Class XII Session 2023-24
Subject - Mathematics
Sample Question Paper - 9

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ’s and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If A is a skew symmetric matrix of order 3, then the value of |A| is **[1]**

a) 0 b) 27

c) 9 d) 3

2. If $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then **[1]**

a) none of these b) a = cos 2θ, b = sin 2θ

c) a = 1, b = 1 d) a = sin 2θ, b = cos 2θ

3. If $A = \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$ and B is square matrix of order 2 such that AB = I then B =? **[1]**

a) $\begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix}$ b) $\begin{bmatrix} 1 & 2 \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$

c) None of these d) $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

4. If x = at^2 ,y = 2 a t, then $\frac{d^2y}{dx^2}$ = **[1]**

a) None of these b) 0

c) $\frac{1}{t^2}$ d) $-\frac{1}{2a\ t^3}$

5. The direction ratios of the line perpendicular to the lines $\frac{x-7}{2} = \frac{y+17}{-3} = \frac{z-6}{1}$ and $\frac{x+5}{1} = \frac{y+3}{2} = \frac{z-4}{-2}$ are proportional to **[1]**

a) 4, 5, 7 b) -4, 5, 7

c) 4, -5, -7 | d) 4, -5, 7

6. The degree of the differential equation $\left\{5+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{5}{3}}=x^5\left(\frac{d^2y}{dx^2}\right)$, is **[1]**

a) 4
b) 5
c) None of these
d) 2

7. By graphical method, the solution of linear programming problem **[1]**

Maximize Z = $3x_1 + 5x_2$

Subject to $3x_1 + 2x_2 \leq 18$

$x_1 \leq 4$

$x_2 \leq 6$

$x_1 \geq 0$, $x_2 \geq 0$, is

a) $x_1 = 2$, $x_2 = 0$, Z = 6
b) $x_1 = 4$, $x_2 = 6$, Z = 42
c) $x_1 = 2$, $x_2 = 6$, Z = 36
d) $x_1 = 4$, $x_2 = 3$, Z = 27

8. Forces $3\,\overrightarrow{OA}, 5\,\overrightarrow{OB}$ act along OA and OB. If their resultant passes through C on AB, then **[1]**

a) 3 AC = 5 CB
b) divides AB in the ratio 2 : 1
c) 2 AC = 3CB
d) C is a mid-point of AB

9. $\int |x|^3 dx$ is equal to **[1]**

a) $\sin\sqrt{x}+C$
b) $\frac{-x^4}{4}+C$
c) none of these
d) $\frac{|x|^4}{4}+c$

10. If A is a square matrix such that $A^2 = A$, then $(I - A)^3 + A$ is equal to: **[1]**

a) I
b) I - A
c) I + A
d) 0

11. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Let F = 4x + 6y be the objective function. Maximum of F – Minimum of F = **[1]**

a) 48
b) 60
c) 42
d) 18

12. If $\vec{a}=(\hat{i}+2\hat{j}-3\hat{k})$ and $\vec{b}=(3\hat{i}-\hat{j}+2\hat{k})$ then the angle between $(2\vec{a}+\vec{b})$ and $(\vec{a}+2\vec{b})$ is **[1]**

a) $\cos^{-1}\left(\frac{31}{50}\right)$
b) none of these
c) $\cos^{-1}\left(\frac{21}{40}\right)$
d) $\cos^{-1}\left(\frac{11}{30}\right)$

13. If $\begin{bmatrix} x & y \\ 3y & x \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}=\begin{bmatrix} 3 \\ 5 \end{bmatrix}$ then find x and y ? **[1]**

a) None of these
b) x = 1, y = 2
c) x = 2, y = 1
d) x = 1, y = 1

14. If A $\subseteq$ B, then which one of the following is not correct? **[1]**

a) P(B/A) = $\frac{P(B)}{P(A)}$
b) P(A/B) = $\frac{P(A)}{P(B)}$

c) $P(A \cap \bar{B}) = 0$

d) $P(A/(A \cup B)) = \frac{P(A)}{P(B)}$

15. The solution of the differential equation $\frac{dy}{dx} = \frac{x^2+xy+y^2}{x^2}$, is **[1]**

a) $\tan^{-1}\left(\frac{y}{x}\right) = \log y + C$

b) $\tan^{-1}\left(\frac{x}{y}\right) = \log x + c$

c) $\tan^{-1}\left(\frac{y}{x}\right) = \log x + C$

d) $\tan^{-1}\left(\frac{x}{y}\right) = \log y + C$

16. If $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}, \vec{a} = 0$, then **[1]**

a) $\vec{b} + \vec{c} = \vec{0}$

b) none of these

c) $\vec{b} = \vec{c}$

d) $\vec{b} = \vec{0}$

17. If x = acos^{3}t, y = asin^{3}t, then $\frac{dy}{dx}$ is equal to **[1]**

a) – tan t

b) cosec t

c) cos t

d) cot t

18. If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular to each other then k = ? **[1]**

a) $\frac{-10}{7}$

b) $\frac{5}{7}$

c) $\frac{-5}{7}$

d) $\frac{10}{7}$

19. **Assertion (A):** If two positive numbers are such that sum is 16 and sum of their cubes is minimum, then numbers are 8, 8. **[1]**

Reason (R): If f be a function defined on an interval I and c $\in$ l and let f be twice differentiable at c, then x = c is a point of local minima if f'(c) = 0 and f''(c) > 0 and f(c) is local minimum value of f.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** A function f: N $\to$ N be defined by $f(n) = \begin{cases} \frac{n}{2} & \text{if } n \text{ is even} \\ \frac{(n+1)}{2} & \text{if } n \text{ is odd} \end{cases}$ for all n $\in$ N; is one-one. **[1]**

Reason (R): A function f: A $\to$ B is said to be injective if a $\neq$ b then f(a) $\neq$ f(b).

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Write the value of $\sin^{-1}\left(\frac{1}{3}\right) - \cos^{-1}\left(-\frac{1}{3}\right)$ **[2]**

OR

Which is greater, tan 1 or tan^{-1} 1?

22. The length x of a rectangle is decreasing at the rate of 2 cm/sec and the width y is increasing at the rate of 2 cm/sec. When x = 12 cm and y = 5 cm, find the rate of change of **[2]**

i. the perimeter and
ii. the area of the rectangle.

23. Find all the point of local maxima and local minima of the function $f(x) = -\frac{3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105$. **[2]**

OR

Prove that the function f given by f(x) = x^2 - x + 1 is neither strictly increasing nor strictly decreasing on (-1, 1).

24. Evaluate $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}} dx$ **[2]**

25. Prove that the function given by f(x) = log sinx is strictly increasing on $\left(0, \frac{\pi}{2}\right)$ and strictly decreasing on $\left(\frac{\pi}{2}, \pi\right)$ **[2]**

Section C

26. Evaluate: $\int \frac{(3\sin\theta-2)\cos\theta}{(5-\cos^2\theta-4\sin\theta)} d\theta$ **[3]**

27. Determine P(E|F): A coin is tossed two times. **[3]**

   i. E : tail appears on one coin, F : one coin shows head.
   ii. E : no tail appears, F : no head appears.

28. Evaluate: $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$ **[3]**

OR

Evaluate $\int_1^5 (|x-1|+|x-2|+|x-4|) dx$:

29. Solve the differential equation: (x+ 2) $\frac{dy}{dx}$ = x^2 + 4x - 9, x $\neq$ -2 **[3]**

OR

Find the general solution: $\frac{dy}{dx} + 3y = e^{-2x}$

30. Maximise and minimise Z = x + 2y **[3]**

subject to the constraints

x + 2y $\geq$100

2x - y $\leq$ 0

2x + y $\leq$200

x, y$\geq$ 0

Solve the above LPP graphically.

OR

Solve the Linear Programming Problem graphically:

Maximize Z = 7x + 10y Subject to

x + y $\leq$ 30000

y $\leq$ 12000

x $\geq$ 6000

x $\geq$ y

x, y $\geq$ 0

31. If y = e^{ax}.cos bx, then prove that: $\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + \left(a^2+b^2\right)y = 0$ **[3]**

Section D

32. Sketch the graph of y = Ix + 3I and evaluate the area under the curve y = Ix + 3I above X - axis and between x = - 6 to x = 0. **[5]**

33. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a - b| is even}, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of {2, 4}. **[5]**

OR

Let A = {1, 2, 3} and R = {(a, b):a, b $\in$ A and $\left|a^2-b^2\right| \leq 5$. Write R as set of ordered pairs. Mention whether R is

   i. reflexive
   ii. symmetric

iii. transitive

Give reason in each case.

34. Two institutions decided to award their employees for the three values of resourcefulness, competence and determination in the form of prizes at the rate of Rs.x, Rs.y, and Rs.z, respectively per person. The first institution decided to award respectively 4, 3 and 2 employees with total prize money of Rs.37000 and the second institution decided to award respectively 5, 3 and 4 employees with total prize money of, Rs.47000. If all the three prizes per person together amount to Rs.12000, then using a matrix method, find the values of x, y, and z. What values are described in this question? **[5]**

35. Find the length and the foot of perpendicular from the point $\left(1, \frac{3}{2}, 2\right)$ to the plane 2x - 2y + 4z + 5 = 0. **[5]**

OR

Find the equation of the plane passing through the line of intersection of the plane $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ and $\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$ and parallel to X-axis.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Akash and Prakash appeared for first round of an interview for two vacancies. The probability of Nisha's selection is $\frac{1}{3}$ and that of Ayushi's selection is $\frac{1}{2}$.

(i) Find the probability that both of them are selected.

(ii) The probability that none of them is selected.

(iii) Find the probability that only one of them is selected.

OR

Find the probability that atleast one of them is selected.

37. **Read the text carefully and answer the questions:** **[4]**

Three slogans on chart papers are to be placed on a school bulletin board at the points A, B and C displaying A (Hub of Learning), B (Creating a better world for tomorrow) and C (Education comes first). The coordinates of these points are (1, 4, 2), (3, -3, -2) and (-2, 2, 6) respectively.

(i) Let $\vec{a}, \vec{b}$ and $\vec{c}$ be the position vectors of points A, B and C respectively, then find $\vec{a} + \vec{b} + \vec{c}$.

(ii) What is the Area of $\triangle$ABC.

(iii) Suppose, if the given slogans are to be placed on a straight line, then find the value of $|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$.

OR

If $\vec{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$, then find the unit vector in the direction of vector $\vec{a}$.

38. **Read the text carefully and answer the questions:** **[4]**

The relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation $y = 4x - \frac{1}{2}x^2$ where x is the number of days exposed to sunlight.

(i) Find the rate of growth of the plant with respect to sunlight.

(ii) What is the number of days it will take for the plant to grow to the maximum height?

Solution

Section A

1. **(a)** 0

   Explanation: A is skew-symmetric means $A^t = -A$. Taking determinant both sides

   $\Delta\left(A^t\right)=\Delta(-A)$

   $\Rightarrow \Delta(A)=(-1)^3 \Delta(A)$

   $\Rightarrow \Delta(A)=-\Delta(A)$

   Which is only possible when $\Delta(A)$ = 0

2. **(b)** a = cos 2θ, b = sin 2θ

   Explanation: $\begin{bmatrix}1 & -\tan\theta \\ \tan\theta & 1\end{bmatrix}\begin{bmatrix}1 & \tan\theta \\ -\tan\theta & 1\end{bmatrix}^{-1}=\begin{bmatrix}a & -b \\ b & a\end{bmatrix}$

   $\begin{bmatrix}1 & -\tan\theta \\ \tan\theta & 1\end{bmatrix}\frac{1}{\sec^2\theta}\begin{bmatrix}1 & -\tan\theta \\ \tan\theta & 1\end{bmatrix}=\begin{bmatrix}a & -b \\ b & a\end{bmatrix}$

   $\frac{1}{\sec^2\theta}\begin{bmatrix}1 & -\tan\theta \\ \tan\theta & 1\end{bmatrix}\begin{bmatrix}1 & -\tan\theta \\ \tan\theta & 1\end{bmatrix}=\begin{bmatrix}a & -b \\ b & a\end{bmatrix}$

   $\begin{bmatrix}\frac{1-\tan^2\theta}{\sec^2\theta} & \frac{-2\tan\theta}{\sec^2\theta} \\ \frac{2\tan\theta}{\sec^2\theta} & \frac{1-\tan^2\theta}{\sec^2\theta}\end{bmatrix}=\begin{bmatrix}a & -b \\ b & a\end{bmatrix}$

   $a=\frac{1-\tan^2\theta}{\sec^2\theta}$ Use tan in terms of sin and cos and simplify, we get,

   a = $\cos^2\theta$ - $\sin^2\theta$

   a = cos 2θ

   and $b=\frac{2\tan\theta}{\sec^2\theta}$ = sin 2θ

3. **(b)** $\begin{bmatrix}1 & 2 \\ \frac{1}{2} & \frac{3}{2}\end{bmatrix}$

   Explanation: B = I

   B = A^{-1} I ....(i)

   $A^{-1}=\frac{1}{|A|}\operatorname{adj} A$ ...(ii)

   |A| = 3 × 2 - (-4) × (-1)

   = 2

   C_{11} = 2, C_{12} = 1

   C_{21} = 4, C_{22} = 3

   Co-factor matrix $A=\begin{pmatrix}2 & 1 \\ 4 & 3\end{pmatrix}$

   $\operatorname{Adj} A=\begin{pmatrix}2 & 1 \\ 4 & 3\end{pmatrix}'$

   $=\begin{pmatrix}2 & 4 \\ 1 & 3\end{pmatrix}$

   Putting in 2

   $A^{-1}=\frac{1}{|2|}\begin{pmatrix}2 & 4 \\ 1 & 3\end{pmatrix}$

   $=\begin{pmatrix}1 & 2 \\ \frac{1}{2} & \frac{3}{2}\end{pmatrix}$

   Put this value in equation (i)

   B = A^{-1} I

$= A^{-1}$

$$= \begin{pmatrix} 1 & 2 \\ \frac{1}{2} & \frac{3}{2} \end{pmatrix}$$

4.

(d) $-\frac{1}{2a\ t^3}$

Explanation: $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \frac{1}{t}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{t}\right) = -\frac{1}{t^2}\frac{dt}{dx} = -\frac{1}{t^2}.\frac{1}{2at} = -\frac{1}{2at^3}$

5. **(a)** 4, 5, 7

Explanation: We have,

$\frac{x-7}{2} = \frac{y+17}{-3} = \frac{z-6}{1}$

$\frac{x+5}{1} = \frac{y+3}{2} = \frac{z-4}{-2}$

The direction ratios of the given lines are proportional to 2, -3, 1 and 1, 2, -2.

The vectors parallel to the given lines are $\overrightarrow{b_1} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\overrightarrow{b_2} = \hat{i} + 2\hat{j} - 2\hat{k}$

Vector perpendicular to the vectors b_1 & b_2 is ,

$\vec{b} = \overrightarrow{b_1} \times \overrightarrow{b_2}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 1 & 2 & -2 \end{vmatrix}$

$= 4\hat{i} + 5\hat{j} + 7\hat{k}$

Hence, the direction ratios of the line perpendicular to the given two lines are proportional to 4, 5, 7.

6.

(c) None of these

Explanation: We have,

$\left\{5 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{5}{3}} = x^5\left(\frac{d^2y}{dx^2}\right)$

$\Rightarrow \left\{5 + \left(\frac{dy}{dx}\right)^2\right\}^5 = \left\{x^5\left(\frac{d^2y}{dx^2}\right)\right\}^3$

Degree is 3.

Therefore none of the given options are matching with answer.

7.

(c) $x_1 = 2$, $x_2 = 6$, Z = 36

Explanation: We need to maximize the function $z = 3x_1 + 5x_2$

First, we will convert the given inequations into equations, we obtain the following equations: $3x_1 + 2x_2 = 18$, $x_1 = 4$, $x_2 = 6$, $x_1 = 0$ and $x_2 = 0$

Region represented by $3x_1 + 2x_2 \leq 18$

The line $3x_1 + 2x_2 = 18$ meets the coordinate axes at A(6, 0) and B(0, 9) respectively. By joining these points we obtain the line $3x_1 +$

$2x_2 = 18$

Clearly (0, 0) satisfies the inequation $3x_1 + 2x_2 = 18$. So, the region in the plane which contain the origin represents the solution set of the inequation $3x_1 + 2x_2 = 18$

Region represented by $x_1 \leq 4$:

The line $x_1 = 4$ is the line that passes through C(4, 0) and is parallel to the Y axis. The region to the left of the line $x_1 = 4$ will satisfy the inequation $x_1 \leq 4$

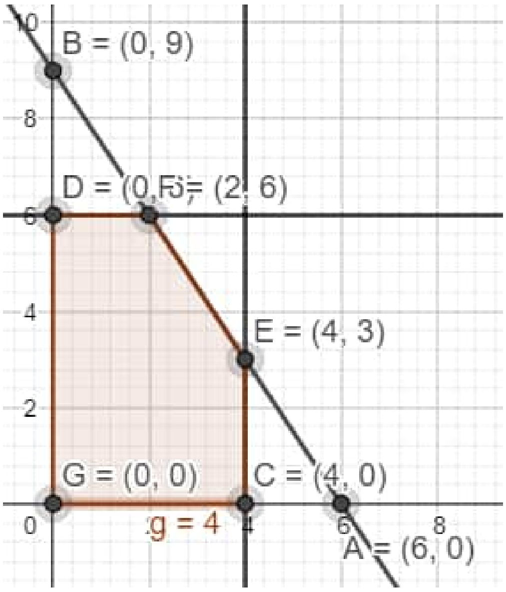
Region represented by $x_2 \leq 6$ The line $x_2 = 6$ is the line that passes through D(0, 6) and is parallel to the x axis. The region below the line $x_2 = 6$ will satisfy the inequation $x_2 \leq 6$.

Region represented by $x_1 \geq 0$ and $x_2 \geq 0$

since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations $x_1 \geq$ oand $x_2 \geq 0$

The feasible region determined by the system of constraints, $3x_1 + 2x_2 \leq 18$, $x_1 \leq 4$, $x_2 \leq 6$, $x_1 \geq 0$, and $x_2 \geq 0$, are as follows :

Corner points are O(0, 0), D(0, 6), F(2, 6), E(4, 3) and C(4, 0).



The values of the objective function at these points are given in the following table

Points : Value of Z

O(0, 0) : 3(0) + 5(0) = 0

D(0, 6) : 3(0) + 5(6) = 30

F(2, 6) : 3(2) + 5(6) = 36

E(4, 3) : 3(4) + 5(3) = 27

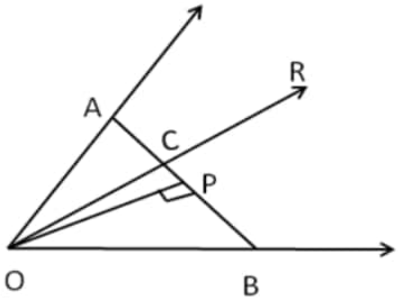
C(4, 0) : 3(4) + 5(0) = 12

We see that the maximum value of the objective function Z is 36 which is at F(2, 6)

8. **(a)** 3 AC = 5 CB

Explanation:

Draw ON, the perpendicular to the lien AB



Let $\vec{i}$ be the unit vector along ON

The resultant force $\vec{R} = 3\overrightarrow{OA} + 5\overrightarrow{OB}$

The angles between $\vec{i}$ and the forces $\vec{R}, 3\overrightarrow{OA}, 5\overrightarrow{OB}$ are $\angle$CON, $\angle$AON, $\angle$BON respectively.

$\Rightarrow R\cos\angle COP = 3OA \times \frac{\mathrm{OP}}{\mathrm{OA}} + 5\mathrm{OB} \times \frac{\mathrm{OP}}{\mathrm{OB}}$

$\frac{R}{OD} = 3 + 5$

$R = 8\overrightarrow{\mathrm{OC}}$

$\overrightarrow{\mathrm{OA}} = \overrightarrow{\mathrm{OC}} + \overrightarrow{\mathrm{CA}}$

$\Rightarrow 3\overrightarrow{OA} = 3\overrightarrow{OC} + 3\overrightarrow{CA}$ ...(i)

$\overrightarrow{OB} = \overrightarrow{OC} + \overrightarrow{CB}$

$\Rightarrow 5\overrightarrow{OB} = 5\overrightarrow{OC} + 5\overrightarrow{CB}$ ...(ii)

$\vec{R} = 8\overrightarrow{OC} + 3\overrightarrow{CA} + 5\overrightarrow{CB}$

$8\overrightarrow{OC} = 3\overrightarrow{CA} + 5\overrightarrow{CB}$

$|\overrightarrow{3CA}| = |\overrightarrow{5CB}|$

$\Rightarrow$ 3CA = 5CB

9.

(c) none of these

Explanation: $\int |x|^3 dx = \int x^3 dx,\ If\ x > 0$

$= \frac{x^4}{4} + c$

And, $\int |x|^3 dx = \int -x^3 dx$, If x < 0

$= -\frac{x^4}{4} + c$

10. **(a)** I

Explanation: Given that $A^2 = A$

Calculating value of $(I - A)^3 + A$:

$(I - A)^3 + A = I^3 - 3I^2A + 3IA^2 - A^3 + A$

$= I - A^2A - 3A + 3A^2 + A$ ($\because I^n = I$ and $IA = A$)

$= I - AA - 3A + 3A + A$ ($\because A^2 = A$)

$= I + A^2 - 3A + 3A + A$

$= I$

Hence, $(I - A)^3 + A = I$

11.

(b) 60

Explanation: Here the objective function is given by : F = 4x +6y .

Corner points	Z = 4x +6 y
(0, 2)	12(Min.)
(3,0)	12.(Min.)
(6,0)	24
(6 , 8)	72
(0 , 5)	30

Maximum of F – Minimum of F = 72 – 12 = 30 .

12. **(a)** $\cos^{-1}\left(\frac{31}{50}\right)$

Explanation: Given vectors $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}\ and\ \vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$

$(2\vec{a} + \vec{b}) = (5\hat{i} + 3\hat{j} - 4\hat{k})$ and $(\vec{a} + 2\vec{b}) = (7\hat{i} + 0\hat{j} + \hat{k})$

$\cos\theta = \frac{(5\times7+3\times0-4\times1)}{\sqrt{50}\times\sqrt{50}} = \frac{31}{50} \Rightarrow \theta = \cos^{-1}\left(\frac{31}{50}\right)$

13.

(d) x = 1, y = 1

Explanation: $\begin{pmatrix} x & y \\ 3y & x \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} x \times 1 + y \times 2 \\ 3y \times 1 + x \times 2 \end{pmatrix}$

$= \begin{pmatrix} x + 2y \\ 3y + 2x \end{pmatrix}$

Comparing with R.H.S
x + 2y = 3 ...(i)
2x + 3y = 5 ...(ii)
(i) x 2 - (ii)
2x + 4y - 2x + 3y = 6 - 5
y = 1
Putting y in (i)
x + 2(1) = 3
x = 1

14. **(a)** P(B/A) = $\frac{P(B)}{P(A)}$

Explanation: As, A $\subseteq$ B, then A $\cup$ B = B and A $\cap$ B = A
Clearly, P(A $\cap \bar{B}$) = P(ϕ) = 0
Now, P $\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)}$
$\Rightarrow P\left(\frac{A}{(A \cup B)}\right) = P\left(\frac{A}{B}\right) = \frac{P(A)}{P(B)}$
but $P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)} = \frac{P(A)}{P(A)}$ = 1

15.

(c) $\tan^{-1}\left(\frac{y}{x}\right) = \log x + C$

Explanation: We have,
$\frac{dy}{dx} = \frac{x^2+xy+y^2}{x^2}$
$\frac{dy}{dx} = 1 + \frac{y}{x} + \frac{y^2}{x^2}$ ...(i)
Let y = vx
$\frac{dy}{dx} = v + x\frac{dv}{dx}$
$1 + v + v^2 = v + x\frac{dv}{dx}$ ....from(i)
$1 + v^2 = x\frac{dv}{dx}$
$\frac{dx}{x} = \frac{dv}{1+v^2}$
$\int \frac{dx}{x} = \int \frac{dv}{1+v^2}$
log |x| = $\tan^{-1}$x + c

16.

(c) $\vec{b} = \vec{c}$

Explanation: $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$
$\Rightarrow \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c} = 0$ and $\vec{a} \times \vec{b} - \vec{a} \times \vec{c} = 0$
$\Rightarrow \vec{a} \cdot (\vec{b} - \vec{c}) = 0$ and $\vec{a} \times (\vec{b} - \vec{c}) = 0$
$\Rightarrow \vec{b} - \vec{c} = 0 \Rightarrow \vec{b} = \vec{c}$
Also,
$\Rightarrow |\vec{a}||\vec{b} - \vec{c}| \cos\theta = 0$ and $|\vec{a}||\vec{b} - \vec{c}| \sin\theta = 0$
$\Rightarrow$ If $\theta = \frac{\pi}{2} \Rightarrow \sin\theta = 1 \Rightarrow \vec{b} - \vec{c} = 0 \Rightarrow \vec{b} = \vec{c}$

17. **(a)** – tan t

Explanation: We have to find: $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3a\sin^2 t \cos t}{3a\cos^2 t(-\sin t)} = -\tan t$

18. **(a)** $\frac{-10}{7}$

Explanation: If the lines are perpendicular to each other then the angle between these lines will be $\frac{\pi}{2}$, then the cosine will be 0
$\vec{a} = -3\hat{i} + 2k\hat{j} + 2\hat{k}$
$\Rightarrow |\vec{a}| = \sqrt{3^2 + (2k)^2 + 2^2}$
$= \sqrt{13 + 4k^2}$

$\vec{b} = 3k\hat{i} + \hat{j} - 5\hat{k}$

$\Rightarrow |\vec{b}| = \sqrt{(3k)^2 + 1 + 5^2}$

$= \sqrt{9k^2 + 26}$

$\cos\left(\frac{\pi}{2}\right) = \frac{(3k\hat{i}+\hat{j}-5\hat{k})\cdot(-3\hat{i}+2k\hat{j}+2\hat{k})}{\sqrt{13+4k^2}\times\sqrt{9k^2+26}}$

$0 = \frac{-9k+2k-10}{\sqrt{13+4k^2}\times\sqrt{9k^2+26}}$

$\Rightarrow k = -\frac{10}{7}$

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Let one number be x, then the other number will be (16 - x).

Let the sum of the cubes of these numbers be denoted by S.

Then, S = $x^3 + (16 - x)^3$

On differentiating w.r.t. x, we get

$\frac{dS}{dx} = 3x^2 + 3(16 - x)^2(-1)$

$= 3x^2 - 3(16 - x)^2$

$\Rightarrow \frac{d^2S}{dx^2} = 6x + 6(16 - x) = 96$

For minima put $\frac{dS}{dx} = 0$.

$\therefore 3x^2 - 3(16 - x)^2 = 0$

$\Rightarrow x^2 - (256 + x^2 - 32x) = 0$

$\Rightarrow 32x = 256$

$\Rightarrow x = 8$

At x = 8, $\left(\frac{d^2S}{dx^2}\right)_{x=8} = 96 > 0$

By second derivative test, x = 8 is the point of local minima of S.

Thus, the sum of the cubes of the numbers is the minimum when the numbers are 8 and 16 - 8 = 8

Hence, the required numbers are 8 and 8.

20.

(d) A is false but R is true.

Explanation: Assertion is false because distinct elements in N has equal images.

for example f(1) = $\frac{(1+1)}{2} = 1$

f(2) = $\frac{2}{2} = 1$

Reason is true because for injective function if elements are not equal then their images should be unequal.

Section B

21. Given $\sin^{-1}\left(\frac{1}{3}\right) - \cos^{-1}\left(-\frac{1}{3}\right)$

We know that $\cos^{-1}(-\theta) = \pi - \cos^{-1}\theta$

$= \sin^{-1}\left(\frac{1}{3}\right) - \left[\pi - \cos^{-1}\left(\frac{1}{3}\right)\right]$

$= \sin^{-1}\left(\frac{1}{3}\right) - \pi + \cos^{-1}\left(\frac{1}{3}\right)$

$= \sin^{-1}\left(\frac{1}{3}\right) + \cos^{-1}\left(\frac{1}{3}\right) - \pi$

$= \frac{\pi}{2} - \pi$

$= -\frac{\pi}{2}$

Therefore we have,

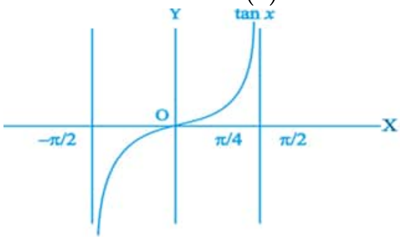
$\sin^{-1}\left(\frac{1}{3}\right) - \cos^{-1}\left(-\frac{1}{3}\right) = -\frac{\pi}{2}$

OR

From Fig. we note that tan x is an increasing function in the interval $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, since $1 > \frac{\pi}{4} \Rightarrow \tan 1 > \tan\frac{\pi}{4}$. This gives tan 1 > 1

$\Rightarrow \tan 1 > 1 > \frac{\pi}{4}$

$\Rightarrow \tan 1 > 1 > \tan^{-1}(1)$



22. Let P be the perimeter and A be the area of the rectangle at any time t. Then,

It is given that x = 12 cm , y = 5 cm, $\frac{dx}{dt} = -2\text{cm/s}$ and $\frac{dy}{dt} = 2\text{cm/s}$

i. We have,

$P = 2(x + y)$

$\Rightarrow \quad \frac{dP}{dt} = 2\left(\frac{dx}{dt} + \frac{dy}{dt}\right) = 2(-2 + 2) = 0\text{cm/sec i.e. the perimeter remains constant.}$

ii. We have,

$A = xy$

$\Rightarrow \quad \frac{dA}{dt} = x\left(\frac{dy}{dt}\right) + y\left(\frac{dx}{dt}\right)$

$\Rightarrow \quad \frac{dA}{dt} = 12 \times 2 - 2 \times 5$

$\Rightarrow \quad \frac{dA}{dt} = 14\text{cm}^2/\text{sec}$

23. $f'(x) = -3x^3 - 24x^2 - 45x$

$= -3x\left(x^2 + 8x + 15\right) = -3x(x + 5)(x + 3)$

$f'(x) = 0 \Rightarrow x = -5, x = -3, x = 0$

$f''(x) = -9x^2 - 48x - 45$

$= -3\left(3x^2 + 16x + 15\right)$

$f''(0) = -45 < 0$. Therefore, x = 0 is point of local maxima

$f''(-3) = 18 > 0$. Therefore, $x = -3$ is point of local minima

$f''(-5) = -30 < 0$. Therefore, $x = -5$ is point of local maxima.

OR

Given: $f(x) = x^2 - x + 1$ f(x) = x^2 - x + 1

$\Rightarrow$ f'(x) = 2x - 1

f(x) is strictly increasing if f'(x) < 0

$\Rightarrow$2x - 1 > 0

$\Rightarrow x > \frac{1}{2}$

i.e., increasing on the interval $\left(\frac{1}{2}, 1\right)$

f(x) is strictly decreasing if f'(x) < 0

$\Rightarrow$ 2x - 1 < 0

$\Rightarrow x < \frac{1}{2}$

i.e., decreasing on the interval $\left(-1, \frac{1}{2}\right)$

hence, f(x) is neither strictly increasing nor decreasing on the interval (-1, 1).

24. Let $I = \int_0^2 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}} dx$ ....(1)

It is known that, $\left(\int_0^0 f(x)dx = \int_0^0 f(a-x)dx\right)$

$I = \int_0^0 \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}} dx$ ....(2)

Adding (1) and (2), we get

$2I = \int_0^x \frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}} dx$

$\Rightarrow 2I = \int_0^a 1dx$

$\Rightarrow 2I = [x]_0^a$

$\Rightarrow 2I = a$

$\Rightarrow I = \frac{a}{2}$

25. $f'(x) = \frac{1}{\sin x} . \cos x$

$f'(x) = \cot x$

$cot\ x > 0\ \forall x \in \left(0, \frac{\pi}{2}\right)$

and

$cot\, x < 0\ \forall x \in \left(\frac{\pi}{2}, \pi\right)$

Hence f(x) = log sinx is strictly increasing on $\left(0, \frac{\pi}{2}\right)$ and decreasing on $\left(\frac{\pi}{2}, \pi\right)$

Section C

26. According to the question, $I = \int \frac{(3\sin\theta - 2)\cos\theta}{5 - \cos^2\theta - 4\sin\theta} d\theta$

$= \int \frac{(3\sin\theta - 2)\cos\theta}{5 - \left(1 - \sin^{2\theta}\right) - 4\sin\theta} d\theta$

Put $\sin\theta = t \Rightarrow \cos\theta d\theta = dt$

$\therefore \quad I = \int \frac{3t - 2}{5 - (1 - t^2) - 4t} dt$

$= \int \frac{3t - 2}{4 + t^2 - 4t} dt = \int \frac{3t - 2}{(t-2)^2} dt$

$= \int \frac{3t - 6 + 4}{(t-2)^2} dt = \int \frac{3(t-2) + 4}{(t-2)^2} dt$

$= \int \frac{3}{(t-2)} dt + \int \frac{4}{(t-2)^2} dt$

$= 3\log|t - 2| + \frac{4(t-2)^{-2+1}}{-2+1} + C$

$\left[\because \int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1\right]$

$= 3\log|t - 2| - \frac{4}{(t-2)} + C$

$= 3\log|\sin\theta - 2| - \frac{4}{(\sin\theta - 2)} + C$

$[Put\ t = \sin\theta]$

27. S = (HH, TH, HT, TT) = 4

i. E : tail appears on one coin

E = (TH, HT) $\Rightarrow$ n(E) = 2

P (E) = $\frac{n(E)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

F : one coin shows head

F = (TH, HT) $\Rightarrow n(F)$ = 2

P(F) = $\frac{n(F)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

$\therefore E \cap F$ = (TH, HT) $\Rightarrow (E \cap F) = 2$

$\therefore P(E \cap F) = \frac{n(E \cap F)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

And $P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$

ii. E : no tail appears

E = (HH) $\Rightarrow$ n(E) = 1

P (E) = $\frac{n(E)}{n(S)} = \frac{1}{4}$

F : no head appears

F = (TT) $\Rightarrow n(F) = 1$ P (F) = $\frac{n(F)}{n(S)} = \frac{1}{4}$

$\therefore E \cap F = \phi \Rightarrow n(E \cap F)$ = 0

$\therefore P(E \cap F) = \frac{n(E \cap F)}{n(S)} = \frac{0}{4} = 0$

And $\therefore P(E|F) = \frac{P(E \cap F)}{n(F)} = \frac{0}{\frac{1}{4}} = 0$

28. Let $x = \tan\theta$

$\Rightarrow \quad dx = \sec^2\theta d\theta$

If $x = 0, \quad \theta = 0$

If $x = 1, \theta = \frac{\pi}{4}$

$\therefore \quad \int_0^1 \frac{\log(1+x)}{1+x^2} dx$

$\Rightarrow \quad I = \int_0^{\frac{\pi}{4}} \log(1 + \tan\theta) d\theta$

[Since 1+tan^{2}x = sec^{2}x]

$\Rightarrow \quad I = \int_0^{\frac{\pi}{4}} \log\left\{1 + \tan\left(\frac{\pi}{4} - \theta\right)\right\} d\theta$

$\Rightarrow \quad I=\int_0^{\frac{\pi}{4}} \log\left\{1+\frac{1-\tan\theta}{1+\tan\theta}\right\} d\theta$

$\Rightarrow \quad I=\int_0^{\frac{\pi}{4}} \log\left(\frac{2}{1+\tan\theta}\right) d\theta$

$\Rightarrow \quad I=\int_0^{\frac{\pi}{4}} (\log 2-\log(1+\tan\theta)) d\theta$

$\Rightarrow \quad 2I=\int_0^{\frac{\pi}{4}} \log 2 \times d\theta=\frac{\pi}{4}\log 2$

$\Rightarrow \quad I=\frac{\pi}{8}\log 2$

OR

$\int_1^5 |x-11+1x-21+1x-4| dx$

$=\int_1^2 (5-x) dx+\int_2^4 (x+1) dx+\int_4^5 (3x-7) dx$

$=\left[5x-\frac{x^2}{2}\right]_1^2+\left[\frac{x^2}{2}+x\right]_2^4+\left[\frac{3x^2}{2}-7x\right]_4^5$

= $\frac{7}{2}$ + 8 + $\frac{13}{2}$ = 18

29. The given differential equation is,

(x+ 2) $\frac{dy}{dx}$ = x^2 + 4x - 9

$\Rightarrow \frac{dy}{dx}=\frac{x^2+4x-9}{x+2}$ [$\because$ x $\neq$ -2]

$\Rightarrow dy=\left(\frac{x^2+4x-9}{x+2}\right) dx$

Integrating both sides, we get,

$\int dy=\int \frac{x^2+4x-9}{x+2} dx$

$\Rightarrow \int dy=\int\left(x+2-\frac{13}{x+2}\right) dx$

$\Rightarrow y=\frac{x^2}{2}$ + 2x - 13 log |x + 2| + C

Clearly, it is defined for all x $\in$ R , except x = -2

Hence, $y=\frac{x^2}{2}$ + 2x - 13 log |x + 2| + C, x $\in$ R - {2} is the solution of the given differential equation.

OR

Given: Differential equation $\frac{dy}{dx}+3y=e^{-2x}$

Comparing with $\frac{dy}{dx}+Py=Q$,

we have P = 2 and $Q=e^{-2x}$

$\therefore \int Pdx=\int 3dx=3\int 1dx=3x$

$\Rightarrow I.F=e^{\int Pdx}=e^{3x}$

Solution is

$\Rightarrow y\,(IF)=\int Q\,(I.F) dx+c$

$\Rightarrow ye^{3x}=\int e^{-2x}e^{3x} dx+c$

$\Rightarrow ye^{3x}=\int e^{-2x+3x} dx+c=\int e^{x} dx+c$

$\Rightarrow ye^{3x}=e^{x}+c$

$\Rightarrow y=\frac{e^x}{e^{3x}}+\frac{c}{e^{3x}}$

$\Rightarrow y=e^{-2x}+ce^{-3x}$

30. Our problem is to mimmise and maximise the given objective function given as Z = x + 2 y .....(i)

Subject to the given constraints,

x + 2y $\geq$100 ....(ii)

2x - y $\leq$ 0 ....(iii)

2x + y $\leq$200 ....(iv)

x$\geq$ 0, y$\geq$ 0 ....(v)

Table for line x + 2y = 100 is

x	0	100
y	50	0

So, the line x + 2y = 100 is passing through the points with coordinates (0, 50) and (100, 0).

On replacing the coordinates of the origin O (0, 0) in the inequality x + 2y $\geq$100, we get

$2\times 0+0\geq 100$

$\Rightarrow \quad 0 \geq 100$ (which is False)

So, the half plane for the inequality of the line (ii) is away from the origin, which means that the point O(0,0) does not lie in the feasible region of the inequality of (ii)

Table for the line (iii) 2x - y = 0 is given as follows.

x	0	10
y	0	20

So, the line 2x - y = 0 is passing through the points (0, 0) and (10, 20).

On replacing the point (5, 0) in the inequality 2x - y$\leq$ 0, we get

$2 \times 5 - 0 \leq 0$

$\Rightarrow \quad 10 \leq 0$ (which is False)

So, the half plane for the inequality of (iii) is towards Y-axis.

Table of values for line 2x + y = 200 is given as follows.

x	0	100
y	200	0

So, the line 2x + y = 200 is passing through the points with coordinates (0, 200) and (100, 0).

On replacing O (0, 0) in the inequality 2 + y $\leq$ 200, we get

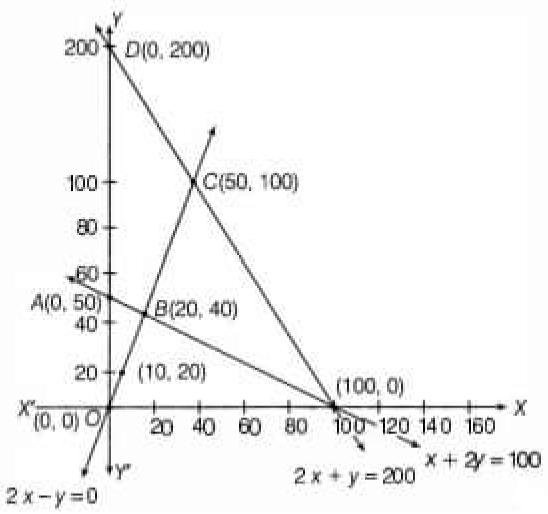
$2 \times 0 + 0 \leq 200$

$\Rightarrow \quad 0 \leq 200$ (which is true)

So, the half plane for the inequality of the line (iv) is towards the origin, which means that the point O (0,0) is a point in the feasible region.

Aslo, x, y$\geq$ 0

So, the region lies in the I quadrant only.



On solving equations 2x - y = 0 and x + 2y = 100, we get the point of intersection as B(20, 40).

Again, solving the equations 2x - y = 0 and 2x + y = 200, we get C(50, 100).

$\therefore$ Feasible region is ABCDA, which is a bounded feasible region.

The coordinates of the corner points of the feasible region are A(0, 50), B(20, 40), C(50, 100) and D(0, 200).

The values of Z at corner points are given below:

Corner points	**Z = x + 2y**
A(0, 50)	Z = 0 + 2$\times$50 = 100
B(20, 40)	Z = 20 + 2$\times$40 = 100
C(50, 100)	Z = 50 + 2$\times$100 = 250
D(0, 200)	Z = 0 + 2$\times$200 = 400

The maximum value of Z is 400 at D(0, 200) and the minimum value of Z is 100 at all the points on the line segment joining A(0, 50) and B(20, 40).

OR

We have to maximize Z = 7x + 10y

First, we will convert the given inequations into equations, we obtain the following equations:

x + y = 30000, y = 12000, x = 6000, x = y, x = 0 andy = 0

Region represented by x + y $\leq$ 30000 :

The line x + y = 30000 meets the coordinate axes at A(30000, 0) and B(0, 30000) respectively.

By joining these points we obtain the line x + y = 30000 Clearly (0, 0) satisfies the inequation x + y $\leq$ 30000.

So, the region containing the origin represents the solution set of the inequationx + y $\leq$ 30000

The line y = 12000 is the line that passes through C(0, 12000) and parallel to x-axis.

The line x = 6000 is the line that passes through (6000, 0) and parallel to y-axis.

Region represented by x $\geq$ y:

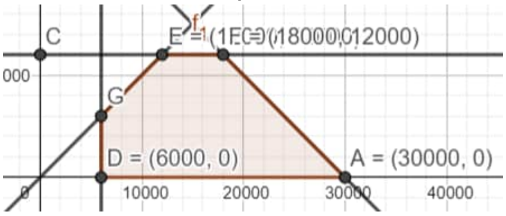
The line x = y is the line that passes through the origin. The points to the right of the line x = y satisfy the inequation x $\geq$ y Like by taking the point (-12000, 6000).

Here, 6000 > -12000 which implies y > x. Hence, the points to the left of line x = y will not satisfy the given in equation x $\geq$ y

Region represented by x $\geq$ 0 and y $\geq$ 0 :

since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the in equations x $\geq$ 0 andy $\geq$ 0

The feasible region determined by subject to the constraints are, x + y $\leq$ 30000, y $\leq$ 12000, x $\geq$ 6000, x$\geq$ y, and non-negative restrictions,x $\geq$ 0 and y $\geq$ 0 are as follows:



The corner points of the feasible region are D(6000, 0), A(3000, 0), F(18000, 12000) and E(12000, 12000).

The values of objective function at the corner points are as follows:

Corner point	Z = 7x + 10y
D(6000, 0)	$7 \times 6000 + 10 \times 0 = 42000$
A(3000, 0)	$7 \times 3000 + 10 \times 0 = 21000$
F(18000, 12000)	$7 \times 18000 + 10 \times 12000 = 246000$
$E(12000, 12000)$	$7 \times 12000 + 10 \times 12000 = 204000$

We see that the maximum value of the objective function Z is 246000 which is at F(18000,12000)

that means at x = 18000 and y = 12000

Thus, the optimal value of objective function z is 246000.

31. Given y = e^{ax}.cos bx

$\Rightarrow \frac{dy}{dx}$ = ae^{ax}cos bx - be^{ax}sin bx

$\Rightarrow \frac{dy}{dx}$ = ay - be^{ax} sin bx

$\Rightarrow \frac{d^2y}{dx^2}$= a$\frac{dy}{dx}$ - b[ae^{ax}sin bx + be^{ax}cos bx]

$\Rightarrow \frac{d^2y}{dx^2}$ = a$\frac{dy}{dx}$ - $a\left(ay - \frac{dy}{dx}\right)$ - b^2y

$\Rightarrow \frac{d^2y}{dx^2}$= 2a $\frac{dy}{dx}$ - a^2y - b^2y

$\therefore \frac{d^2y}{dx^2}$ - 2a $\frac{dy}{dx}$ + (a^2 + b^2)y = 0, Hence proved.

Section D

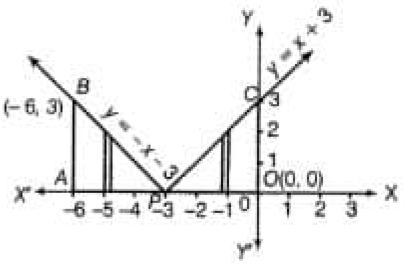
32. First, we sketch the graph of $y = |x + 3|$

$$\therefore y = |x+3| = \begin{cases} x+3, & \text{if} \quad x+3 \geq 0 \\ -(x+3), & \text{if} \quad x+3 < 0 \end{cases}$$

$$\Rightarrow y = |x+3| = \begin{cases} x+3, & \text{if} \quad x \geq -3 \\ -x-3, & \text{if} \quad x < -3 \end{cases}$$

So, we have $y = x + 3$ for x $\geq$ - 3 and y = -x - 3 for x < -3

A sketch of $y = |x + 3|$ is shown below:



$y=x+3$ is the straight line which cuts X and Y-axes at (-3, 0 and (0, 3), respectively.

$\therefore y=x+3$ for x $\geq$ -3 represents the part of the line which lies on the right side of x = -3.

Similarly,$y=-x-3$, x < -3 represents the part of line $y=-x-3$, which lies on left side of x = -3

Clearly, required area = Area of region ABPA + Area of region PCOP

$=\int_{-6}^{-3}(-x-3)dx+\int_{-3}^{0}(x+3)dx$

$=\left[-\frac{x^2}{2}-3x\right]_{-6}^{-3}+\left[\frac{x^2}{2}+3x\right]_{-3}^{0}$

$=\left[\left(-\frac{9}{2}+9\right)-(-18+18)\right]+\left[0-\left(\frac{9}{2}-9\right)\right]$

$=\left(-\frac{9}{2}-\frac{9}{2}\right)+(9+9)$

= 18 - 9

= 9 sq. units

33. A = {1, 2, 3, 4, 5} and R = {(a, b) : |a - b| is even}, then R = {(1, 3), (1, 5), (3, 5), (2, 4)}

1. For (a, a), |a - a| = 0 which is even. $\therefore$ R is reflexive.
   If |a - b| is even, then |b - a| is also even. $\therefore$ R is symmetric.
   Now, if |a - b| and |b - c| is even then |a - b + b - c| is even
   $\Rightarrow$ |a - c| is also even. $\therefore$ R is transitive.
   Therefore, R is an equivalence relation.
2. Elements of {1, 3, 5} are related to each other.
   Since |1 - 3| = 2, |3 - 5| = 2, |1 - 5| = 4 all are even numbers
   $\Rightarrow$ Elements of {1, 3, 5} are related to each other.
   Similarly elements of (2, 4) are related to each other.
   Since |2 - 4| = 2 an even number, then no element of the set {1, 3, 5} is related to any element of (2, 4).
   Hence no element of {1, 3, 5} is related to any element of {2, 4}.

OR

Given that

Let A = {1, 2, 3} and R = {(a, b) : a, b $\in$ A and $\left|a^2-b^2\right|\leq 5$

Put a = 1, b = 1 $\left|1^2-1^2\right|\leq 5,(1,1)$ is an ordered pair.

Put a = 1, b = 2 $\left|1^2-2^2\right|\leq 5,(1,2)$ is an ordered pair.

Put a = 1, b = 3 $\left|1^2-3^2\right|>5,(1,3)$ is not an ordered pair.

Put a = 2, b = 1 $\left|2^2-1^2\right|\leq 5,(2,1)$ is an ordered pair.

Put a = 2, b = 2 $\left|2^2-2^2\right|\leq 5,(2,2)$ is an ordered pair.

Put a = 2, b = 3 $\left|2^2-3^2\right|\leq 5,(2,3)$ is an ordered pair.

Put a = 3, b = 1 $\left|3^2-1^2\right|>5,(3,1)$ is not an ordered pair.

Put a = 3, b = 2 $\left|3^2-2^2\right|\leq 5,(3,2)$ is an ordered pair.

Put a = 3, b = 3 $\left|3^2-3^2\right|\leq 5,(3,3)$ is an ordered pair.

R = {(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 2), (3, 3)}

i. For (a, a) $\in$ R
   $\left|a^2-a^2\right|=0\leq 5$. Thus, it is reflexive.
ii. Let (a, b) $\in$ R
   $(a,b)\in R,\left|a^2-b^2\right|\leq 5$
   $\left|b^2-a^2\right|\leq 5$
   (b, a) $\in$ R
   Hence, it is symmetric
iii. Put a = 1, b = 2, c = 3
   $\left|1^2-2^2\right|\leq 5$

$\left|2^2-3^2\right| \le 5$
But $\left|1^2-3^2\right| > 5$
Thus, it is not transitive

34. Let the numbers are x, y, z be the cash awards for Resourcefulness, Competence, and Determination respectively
4x + 3y + 2z = 37000 …… (i)
Also,
5x + 3y + 4z = 47000 …… (ii)
Again,
x + y + z = 12000 …… (iii)
$$\begin{bmatrix}4 & 3 & 2\\ 5 & 3 & 4\\ 1 & 1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}37000\\ 47000\\ 12000\end{bmatrix}$$
A X = B
|A| = 4(3 - 4) - 3(5 - 4) + 2(5 - 3)
= 4(-1) - 3(1) + 2(2)
= -4 - 3 + 4
= -3
Hence, the unique solution given by $X = A^{-1}B$
$C_{11} = (-1)^{1+1}(3-4) = -1$
$C_{12} = (-1)^{1+2}(5-4) = -1$
$C_{13} = (-1)^{1+3}(5-3) = 2$
$C_{21} = (-1)^{2+1}(3-2) = -1$
$C_{22} = (-1)^{2+2}(4-2) = 2$
$C_{23} = (-1)^{2+3}(4-3) = -1$
$C_{31} = (-1)^{3+1}(12-6) = 6$
$C_{32} = (-1)^{3+2}(16-10) = -6$
$C_{33} = (-1)^{3+3}(12-15) = -3$
$$\text{Adj } A = \begin{bmatrix}-1 & -1 & 2\\ -1 & 2 & -1\\ 6 & -6 & -3\end{bmatrix}^T = \begin{bmatrix}-1 & -1 & 6\\ -1 & 2 & -6\\ 2 & -1 & -3\end{bmatrix}$$
$X = A^{-1}B = \frac{1}{|A|}(\text{Adj } A)B$
$$X = \frac{1}{-3}\begin{bmatrix}-1 & -1 & 6\\ -1 & 2 & -6\\ 2 & -1 & -3\end{bmatrix}\begin{bmatrix}37000\\ 47000\\ 12000\end{bmatrix}$$
$$X = \frac{1}{-3}\begin{bmatrix}-37000-47000+72000\\ -37000+94000-72000\\ 74000-47000-36000\end{bmatrix}$$
$$\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}4000\\ 5000\\ 3000\end{bmatrix}$$
Hence, x = 4000, y = 5000 and z = 3000
Thus, the value x, y, z describes the number of prizes per person for Resourcefulness, Competence, and Determination.

35. Equation of the given plane is 2x - 2y + 4z + 5 = 0 .... (i)
$\Rightarrow \vec{n} = 2\hat{i} - 2\hat{j} + 4\hat{k}$
So, the equation of line through $\left(1, \frac{3}{2}, 2\right)$ and parallel to $\vec{n}$ is given by
$$\frac{x-1}{2} = \frac{y-\frac{3}{2}}{-2} = \frac{z-2}{4} = \lambda$$
$\Rightarrow x = 2\lambda + 1, y = -2\lambda + \frac{3}{2}$ and $z = 4\lambda + 2$
If this point lies on the given plane, then

$2\,(2\lambda+1)-2\left(-2\lambda+\frac{3}{2}\right)+4\,(4\lambda+2)+5=0$ [using Eq. (i)]
$\Rightarrow 4\lambda+2+4\lambda-3+16\lambda+8+5=0$
$\Rightarrow 24\lambda=-12 \Rightarrow \lambda=\frac{-1}{2}$
$\therefore$ Required foot of perpendicular
$=\left[2\times\left(\frac{-1}{2}\right)+1,-2\times\left(\frac{-1}{2}\right)+\frac{3}{2},4\times\left(\frac{-1}{2}\right)+2\right]$ i.e., $\left(0,\frac{5}{2},0\right)$
$\therefore$ Required length of perpendicular $=\sqrt{(1-0)^2+\left(\frac{3}{2}-\frac{5}{2}\right)^2+(2-0)^2}$
$=\sqrt{1+1+4}=\sqrt{6}\ units$

OR

According to the question, equation of planes are
$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=1$ and $\vec{r}.(2\hat{i}+3\hat{j}-\hat{k})=-4$
The intersection equation of two planes is
$\vec{r}.(\hat{i}+\hat{j}+\hat{k})+\vec{r}\cdot\lambda(2\hat{i}+3\hat{j}-\hat{k})=1+\lambda(-4)$ $[\because \vec{r}_1.\vec{n}_1+\lambda\vec{r}_2.\vec{n}_2=d_1+\lambda d_2]$
$\Rightarrow \quad \vec{r}\cdot[\hat{i}+\hat{j}+\hat{k}+\lambda(2\hat{i}+3\hat{j}-\hat{k})]=1-4\lambda$
$\Rightarrow \vec{r}.[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1-\lambda)\hat{k})]=1-4\lambda$ ......(i)
$\overrightarrow{n_1}=(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1-\lambda)\hat{k}$
Normal vector of X-axis is
$\vec{n}_2=\hat{i}$
The required plane is parallel to X-axis, therefore normal of the plane is perpendicular to the X-axis.
$\vec{n}_1.\vec{n}_2=0$
$\Rightarrow [(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1-\lambda)\hat{k})]\cdot(\hat{i})=0$
$\Rightarrow \quad 1+2\lambda=0 \Rightarrow \lambda=-\frac{1}{2}$
Putting $\lambda=-\frac{1}{2}$ in Equation (i),
$\vec{r}\cdot\left[\left(1-\frac{2}{2}\right)\hat{i}+\left(1-\frac{3}{2}\right)\hat{j}+\left(1+\frac{1}{2}\right)\hat{k}\right]=1+\frac{4}{2}$
$\Rightarrow \vec{r}\cdot\left[0\hat{i}-\frac{1}{2}\hat{j}+\frac{3}{2}\hat{k}\right]=3$
$\Rightarrow \vec{r}.[-\hat{j}+3\hat{k}]=3\times 2$
$\therefore \vec{r}.[\hat{j}-3\hat{k}]+6=0$

Section E

36. **Read the text carefully and answer the questions:**
Akash and Prakash appeared for first round of an interview for two vacancies. The probability of Nisha's selection is $\frac{1}{3}$ and that of Ayushi's selection is $\frac{1}{2}$.

(i) P(A) = $\frac{1}{3}$, P(A') = $1-\frac{1}{3}=\frac{2}{3}$
P(B) = $\frac{1}{2}$, P(b') = $1-\frac{1}{3}=\frac{1}{2}$
P(Both are selected) = $P(A\cap B)=P(A)\cdot P(B)$ = $\frac{1}{3}.\frac{1}{2}$
P(Both are selected) = $\frac{1}{6}$
(ii) P(A) = $\frac{1}{3}$, P(A') = $1-\frac{1}{3}=\frac{2}{3}$
P(B) = $\frac{1}{2}$, P(b') = $1-\frac{1}{3}=\frac{1}{2}$

P(none of them selected) = $P(A' \cap B') = P(A') \cdot P(B')$ = $\frac{2}{3} \cdot \frac{1}{2}$

P(Both are selected) = $\frac{1}{3}$

(iii)P(A) = $\frac{1}{3}$, P(A') = $1 - \frac{1}{3} = \frac{2}{3}$

P(B) = $\frac{1}{2}$, P(b') = $1 - \frac{1}{3} = \frac{1}{2}$

P(none of them selected) = $P(A') \cdot P(B) + P(A) \cdot P(B')$ = $\frac{2}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{1}{2}$

P(Both are selected) = $\frac{3}{6} = \frac{1}{2}$

OR

P(A) = $\frac{1}{3}$, P(A') = $1 - \frac{1}{3} = \frac{2}{3}$

P(B) = $\frac{1}{2}$, P(b') = $1 - \frac{1}{3} = \frac{1}{2}$

P(atleast one of them selected) = 1 - P(none selected) = $1 - \frac{1}{3}$

P(atleast one of them selected) = $\frac{2}{3}$

37. **Read the text carefully and answer the questions:**

Three slogans on chart papers are to be placed on a school bulletin board at the points A, B and C displaying A (Hub of Learning), B (Creating a better world for tomorrow) and C (Education comes first). The coordinates of these points are (1, 4, 2), (3, -3, -2) and (-2, 2, 6) respectively.

(i) $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 3\hat{j} - 2\hat{k}$ and $\vec{c} = -2\hat{i} + 2\hat{j} + 6\hat{k}$

$\therefore \vec{a} + \vec{b} + \vec{c} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

(ii) We have, A(1, 4, 2), B(3, -3, -2) and C(-2, 2, 6)

Now, $\overrightarrow{AB} = \vec{b} - \vec{a} = 2\hat{i} - 7\hat{j} - 4\hat{k}$ and $\overrightarrow{AC} = \vec{c} - \vec{a} = -3\hat{i} - 2\hat{j} + 4\hat{k}$

$$\therefore \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & -4 \\ -3 & -2 & 4 \end{vmatrix}$$

= $\hat{i}$(-28 - 8) - $\hat{j}$(8 - 12) + $\hat{k}$(-4 - 21) = -36$\hat{i}$ +4$\hat{j}$ - 25$\hat{k}$

Now, $|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(-36)^2 + 4^2 + (-25)^2}$

= $\sqrt{1296 + 16 + 625} = \sqrt{1937}$

$\therefore$ Area of $\triangle$ABC = $\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}\sqrt{1937}$ sq. units

(iii)If the given points lie on the straight line, then the points will be collinear and so area of $\triangle$ABC = 0

$\Rightarrow |\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$ = 0 [$\because$ If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the three vertices A, B and C of $\triangle$ABC, then area of triangle = $\frac{1}{2}|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$]

OR

Here, $\overrightarrow{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$|\overrightarrow{a}| = \sqrt{4 + 9 + 36} = \sqrt{49} = 7$

Now unit vector $\widehat{a} = \frac{2\hat{i} + 3\hat{j} + 6\hat{k}}{7}$

$\widehat{a} = \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$

38. **Read the text carefully and answer the questions:**

The relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation y = 4x - $\frac{1}{2}$x^2 where x is the number of days exposed to sunlight.

(i) The rate of growth = $\frac{dy}{dx}$

= $\frac{d(4x-\frac{1}{2}x^2)}{dx}$

= 4 - x

(ii) For the height to be maximum or minimum

$\frac{dy}{dx}$ = 0

$\Rightarrow \frac{dy}{dx} = \frac{d\left(4x-\frac{1}{2}x^2\right)}{dx} = 4 - \frac{1}{2} \cdot 2x$ = 0

$\frac{dy}{dx}$ = 4 - x = 0

$\Rightarrow$ x = 4

$\therefore$ Number of required days = 4