Class XII Session 2023-24
Subject - Mathematics
Sample Question Paper - 7

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If A = $\begin{bmatrix} 2 & -1 & 3 \\ -4 & 5 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 3 \\ 4 & -2 \\ 1 & 5 \end{bmatrix}$ then **[1]**

a) only BA is defined
b) only AB is defined
c) AB and BA both are not defined
d) AB and BA both are defined

2. $A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$, then the value of |adj A| is: **[1]**

a) 16
b) -8
c) 0
d) 64

3. If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and A_{ij} is Cofactors of a_{ij}, then value of Δ is given by **[1]**

a) $a_{11} A_{31} + a_{12} A_{32} + a_{13} A_{33}$
b) $a_{21} A_{11} + a_{22} A_{12} + a_{23} A_{13}$
c) $a_{11} A_{11} + a_{21} A_{21} + a_{31} A_{31}$
d) $a_{11} A_{11} + a_{12} A_{21} + a_{13} A_{31}$

4. If $e^{x + y} = xy$ then $\frac{dy}{dx} = ?$ **[1]**

a) $\frac{(x-xy)}{(xy-y)}$
b) none of these
c) $\frac{y(1-x)}{x(y-1)}$
d) $\frac{x(1-y)}{y(x-1)}$

5. If the points A(-1, 3, 2), B(-4, 2, -2) and C(5, 5, λ) are collinear then the value of λ is **[1]**

a) 5

b) 10

c) 8

d) 7

6. Consider a differential equation of order m and degree n. Which one of the following pairs is not feasible? **[1]**

a) $(2, \frac{3}{2})$

b) (2, 4)

c) (3, 2)

d) (2, 2)

7. Maximize Z = – x + 2y, subject to the constraints: x ≥ 3, x + y ≥ 5, x + 2y ≥ 6, y ≥ 0. **[1]**

a) Z has no maximum value

b) Maximum Z = 14 at (2, 6)

c) Maximum Z = 12 at (2, 6)

d) Maximum Z = 10 at (2, 6)

8. The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ. **[1]**

a) λ = 1

b) λ = -2

c) λ = 2

d) λ = -1

9. $\int_{\pi}^{2\pi}|\sin x|dx=$? **[1]**

a) None of these

b) 0

c) 2

d) 1

10. If $2\begin{bmatrix}3 & 4\\5 & x\end{bmatrix}+\begin{bmatrix}1 & y\\0 & 1\end{bmatrix}=\begin{bmatrix}7 & 0\\10 & 5\end{bmatrix}$ then **[1]**

a) (x = 2, y = 8)

b) (x = 3, y = -6)

c) (x = -3, y = 6)

d) (x = 2, y = -8)

11. Corner points of the feasible region determined by the system of linear constraints are (0, 3), (1, 1) and (3, 0). Let Z = px+qy, where p, q > 0. Condition on p and q so that the minimum of Z occurs at (3, 0) and (1, 1) is **[1]**

a) p = 3q

b) p = 2q

c) p = q

d) $p=\frac{q}{2}$

12. Find $\left|\vec{a}\times\vec{b}\right|$, if $\vec{a}=3\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}-2\hat{j}+4\hat{k}$ **[1]**

a) $8\sqrt{3}$

b) $19\sqrt{3}$

c) $19\sqrt{5}$

d) $17\sqrt{2}$

13. If A is an invertible square matrix and k is a non-negative real number then $(kA)^{-1}$ = ? **[1]**

a) $\frac{1}{k}\cdot A^{-1}$

b) $-k.A^{-1}$

c) $k.A^{-1}$

d) None of these

14. If the events A and B are independent, then $P(A\cap B)$ is equal to **[1]**

a) $P(A).P(B)$

b) $P(A)/P(B)$

c) P(A) − P(B)

d) P(A) + P(B)

15. The degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}=\frac{d^2y}{dx^2}$ is **[1]**

a) 2

b) $\frac{3}{2}$

c) not defined

d) 4

16. If $\vec{a} \cdot \hat{i} = \vec{a} \cdot (\hat{i} + \hat{j}) = \vec{a} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1,$ then $\vec{a}$= **[1]**

a) $\hat{i}$

b) $\overrightarrow{0}$

c) $\hat{j}$

d) $\hat{i} + \hat{j} + \hat{k}$

17. If $y = x\sqrt{1 - x^2} + \sin^{-1}x,$ then $\frac{dy}{dx}$ is equal to **[1]**

a) $\frac{1}{\sqrt{1-x^2}}$

b) $\sqrt{1 - x^2}$

c) $2\sqrt{1 - x^2}$

d) None of these

18. The straight line $\frac{x-2}{3} = \frac{y-3}{1} = \frac{z+1}{0}$ is **[1]**

a) parallel to the y-axis

b) perpendicular to the z-axis

c) parallel to the x-axis

d) parallel to the z-axis

19. **Assertion (A):** The function f(x) = sin x decreases on the interval (0, $\frac{\pi}{2}$). **[1]**

Reason (R): The function f(x) = cos x decreases on the interval (0, $\frac{\pi}{2}$).

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. Consider the function f : R $\rightarrow$ R defined as f(x) = x^3 **[1]**

Assertion (A): f(x) is a one-one function.

Reason (R): f(x) is a one-one function if co-domain = range.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Write the value of $\sin^{-1}\left(\frac{1}{3}\right) - \cos^{-1}\left(-\frac{1}{3}\right)$ **[2]**

OR

$\tan^{-1}\left(\tan\frac{3\pi}{4}\right) = ?$

22. Find the points of local maxima or local minima, if any, using the first derivative test. Also find the local maximum or local minimum values, as the case may be: f(x) = sin2x, 0 < x < π. **[2]**

23. Find the interval in function f(x) = log (2 +x) - $\frac{2x}{2+x}, x \in R$ is increasing or decreasing. **[2]**

OR

Show that f(x) = tan x is an increasing function on $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

24. Find the integral of the function $\frac{1}{\cos(x-a)\cos(x-b)}$ **[2]**

25. Find the maximum or minimum values, if any, without using derivatives, of the function: f(x) = $|\sin 4x + 3|$ **[2]**

Section C

26. By using the properties of definite integrals, evaluate the integral $\int_{-5}^{5} |x+2|\,dx$ **[3]**

27. In a factory which manufactures bolts, machines. A, B and C manufacture respectively 25%, 35% and 40% of the bolts. Of their output 5, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from the **[3]**

product and is found to be defective. What is the probability that it is manufactured by the machine B?

28. Evaluate: $\int \cos(\log x)\, dx$ **[3]**

OR

Evaluate the integral: $\int_2^4 \frac{x^2+x}{\sqrt{2x+1}} dx$

29. In the differential equation show that it is homogeneous and solve it: $(x^2 - y^2)dx + 2xy\, dy = 0$ **[3]**

OR

Find the equation of the family of curves for which the slope of tangent at any point (x, y) on it, is $\frac{x^2+y^2}{2xy}$.

30. Solve the following LFP graphically: **[3]**

Maximize Z = 20x + 10y

Subject to the following constraints

$x + 2y \leq 28$

$3x + y \leq 24$

$x \geq 2$

$x, y \geq 0$

OR

The feasible region for a LPP is shown in Figure. Evaluate Z = 4x + y at each of the corner points of this region. Find the minimum value of Z, if it exists.

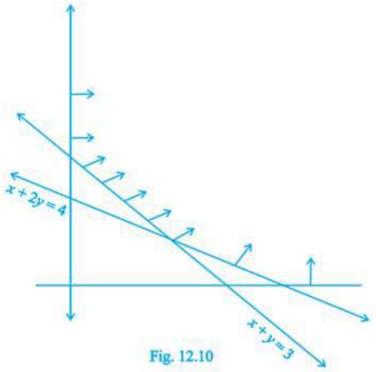


Fig. 12.10

31. If x = a cosθ, y = b sinθ, show that $\frac{d^2y}{dx^2} = -\frac{b^4}{a^2y^3}$ **[3]**

Section D

32. Find the area of the region enclosed by the parabola $y^2 = x$ and the line x + y = 2. **[5]**

33. Show the relation R in the set A = {x $\in$ Z : 0 $\leq$ x $\leq$12}, given by R = {(a, b) : a = b}, is an equivalence relation. Find the set of all elements related to 1 in each case. **[5]**

OR

Let A = {1, 2, 3} and R = {(a, b):a, b $\in$ A and $|a^2 - b^2| \leq 5$. Write R as set of ordered pairs. Mention whether R is

i. reflexive
ii. symmetric
iii. transitive

Give reason in each case.

34. If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find the value of A^{-1}. **[5]**

Using A^{-1}, solve the system of linear equations:

x - 2y = 10,

2x - y - z = 8,

-2y +z = 7

35. Find the length shortest distance between the lines: $\frac{x-3}{3} = \frac{y-8}{-1} = z-3$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ **[5]**

OR

Find the vector equation of the line passing through (1,2,3) and || to the plane $\vec{r}.\left(\hat{i}-\hat{j}+2\hat{k}\right)=5$ and $\vec{r}.\left(3\hat{i}+\hat{j}+\hat{k}\right)=6$

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Shama is studying in class XII. She wants do graduate in chemical engineering. Her main subjects are mathematics, physics, and chemistry. In the examination, her probabilities of getting grade A in these subjects are 0.2, 0.3, and 0.5 respectively.

(i) Find the probability that she gets grade A in all subjects.

(ii) Find the probability that she gets grade A in no subjects.

(iii) Find the probability that she gets grade A in two subjects.

OR

Find the probability that she gets grade A in at least one subject.

37. **Read the text carefully and answer the questions:** **[4]**

If two vectors are represented by the two sides of a triangle taken in order, then their sum is represented by the third side of the triangle taken in opposite order and this is known as triangle law of vector addition.

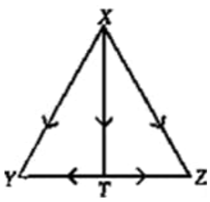
(i) If $\vec{p}, \vec{q}, \vec{r}$ are the vectors represented by the sides of a triangle taken in order, then find $\vec{q}+\vec{r}$.

(ii) If ABCD is a parallelogram and AC and BD are its diagonals, then find the value of $\overrightarrow{AC}+\overrightarrow{BD}$.

(iii) If ABCD is a parallelogram, where $\overrightarrow{AB} = 2\vec{a}$ and $\overrightarrow{BC} = 2\vec{b}$, then find the value of $\overrightarrow{AC}-\overrightarrow{BD}$.

OR

If T is the mid point of side YZ of △XYZ, then what is the value of $\overrightarrow{XY}+\overrightarrow{XZ}$.



38. **Read the text carefully and answer the questions:** **[4]**

Mrs. Maya is the owner of a high-rise residential society having 50 apartments. When he set rent at ₹10000/month, all apartments are rented. If he increases rent by ₹250/ month, one fewer apartment is rented.

The maintenance cost for each occupied unit is ₹500/month.

(i) If P is the rent price per apartment and N is the number of rented apartments, then find the profit.

(ii) If x represents the number of apartments which are not rented, then express profit as a function of x.

Solution

Section A

1.

(d) AB and BA both are defined

Explanation: In given matrix

order of A = 2×3

order of B $= 3 \times 2$

AB will be defined if the number of column in A is equal to the number of rows in B

so, $(A_{2\times3})(B_{3\times2}) = AB_{2\times2}$

Similarly $(B_{3\times2})(A_{2\times3}) = BA_{3\times3}$

Thus, Both AB and BA are defined.

2.

(d) 64

Explanation: $A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

|A| = -2[-4 - 0] - 0 + 0

= -8

Now, |adj A| = $|A|^{n-1}$ ...(where n is the order of matrix n)

= $(-8)^{3-1}$

= $(-8)^2$

= 64

3.

(c) $a_{11} A_{11} + a_{21} A_{21} + a_{31} A_{31}$

Explanation: $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

Expanding along Column 1

$\Delta = (-1)^{1+1} \times a_{11} \times \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{2+1} \times a_{21} \times \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{3+1} \times a_{31} \times \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$

$\Delta = a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

4.

(c) $\frac{y(1-x)}{x(y-1)}$

Explanation: Given that $xy = e^{x + y}$

Taking log both sides, we get

$\log_e xy = x + y$ (Since $\log_a b^c = c\log_a b$)

Since $\log_a bc = \log_a b + \log_a c$, we get

$\log_e x + \log_e y = x + y$

Differentiating with respect to x, we obtain

$\frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = 1 + \frac{dy}{dx}$

Or

$\frac{dy}{dx}\left(\frac{y-1}{y}\right) = \frac{1-x}{x}$

Therefore, $\frac{dy}{dx} = \frac{y(1-x)}{x(y-1)}$

5.

(b) 10

Explanation: Determinant of these point should be zero

$$\begin{vmatrix} -1 & 3 & 2 \\ -4 & 2 & -2 \\ 5 & 5 & \lambda \end{vmatrix} = 0$$

$-1(2\lambda + 10) - 3(-4\lambda + 10)$ + 2(-20 - 10) = 0

10λ = 10 +30 +60= 100

λ = 10

6. **(a)** (2, $\frac{3}{2}$)

   Explanation: The pairs $\left(2, \frac{3}{2}\right)$ is not feasible. Because the degree of any differential equation cannot be rational type. If so, then we use rationalization and convert it into an integer.

7. **(a)** Z has no maximum value

   Explanation: Objective function is Z = - x + 2 y ……………………(1).

   The given constraints are : x ≥ 3, x + y ≥ 5, x + 2y ≥ 6, y ≥ 0.

Corner points	Z = - x + 2y
D(6,0)	-6
A(4,1)	-2
B(3,2)	1

   Here, the open half plane has points in common with the feasible region.

   Therefore, Z has no maximum value.

8. **(a)** λ = 1

   Explanation: Let $\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}$, $\overrightarrow{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\overrightarrow{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$,

   $\vec{b} + \vec{c} = (\lambda + 2)\hat{i} + 6\hat{j} - 2\hat{k}$

   $\Rightarrow |\vec{b} + \vec{c}| = \sqrt{(\lambda + 2)^2 + 40}$

   Therefore, a unit vector along

   $\overrightarrow{b} + \overrightarrow{c}$ is given by:

   $\frac{\vec{b}+\vec{c}}{|\vec{b}+\vec{c}|} = \frac{(\lambda+2)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(\lambda+2)^2+40}} = 1$

   Also, scalar product of $\hat{i} + \hat{j} + \hat{k}$ with above unit vector is 1.

   $\therefore (\hat{i} + \hat{j} + \hat{k}). \frac{(\lambda+2)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(\lambda+2)^2+40}} = 1$

   $\Rightarrow (\lambda + 6)^2 = (\lambda + 2)^2 + 40 \Rightarrow \lambda = 1$

9. **(c)** 2

   Explanation: The given integral is $\int_{\pi}^{2\pi} |\sin x| dx$

   sin x = 0

   $as\ x = 0, \pi, 2\pi$ ...

   So $\pi, 2\pi$ are the limits so no breaking points for the integral,

   $\therefore \int_{\pi}^{2\pi} - \sin x dx = -\cos x(\pi \text{ to } 2\pi)$

   = 2

10. **(d)** (x = 2, y = -8)

   Explanation: $2\begin{pmatrix} 3 & 4 \\ 5 & x \end{pmatrix} + \begin{pmatrix} 1 & y \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 0 \\ 10 & 5 \end{pmatrix}$

   To solve this problem we will use the comparison that is we will use that all the elements of L.H.S. are equal to R.H.S.

   $= \begin{pmatrix} 6 & 8 \\ 10 & 2x \end{pmatrix} + \begin{pmatrix} 1 & y \\ 0 & 1 \end{pmatrix}$

   $= \begin{pmatrix} 7 & 8 + y \\ 10 & 2x + 1 \end{pmatrix}$

   Comparing with R.H.S.

8 + y = 0
y = -8
2x + 1 = 5
2x = 4
x = 2

11.

(d) $p = \frac{q}{2}$
Explanation: We have Z = px + qy , At (3, 0) Z = 3p ……………………………….(1)
At (1 , 1) Z = p + q …………………………(2) Therefore , from (1) and (2) : We have : p = q/2 .

12. **(a)** $8\sqrt{3}$
Explanation: $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 2 & -2 & 4 \end{vmatrix} = 8\hat{i} - 8\hat{j} - 8\hat{k}$
$\therefore\ |\vec{a} \times \vec{b}| = \sqrt{8^2 + (-8)^2 + (-8)^2} = 8\sqrt{3}$

13. **(a)** $\frac{1}{k}$ **.** A^{-1}
Explanation: by the property of inverse
$(AB)^{-1} = B^{-1}A^{-1}$
$(KA)^{-1} = A^{-1}K^{-1}$
$= \frac{1}{K}A^{-1}$

14. **(a)** $P(A).P(B)$
Explanation: If A and B are independent, then $P(A \cap B) = P(A) \cdot P(B)$

15. **(a)** 2
Explanation: In general terms for a polynomial the degree is the highest power.
The differential equation is $\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}} = \frac{d^2y}{dx^2}$
Square both the sides
$\Rightarrow \left(1 + \left(\frac{dy}{dx}\right)^2\right)^3 = \left(\frac{d^2y}{dx^2}\right)^2$
Now for degree to exist the given differential equation must be a polynomial in some differentials.
Here differentials mean $\frac{dy}{dx}$ or $\frac{d^2y}{dx^2}$ or ....$\frac{d^ny}{dx^n}$
The given differential equation is polynomial in differentials $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$
Degree of differential equation is defined as the highest integer power of highest order derivative in the equation.
Here the highest derivative is $\frac{d^2y}{dx^2}$ and there is only one term of highest order derivative in the equation which is $(\frac{d^2y}{dx^2})^2$ whose power is 2 hence degree is 2.

16. **(a)** $\hat{i}$
Explanation: we know that $\hat{i}.\hat{i} = 1$ and $\hat{i}.\hat{j} = \hat{i}.\hat{k} = \hat{j}.\hat{k} = 0$ so the answer is $\vec{a} = \hat{i}$

17.

(c) $2\sqrt{1 - x^2}$
Explanation: $y = x\sqrt{1 - x^2} + \sin^{-1}(x)$
$\Rightarrow \frac{dy}{dx} = x\left\{\frac{1}{2\sqrt{1-x^2}}(-2x)\right\} + \sqrt{1 - x^2} + \frac{1}{\sqrt{1-x^2}}$
$\Rightarrow \frac{dy}{dx} = \frac{-x^2}{\sqrt{1-x^2}} + \sqrt{1 - x^2} + \frac{1}{\sqrt{1-x^2}}$
$\Rightarrow \frac{dy}{dx} = \frac{-x^2+1-x^2+1}{\sqrt{1-x^2}}$
$\Rightarrow \frac{dy}{dx} = \frac{-2x^2+2}{\sqrt{1-x^2}}$
$\Rightarrow \frac{dy}{dx} = 2\sqrt{1 - x^2}$

18.

(b) perpendicular to the z-axis

Explanation: It is perpendicular to z-axis.

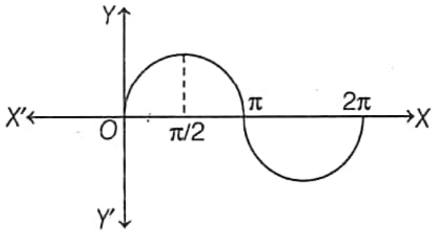
Given, direction ratios of the line : a_1 =3 ,a_2=1, a_3=0 & direction ratios of z-axis is b_1=0, b_2=0,b_3=1.

Now, $a_1a_2+b_1b_2+c_1c_2$= 3.0+1.0+0.1=0 which implies that line is perpendicular to z-axis.

19.

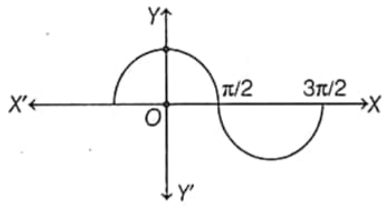
(d) A is false but R is true.

Explanation: Assertion: Given, function f(x) = sin x



From the graph of sin x, we observe that f(x) increases on the interval (0, $\frac{\pi}{2}$).

Reason: Given function is f(x) = cos x.



From the graph of cos x, we observe that, f(x) decreases on the interval (0, $\frac{\pi}{2}$).

Hence, Assertion is false and Reason is true.

20.

(c) A is true but R is false.

Explanation: f(x) is a one-one function if

$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

Hence R is false.

Let f $(x_1) = f(x_2)$ for some $x_1, x_2 \in R$

$\Rightarrow (x_1)^3 = (x_2)^3$

$\Rightarrow x_1 = x_2$

Hence f(x) is one-one.

Section B

21. Given $\sin^{-1}\left(\frac{1}{3}\right) - \cos^{-1}\left(-\frac{1}{3}\right)$

We know that $\cos^{-1}(-\theta) = \pi - \cos^{-1}\theta$

$= \sin^{-1}\left(\frac{1}{3}\right) - \left[\pi - \cos^{-1}\left(\frac{1}{3}\right)\right]$

$= \sin^{-1}\left(\frac{1}{3}\right) - \pi + \cos^{-1}\left(\frac{1}{3}\right)$

$= \sin^{-1}\left(\frac{1}{3}\right) + \cos^{-1}\left(\frac{1}{3}\right) - \pi$

$= \frac{\pi}{2} - \pi$

$= -\frac{\pi}{2}$

Therefore we have,

$\sin^{-1}\left(\frac{1}{3}\right) - \cos^{-1}\left(-\frac{1}{3}\right) = -\frac{\pi}{2}$

OR

$\tan^{-1}\left(\tan\frac{3\pi}{\pi}\right) \neq \frac{3\pi}{4}$ as $\frac{3\pi}{4} \notin \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$\because \tan^{-1}\left(\tan\frac{3\pi}{4}\right) = \tan^{-1}\left[\tan\left(\pi - \frac{\pi}{4}\right)\right]$

$= \tan^{-1}\left[-\tan\left(\frac{\pi}{4}\right)\right]$
$= -\frac{\pi}{4}$

22. Given : f(x) = sin2x
$\Rightarrow$ f'(x) = 2cos 2x
For a local maximum or a local minimum we must have
f'(x) = 0
$\Rightarrow$ 2 cos 2x = 0
$\Rightarrow$ cos2x = 0
$\Rightarrow x = \frac{\pi}{4}$ or $\frac{3\pi}{4}$

Since f'(x) changes from positive to negative when x increases through $\frac{\pi}{4}, x = \frac{\pi}{4}$ is the point of maxima.
Hence, the local maximum value of f(x) at x = $\frac{\pi}{4}$ is given by $\sin\left(\frac{\pi}{2}\right) = 1$
Sicnce f'(x) changes from negative to positive when x increases through $\frac{3\pi}{4}, x = \frac{3\pi}{4}$ is the point of minima.
Hence, The local minimum value of f(x) at x = $\frac{3\pi}{4}$ is given by $\sin\left(\frac{3\pi}{2}\right) = -1$.

23. Given: $f(x) = \log(2+x) - \frac{2x}{2+x}, x \in R$
$\Rightarrow f'(x) = \frac{1}{2+x} - \frac{(2+x)2-2x\times1}{(2+x)^2}$
$\Rightarrow f'(x) = \frac{1}{2+x} - \frac{4+2x-2x}{(2+x)^2}$
$\Rightarrow f'(x) = \frac{1}{2+x} - \frac{4}{(2+x)^2}$
$\Rightarrow f'(x) = \frac{2+x-4}{(2+x)^2}$
$\Rightarrow f'(x) = \frac{x-2}{(2+x)^2}$
for (x) to be increasing, we must have
f(x) > 0
= (x) - 2 > 0
= 2 < x < 0
For (x) to be decreasing, we must have,
f(x) < 0
= x - 2 < 0
$\Rightarrow -\infty < x < 2$
$\Rightarrow x \in (-\infty, 2)$
So, f(x) is decreasing in $(-\infty, 2)$

OR

Given function: f(x) = tan x
$\Rightarrow f(x) = \frac{d}{dx}(\tan x)$
f'(x) = $\sec^2$x
Now, as given
x $\in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
That is 4th quadrant, where
= $\sec^2$x > 0
= f'(x) > 0
Hence, Condition for f(x) to be increasing
Thus, f(x) is increasing on the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

24. Clearly, $\frac{1}{\cos(x-a)\cos(x-b)} = \frac{1}{\sin(a-b)}\left[\frac{\sin(a-b)}{\cos(x-a)\cos(x-b)}\right]$
$= \frac{1}{\sin(a-b)}\left[\frac{\sin[(x-b)-(x-a)]}{\cos(x-a)\cos(x-b)}\right]$
$= \frac{1}{\sin(a-b)}\left[\frac{\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)}{\cos(x-a)\cos(x-b)}\right]$
$= \frac{1}{\sin(a-b)}[\tan(x-b) - \tan(x-a)]$

Now, $\int \frac{1}{\cos(x-a)\cos(x-b)}dx = \frac{1}{\sin(a-b)}\int[\tan(x-b)-\tan(x-a)]dx$

$= \frac{1}{\sin(a-b)}[-\log|\cos(x-b)|+\log|\cos(x-a)|]$

$= \frac{1}{\sin(a-b)}\left[\log\left|\frac{\cos(x-a)}{\cos(x-b)}\right|\right]+C$

25. Maximum value = 4, Minimum value = 2

We know that

$-1 \leq \sin\theta \leq 1$

$\therefore\ -1 \leq \sin 4x \leq 1$

Adding 3, on both sides, of above

We get

$-1+3 \leq \sin 4x + 3 \leq 1+3$

$2 \leq |\sin 4x + 3| \leq 4$

Hence min.Value is 2 and max value is 4.

Section C

26. Let $I = \int\limits_{-5}^{5}|x+2|\,dx$ ...(i)

Putting x + 2 = 0

$\Rightarrow x = -2 \in (-5, 5)$

$\therefore$ From eq. (i),

$I = \int\limits_{-5}^{-2}|x+2|dx + \int\limits_{-2}^{5}|x+2|\,dx$

$= \int\limits_{-5}^{-2}-(x+2)\,dx + \int\limits_{-2}^{5}(x+2)\,dx$

$= -\left(\frac{x^2}{2}+2x\right)_{-5}^{-2} + \left(\frac{x^2}{2}+2x\right)_{-2}^{5}$

$= -\left[\left(\frac{4}{2}-4\right)-\left(\frac{25}{2}-10\right)\right] + \left[\left(\frac{25}{2}+10\right)-\left(\frac{4}{2}-4\right)\right]$

$= -\left(-2-\frac{5}{2}\right)+\left(\frac{45}{2}+2\right)$

$= 2+\frac{5}{2}+\frac{45}{2}+2$

$= 4+25 = 29$

27. Let B_1 = bolt is manufactured by A

B_2 = bolt is manufactured by B

B_3 = bolt is manufactured by C

Let E denote the event that bolt is defective.

The event E occurs with B_1 or with B_2 or with B_3. Given that,

P (B_1) = 25% = 0.25

P (B_2) = 0.35

P (B_3) = 0.40

P (E|B_1) =Probability that the bolt drawn is defective given that it is manu-factured by machine A = 5% = 0.05.

Similarly, P (E|B_2) = 0.04

P (E|B_3) = 0.02

Hence, by Bayes theorm, we have,

$P(B_2/E) = \frac{P(B_2)P(E/B_2)}{P(B_1)P(E/B_1)+P(B_2)P(E/B_2)+P(B_3)P(E/B_3)}$

$= \frac{0.35\times0.04}{0.25\times0.05+0.35\times0.04+0.40\times0.02}$

$= \frac{0.0140}{0.0345}$

$= \frac{28}{69}$

28. Let the given integral be,

I = $\int$ cos (log x) dx

Let log x = t

$\Rightarrow$ x = e^t

$\Rightarrow$ dx = e^t dt

$\therefore$ I = $\int e^t \cos(t)\, dt$

Now solving by parts

Considering cos(t) as first function and e^t as second function

I = $\cos t\, e^t - \int (-\sin t)\, e^t\, dt$

$\Rightarrow$ I = $\cos t\, e^t + \int \sin t\, e^t\, dt$

$\Rightarrow$ I = $\cos t\, e^t + I_1$ ...(i)

where $I_1 = \int e^t \sin t\, dt$

$I_1 = \int e^t \sin t\, dt$

$I_1 = \int e^t \sin t\, dt$

Cosidering sin t as first function and e^t as second function

$I_1 = \sin t\, e^t - \int \cos t\, e^t\, dt$

$\Rightarrow I_1 = \sin t\, e^t - I$ ...(ii)

from (i) and (ii)

I = $\cos t\, e^t + \sin t\, e^t - I$

$\Rightarrow$ 2I = $e^t (\sin t + \cos t)$

$\Rightarrow I = \frac{e^t(\sin t+\cos t)}{2} + C$

$\Rightarrow I = \frac{e^{\log x}[\sin(\log x)+\cos(\log x)]}{2} + C$

$\Rightarrow I = \frac{x}{2}$ [sin (log x) + cos (log x)] + C

OR

Let the given integral be,

$I = \int_2^4 \frac{x^2+x}{\sqrt{2x+1}} dx$

Put $2x + 1 = z^2$

$\Rightarrow$ 2dx = 2zdz

$\Rightarrow$ dx = zdz

When

$x \to 2, z \to \sqrt{5}$

When

$x \to 4, z \to 3$

$\therefore I = \int_{\sqrt{5}}^{3} \frac{\left(\frac{z^2-1}{2}\right)^2 + \frac{z^2-1}{2}}{z} \times zdz$

$\Rightarrow I = \int_{\sqrt{5}}^{3} \frac{z^4-2z^2+1+2z^2-2}{4} dz$

$\Rightarrow I = \frac{1}{4} \int_{\sqrt{5}}^{3} \left(z^4 - 1\right) dz$

$\Rightarrow I = \frac{1}{4} \times \left(\frac{z^5}{5} - z\right)\Big|_{\sqrt{5}}^{3}$

$\Rightarrow I = \frac{1}{4}\left[\left(\frac{243}{5} - 3\right) - \left(\frac{25\sqrt{5}}{5} - \sqrt{5}\right)\right]$

$\Rightarrow I = \frac{1}{4} \times \frac{228}{5} - \frac{1}{4} \times 4\sqrt{5}$

$\Rightarrow I = \frac{57}{5} - \sqrt{5}$

29. The given differential equation is,

$x^2 - y^2)dx + 2xy\, dy = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2 - x^2}{2xy} = \frac{y}{2x} - \left(\frac{2y}{x}\right)^{-1}$$

$$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

$\Rightarrow$ the given differential equation is a homogenous equation.

The solution of the given differential equation is:

Put y = vx

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow \mathrm{v}+\mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}}=\frac{\mathrm{vx}}{2\mathrm{x}}-\left(\frac{2\mathrm{vx}}{\mathrm{x}}\right)^{-1}$

$\Rightarrow \mathrm{v}+\mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}}=\frac{\mathrm{v}}{2}-(2\mathrm{v})^{-1}$
$\Rightarrow \mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}}=\frac{\mathrm{v}}{2}-\frac{1}{2\mathrm{v}}-\mathrm{v}$

$\Rightarrow \mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}}=-\frac{\mathrm{v}}{2}-\frac{1}{2\mathrm{v}}$
$\Rightarrow \mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}}=-\left(\frac{2\mathrm{v}^2+2}{4\mathrm{v}}\right)$

$\Rightarrow \frac{2\mathrm{v}}{\mathrm{v}^2+1}=-\frac{\mathrm{dx}}{\mathrm{x}}$
Integrating both the sides we get:
$\int\frac{2\mathrm{v}}{\mathrm{v}^2+1}\mathrm{dv}=-\int\frac{\mathrm{dx}}{\mathrm{x}}+\mathrm{c}$

$\Rightarrow$ In $|v^2 + 1|$ = -In |x| + Inc
Resubstituting the value of y = vx we get
$\Rightarrow \ln\left|\left(\frac{y}{x}\right)^2+1\right|$ + In|x| = Inc
$\Rightarrow \left(\left(\frac{y}{x}\right)^2+1\right)$(x) = c

$\Rightarrow$ $x^2 + y^2$ = cx, which is the required solution.

OR

We know that the slope of the tangent at any point (x, y) of the curve is $\frac{dy}{dx}$
$\therefore \quad \frac{dy}{dx}=\frac{x^2+y^2}{2xy}$

On dividing the Numerator and denominator of RHS of (i) by x^2, we get,

$\frac{dy}{dx}=\frac{\left\{1+\left(\frac{y}{x}\right)^2\right\}}{2\left(\frac{y}{x}\right)}=f\left(\frac{y}{x}\right)$

Therefore, the given differential equation is homogeneous. Putting y = vx and $\frac{dy}{dx}=v+x\frac{dv}{dx}$ in (i), we have,
$v+x\frac{dv}{dx}=\frac{1+v^2}{2v}$
$\Rightarrow \quad x\frac{dv}{dx}=\left\{\frac{(1+v^2)}{2v}-v\right\}=\frac{(1-v^2)}{2v}$
$\Rightarrow \frac{2v}{(1-v^2)}dv=\frac{1}{x}dx$
$\Rightarrow \int\frac{2v}{(v^2-1)}dv=-\int\frac{1}{x}dx$ [on integrating both sides]

$\Rightarrow$ log $|v^2$ - 1| = -log |x| + log $|C_1|$

where C_1 is an arbitrary constant

$\Rightarrow$ log $|v^2$ - 1| + log |x| = log| $C_1|$

$\Rightarrow$ log $|(v^2$ - 1) x| = log $|C_1|$

$\Rightarrow$ $(v^2$ - 1) x = $\pm$ C_1

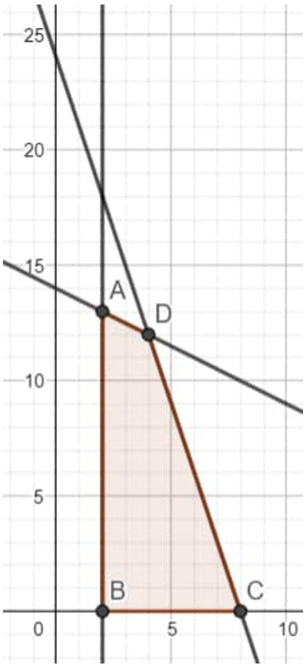
$\Rightarrow$ $(\frac{y^2}{x^2}$-1) x = $\pm$ C_1

$\Rightarrow$ $(y^2 - x^2)$ = $\pm$ C_1x

$\Rightarrow$ $(x^2 - y^2)$ = Cx, where $\pm$ C_1 = C

Therefore, $(x^2 - y^2)$ = Cx is the equation of the required family of curves.

30. Subject to the constraints are
x + 2y $\leq$ 28
3x + y $\leq$ 24
x $\geq$ 2
and the non-negative restrictions x, y $\geq$ 0
Converting the given inequations into equations, we get
x + 2y = 28, 3x + y = 24, x = 2, x = 0 and y = 0
These lines are drawn on the graph and the shaded region ABCD represents the feasible region of the given LPP.



It can be observed that the feasible region is bounded. The coordinates of the corner points of the feasible region are A(2, 13), B(2, 0), C(4, 12) and D(8, 0)

The values of the objective function, Z at these corner points are given in the following table:

Corner Point Value of the Objective Function Z = 20x + 10y

A(2, 13) : $Z = 20 \times 2 + 10 \times 13 = 170$

B(2, 0) : $Z = 20 \times 2 + 10 \times 0 = 40$

C(8, 0) : $Z = 20 \times 8 + 10 \times 0 = 160$

D(4, 12) : $Z = 20 \times 4 + 10 \times 12 = 200$

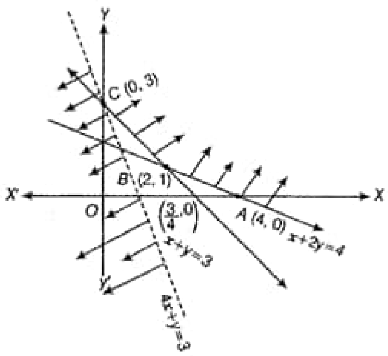
From the table, Z is maximum at x = 4 and y = 12 and the maximum value of objective function Z is 200.

OR

From the shaded region, it is clear that feasible region is unbounded with the corner points A(4, 0), B(2, 1) and C(0, 3).

Also, we have Z = 4x + y.

[Since, x + 2y = 4 and x + y = 3 ⇒ y = 1 and x = 2]



Corner Points	Corresponding value of Z
(4, 0)	16
(2, 1)	9
(0, 3)	3 (minimum)

Now, we see that 3 is the smallest value of Z at the corner point (0, 3). Note that here we see that the region is unbounded, therefore 3 may or may not be the minimum value of Z.

To decide this issue, we graph the inequality 4x + y < 3 and check whether the resulting open half plan has no point in common with feasible region otherwise, Z has no minimum value.

From the shown graph above, it is clear that there is no point in common with feasible region and hence Z has minimum value of 3 at (0, 3).

31. Given,

x = a cos θ ...(i)

y = b sin θ ...(ii)

To prove: $\frac{d^2y}{dx^2} = -\frac{b^4}{a^2y^3}$

To prove the above we will differentiate the function y wrt x two times .

As $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

So, lets first find dy/dx using parametric form and differentiate it again.

$\frac{dx}{d\theta} = \frac{d}{d\theta}a\cos\theta = -a\sin\theta$ ...(iii)

Similarly, $\frac{dy}{d\theta} = b\cos\theta$ ...(iv)

$\left[\because \frac{\mathrm{d}}{\mathrm{dx}}\cos\mathrm{x} = -\sin\mathrm{x}\tan\mathrm{x}, \frac{\mathrm{d}}{\mathrm{dx}}\sin\mathrm{x} = \cos\mathrm{x}\right]$

$\therefore \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\frac{\mathrm{dy}}{\mathrm{d}\theta}}{\frac{\mathrm{dx}}{\mathrm{d}\theta}} = -\frac{\mathrm{b}\cos\theta}{\mathrm{a}\sin\theta} = -\frac{\mathrm{b}}{\mathrm{a}}\cot\theta$

Differentiating again w.r.t x:

$\frac{\mathrm{d}}{\mathrm{dx}}\left(\frac{\mathrm{dy}}{\mathrm{dx}}\right) = \frac{\mathrm{d}}{\mathrm{dx}}\left(-\frac{\mathrm{b}}{\mathrm{a}}\cot\theta\right)$

$\frac{d^2y}{dx^2} = \frac{b}{a}\ cosec^2\theta\frac{d\theta}{dx}$ ...(v)

[using chain rule and $\frac{\mathrm{d}}{\mathrm{dx}}$ cot x = -cosec^{2}x]

From equation ...(iii)

$\frac{dx}{d\theta} = -\mathrm{a}\sin\theta$

$\therefore \frac{d\theta}{dx} = \frac{-1}{a\sin\theta}$

Putting the value in equation ...(v)

$\frac{d^2y}{dx^2} = -\frac{b}{a}\ cosec^2\theta\frac{1}{a\sin\theta}$

$\frac{d^2y}{dx^2} = \frac{-b}{a^2\sin^3\theta}$

From equation ...(i)

y = b sin θ

$\therefore \frac{d^2y}{dx^2} = \frac{-b}{\frac{a^2y^3}{b^3}} = -\frac{b^4}{a^2y^3}$

Hence proved

Section D

32. According to the question ,

Given parabola is y^2 = x......(i)

vertex of parabola is (0, 0)

axis of parabola lies along X-axis.

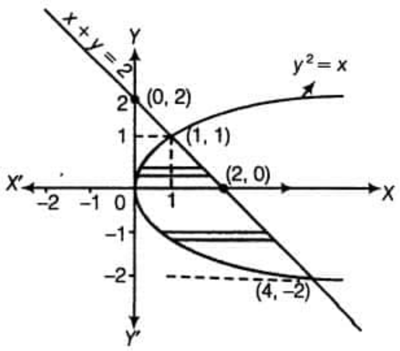
Given equation of line is x + y = 2.......(ii)

For, x + y = 2

x	2	0
y	0	2

So, line passes through the points (2, 0) and (0, 2).

Now, let us sketch the graph of given curve and line as shown below:



On putting x = 2 - y from Eq. (ii) in Eq. (i), we get

$y^2 = 2 - y$

$\Rightarrow y^2 + y - 2 = 0$

$\Rightarrow y^2+2y-y-2=0$
$\Rightarrow y(y+2)-1(y+2)=0$
$\Rightarrow (y-1)(y+2)=0$
$\therefore$ y = 1 or -2
When y = 1, then x = 2 - y = 1
When y = -2, then x = 2 - y = 2 - (- 2) = 4
So, points of intersection are (1, 1) and (4, - 2).
Now, required area = $\int_{-2}^{1}\left[x_{(line)}-x_{\text{(parabola)}}\right]dy$
$=\int_{-2}^{1}\left(2-y-y^2\right)dy$
$=\left[2y-\frac{y^2}{2}-\frac{y^3}{3}\right]_{-2}^{1}$
$=\left(2-\frac{1}{2}-\frac{1}{3}\right)-\left(-4-2+\frac{8}{3}\right)$
$=2-\frac{5}{6}+6-\frac{8}{3}$
$=8-\frac{5}{6}-\frac{8}{3}$
$=\frac{48-5-16}{6}$
$=\frac{48-21}{6}$
$=\frac{27}{6}$
$=\frac{9}{2}$ sq units.

33. We have, $A=\{x\in Z:0\leq x\leq 12\}$ be a set and
R = {(a, b) : a = b} be a relation on A
Now,
Reflexivity: Let a $\in$ A
$\Rightarrow$ a = a
$\Rightarrow$ (a, a) $\in$ R
$\Rightarrow$ R is reflexive
Symmetric: Let a, b, $\in$ A and (a, b) $\in$ R
$\Rightarrow$ a = b
$\Rightarrow$ b = a
$\Rightarrow$ (b, a) $\in$ R
$\Rightarrow$ R is symmetric
Transitive: Let a, b & c $\in$ A
and let (a, b) $\in$ R and (b, c) $\in$ R
$\Rightarrow$ a = b and b = c
$\Rightarrow$ a = c
$\Rightarrow$ (a, c) $\in$ R
$\Rightarrow$ R is transitive
Since R is being reflexive, symmetric and transitive, so R is an equivalence relation.
Also we need to find the set of all elements related to 1.
Since the relation is given by, R = {(a, b): a = b}, and 1 is an element of A.
R = {(1, 1): 1 = 1}
Thus, the set of all elements related to 1 is {1}.

OR

Given that
Let A = {1, 2, 3} and R = {(a, b) : a, b $\in$ A and $\left|a^2-b^2\right|\leq 5$
Put a = 1, b = 1 $\left|1^2-1^2\right|\leq 5,(1,1)$ is an ordered pair.
Put a = 1, b = 2 $\left|1^2-2^2\right|\leq 5,(1,2)$ is an ordered pair.
Put a = 1, b = 3 $\left|1^2-3^2\right|>5,(1,3)$ is not an ordered pair.
Put a = 2, b = 1 $\left|2^2-1^2\right|\leq 5,(2,1)$ is an ordered pair.
Put a = 2, b = 2 $\left|2^2-2^2\right|\leq 5,(2,2)$ is an ordered pair.
Put a = 2, b = 3 $\left|2^2-3^2\right|\leq 5,(2,3)$ is an ordered pair.
Put a = 3, b = 1 $\left|3^2-1^2\right|>5,(3,1)$ is not an ordered pair.
Put a = 3, b = 2 $\left|3^2-2^2\right|\leq 5,(3,2)$ is an ordered pair.

Put a = 3, b = 3 $\left|3^2 - 3^2\right| \leq 5, (3, 3)$ is an ordered pair.

R = {(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 2), (3, 3)}

i. For (a, a) ∈ R
   $\left|a^2 - a^2\right| = 0 \leq 5$. Thus, it is reflexive.

ii. Let (a, b) ∈ R
   $(a, b) \in R, \left|a^2 - b^2\right| \leq 5$
   $\left|b^2 - a^2\right| \leq 5$
   (b, a) ∈ R
   Hence, it is symmetric

iii. Put a = 1, b = 2, c = 3
   $\left|1^2 - 2^2\right| \leq 5$
   $\left|2^2 - 3^2\right| \leq 5$
   But $\left|1^2 - 3^2\right| > 5$
   Thus, it is not transitive

34. We have, $A = \begin{vmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{vmatrix}$ ...(i)

$\therefore |A| = 1(-3) - 2(-2) + 0 = 1 \neq 0$

Now, $A_{11} = -3, A_{12} = 2, A_{13} = 2,\ A_{21} = -2, A_{22} = 1, A_{23} = 1,\ A_{31} = -4, A_{32} = 2$ and $A_{33} = 3$

$\therefore adj\,(A) = \begin{vmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{vmatrix}^T = \begin{vmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{vmatrix}$

$\therefore A^{-1} = \frac{adjA}{|A|}$

$= \frac{1}{1}\begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$

$\Rightarrow A^{-1} = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ ...(i)

Also, we have the system of linear equations as

$x - 2y = 10$

$2x - y - z = 8$

and $-2y + z = 7$

Now, the given system of equations can be rewritten in the form AX=B,

where, $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$

Since A is non singular , therefore given system of equations has a unique solution given by,

$\therefore X = A^{-1}B$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$

$= \begin{bmatrix} -30 + 60 + 14 \\ -20 + 8 + 7 \\ -40 + 16 + 21 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$

$\therefore x = 0, y = -5$ and $z = -3$

35. Here,it is given that the equation of lines

L1 : $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$

L2 $= \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$

Direction ratios of L_1 and L_2 are (3, -1, 1) and (-3, 2, 4) respectively.

Suppose general point on line L_1 is P = (x_1, y_1, z_1)

x_1 = 3s + 3, y_1 = -s + 8, z_1 = s + 3 and suppose general point on line L_2 is Q = (x_2, y_2, z_2)

x_2 = -3t - 3, y_2 = 2t - 7, Z_2 = 4t + 6

$\therefore \overrightarrow{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

$= (-3t - 3 - 3s - 3)\hat{i} + (2t - 7 + s - 8)j + (4t + 6 - s - 3)\hat{k}$

$\therefore \overrightarrow{PQ} = (-3t - 3s - 6)\hat{i} + (2t + s - 15)j + (4t - s + 3)\hat{k}$

Direction ratios of $\overrightarrow{PQ}$ are ((-3t - 3s - 6, (2t + s - 15), (4t - s + 3))

PQ will be the shortest distance if it perpendicular to both the given lines

Thus, by the condition of perpendicularity,

$\implies$ 3(-3t - 3s - 6) - 1(2t + s - 15) + 1(4t - s + 3) = 0 and

$\implies$ -3(-3t - 3s - 6) + 2(2t + s - 15) + 4(4t - s + 3) = 0

⇒ -9t - 9s - 18 -2t - s + 15 + 4t - s + 3 = 0 and

9t + 9s + 18 + 4t + 2s - 30 + 16t - 4s + 12 = 0

⇒ -7t - 11s = 0 and

29t + 7s = 0

Solving above two equations, we get,

t = 0 and s = 0

therefore,

P = (3, 8, 3) and Q = (-3, -7, 6)

Now distance between points P and Q is

$d = \sqrt{(3+3)^2 + (8+7)^2 + (3-6)^2}$

$= \sqrt{(6)^2 + (15)^2 + (-3)^2}$

$= \sqrt{36 + 225 + 9}$

$= \sqrt{270}$

$= 3\sqrt{30}$

Thus, the shortest distance between two given lines is

$d = 3\sqrt{30}$ units

Now, equation of line passing through points P and Q is,

$\frac{x-x_1}{x_1-x_2} = \frac{y-y_1}{y_1-y_2} = \frac{z-z_1}{z_1-z_2}$

$\therefore \frac{x-3}{3+3} = \frac{y-8}{8+7} = \frac{z-3}{3-6}$

$\therefore \frac{x-3}{6} = \frac{y-8}{15} = \frac{z-3}{-3}$

$\therefore \frac{x-3}{2} = \frac{y-8}{5} = \frac{z-3}{-1}$

Thus, equation of line of shortest distance between two given lines is

$\frac{x-3}{2} = \frac{y-8}{5} = \frac{z-3}{-1}$

OR

Line passing through (1, 2, 3)

ie $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and parallel to the given planes is perpendicular to the vectors

$\vec{b}_1 = \hat{i} - \hat{j} + 2\hat{k}$ and

$\vec{b}_2 = 3\hat{i} + \hat{j} + \hat{k}$

Required line is parallel to $\vec{b}_1 \times \vec{b}_2$

$\vec{b} = \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 3 & 1 & 1 \end{vmatrix} = \vec{i}(-1-2) - \vec{j}(1-6) + \vec{k}(1+3) = -3\vec{i} + 5\vec{j} + 4\vec{k}$

Required education of line is : -

$\vec{r} = \vec{a} + \lambda\vec{b}$

$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda\left(-3\hat{i} + 5\hat{j} + 4\hat{k}\right)$

Section E

36. **Read the text carefully and answer the questions:**

Shama is studying in class XII. She wants do graduate in chemical engineering. Her main subjects are mathematics, physics, and chemistry. In the examination, her probabilities of getting grade A in these subjects are 0.2, 0.3, and 0.5 respectively.

(i) P (Grade A in Maths) = P(M) = 0.2

P (Grade A in Physics) = P(P) = 0.3

P (Grade A in Chemistry) = P(C) = 0.5

P(not A garde in Maths) = P($\overline{M}$) = 1 - 0.2 = 0.8

P(not A garde in Physics) = P($\overline{P}$) = 1 - 0.3 = 0.7

P(not A garde in Chemistry) = P($\overline{C}$) = 1 - 0.5 = 0.5

P(getting grade A in all subjects) = P(M $\cap P \cap$ C)

= P(M) $\times$ P(P) $\times$ P(C)

= 0.2 $\times$ 0.3 $\times$ 0.5 = 0.03

(ii) P (Grade A in Maths) = P(M) = 0.2

P (Grade A in Physics) = P(P) = 0.3

P (Grade A in Chemistry) = P(C) = 0.5

P(not A garde in Maths) = P($\overline{M}$) = 1 - 0.2 = 0.8

P(not A garde in Physics) = P($\overline{P}$) = 1 - 0.3 = 0.7

P(not A garde in Chemistry) = P($\overline{C}$) = 1 - 0.5 = 0.5

P(getting grade A in on subjects) = $P(\overline{M} \cap \overline{P} \cap \overline{C})$

= $P(\overline{M}) \times P(\overline{P}) \times P(\overline{C})$

= 0.8 $\times$ 0.7 $\times$ 0.5 = 0.280

(iii) P (Grade A in Maths) = P(M) = 0.2

P (Grade A in Physics) = P(P) = 0.3

P (Grade A in Chemistry) = P(C) = 0.5

P(not A garde in Maths) = P($\overline{M}$) = 1 - 0.2 = 0.8

P(not A garde in Physics) = P($\overline{P}$) = 1 - 0.3 = 0.7

P(not A garde in Chemistry) = P($\overline{C}$) = 1 - 0.5 = 0.5

P(getting grade A in 2 subjects)

$\Rightarrow$ P(grade A in M and P not in C) + P(grade A in P & C not in M) + P(grade A in M & C not in P)

$\Rightarrow$ P$\left(M \cap P \cap \overline{C}\right)$ + P$\left(P \cap C \cap \overline{M}\right)$ + P$(M \cap C \cap \overline{P})$

$\Rightarrow$ 0.2 $\times 0.3 \times$ 0.5 + 0.3 $\times 0.5 \times$ 0.8 + 0.2 $\times 0.5 \times$ 0.7 = 0.03 + 0.12 + 0.07

P(getting grade A in 2 subjects) = 0.22

OR

P (Grade A in Maths) = P(M) = 0.2

P (Grade A in Physics) = P(P) = 0.3

P (Grade A in Chemistry) = P(C) = 0.5

P(not A garde in Maths) = P($\overline{M}$) = 1 - 0.2 = 0.8

P(not A garde in Physics) = P($\overline{P}$) = 1 - 0.3 = 0.7

P(not A garde in Chemistry) = P($\overline{C}$) = 1 - 0.5 = 0.5

P(getting grade A in 1 subjects)

$\Rightarrow$ P(grade A in M not in P and C) + P(grade A in P not in M and C) + P(grade A in C not in P and M)

$\Rightarrow$ P$\left(M \cap \overline{P} \cap \overline{C}\right)$ + P$\left(P \cap \overline{C} \cap \overline{M}\right)$ + P$(C \cap \overline{M} \cap \overline{P})$

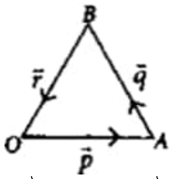
$\Rightarrow$ 0.2 $\times 0.7 \times$ 0.5 + 0.3 $\times 0.5 \times$ 0.8 + 0.5 $\times 0.8 \times$ 0.7 = 0.07 + 0.12 + 0.028

P(getting grade A in 1 subjects) = 0.47

37. **Read the text carefully and answer the questions:**

If two vectors are represented by the two sides of a triangle taken in order, then their sum is represented by the third side of the triangle taken in opposite order and this is known as triangle law of vector addition.

(i) Let OAB be a triangle such that



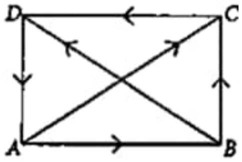
$\overrightarrow{AO} = -\vec{p}, \overrightarrow{AB} = \vec{q}, \overrightarrow{BO} = \vec{r}$

Now, $\vec{q} + \vec{r} = \overrightarrow{AB} + \overrightarrow{BO}$

$= \overrightarrow{AO} = -\vec{p}$

(ii) From triangle law of vector addition,

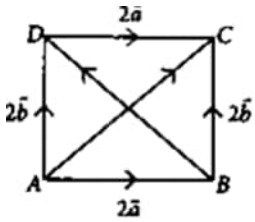
$\overrightarrow{AC} + \overrightarrow{BD} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{BC} + \overrightarrow{CD}$



= $\overrightarrow{AB} + 2\overrightarrow{BC} + \overrightarrow{CD}$

= $\overrightarrow{AB} + 2\overrightarrow{BC} - \overrightarrow{AB} = 2\overrightarrow{BC}$ $[\because \overrightarrow{AB} = -\overrightarrow{CD}]$

(iii) In $\triangle$ABC, $\overrightarrow{AC}$ = $2\vec{a}$ + $2\vec{b}$ ...(i)



and in $\triangle$ABD, $2\vec{b}$ = $2\vec{a}$ + $\overrightarrow{BD}$ ...(ii) [By triangle law of addition]

Adding (i) and (ii), we have $\overrightarrow{AC} + 2\vec{b} = 4\vec{a} + \overrightarrow{BD} + 2\vec{b}$

$\Rightarrow \overrightarrow{AC} - \overrightarrow{BD} = 4\vec{a}$

OR

Since T is the mid point of YZ

So, $\overrightarrow{YT} = \overrightarrow{TZ}$

Now, $\overrightarrow{XY} + \overrightarrow{XZ} = (\overrightarrow{XT} + \overrightarrow{TY}) + (\overrightarrow{XT} + \overrightarrow{TZ})$ [By triangle law]

= $2\overrightarrow{XT} + \overrightarrow{TY} + \overrightarrow{TZ} = 2\overrightarrow{XT}$ $[\because \overrightarrow{TY} = -\overrightarrow{YT}]$

38. **Read the text carefully and answer the questions:**

Mrs. Maya is the owner of a high-rise residential society having 50 apartments. When he set rent at ₹10000/month, all apartments are rented. If he increases rent by ₹250/ month, one fewer apartment is rented. The maintenance cost for each occupied unit is ₹500/month.

(i) If P is the rent price per apartment and N is the number of rented apartments, the profit is given by NP - 500 N = N(P - 500)

[∵ ₹500/month is the maintenance charge for each occupied unit]

(ii) Let R be the rent price per apartment and N is the number of rented apartments.

Now, if x be the number of non-rented apartments, then N(x) = 50 - x and R(x) = 10000 + 250x

Thus, profit = $P(x) = NR = (50 - x)(10000 + 250x - 500)$

$= (50 - x)(9500 + 250x) = 250(50 - x)(38 + x)$