frac{\pi}{3}\right) \ [\because \tan^{-1}(-x) = -\tan^{-1}x]$
$= \tan^{-1} \tan \ \left(\frac{-\pi}{3}\right) = -\frac{\pi}{3} \ \left[\because \tan^{-1}(\tan x) = x, \ x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)\right]$
Note: Remember that, $\tan^{-1}\left(\tan \frac{2\pi}{3}\right) \neq \frac{2\pi}{3}$
Since, $\tan^{-1}(\tan x) = x$, if $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\frac{2\pi}{3} \notin \left(\frac{-\pi}{2}, \ \frac{\pi}{2}\right)$

22. Consider the function
$f(x) = x^3 + ax^2 + bx + c$
Then $f'(x) = 3x^2 + 2ax + b$
It is given that f(x) is maximum at x = -1.
$\because f'(-1) = 3(-1)^2 + 2a(-1) + b = 0$
$\Rightarrow f'(-1) = 3 - 2a + b = 0$ ....(1)
It is given that f(x) is minimum at x = 3
$\because f'(3) = 3(3)^2 + 2a(3) + b = 0$
$\Rightarrow f'(3) = 27 + 6a + b = 0$ ....(2)
Solving equations(1) and (2), we have
a = -3 and b = - 9
Since f'(x) is independent of constant c, it can be any real number.

23. Given: $f(x) = (x + 2)e^{-x}$
$f'(x) = e^{-x} - e^{-x}(x+2)$
$= e^{-x}(1 - x - 2)$
$= -e^{-x}(x+1)$
For Critical points
$f'(x) = 0$
$\Rightarrow -e^{-x}(x + 1) = 0$
$\Rightarrow x = -1$
Clearly $f'(x) > 0$ if $x < -1$

$f'(x) < 0$ if $x > -1$
Hence f(x) increases in (–∞,–1), decreases in (–1, ∞)

OR

Here

f(x) = a^x

f(x) = a^x log a

Given : F (x) is decreasing on R

= f (x) < 0 $\forall x \in R$

$\Rightarrow a^x \log a < 0, \forall x \in R$

Here logaritmic function is not defined for negative values in R

$\Rightarrow a^x > 0$

$\Rightarrow 0 < a < 1$

24. Let $I = \int\limits_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^7 x dx$

Here f(x) = $\sin^7$x

$\therefore f(-x) = \sin^7(-x)$

(-sin x)^7

= -$\sin^7$x = -f(x)

$\therefore$ f(x) is an odd function of x.

$\therefore I = \int\limits_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^7 x dx = 0$

$\left[\because \int\limits_{-a}^{a} f(x)\,dx = 0\right.$ when f(x) is an odd function]

25. Given: $f(x) = \frac{x^4}{4} + \frac{2}{3}x^3 - \frac{5}{2}x^2 - 6x + 7$

⇒ f'(x) = x^3 + 2x^2 – 5x – 6

To find critical point for f(x), we must have

⇒ f'(x) = 0

⇒ x^3 + 2x^2 – 5x – 6 = 0

⇒ (x + 1)(x – 2)(x + 3) = 0

⇒ x = –1, 2 , –3

clearly, f'(x) > 0 if –3 < x < –1 and x > 2

and f'(x) < 0 if x < –3 and –3 < x < –1

Thus, f(x) increases on (–3, –1) ∪ (2, ∞)

and f(x) is decreasing on interval (∞, –3) ∪ (–1, 2)

Section C

26. Here we have,

$\int_1^2 \frac{x+3}{x(x+2)} dx$

$= \int_1^2 \frac{x}{x(x+2)} dx + \int_1^2 \frac{3}{x(x+2)} dx$

$= \int_1^2 \frac{dx}{(x+2)} + \int_1^2 \frac{3}{x(x+2)} dx$

$= [\log(x+2)]_1^2 + \frac{3}{2}\int_1^2 \frac{1}{x} - \frac{1}{x+2} dx$ [using partial fraction]

$= [\log(x+2)]_1^2 + \left[\frac{3}{2}\log x - \frac{3}{2}\log(x+2)\right]_1^2$

$= \left[\frac{3}{2}\log x - \frac{1}{2}\log(x+2)\right]_1^2$

$= \frac{1}{2}[3\log 2 - \log 4 + \log 3]$

$= \frac{1}{2}[3\log 2 - 2\log 2 + \log 3]$ $[\because \log 4 = 2\log 2]$

$= \frac{1}{2}[\log 2 + \log 3]$

$= \frac{1}{2}[\log 6]$

$= \frac{1}{2}\log 6$
$\therefore \int_1^2 \frac{x+3}{x(x+2)}dx = \frac{1}{2}\log 6$

27. Let E be the event that the man reports that six occurs in the throwing of the dice and let S_1 be the event that six occurs and S_2 be the event six does not occur.

Then $P(S_1)$ =Probability that six occurs = $\frac{1}{6}$

$P(S_2)$ =Probability that six does not occur = $\frac{5}{6}$

$P(E|S_1)$ = Probability that the man reports that six occurs when six has actually occurred on the die

= Probability that the man speaks the truth = $\frac{3}{4}$

$P(E|S_2)$ = Probability that the man reports that six occurs when six hasn't actually occurred on the die

= Probability that the man does not speak the truth $1 - \frac{3}{4} = \frac{1}{4}$

Thus, by Bayes' theorem, we get

$P(S_1|E)$ = Probability that the report of the man that six has occurred is actually a six

$= \frac{P(S_1)P(E/S_2)}{P(S_1)P(E/S_1)+P(S_2)P(E/S_2)}$
$= \frac{1/6\times3/4}{1/6\times3/4+5/6\times1/4}$ = $\frac{1}{8} \times \frac{24}{8} = \frac{3}{8}$

28. Given $I = \int_0^1 \log\left|\frac{1}{x} - 1\right|dx$

$\Rightarrow I = \int_0^1 \log\left|\frac{1-x}{x}\right|dx$ ...(i)

$\Rightarrow \quad I = \int_0^1 \log\left|\frac{1-(1-x)}{1-x}\right|dx \left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$

$\Rightarrow \quad I = \int_0^1 \log\left|\frac{x}{1-x}\right|dx$ ...(ii)

Adding Equations. (i) and (ii),

$2I = \int_0^1 \log\left|\frac{1-x}{x}\right|dx + \int_0^1 \log\left|\frac{x}{1-x}\right|dx$

$\Rightarrow \quad 2I = \int_0^1 \left[\log\left|\frac{1-x}{x}\right| + \log\left|\frac{x}{1-x}\right|dx\right]$

$\Rightarrow \quad 2I = \int_0^1 \log\left|\left(\frac{1-x}{x} \times \frac{x}{1-x}\right)\right|dx$ $[\because \log m + \log n = \log(m \times n)]$

$\Rightarrow \quad 2I = \int_0^1 \log 1dx$ [∵log 1 = 0]

$\Rightarrow \quad 2I = \int_0^1 0dx = 0$

$\therefore I = 0$

OR

According to the question , $I = \int_0^4 [|x| + |x-2| + |x-4|]dx$

For,

0< x <4, |x| = x

$0 < x \leq 2, |x-2| = -(x-2)$

$2 \leq x < 4, |x-2| = (x-2)$

$0 < x < 4, |x-4| = -(x-4)$

$\therefore \quad I = \int_0^4 xdx + \int_0^2 (2-x)dx + \int_2^4 (x-2)dx + \int_0^4 (4-x)dx$

$= \left[\frac{x^2}{2}\right]_0^4 + \left[2x - \frac{x^2}{2}\right]_0^2 + \left[\frac{x^2}{2} - 2x\right]_2^4 + \left[4x - \frac{x^2}{2}\right]_0^4$

$= (8) + [(4-2) - 0] + [(8-8) - (2-4)] + \left[16 - \frac{16}{2}\right]$

$= 8 + 2 + 2 + (16 - 8)$

= 20

$\therefore I = 20$ sq units.

29. The given differential equation is,

$(1 + x^2)\frac{dy}{dx} - x = 2\tan^{-1} x$

$\Rightarrow (1 + x^2)\frac{dy}{dx} = x + 2\tan^{-1} x$

$\Rightarrow dy = \left\{\frac{x}{1+x^2} + \left(\frac{2}{1+x^2}\right)\tan^{-1} x\right\}dx$

$\Rightarrow dy = \left\{\frac{1}{2} \times \frac{2x}{1+x^2} + \left(\frac{2}{1+x^2}\right)\tan^{-1} x\right\}dx$

Integrating both sides, we get,

$\int dy = \int \left\{\frac{1}{2} \times \frac{2x}{1+x^2} + \left(\frac{2}{1+x^2}\right)\tan^{-1} x\right\}dx$

$\Rightarrow y=\frac{1}{2}\int \frac{2x}{1+x^2}dx+2\int\left[\frac{1}{1+x^2}\tan^{-1}x\right]dx$

$\Rightarrow y=\frac{1}{2}\log\left|1+x^2\right|+2\int\left[\frac{1}{1+x^2}\tan^{-1}x\right]dx$

Putting $\tan^{-1} x = t$

$\Rightarrow \frac{1}{1+x^2}dx=dt$

$\therefore y=\frac{1}{2}\log\left|1+x^2\right|+2\int t dt$

$= \frac{1}{2} \log |1 + x^2| + t^2 + C$

$= \frac{1}{2} \log |1 + x^2| + (\tan^{-1} x)^2 + C$

Hence, $y = \frac{1}{2} \log |1 + x^2| + (\tan^{-1} x)^2 + C$ is the solution to the given differential equation.

OR

Given differential equation may be rewritten as,

$\frac{dy}{dx} = (1 + x^2)(1 + y^2)$

$\int \frac{dy}{1+y^2}=\int\left(x^2+1\right)dx$

$\tan^{-1} y = \frac{x^3}{3} + x + C$

$x = 0, y = 1 \Rightarrow C = \frac{\pi}{4}$

Therefore, particular solution is, $\tan^{-1} y = \frac{x^3}{3} + x + \frac{\pi}{4}$

30. Here, $x + y = 1$ meets the axes at A (1, 0) and B(0,1).

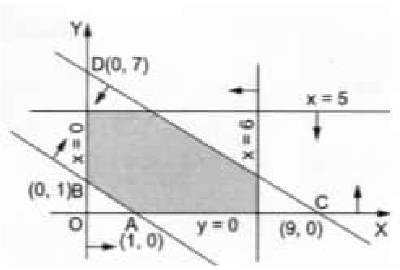
Join these points by a thick line. We note that the portion not containing O(0, 0) is the solution set of $x + y > 1$.So,

$7x+9y=63\Rightarrow \frac{x}{9}+\frac{y}{7}=1$

This line meets the axes at C (9,0) and D(0, 7). Join these points by a thick line. We note that the portion containing (0, 0) is the solution set of $7x + 9y < 63$

$y = 5$ is a line parallel to the x-axis at a distance 5 from the x-axis and the portion containing O(0,0) is the solution set of the inequation $y < 5$. $x = 6$ is a line parallel to the y-axis at a distance 6 from the y-axis and the portion containing (0,0) is the solution set of $x < 6$

We note that, $x > 0$ has a solution represented by the y-axis and the portion on its right. Also, $y > 0$ has a solution represented by the x-axis and the portion above it. The shaded region represents the solution set of the given system of inequations



OR

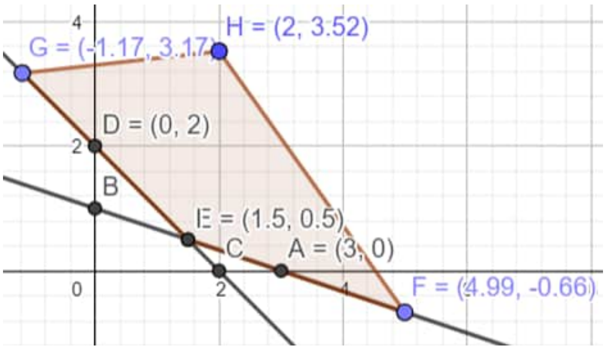
First, we will convert the given inequations into equations, we obtain the following equations:

$x_1 + 3x_2 = 3$, $x_1 + x_2 = 2$, $x_1 = 0$ and $x_2 = 0$

Region represented by $x_1 + 3 x_2 \geq 3$:

The line $x_1 + 3 x_2 = 3$ meets the coordinate axes at A(3,0) and B(0,1) respectively. By joining these points we obtain the line $x_1 + 3 x_2 = 3$

Clearly (0,0) does not satisfies the inequation $x_1 + 3 x_2 \geq 3$. So, the region in the plane which does not contain the origin represents the solution set of the inequation $x_1 + 3 x_2 \geq 3$ Region represented by $x_1 + x_2 \geq 2$ The line $x_1 + x_2 = 2$ meets the coordinate axes at C(2,0) and D(0,2) respectively. By joining these points we obtain the
line $x_1 + x_2 = 2$ Clearly (0,0) does not satisfies the inequation $x_1 + x_2 \geq 2$. So, the region containing the origin represents the solution set of the inequation $x_1 + x_2 \geq 2$ Region represented by $x_1 \geq 0$ and $x_2 \geq 0$ since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations $x_1 \geq 0$ and $x_2 \geq 0$ The feasible region determined by subject to the constraints are, $x_1 + 3 x_2 \geq 3$, $x_1 + x_2 \geq 2$,and the non-negative restrictions $x_1 \geq 0$, and $x_2 \geq 0$, are as follows



The corner points of the feasible region are O(0,0), B(0,1), $E\left(\frac{3}{2}, \frac{1}{2}\right)$ and C(2,0)

The values of objective function at the corner points are as follows:

Corner point : $z = 3x_1 + 5x_2$

O(0, 0) : $3 \times 0 + 5 \times 0 = 0$

B(0, 1) : $3 \times 0 + 5 \times 1 = 5$

$E\left(\frac{3}{2}, \frac{1}{2}\right)$: $\frac{3}{2} + 5 \times \frac{1}{2} = 7$

C(2, 0) : $3 \times 2 + 5 \times 0 = 6$

Therefore, the minimum value of objective function Z is 0 at the point O(0,0). Hence, $x_1 = 0$ and $x_2 = 0$ is the optimal solution of the given LPP.

Thus, the optimal value of objective function Z is 0.

31. Given $f(x) = \begin{cases} \frac{\sin x}{x}, & x < 0 \\ x + 1, & x \geq 0 \end{cases}$

When x < 0, we have

$f(x) = \frac{\sin x}{x}$

We know that sin x, as well as the identity function x, are everywhere continuous

So is the quotient function $\frac{\sin x}{x}$

When x > 0, we have

f(x) = x + 1

$\therefore$ f(x) is continuous at each x > 0

We have

(LHL at x = 0) = $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{h \to 0} f(0 - h) = \lim\limits_{h \to 0} f(-h)$

$= \lim\limits_{h \to 0} \left(\frac{\sin(-h)}{-h}\right) = \lim\limits_{h \to 0} \left(\frac{\sin(h)}{h}\right) = 1$

(RHL at x = 0) = $\lim\limits_{x \to 0^+} f(x) = \lim\limits_{h \to 0} f(0 + h) = \lim\limits_{h \to 0} f(h) = \lim\limits_{h \to 0} (h + 1) = 1$

Also,

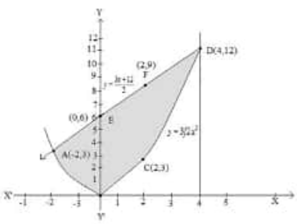
f(0) = 0 + 1 = 1

$\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^+} f(x) = f(0)$

Thus, f(x) is continuous at x = 0

Hence, f(x) is everywhere continuous.

Section D

32. 

$4y = 3x^2$ ......(1)

2y = 3x + 12......(2)

From (2), $y = \frac{3x+12}{2}$

Using this value of y in (1), we get,

$x^2 - 6x - 8 = 0$

$\Rightarrow$ (x + 2) (x - 4) = 0

$\Rightarrow$ x = -2, 4

From (2),

When, x = -2, y = 3

When, x = 4, y = 12

Thus, points of intersection are, (-2, 3) and (4, 12).

Area $= \int_{-2}^{4} \frac{3x+12}{2} dx - \int_{-2}^{4} \frac{3}{4} x^2 dx$

$= \frac{1}{2}[\frac{3x^2}{2} + 12x]_{-2}^{4} - \frac{3}{4}[\frac{x^3}{3}]_{-2}^{4}$

$\frac{1}{2}[(24+48) - (6-24)] - \frac{1}{4}[64 - (-8)]$

= 45 - 18 = 27 sq units.

33. Let x_1 x_2 $\in$ A

Such that $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$

$\Rightarrow (x_1-2)(x_2-3) = (x_2-2)(x_1-3)$

$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6$

$= x_1x_2 - 2x_1 - 3x_2 + 6$

$\Rightarrow -3x_1 - 2x_2 = -2x_1 - 3x_2$

$\Rightarrow x_1 = x_2$

$\therefore f$ is one - one function.

Let y be an arbitrary element of B.Then,

f(x) = y implies,

$y = \frac{(x-2)}{(x-3)}$

(x - 3)y = x - 2

xy - 3y = x - 2

xy - x = 3y - 2

$x = \frac{(3y-2)}{(y-1)}$

Clearly, x = $\frac{3y-2}{y-1}$ is a real number for all $y \neq 1$.

Also, $\frac{3y-2}{y-1} \neq 3$,for if we take $\frac{3y-2}{y-1} = 3$, we get,1 = 0, which is wrong.

Thus, every element y in B has it's pre-image in A given by x = $\frac{3y-2}{y-1}$.

Hence f is onto.

OR

Given that A = {1, 2, 3, 4},

a. Let R_1 = {(1,1), (1,2), (2,3), (2,2), (1,3), (3,3)}

   R_1 is reflexive, since, (1,1) (2,2) (3,3) lie is R_1

   Now, (1,2) $\in R_1$ (2,3) $\in R_1 \Rightarrow$ (1, 3) $\in R_1$

   Hence, R_1 is also transitive but (1,2), $\in R_1 \Rightarrow$ (2,1) $\notin R_1$

   So, it is not symmetric.

b. Let R_2 = {(1,2),(2,1)}. Here,1,2,3 $\in$ {1,2,3} but (1,1),(2,2),(3,3) are not in R.

   Therefore, R is not reflexive.Now, (1,2) $\in R_2$, (2,1) $\in R_2$

   So, it is symmetric.

   Now (1,2) $\in$ R (2,1) $\in$ R, but (1,1) $\notin$ R,

   therefore, R is not transitive.

c. Let R3 = {(1,2), (2,1),(1,1)(2,2),(3,3),(1,3),(3,1),(2,3)}

   Clearly, R_3 is reflexive, symmetric and transitive.

34. Given: $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

$\therefore A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

$$\Rightarrow A^2=\begin{bmatrix}1+1+2 & 1+2-1 & 1-3+3\\ 1+2-6 & 1+4+3 & 1-6-9\\ 2-1+6 & 2-2-3 & 2+3+9\end{bmatrix}$$

$$=\begin{bmatrix}4 & 2 & 1\\ -3 & 8 & -14\\ 7 & -3 & 14\end{bmatrix}$$

Now $A^3=A^2A=\begin{bmatrix}4 & 2 & 1\\ -3 & 8 & -14\\ 7 & -3 & 14\end{bmatrix}\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{bmatrix}$

$$=\begin{bmatrix}4+2+2 & 4+4+1 & 4-6+3\\ -3+8-28 & -3+16+14 & -3-24-42\\ 7-3+28 & 7-6-14 & 7+9+42\end{bmatrix}$$

$$=\begin{bmatrix}8 & 7 & 1\\ -23 & 27 & -69\\ 32 & -13 & 58\end{bmatrix}$$

L.H.S. $= A^3-6A^2+5A+11I$

$$=\begin{bmatrix}8 & 7 & 1\\ -23 & 27 & -69\\ 32 & -13 & 58\end{bmatrix}-6\begin{bmatrix}4 & 2 & 1\\ -3 & 8 & -14\\ 7 & -3 & 14\end{bmatrix}+5\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{bmatrix}+11\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}8 & 7 & 1\\ -23 & 27 & -69\\ 32 & -13 & 58\end{bmatrix}-\begin{bmatrix}24 & 12 & 6\\ -18 & 48 & -84\\ 42 & -18 & 84\end{bmatrix}+\begin{bmatrix}5 & 5 & 5\\ 5 & 10 & -15\\ 10 & -5 & 15\end{bmatrix}+\begin{bmatrix}11 & 0 & 0\\ 0 & 11 & 0\\ 0 & 0 & 11\end{bmatrix}$$

$$=\begin{bmatrix}8-24+5 & 7-12+5 & 1-6+5\\ -23+18+5 & 27-48+10 & -69+84-15\\ 32-42+10 & -13+18-5 & 58+84+15\end{bmatrix}+\begin{bmatrix}11 & 0 & 0\\ 0 & 11 & 0\\ 0 & 0 & 11\end{bmatrix}$$

$$=\begin{bmatrix}-11 & 0 & 0\\ 0 & -11 & 0\\ 0 & 0 & -11\end{bmatrix}+\begin{bmatrix}11 & 0 & 0\\ 0 & 11 & 0\\ 0 & 0 & 11\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}=0$$ = R.H.S.

Now, to find A^{-1}, multiplying $A^3-6A^2+5A+11I=0$ by A^{-1}

$\Rightarrow A^3A^{-1}-6A^2A^{-1}+5AA^{-1}+11I.\,A^{-1}=0.A^{-1}$

$\Rightarrow A^2-6A+5I+11A^{-1}=0$

$\Rightarrow 11A^{-1}=6A-5I-A^2$

$$\Rightarrow 11A^{-1}=6\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{bmatrix}-5\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}-\begin{bmatrix}4 & 2 & 1\\ -3 & 8 & -14\\ 7 & -3 & 14\end{bmatrix}$$

$$\Rightarrow 11A^{-1}=\begin{bmatrix}6 & 6 & 6\\ 6 & 12 & -18\\ 12 & -6 & 18\end{bmatrix}-\begin{bmatrix}5 & 0 & 0\\ 0 & 5 & 0\\ 0 & 0 & 5\end{bmatrix}-\begin{bmatrix}4 & 2 & 1\\ -3 & 8 & -14\\ 7 & -3 & 14\end{bmatrix}$$

$$\Rightarrow 11A^{-1}=\begin{bmatrix}6-5-4 & 6-2 & 6-1\\ 6+3 & 12-5-8 & -18+14\\ 12-7 & -6+3 & 18-5-14\end{bmatrix}$$

$$\Rightarrow 11A^{-1}=\begin{bmatrix}-3 & 4 & 5\\ 9 & -1 & -4\\ 5 & -3 & -1\end{bmatrix}$$

$$\Rightarrow A^{-1}=\frac{1}{11}\begin{bmatrix}-3 & 4 & 5\\ 9 & -1 & -4\\ 5 & -3 & -1\end{bmatrix}$$

35. The equation of the plane in the intercept form is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

Distance of this plane from the origin is given to be p.

$$\therefore p=\frac{\left|\frac{1}{a}\times 0+\frac{1}{b}\times 0+\frac{1}{c}\times 0-1\right|}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2+\left(\frac{1}{c}\right)^2}}$$

$$\Rightarrow p=\frac{1}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2+\left(\frac{1}{c}\right)^2}}$$

$$\Rightarrow \frac{1}{p}=\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2+\left(\frac{1}{c}\right)^2}$$

$\Rightarrow \frac{1}{p^2} = \left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2$
$\Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$

OR

We have equation of the line as $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9} = \lambda$.
$\Rightarrow x = \lambda - 5, y = 4\lambda - 3, z = 6 - 9\lambda$
Let the coordinates of L be $(\lambda - 5, 4\lambda - 3, 6 - 9\lambda)$, then Dr’s of PL are $(\lambda - 7, 4\lambda - 7, 7 - 9\lambda)$.
Also, the direction ratios of given line are proportional to 1, 4, -9.
Since, P L is perpendicular to the given line.
$\therefore (\lambda - 7) \cdot 1 + (4\lambda - 7) \cdot 4 + (7 - 9\lambda) \cdot (-9) = 0$
$\Rightarrow \lambda - 7 + 16\lambda - 28 + 81\lambda - 63 = 0$
$\Rightarrow 98\lambda = 98 \Rightarrow \lambda = 1$
So, the coordinates of L are (-4, 1, -3).
$\therefore$ Required distance, PL= $\sqrt{(-4-2)^2 + (1-4)^2 + (-3+1)^2}$
$= \sqrt{36+9+4} = 7\ units$

Section E

36. **Read the text carefully and answer the questions:**

To hire a marketing manager, it's important to find a way to properly assess candidates who can bring radical changes and has leadership experience.
Ajay, Ramesh and Ravi attend the interview for the post of a marketing manager. Ajay, Ramesh and Ravi chances of being selected as the manager of a firm are in the ratio 4 : 1 : 2 respectively. The respective probabilities for them to introduce a radical change in marketing strategy are 0.3, 0.8, and 0.5. If the change does take place.

(i) Let E_1: Ajay (A) is selected, E_2: Ramesh (B) is selected, E_3: Ravi (C) is selected

Let A be the event of making a change
$P(E_1) = \frac{4}{7},\ P(E_2) = \frac{1}{7},\ P(E_3) = \frac{2}{7}$
$P(A/E_1) = 0.3, P(A/E_2) = 0.8, P(A/E_3) = 0.5$
$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$
$= \frac{\frac{4}{7}\times 0.3}{\frac{4}{7}\times 0.3+\frac{1}{7}\times 0.8+\frac{2}{7}\times 0.5} = \frac{\frac{1.2}{7}}{\frac{1.2}{7}+\frac{0.8}{7}+\frac{1}{7}} = \frac{\frac{1.2}{7}}{\frac{3}{7}}$
$= \frac{1.2}{3} = \frac{12}{30} = \frac{2}{5}$

(ii) Let E_1: Ajay(A) is selected, E_2: Ramesh(B) is selected, E_3: Ravi (C) is selected

Let A be the event of making a change
$P(E_1) = \frac{4}{7},\ P(E_2) = \frac{1}{7},\ P(E_3) = \frac{2}{7}$
$P(A/E_1) = 0.3, P(A/E_2) = 0.8, P(A/E_3) = 0.5$
$P(E_2/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$
$= \frac{\frac{1}{7}\times 0.8}{\frac{4}{7}\times 0.3+\frac{1}{7}\times 0.8+\frac{2}{7}\times 0.5} = \frac{\frac{0.8}{7}}{\frac{1.2}{7}+\frac{0.8}{7}+\frac{1}{7}} = \frac{\frac{0.8}{7}}{\frac{3}{7}}$
$= \frac{0.8}{3} = \frac{8}{30} = \frac{4}{15}$

(iii) Let E_1: Ajay (A) is selected, E_2: Ramesh (B) is selected, E_3: Ravi (C) is selected

Let A be the event of making a change
$P(E_1) = \frac{4}{7},\ P(E_2) = \frac{1}{7},\ P(E_3) = \frac{2}{7}$
$P(A/E_1) = 0.3, P(A/E_2) = 0.8, P(A/E_3) = 0.5$

$$P(E_3/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$$

$$= \frac{\frac{2}{7}\times 0.5}{\frac{4}{7}\times 0.3+\frac{1}{7}\times 0.8+\frac{2}{7}\times 0.5} = \frac{\frac{1}{7}}{\frac{1.2}{7}+\frac{0.8}{7}+\frac{1}{7}} = \frac{1}{3}$$

OR

Let E_1: Ajay (A) is selected, E_2: Ramesh (B) is selected, E_3: Ravi (C) is selected

Let A be the event of making a change

$P(E_1) = \frac{4}{7},\ P(E_2) = \frac{1}{7},\ P(E_3) = \frac{2}{7}$

$P(A/E_1) = 0.3$, $P(A/E_2) = 0.8$, $P(A/E_3) = 0.5$

Ramesh or Ravi

$\Rightarrow P(E_2/A) + P(E_3/A) = \frac{4}{15} + \frac{1}{3} = \frac{9}{15} = \frac{3}{5}$

37. **Read the text carefully and answer the questions:**

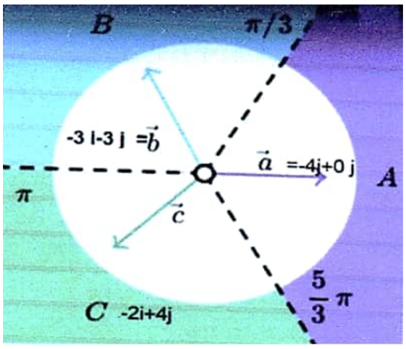
Three friends Ganesh, Dinesh and Ramesh went for playing a Tug of war game. Team A, B, and C belong to Ganesh, Dinesh and Ramesh respectively.

Teams A, B, C have attached a rope to a metal ring and is trying to pull the ring into their own area (team areas shown below).

Team A pulls with $F_1 = 4\hat{i} + 0\hat{j}$ KN

Team B $\rightarrow$ $F_2 = -2\hat{i} + 4\hat{j}$ KN

Team C $\rightarrow$ $F_3 = -3\hat{i} - 3\hat{j}$ KN



(i) Force applied by team A

$= \sqrt{4^2 + 0^2}$

$= 4$ N

Force applied by team B

$= \sqrt{(-2)^2 + 4^2}$

$= \sqrt{4 + 16}$

$= \sqrt{20}$

$= 2\sqrt{5}$ N

Force applied by team C

$= \sqrt{(-3)^2 + (-3)^2}$

$= \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$

Hence, the force applied by team B is maximum.

So, Team 'B' will win.

(ii) Sum of force applied by team A, B and C

$= (4 + (-2) + (-3))\hat{i} + (0 + 4 + (-3))\hat{j}$

$= -\hat{i} + \hat{j}$

Magnitude of team combine force

$= \sqrt{(-1)^2 + (1)^2}$

$= \sqrt{2}N$

(iii)Force applied by team B

$= \sqrt{(-2)^2 + 4^2}$

$= \sqrt{4 + 16}$

$= \sqrt{20}$

$= 2\sqrt{5}$ N

OR

Force applied by team A

$= \sqrt{4^2 + 0^2}$

= 4 N

38. **Read the text carefully and answer the questions:**

A tin can manufacturer designs a cylindrical tin can for a company making sanitizer and disinfectors. The tin can is made to hold 3 litres of sanitizer or disinfector. The cost of material used to manufacture the tin can is ₹100/m^2.

(i) Given, r cm is the radius and h cm is the height of required cylindrical can.

Given that, volume of cylinder = 3l = 3000 cm^3 ($\because$ 1l = 1000 cm^3)

$\Rightarrow \pi r^2 h$ = 3000 $\Rightarrow$ h = $\frac{3000}{\pi r^2}$

Now, the surface area, as a function of r is given by

S(r) = $2\pi r^2 + 2\pi r h = 2\pi r^2 + 2\pi r\left(\frac{3000}{\pi r^2}\right)$

= $2\pi r^2 + \frac{6000}{r}$

(ii) Now, S(r) = $2\pi r^2 + \frac{6000}{r}$

$\Rightarrow$ S'(r) = $4\pi r - \frac{6000}{r^2}$

To find critical points, put S'(r) = 0

$\Rightarrow \frac{4\pi r^3 - 6000}{r^2}$ = 0

$\Rightarrow$ r^3 = $\frac{6000}{4\pi} \Rightarrow r = \left(\frac{1500}{\pi}\right)^{1/3}$

Also, $S''(r)\big|_{r = \sqrt[3]{\frac{1500}{\pi}}} = 4\pi + \frac{12000 \times \pi}{1500}$

= $4\pi + 8\pi$ = 12π > 0

Thus, the critical point is the point of minima.