Class XII Session 2025-26
Subject - Mathematics
Sample Question Paper - 5

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If A = $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, and A + A' = I, if the value of α is **[1]**

   a) π b) $\frac{3\pi}{2}$

   c) $\frac{\pi}{6}$ d) $\frac{\pi}{3}$

2. For the set of linear equations **[1]**

   λx - 3y+ z = 0

   x + λy + 3z = 1

   3x + y + 5z = 2

   the value of λ, for which the equations does not have unique solution is

   a) 1, $\frac{11}{5}$ b) -1, $\frac{-11}{5}$

   c) -1, $\frac{11}{5}$ d) $-\frac{11}{5}$, 1

3. Family y = Ax + A^3 of curves is represented by the differential equation of degree: **[1]**

   a) 2 b) 1

   c) 4 d) 3

4. Differentiation of the following w.r.t. xy = e^{x^3} **[1]**

   a) $x^2e^{x^2}$ b) $x^3e^{x^3}$

   c) $3x^2e^{x^3}$ d) $x^2e^{x^3}$

5. The direction ratios of the line x - y + z - 5 = 0 = x - 3y - 6 are proportional to **[1]**

   a) 2, - 4, 1 b) $\frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}, \frac{-2}{\sqrt{14}}$

c) 3, 1, - 2 | d) $\frac{2}{\sqrt{41}}, \frac{-4}{\sqrt{41}}, \frac{1}{\sqrt{41}}$

6. The order and degree (if defined) of the differential equation, $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 = x \sin\left(\frac{dy}{dx}\right)$ respectively are: **[1]**

a) 2, degree not defined

b) 2, 2

c) 2, 3

d) 1, 3

7. Solution of LPP maximize Z = 2x - y subject to x + y $\leq$ 2, x, y $\geq$ 0 **[1]**

a) 4

b) 2

c) 0

d) 1

8. What is the domain of the function $\cos^{-1}(2x - 3)$? **[1]**

a) (-1, 1)

b) [1, 2]

c) [-1, 1]

d) [1, 3]

9. $\int x\sqrt{x^2-1}dx = ?$ **[1]**

a) $\frac{1}{3}\left(x^2-1\right)^{3/2}+C$

b) $\frac{1}{2\sqrt{x^2-1}}+C$

c) $\frac{2}{3}\left(x^2-1\right)^{3/2}+C$

d) $\frac{1}{\sqrt{x^2-1}}+C$

10. The matrix $\begin{bmatrix} 0 & 5 & -7 \\ -5 & 0 & 11 \\ 7 & -11 & 0 \end{bmatrix}$ is **[1]**

a) an upper triangular matrix

b) a skew-symmetric matrix

c) a symmetric matrix

d) a diagonal matrix

11. The corner points of the feasible region determined by the following system of linear inequalities: 2x + y $\leq$ 10, x + 3y $\leq$ 15, x, y $\geq$ 0 are (0, 0), (5, 0), (3, 4) and (0, 5). Let Z = px + qy, where p, q $\geq$ 0. Condition on p and q so that the maximum of Z occurs at both (3, 4) and (0, 5) is **[1]**

a) p = q

b) q = 3p

c) p = 3q

d) p = 2q

12. Let f(x) = x^2lxl then the set of values, where f(x) is three times differentiable, is **[1]**

a) 2

b) Infinite

c) Definite

d) 3

13. Find the angle between the following pairs of lines: $\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k}$ $+\lambda\left(3\hat{i}+2\hat{j}+6\hat{k}.\right)$ and $\vec{r} = 7\hat{i} - 6\hat{k}$ + $\mu\left(\hat{i}+2\hat{j}+2\hat{k}.\right)$ $,\lambda,\mu \in R$ **[1]**

a) $\theta = \cos^{-1}(\frac{19}{21})$

b) $\theta = \sin^{-1}\left(\frac{19}{21}\right)$

c) $\theta = \cot^{-1}\left(\frac{19}{21}\right)$

d) $\theta = \tan^{-1}\left(\frac{19}{21}\right)$

14. In a certain town, 40% persons have brown hair, 25% have brown eyes, and 15% have both. If a person selected at random has brown hair, the chance that a person selected at random with brown hair is with brown eyes **[1]**

a) $\frac{2}{3}$

b) $\frac{1}{3}$

c) $\frac{3}{8}$

d) $\frac{3}{20}$

15. The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is **[1]**

a) $e^x + e^y = C$
b) $e^x + e^{-y} = C$
c) $e^{-x} + e^y = C$
d) $e^{-x} + e^{-y} = C$

16. The values of a for which the function f(x) = sin x - ax + b increases on R are **[1]**

a) (-∞, -1)
b) $(-\infty, \infty)$
c) [-1, 1]
d) [1, 1]

17. If x = at^2 ,y = 2 a t, then $\frac{d^2y}{dx^2}$ = **[1]**

a) $\frac{1}{t^2}$
b) 0
c) $-\frac{1}{2a\ t^3}$
d) $\frac{1}{2t^2}$

18. The equation of a line passing through point (2, -1, 0) and parallel to the line $\frac{x}{1} = \frac{y-1}{2} = \frac{2-z}{2}$ is: **[1]**

a) $\frac{x-2}{1} = \frac{y-1}{2} = \frac{z}{2}$
b) $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z}{-2}$
c) $\frac{x+2}{1} = \frac{y-1}{2} = \frac{z}{-2}$
d) $\frac{x+2}{1} = \frac{y-1}{2} = \frac{z}{2}$

19. **Assertion (A):** The Points A(2, 3, -4), B(1, -2, 3) and C(3, 8, -11) are conllinear **[1]**
Reason (R): It direction ratios of AB and BC are proportional, A, BC are collinear points.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** $\int \frac{dx}{16+9x^2} = \frac{1}{12}\tan^{-1}\left(\frac{3x}{4}\right)$ **[1]**
Reason (R): $\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. If A = $\begin{bmatrix} p & 2 \\ 2 & p \end{bmatrix}$ and $|A^3|$ = 125, then find the values of p. **[2]**

OR

Using determinant show that the (5, 5), (- 5, 1) and (10, 7) points are collinear.

22. Evaluate: $\int e^x \frac{(2-x)}{(1-x)^2} dx.$ **[2]**

23. Find the absolute maximum and minimum values of a function f given by **[2]**
f (x) = $2x^3 - 15x^2 + 36x + 1$ on the interval [1, 5].

OR

Prove that the function $f(x) = \tan x - 4x$ is strictly decreasing on $\left(\frac{-\pi}{3}, \frac{\pi}{3}\right)$.

24. If A and B are two independent events such that P(A ∪ B) = 0.60 and P(A) = 0.2, find P (B). **[2]**

25. Show that: $\frac{d}{dx}(|x|) = \frac{x}{|x|}, x \neq 0.$ **[2]**

Section C

26. Evaluate: $\int_0^{\pi/2} \frac{1}{2\cos x + 4\sin x} dx$ **[3]**

27. Differentiate the function $\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right], 0 < x < \frac{\pi}{2}$ w.r.t. x. **[3]**

28. Evaluate $\int \frac{dx}{x(x^5+3)}$. **[3]**

OR

Evaluate the integral: $\int \frac{1}{\sin x(2+3\cos x)}dx$

29. Solve the differential equation $ye^{\frac{x}{y}}dx = \left(xe^{\frac{x}{y}} + y^2\right)dy(y \neq 0)$ **[3]**

OR

Solve the differential equation: $(x^2 - 1)\frac{dy}{dx} + 2(x + 2)y = 2(x + 1)$

30. By computing the shortest distance determine whether the pairs of lines intersect or not: **[3]**

$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$

OR

If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors such that each one is perpendicular to the vector obtained by sum of the other two and $|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{c}| = 5$, then prove that $|\vec{a} + \vec{b} + \vec{c}| = 5\sqrt{2}$.

31. Maximize Z = 100x + 170y subject to **[3]**

$3x+2y \leq 3600$

$x+4y \leq 1800$

$x \geq 0, y \geq 0$

Section D

32. Find the area of the region bounded by the curve $y = x^2$ and the line y = 4. **[5]**

33. The scalar product of the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ and hence, find the unit vector along $\vec{b} + \vec{c}$. **[5]**

OR

Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{p}$, which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{p} \cdot \vec{c} = 18$.

34. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, prove that $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$ **[5]**

35. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\cos^{-1}\frac{1}{\sqrt{3}}$ **[5]**

OR

A right circular cylinder is inscribed in a cone. Show that the curved surface area of the cylinder is maximum when the diameter of the cylinder is equal to the radius of the base of the cone.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

In an office three employees Govind, Priyanka and Tahseen process incoming copies of a certain form. Govind process 50% of the forms, Priyanka processes 20% and Tahseen the remaining 30% of the forms. Govind has an error rate of 0.06, Priyanka has an error rate of 0.04 and Tahseen has an error rate of 0.03.

i. The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, find the probability that the form is NOT processed by Govind. (1)

ii. Find the probability that Priyanka processed the form and committed an error. (1)

iii. Find the total probability of committing an error in processing the form. (2)

OR

iv. Let A be the event of committing an error in processing the form and let E_1, E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form. The value of $\sum_{i=1}^{3} P(E_i \mid A)$? (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Priyanka and Renu are playing Ludo at home during Covid-19. While rolling the dice, Priyanka's sister Pummy observed and noted the possible outcomes of the throw every time belongs to set {1, 2, 3, 4, 5, 6}. Let A be the set of players while B be the set of all possible outcomes.

A = {S, D}, B = {1, 2, 3, 4, 5, 6}

i. Pummy wants to know the number of functions from A to B. How many number of functions are possible? (1)

ii. Pummy wants to know the number of relations possible from A to B. How many number of relations are possible? (1)

iii. Let R be a relation on B defined by R = {(1, 2), (2, 2),(1, 3), (3, 4), (3, 1), (4, 3), (5, 5),}. Is R an equivalence relation? (2)

OR

Show that a relation, R : B $\rightarrow$ B be defined by R = {(x, y) : y is divisible by x} is a reflexive and transitive but not symmetric. (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Let f(x) be a real valued function. Then its

- Left Hand Derivative (L.H.D.): Lf'(a) = $\lim_{h \to 0} \frac{f(a-h)-f(a)}{-h}$
- Right Hand Derivative (R.H.D.): Rf'(a) = $\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$

Also, a function f(x) is said to be differentiable at x = a if its L.H.D. and R.H.D. at x = a exist and both are equal.

For the function f(x) = $\begin{cases} |x-3|, x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, x < 1 \end{cases}$

i. What is R.H.D. of f(x) at x = 1? (1)

ii. What is L.H.D. of f(x) at x = 1? (1)

iii. Check if the function f(x) is differentiable at x = 1. (2)

OR

Find f'(2) and f'(-1). (2)

Solution

Section A

1.

(d) $\frac{\pi}{3}$

Explanation:

Given $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

Therefore, $A'=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$

Also given that A + A' = I ...(1)

(Putting the values in equation (1))

$\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}+\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\Rightarrow\begin{bmatrix}\cos\alpha+\cos\alpha & -\sin\alpha+\sin\alpha\\ \sin\alpha-\sin\alpha & \cos\alpha+\cos\alpha\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\Rightarrow\begin{bmatrix}2\cos\alpha & 0\\ 0 & 2\cos\alpha\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

We know the two matrices are equal only when all their corresponding elements or entries are equal i.e. if A = B, then a_{ij} and b_{ij} for all i and j.

This implies,

$2\cos\alpha=1$

$\Rightarrow\cos\alpha=\frac{1}{2}$

$\Rightarrow\cos\alpha=\cos\frac{\pi}{3}\quad\ldots\left(\because\cos\frac{\pi}{3}=\frac{1}{2}\right)$

$\Rightarrow\alpha=\frac{\pi}{3}$

2.

(c) -1, $\frac{11}{5}$

Explanation:

-1, $\frac{11}{5}$

3.

(d) 3

Explanation:

y = Ax + A^3

Let us find the differential equation representing it so we have to eliminate the constant A

Differentiate with respect to x

$\Rightarrow\frac{dy}{dx}=A$

Put back value of A in y

$\Rightarrow \mathrm{y}=\frac{\mathrm{dy}}{\mathrm{dx}}\mathrm{x}+\left(\frac{\mathrm{dy}}{\mathrm{dx}}\right)^3$

Now for the degree to exist the differential equation must be a polynomial in some differentials

Here differentials mean $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{d^2y}{dx^2}$ or .... $\frac{\mathrm{d^n y}}{\mathrm{dx^n}}$

The given differential equation is polynomial in differentials $\frac{\mathrm{d}y}{\mathrm{d}x}$

The degree of a differential equation is defined as the highest integer power of highest order derivative in the equation

The highest derivative is $\frac{\mathrm{d}y}{\mathrm{d}x}$ and highest power to it is 3

Hence degree is 3.

4.

(c) $3x^2 e^{x^3}$

Explanation:

Let y = e^{x^3}

On differentiating both sides w.r.t. x, we get

$\frac{dy}{dx}$ = $e^{x^3} \frac{d}{dx}\left(x^3\right)$[by chain rule of derivative]

= $e^{x^3} \cdot$ 3x^2 = 3x^2 e^{x^3}

5.

(c) 3, 1, - 2

Explanation:

We have,

x - y + z - 5 = 0 = x - 3y - 6

⇒ x - 3y - 6 = 0

&, x - y + z - 5 = 0

⇒ x = 3y + 6 ....(i)

x - y + z - 5 = 0 ...(ii)

From (i) and (ii)

We get, 3y + 6 - y + z - 5 = 0

⇒ 2y + z + 1 = 0

⇒ y = $\frac{-z-1}{2}$

y = $\frac{x-6}{3}$ [from (i)]

$\therefore \frac{x-6}{3} = y = \frac{-z-1}{2}$

So, the given equation can be re-written as

$\frac{x-6}{3} = \frac{y}{1} = \frac{z+1}{-2}$

Hence, the direction ratios of the given line are proportional to 3, 1, -2.

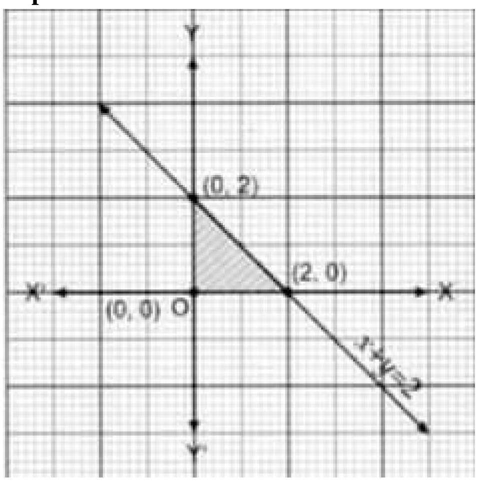
6. **(a)** 2, degree not defined

Explanation:

2, degree not defined

7. **(a)** 4

Explanation:



Feasible region is shaded region with corner points (0, 0), (2, 0) and (0, 2).

Z(0, 0) = 0

Z (2, 0) = 4 ⟵ maximum

Z(0, 2) = - 2

Z_{max} = 4 and obtained at (2, 0)

8.

(b) [1, 2]

Explanation:

Let, f(x) = $\cos^{-1}(2x - 3)$

$-1 \leq 2x - 3 \leq 1$

$\Rightarrow 2 \leq 2x \leq 4$

$\Rightarrow 1 \leq x \leq 2$

$\therefore x \in [1, 2]$ or domain of x is [1, 2].

9. **(a)** $\frac{1}{3}\left(x^2-1\right)^{3/2}+C$

Explanation:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c; \int \frac{1}{t\sqrt{t^2-1}} dt = \sec^{-1} t + c$

Therefore ,

$\Rightarrow \int x\sqrt{x^2-1}dx$

Put x^2-1=t $\Rightarrow$2 xdx=dt

$\Rightarrow \int \sqrt{t}\frac{dt}{2} = \frac{1}{2}\int \frac{t^{\frac{3}{2}}}{\frac{3}{2}}dt$

$= \frac{t^{\frac{2}{2}}}{3} + c = \frac{(x^2-1)^{\frac{3}{2}}}{3} + c$

$= \frac{1}{3}\left(x^2-1\right)^{\frac{3}{2}} + c$

10.

(b) a skew-symmetric matrix

Explanation:

A = $\begin{bmatrix} 0 & 5 & -7 \\ -5 & 0 & 11 \\ 7 & -11 & 0 \end{bmatrix}$

A^T = $\begin{bmatrix} 0 & -5 & 7 \\ 5 & 0 & -11 \\ -7 & 11 & 0 \end{bmatrix}$

-A = $\begin{bmatrix} 0 & 5 & -7 \\ -5 & 0 & 11 \\ 7 & -11 & 0 \end{bmatrix}$

$\because A^T = -A$

Then, the given matrix is a skew–symmetric matrix.

11.

(b) q = 3p

Explanation:

The maximum value of Z is unique.

It is given that the maximum value of Z occurs at two points (3,4) and (0,5)

$\therefore$ Value of Z at (3, 4) = Value of Z at (0, 5)

$\Rightarrow$ p(3) + q(4) = p(0) + q(5)

$\Rightarrow$ 3p + 4q = 5q

$\Rightarrow$ q = 3p

12.

(b) Infinite

Explanation:

Infinite

13. **(a)** $\theta = \cos^{-1}(\frac{19}{21})$

Explanation:

If θ is the acute angle between $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$, then cosine of the angle between these two lines is given by : $\cos\theta = \left|\frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{\left|\overrightarrow{b_1}\right|\left|\overrightarrow{b_2}\right|}\right|$

Here, $\overrightarrow{b_1} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, $\overrightarrow{b_2} = \hat{i} + 2\hat{j} + 2\hat{k}$

Then,

$\cos\theta = \left|\frac{(3\hat{i}+2\hat{j}+6\hat{k}).(\hat{i}+2\hat{j}+2\hat{k})}{\left|(3\hat{i}+2\hat{j}+6\hat{k})\right|\left|(\hat{i}+2\hat{j}+2\hat{k})\right|}\right|$

$\cos\theta = \left|\frac{19}{\sqrt{49}\sqrt{9}}\right| = \left|\frac{19}{21}\right| \Rightarrow \theta = \cos^{-1}\left(\left|\frac{19}{21}\right|\right)$

14.

(c) $\frac{3}{8}$

Explanation:

Let A be the event that a person has brown hair, B be the event that a person has brown eyes. Then,

P(A) = $\frac{40}{100}$, P(B) = $\frac{25}{100}$, $P(A \cap B) = \frac{15}{100}$

Required probability = $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{15}{100}}{\frac{40}{100}} = \frac{3}{8}$

15.

(b) $e^x + e^{-y} = C$

Explanation:

We have, $\frac{dy}{dx} = e^{x+y}$

$\Rightarrow \frac{dy}{dx} = e^x \times e^y$

separating variables

$\Rightarrow \mathrm{e}^{-\mathrm{y}}\mathrm{dy} = \mathrm{e}^{\mathrm{x}}\mathrm{dx}$

Integrating both sides

$\Rightarrow \int \mathrm{e}^{-\mathrm{y}}\mathrm{dy} = \int \mathrm{e}^{\mathrm{x}}\mathrm{dx}$

$\Rightarrow -e^{-y} = e^x + c$

$\Rightarrow e^x + e^{-y} = -c$

Or,

$e^x + e^{-y} = c$ (c is a constant)

16. **(a)** (-∞, -1)

Explanation:

f(x) = sin x - ax + b

$\Rightarrow$ f'(x) = cos x - a

For increasing function

f'(x) ≥ 0

cos x - a $\geq 0 \Rightarrow$ cos x $\geq$ a

i.e. a $\leq$ cos x a $\leq$ min (cos x) = -1

$\therefore$ a $\in$ (-∞, -1)

17.

(c) $-\frac{1}{2a\ t^3}$

Explanation:

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \frac{1}{t}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{t}\right) = -\frac{1}{t^2}\frac{dt}{dx} = -\frac{1}{t^2}.\frac{1}{2at} = -\frac{1}{2at^3}$

18.

(b) $\frac{\mathrm{x}-2}{1} = \frac{\mathrm{y}+1}{2} = \frac{\mathrm{z}}{-2}$

Explanation:

$\frac{x-2}{1}=\frac{y+1}{2}=\frac{z}{-2}$

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Let I = $\int \frac{dx}{16+9x^2}$

Let 3x = t $\Rightarrow$ 3dx = dt

$\Rightarrow dx=\frac{dt}{3}$

$\therefore$ I = $\frac{1}{3}\int \frac{dt}{(4)^2+t^2}$

$=\frac{1}{3}\times\frac{1}{4}\tan^{-1}\left(\frac{t}{4}\right)+C$

$=\frac{1}{12}\tan^{-1}\left(\frac{3x}{4}\right)+C$

Section B

21. $|A| = p^2 - 4$

$|A^3| = 125 \Rightarrow |A|^3 = 125 \Rightarrow |A| = 5$

$\therefore p^2 - 4 = 5 \Rightarrow p = \pm 3$

OR

Here, (5, 5), (–5, 1) and (10, 7) are three points ...(Given)

Now, we know that,

Vertices of a triangle are (x_1, y_1), (x_2, y_2) and (x_3, y_3), then the area of the triangle is given by:

$$\Delta=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}=0$$

Now,

Substituting given value in above formula

$$\Delta=\frac{1}{2}\begin{vmatrix} 5 & 5 & 1\\ -5 & 1 & 1\\ 10 & 7 & 1\end{vmatrix}=0$$

R.H.S

$$\frac{1}{2}\begin{vmatrix} 5 & 5 & 1\\ -5 & 1 & 1\\ 10 & 7 & 1\end{vmatrix}$$

Expanding along R_1

$=\frac{1}{2}\left[5\begin{vmatrix}1 & 1\\ 7 & 1\end{vmatrix}-5\begin{vmatrix}-5 & 1\\ 10 & 1\end{vmatrix}+1\begin{vmatrix}-5 & 1\\ 10 & 7\end{vmatrix}\right.$

= $\frac{1}{2}$ [5(-6) - 5(-15) + 1(-45)]

= $\frac{1}{2}$ [-35 + 75 - 45]

= 0

= LHS

Since, Area of triangle is zero

Hence, points are collinear

22. USING PARTIAL FRACTION

$\frac{2-x}{(1-x)^2}=\frac{A}{(1-x)}+\frac{B}{(1-x)^2}$

$\Rightarrow$ 2 - x = A(1 - x) + B

For x = 1, equation: 1 = B i.e. B = 1

For x = 2, equation: 0 = -A + 1 i.e. A = 1

$\therefore \frac{2-x}{(1-x)^2}$

$= \frac{1}{(1-x)} + \frac{1}{(1-x)^2}$

The given equation becomes

$\int e^x \left[\frac{1}{(1-x)} + \frac{1}{(1-x)^2}\right] dx$

$= \int e^x \times \frac{1}{(1-x)^2} dx + \int e^x \times \frac{1}{1-x} dx$

Taking $f_1(x) = 1/(1-x)$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$\int \frac{e^x}{(1-x)^2} dx + \int \frac{e^x}{1-x} dx$

$= \int \frac{e^x}{(1-x)^2} dx + \frac{1}{1-x} \int e^x dx - \int \left[\frac{d}{dx}\left(\frac{1}{1-x}\right) \int e^x dx\right] dx$

$= \int \frac{e^x}{(1-x)^2} dx + \frac{e^x}{1-x} - \int \frac{e^x}{(1-x)^2} dx + c$

$= \frac{e^x}{1-x} + c$, where c is the integrating constant

23. We have

$f(x) = 2x^3 – 15x^2 + 36x + 1$

or $f'(x) = 6x^2 – 30x + 36 = 6 (x – 3) (x – 2)$

Note that $f'(x) = 0$ gives $x = 2$ and $x = 3$.

We shall now evaluate the value of f at these points and at the end points of the interval [1, 5], i.e., at x = 1, x = 2, x = 3 and at x = 5. So

$f(1) = 2(1^3) – 15(1^2) + 36 (1) + 1 = 24$

$f(2) = 2(2^3) – 15(2^2) + 36 (2) + 1 = 29$

$f(3) = 2(3^3) – 15(3^2) + 36 (3) + 1 = 28$

$f(5) = 2(5^3) – 15(5^2) + 36 (5) + 1 = 56$

Thus, we conclude that absolute maximum value of f on [1, 5] is 56, occurring at x =5, and absolute minimum value of f on [1, 5] is 24 which occurs at x = 1.

OR

Given function, $f(x) = \tan x - 4x \Rightarrow f'(x) = \sec^2 x - 4$

When $\frac{-\pi}{3} < x < \frac{\pi}{3}, 1 < \sec x < 2$

Therefore, $1 < \sec^2 x < 4 \Rightarrow -3 < \left(\sec^2 x - 4\right) < 0$

Thus for $\frac{-\pi}{3} < x < \frac{\pi}{3}, f'(x) < 0$

Hence, f is strictly decreasing on $\left(\frac{-\pi}{3}, \frac{\pi}{3}\right)$.

24. By Addition theorem of probability, we have

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = P(A) + P(B) - P(A) \times P(B)$ [$\because$ A and B are independent events]

$\Rightarrow$ 0.6 = 0.2 + P(B) - 0.2 $\times$ P(B)

$\Rightarrow$ 0.6 - 0.2 = P(B)(1 - 0.2)

$\Rightarrow$ P(B) = $\frac{0.6-0.2}{1-0.2}$

$\Rightarrow$ P(B) = $\frac{0.4}{0.8}$

$\Rightarrow$ P(B) = $\frac{1}{2}$

$\Rightarrow$ P(B) = 0.5

25. $\frac{d(|x|)}{dx} = \frac{d\left(\sqrt{x^2}\right)}{dx}, x \neq 0$

$= \frac{1}{2}\left(x^2\right)^{-\frac{1}{2}} \times \frac{d\left(x^2\right)}{dx}$

$= \frac{1}{2\sqrt{x^2}} 2x = \frac{x}{|x|}$

Section C

26. Let $I = \int_0^{\pi/2} \frac{1}{2\cos x + 4\sin x} dx.$ Then,

$I = \int_0^{\pi/2} \frac{1}{2\left(1-\tan^2 \frac{x}{2}\right)} + \frac{4\left(2\tan \frac{x}{2}\right)}{1+\tan^2 \frac{x}{2}} dx$

$\Rightarrow \quad I = \int_0^{\pi/2} \frac{1+\tan^2 \frac{x}{2}}{2-2\tan^2 \frac{x}{2}+8\tan \frac{x}{2}} dx = \int_0^{\pi/2} \frac{\sec^2 \frac{x}{2}}{2-2\tan^2 \frac{x}{2}+8\tan \frac{x}{2}} dx$

Using substitution

Let $\tan\frac{x}{2}=t$. Then, $d\left(\tan\frac{x}{2}\right)= dt \Rightarrow \frac{1}{2}\sec^2\frac{x}{2}dx=dt \Rightarrow dx=2\frac{dt}{\sec^2\frac{x}{2}}$

Also, $x=0 \Rightarrow t=\tan 0=0,$ and $x=\frac{\pi}{2} \Rightarrow t=\tan\frac{\pi}{4}=1$

$\therefore \quad I=\int_0^1 \frac{\sec^2\frac{x}{2}}{2-2t^2+8t}\times\frac{2dt}{\sec^2\frac{x}{2}} = \int_0^1\frac{1}{1-t^2+4t}dt=\int_0^1\frac{1}{-(t^2-4t-1)}dt$

$\Rightarrow \quad I=\int_0^1\frac{1}{-(t^2-4t+4-4-1)} = \int_0^1\frac{1}{-\left\{(t-2)^2-5\right.}\cdot dt=\int_0^1\frac{1}{(\sqrt{5})^2-(t-2)^2}dt$

$\Rightarrow \quad I=\frac{1}{2\sqrt{5}}\left[\log\left|\frac{\sqrt{5}+t-2}{\sqrt{5}-t+2}\right|\right]_0^1 = \frac{1}{2\sqrt{5}}\left[\log\left(\frac{\sqrt{5}-1}{\sqrt{5}+1}\right)-\log\left(\frac{\sqrt{5}-2}{\sqrt{5}+2}\right)\right]$

$\Rightarrow \quad I=\frac{1}{2\sqrt{5}}\left[\log\left\{\frac{(\sqrt{5}-1)(\sqrt{5}+2)}{(\sqrt{5}+1)(\sqrt{5}-2)}\right\}\right]=\frac{1}{2\sqrt{5}}\log\left(\frac{3+\sqrt{5}}{3-\sqrt{5}}\right)$

$\Rightarrow \quad I=\frac{1}{2\sqrt{5}}\log\left(\frac{3+\sqrt{5}}{3-\sqrt{5}}\times\frac{3+\sqrt{5}}{3+\sqrt{5}}\right)=\frac{1}{2\sqrt{5}}\log\left(\frac{3+\sqrt{5}}{2}\right)^2=\frac{2}{2\sqrt{5}}\log\left(\frac{3+\sqrt{5}}{2}\right)$

$\Rightarrow \quad I=\frac{1}{\sqrt{5}}\log\left(\frac{3+\sqrt{5}}{2}\right)$

Hence the result

27. $y=\cot^{-1}\left[\frac{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}+\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}-\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}\right]$ $\left[\because\sqrt{1\pm\sin x}=\sqrt{\left(\cos\frac{x}{2}\pm\sin\frac{x}{2}\right)^2}\right]$

$=\cot^{-1}\frac{\cos\frac{x}{2}+\sin\frac{x}{2}+\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}-\cos\frac{x}{2}+\sin\frac{x}{2}}$

$=\cot^{-1}\left(\frac{2\cos\frac{x}{2}}{2\sin\frac{x}{2}}\right)$

$=\cot^{-1}\left(\cot\frac{x}{2}\right)$

$=\frac{x}{2}$

$y=\frac{x}{2}$

$\frac{dy}{dx}=\frac{1}{2}$

28. According to the question , $I=\int\frac{dx}{x\left(x^5+3\right)}$

Multiplying the numerator and denominator by x^4 , we get

$=\int\frac{x^4}{x^5\left(x^5+3\right)}dx$

Put $t=x^5$

$\Rightarrow dt=5x^4dx$

$=\int\frac{dt}{5t(t+3)}$

$\therefore \quad I=\frac{1}{5}\int\frac{dt}{t(t+3)}$ ...(i)

By using partial fractions ,

let $\frac{1}{t(t+3)}=\frac{A}{t}+\frac{B}{t+3}$ ...(ii)

$1=A(t+3)+B(t)$

$1=t(A+B)+3A$

By comparing the coefficient of t and constant terms , we get

$3A=1\Rightarrow A=\frac{1}{3}$

$A+B=0$

$\frac{1}{3}+B=0$

$B=-\frac{1}{3}$

from (i) and (ii) , we get

$=\frac{1}{5}\int\frac{1}{3}\left[\frac{1}{t}-\frac{1}{t+3}\right]dt$

$=\frac{1}{15}[\log|t|-\log|t+3|]+C$

$=\frac{1}{15}\log\left|\frac{t}{t+3}\right|+C\ \left[\because log\ m-log\ n=log\ \frac{m}{n}\right]$

$=\frac{1}{15}\log\left|\frac{x^5}{x^5+3}\right|+C$ [$\because$t = x^5]

OR

Let $\mathrm{I}=\int\frac{1}{\sin x(2+3\cos x)}\mathrm{dx}$

multiply and divide numerator by sin x.

$= \int \frac{\sin x}{\sin^2 x(2+3\cos x)}dx$

$= \int \frac{\sin x}{(1-\cos^2 x)(2+3\cos x)}dx$

$= \int \frac{\sin x}{(1-\cos x)(1+\cos x)(2+3\cos x)}dx$

Putting cos x = t

⇒ cos x = t

⇒ -sin x dx = dt

$\therefore I = \int \frac{-1}{(1-t)(1+t)(2+3t)}dt$

$= \int \frac{1}{(t-1)(t+1)(3t+2)}dt$

Let $\frac{1}{(t-1)(t+1)(3t+2)} = \frac{A}{t-1} + \frac{B}{t+1} + \frac{C}{3t+2}$

$\Rightarrow \frac{1}{(t-1)(t+1)(3t+2)} = \frac{A(t+1)(3t+2)+B(t-1)(3t+2)+C(t+1)(t-1)}{(t-1)(t+1)(3t+2)}$

⇒ 1 = t + 1)(3t + 2) + B(t - 1)(3t + 2) + C(t + 1)(t - 1)

Putting t + 1 = 0 or t = -1

⇒ 1 = A × 0 + B(-1 - 1)(3 × -1 + 2) + C × 0

$\therefore B = \frac{1}{2}$

Now, putting t - 1 = 0 or t = 1

⇒ 1 = A(1 + 1)(3 + 2) + B ×0 + C × 0

$\therefore A = \frac{1}{10}$

Now, putting 3t + 2 = 0 or t $= \frac{-2}{3}$

$\Rightarrow 1 = A \times 0 + B \times 0 + C\left(-\frac{2}{3}+1\right)\left(-\frac{2}{3}-1\right)$

$\Rightarrow 1 = C\left(\frac{1}{3}\right)\left(\frac{-5}{3}\right)$

$\therefore C = \frac{-9}{5}$

$\therefore I = \int \frac{1}{10(t-1)}dt + \frac{1}{2}\int \frac{1}{t+1}dt - \frac{9}{5}\int \frac{1}{3t+2}dt$

$= \frac{1}{10}\ln|t-1| + \frac{1}{2}\ln|t+1| - \frac{9}{5}\ln\frac{|3t+2|}{3} + C$

$= \frac{1}{10}\ln|t-1| + \frac{1}{2}\log|t+1| - \frac{3}{5}\ln|3t+2| + C$

$= \frac{1}{10} + \ln|\cos x - 1| + \frac{1}{2}\ln|\cos x + 1| - \frac{3}{5}\ln|3\cos x + 2| + C$ [∵ t = cos x]

29. It is given that ye $^{\frac{x}{y}} dx = \left(xe^{\frac{x}{y}} + y^2\right)dy$

⇒ ye $^{\frac{x}{y}}\frac{dx}{dy} = xe^{\frac{x}{y}} + y^2$

⇒ e $^{\frac{x}{y}}\left[y \cdot \frac{dx}{dy} - x\right] = y^2$

$\Rightarrow e^{\frac{x}{y}} \cdot \frac{\left[y\cdot\frac{dx}{dy}-x\right]}{y^2} = 1$ ....(i)

Let $e^{\frac{x}{y}} = z$

Differentiating it w.r.t. y, we get,

$\frac{d}{dy}\left(e^{\frac{x}{y}}\right) = \frac{dz}{dy}$

$\Rightarrow e^{\frac{x}{y}} \cdot \frac{d}{dy}\left(\frac{x}{y}\right) = \frac{dz}{dy}$

$\Rightarrow e^{\frac{x}{y}} \cdot \left[\frac{y\cdot\frac{dx}{dy}-x}{y^2}\right] = \frac{dz}{dy}$ ......(ii)

From equation (i) and equation (ii), we get,

$\frac{\mathrm{dz}}{\mathrm{dy}} = 1$

⇒ dz = dy

On integrating both sides, we get,

z = y + C

$\Rightarrow \mathrm{e}^{\frac{\mathrm{x}}{\mathrm{y}}} = \mathrm{y} + \mathrm{C}$

OR

The given differential equation is,

$(x^2 - 1)\frac{dy}{dx} + 2(x + 2)y = 2(x + 1)$

$\frac{dy}{dx} + \frac{2(x+2)}{(x^2-1)}y = \frac{2(x+1)}{(x^2-1)}$

It is a linear differential equation. Comparing it with,

$\frac{dy}{dx}$ + Py = Q

P = $\frac{2(x+2)}{x^2-1}$, Q = $\frac{2(x+1)}{(x^2-1)}$

I.F. $= e^{\int pdx}$

$= e^{2\int \frac{(x+2)}{\left(x^2-1\right)}dx}$

$= e^{\int \frac{2x}{x^2-1}dx + 4\int \frac{1}{x^2-1}\,dx}$

$= e^{\log\left|x^2-1\right| + 4x\frac{1}{2}\log\left|\frac{x-1}{x+1}\right|}$

$= e^{log\left|x^2-1\right| + \log\left|\frac{x-1}{x+1}\right|^2}$

$= e^{\log\frac{\left(x^2-1\right)(x-1)^2}{(x+1)^2}}$

I.F. $= \frac{(x+1)(x-1)(x-1)^2}{(x+1)^2}$

$= \frac{(x-1)^3}{(x+1)}$

Solution of the equation is given by,

4 × (I.F.) = $\int$ Q × (I.F.) dx + c

$\frac{y(x-1)^3}{(x+1)} = 2\frac{x^2}{2}$ - 6x + 8 log |x + 1| + c

$\frac{y(x-1)^3}{x+1}$ = x^2 - 6x + 8 log |x + 1| + c

y = $\frac{x+1}{(x-1)^3}$ [x^2 - 6x + 8 log |x + 1| + c]

30. Given that,

$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$

Comparing the given equations with the equations

$\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$

We get,

$\overrightarrow{a_1} = \hat{i} + \hat{j} - \hat{k}$

$\overrightarrow{a_2} = 4\hat{i} - \hat{k}$

$\overrightarrow{b_1} = 3\hat{i} - \hat{j}$

$\overrightarrow{b_2} = 2\hat{i} + 3\hat{k}$

$\therefore \overrightarrow{a_2} - \overrightarrow{a_1} = 3\hat{i} - \hat{j}$

and $\overrightarrow{b_1} \times \overrightarrow{b_2}$ = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix}$

= $-3\hat{i} - 9\hat{j} + 2\hat{k}$

$(\overrightarrow{a_2} - \overrightarrow{a_1}) \cdot (\overrightarrow{b_1} \times \overrightarrow{b_2})$ = $(3\hat{i} - \hat{j}) \cdot (-3\hat{i} - 9\hat{j} + 2\hat{k})$

= -9 + 9

= 0

We observe

$(\overrightarrow{a_2} - \overrightarrow{a_1}) \cdot (\overrightarrow{b_1} \times \overrightarrow{b_2})$ = 0

Thus, the given lines intersect.

OR

According to the question ,

$\vec{a} \perp (\vec{b} + \vec{c}), \vec{b} \perp (\vec{c} + \vec{a}), \vec{c} \perp (\vec{a} + \vec{b})$

and $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 5$

To prove $|\vec{a} + \vec{b} + \vec{c}| = 5\sqrt{2}$

Consider, $|\vec{a} + \vec{b} + \vec{c}|^2 = (\vec{a} + \vec{b} + \vec{c}).(\vec{a} + \vec{b} + \vec{c})$ $\left[\because |\vec{x}|^2 = \vec{x}.\vec{x}\right]$

$= \vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{a} \cdot \vec{c} + \vec{b}.\vec{a} + \vec{b}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} + \vec{c} \cdot \vec{b} + \vec{c}.\vec{c}$

$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + \vec{a} \cdot (\vec{b} + \vec{c}) + \vec{b} \cdot (\vec{a} + \vec{c}) + \vec{c}. (\vec{a} + \vec{c})$

$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 0 + 0 + 0$

$[\because \vec{a} \perp (\vec{b} + \vec{c})]$

$\therefore \vec{a}. (\vec{b} + \vec{c}) = 0$

Similarly, $\vec{b}. (\vec{a} + \vec{c}) = 0$

and $\vec{c}. (\vec{a} + \vec{b}) = 0$

= $3^2 + 4^2 + 5^2$ = 9 + 16 + 25

$\Rightarrow |\vec{a} + \vec{b} + \vec{c}|^2 = 50$

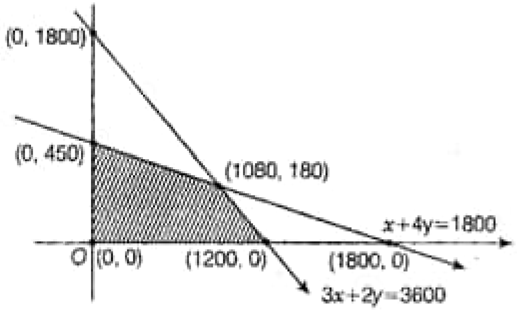
$\Rightarrow \quad |\vec{a} + \vec{b} + \vec{c}| = 5\sqrt{2}$

31. Maximise Z= 100x + 170y subject to

$3x + 2y \leqslant 3600, x + 4y \leqslant 1800, x \geqslant 0, y \geqslant 0$

From the shaded feasible region it is clear that the coordinates of corner points are (0,0),(1200,0),(1080,180) and (1080,180) and (0,450).

On solving x + 4y = 1800 and 3x + 2y = 3600 we get x = 1080 and y = 180.

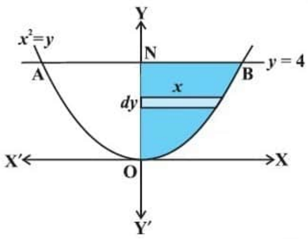


Corner Points	**Corresponding value of Z = 100x + 170y**
(0, 0)	0
(1200, 0)	$1200 \times 100 = 12000$
(1080, 180)	$100 \times 1080 + 170 \times 180 = 138600 \; (\max imum)$
(0, 450)	$0 + 170 \times 450 = 76500$

Hence, the maximum is 138600.

Section D

32. The required area is shown in fig below by shaded region;



The points of intersection of two curves can be calculated and are (-1,1) and (2,4) as shown in fig.

The required area is given as;

$2 \int_0^4 x dy = 2$ (area of the region BONB bounded by curve, y - axis and the lines y = 0 and y = 4)

$= 2 \int_0^4 \sqrt{y} dy = 2 \times \frac{2}{3} \left[y^{\frac{3}{2}} \right]_0^4 = \frac{4}{3} \times 8 = \frac{32}{2}$

which is the required area.

33. According to the question, $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$

and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$.

Now, $\vec{b} + \vec{c} = 2\hat{i} + 4\hat{j} - 5\hat{k} + \lambda\hat{i} + 2\hat{j} + 3\hat{k}$

$= (2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}$

$\therefore \quad |\vec{b} + \vec{c}| = \sqrt{(2 + \lambda)^2 + (6)^2 + (-2)^2}$

$= \sqrt{4 + \lambda^2 + 4\lambda + 36 + 4}$

$= \sqrt{\lambda^2 + 4\lambda + 44}$

The unit vector along $\vec{b}+\vec{c}$

$= \frac{\vec{b}+\vec{c}}{|\vec{b}+\vec{c}|} = \frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}$ ...(i)

According to the question, the scalar product of $(\hat{i}+\hat{j}+\hat{k})$with unit vector $\vec{b}+\vec{c}$ is 1.

$\therefore \quad (\hat{i}+\hat{j}+\hat{k}) \cdot \frac{\vec{b}+\vec{c}}{|\vec{b}+\vec{c}|} = 1$

$\Rightarrow \quad (\hat{i}+\hat{j}+\hat{k}) \cdot \frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}} = 1$

$\Rightarrow \quad \frac{1(2+\lambda)+1(6)+1(-2)}{\sqrt{\lambda^2+4\lambda+44}} = 1$

$\Rightarrow \frac{(2+\lambda)+6-2}{\sqrt{\lambda^2+4\lambda+44}} = 1$

$\Rightarrow \quad \lambda+6 = \sqrt{\lambda^2+4\lambda+44}$

$\Rightarrow \quad (\lambda+6)^2 = \lambda^2+4\lambda+44$

[squaring both sides]

$\Rightarrow \quad \lambda^2+36+12\lambda = \lambda^2+4\lambda+44$

$\Rightarrow \quad 8\lambda = 8$

$\Rightarrow \quad \lambda = 1$

The value of λ is 1.

Substituting the value of λ in Eq. (i), we get unit vector along $\vec{b}+\vec{c}$

$= \frac{(2+1)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(1)^2+4(1)+44}}$

$= \frac{3\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{1+4+44}}$

$= \frac{3\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{49}} = \frac{3}{7}\hat{i}+\frac{6}{7}\hat{j}-\frac{2}{7}\hat{k}$

OR

According to the question vectors are

$\vec{a} = \hat{i}+4\hat{j}+2\hat{k}$,

$\vec{b} = 3\hat{i}-2\hat{j}+7\hat{k}$

$\vec{c} = 2\hat{i}-\hat{j}+4\hat{k}$

Suppose, $\vec{p} = x\hat{i}+y\hat{j}+z\hat{k}$

We have, $\vec{p}$ is perpendicular to both $\vec{a}$ and $\vec{b}$.

$\vec{p}\cdot\vec{a} = 0$

$\Rightarrow \quad (x\hat{i}+y\hat{j}+z\hat{k})\cdot(\hat{i}+4\hat{j}+2\hat{k}) = 0$

$\Rightarrow \quad x+4y+2z = 0$ ...(i)

and $\vec{p}.\vec{b} = 0$

$\Rightarrow \quad (x\hat{i}+y\hat{j}+z\hat{k})\cdot(3\hat{i}-2\hat{j}+7\hat{k}) = 0$

$\Rightarrow \quad 3x-2y+7z = 0$ ...(ii)

Also, given $\vec{p}\cdot\vec{c} = 18$

$\Rightarrow \quad \left(x\hat{i}+y\hat{j}+z\hat{k}\right)\cdot(2\hat{i}-\hat{j}+4\hat{k}) = 0$

$\Rightarrow \quad 2x-y+4z = 18$ ...(iii)

Multiplying Eq. (i) by 3 and subtracting it from Eq. (ii), we get

$-14y+z = 0$

Multiplying Eq. (i) by 2 and subtracting it from Eq. (iii), we get

$-9y = 18$

$\Rightarrow y = -2$

On putting y = -2 and z= -28 in Eq. (i), we get

$x+4(-2)+2(-28) = 0$

$\Rightarrow x-8-56 = 0$

$\Rightarrow x = 64$

Hence, the required vector is

$\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$

i.e. $\vec{p} = 64\hat{i} - 2\hat{j} - 28\hat{k}$

34. For n = 1

$$A^1 = \begin{bmatrix} 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \end{bmatrix} = \begin{bmatrix} 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Result is true for n = 1

Let it be true for n = k

$$A^k = \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}$$

Therefore $A^{k+1} = A.A^k$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}$$

$$= \begin{bmatrix} 3.3^{k-1} & 3.3^{k-1} & 3.3^{k-1} \\ 3.3^{k-1} & 3.3^{k-1} & 3.3^{k-1} \\ 3.3^{k-1} & 3.3^{k-1} & 3.3^{k-1} \end{bmatrix}$$

$$= \begin{bmatrix} 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \end{bmatrix}$$

Thus, result is true for n = k+1

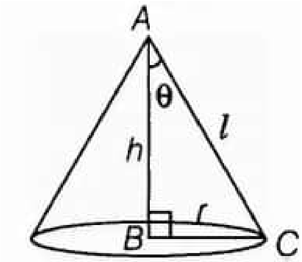
Whenever it is true for n = k.

Hence proved.

35. Let θ be the semi-vertical angle of the cone.

It is clear that $\theta \in \left(0, \frac{\pi}{2}\right)$

Let r, h and l be the radius, height and the slant height of the cone, respectively.



Since, slant height of the cone is given, so consider it as constant.

Now, in ΔABC, r=l sin $\theta$and h=l cos θ

Let V be the volume of the cone.

Then, $V = \frac{\pi}{3} r^2 h$

$\Rightarrow \quad V = \frac{1}{3}\pi\left(l^2 \sin^2\theta\right)(l\cos\theta)$

$\Rightarrow \quad V = \frac{1}{3}\pi l^3 \sin^2\theta\cos\theta$

On differentiating both sides w.r.t. θ , we get,

$\frac{dV}{d\theta} = \frac{l^3\pi}{3}\left[\sin^2\theta(-\sin\theta) + \cos\theta(2\sin\theta\cos\theta)\right]$

$= \frac{l^3\pi}{3}\left(-\sin^3\theta + 2\sin\theta\cos^2\theta\right)$

Again,differentiating both sides w.r.t θ,we get,

$\frac{d^2V}{d\theta^2} = \frac{l^3\pi}{3}\left(-3\sin^2\theta\cos\theta + 2\cos^3\theta - 4\sin^2\theta\cos\theta\right)$

$\Rightarrow \frac{d^2V}{d\theta^2} = \frac{l^3\pi}{3}\left(2\cos^3\theta - 7\sin^2\theta\cos\theta\right)$

For maxima or minima, put $\frac{dV}{d\theta} = 0$

$\Rightarrow \quad \sin^3\theta = 2\sin\theta\cos^2\theta$

$\Rightarrow \quad \tan^2\theta = 2$

$\Rightarrow \quad \tan\theta = \sqrt{2} \Rightarrow \theta = \tan^{-1}\sqrt{2}$

Now, when $\theta = \tan^{-1}\sqrt{2}$ then $\tan^2\theta = 2$

$\Rightarrow \quad \sin^2\theta = 2\cos^2\theta$

Now, we have

$\frac{d^2V}{d\theta^2} = \frac{l^3\pi}{3}(2\cos^3\theta - 14\cos^3\theta)$

$= -4\pi l^3 \cos^3\theta < \theta,\ \text{for}\ \theta \in \left(0, \frac{\pi}{2}\right)$

$\therefore$ V is maximum, when $\theta = \tan^{-1}\sqrt{2}$

or $\theta = \cos^{-1}\frac{1}{\sqrt{3}}$ $\left[\because \cos\theta = \frac{1}{\sqrt{1+\tan^2\theta}} = \frac{1}{\sqrt{1+2}} = \frac{1}{\sqrt{3}}\right]$

Hence, for given slant height, the semi-vertical angle of the cone of maximum volume is $\cos^{-1}\frac{1}{\sqrt{3}}$.

OR

Here,it is given that A right circular cylinder is inscribed inside a cone.

The curved surface area is maximum, suppose that

'r_1' be the radius of the cone.

'h_1' be the height of the cone.

'r' be the radius of the inscribed cylinder.

'h' be the height of the inscribed cylinder.

DF = r, and AD = AL – DL = h_1 – h

Now, here $\triangle$ADF and $\triangle$ALC are similar,

Then,

$\frac{AD}{AL} = \frac{DF}{LC} \Rightarrow \frac{h_1-h}{h_1} = \frac{r}{r_1}$

$h_1 - h = \frac{rh_1}{r_1}$

$h = h_1 - \frac{rh_1}{r_1} = h_1\left(1 - \frac{r}{r_1}\right)$

$h = h_1\left(1 - \frac{r}{r_1}\right)$ ... (i)

Now let us consider the curved surface area of the cylinder,

S = 2πrh

Putting, h in the formula,

$S = 2\pi r\left[h_1\left(1 - \frac{r}{r_1}\right)\right]$

$S = 2\pi r h_1 - \frac{2\pi h_1 r^2}{r_1}$ ... (ii)

For finding the maximum/ minimum of given function, we can find it by differentiating it with r and then equating it to zero. This is because if the function S(r) has a maximum/minimum at a point c then S'(c) = 0.

Differentiating the equation (ii) with respect to r,we get

$\frac{dS}{dr} = \frac{d}{dr}\left[2\pi r h_1 - \frac{2\pi h_1 r^2}{r_1}\right]$

$\frac{dS}{dr} = 2\pi h_1 - \frac{2\pi h_1(2r)}{r_1}$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

$\frac{dS}{dr} = 2\pi h_1 - \frac{4\pi h_1 r}{r_1}$ ... (iii)

To find the critical point, we need to equate equation (iii) to zero,we get

$\frac{dS}{dr} = 2\pi h_1 - \frac{4\pi h_1 r}{r_1} = 0$

$\frac{4\pi h_1 r}{r_1} = 2\pi h_1$

$r = \frac{2\pi h_1 r_1}{4\pi h_1}$

$r = \frac{r_1}{2}$

Now to check if this critical point will determine the maximum volume of the inscribed cylinder, we need to check with second differential which needs to be negative.

Consider differentiating the equation (iii) with r,we get

$\frac{d^2S}{dr^2} = \frac{d}{dr}\left[2\pi h_1 - \frac{4\pi h_1 r}{r_1}\right]$

$\frac{d^2S}{dr^2} = 0 - \frac{4\pi h_1}{r_1} = -\frac{4\pi h_1}{r_1}$ ... (iv)

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

Now let us find the value of

$\frac{d^2S}{dr^2}_{r=\frac{r_1}{2}} = -\frac{4\pi h_1}{r_1}$

As $\frac{d^2S}{dr^2}r = \frac{r_1}{2} = -\frac{4\pi h_1}{r_1} < 0$, thus,the function S is maximum at $r = \frac{r_1}{2}$ therefore,

Putting r in equation (i)

$h = h_1\left(1 - \frac{\frac{r_1}{2}}{r_1}\right)$

$h = h_1\left(1 - \frac{1}{2}\right) = \frac{h_1}{2}$ ... (v)

As S is maximum, from (v) we can clearly say that h_1 = 2h and

r_1 = 2r

this means the radius of the cone is twice the radius of the cylinder or equal to diameter of the cylinder.

Section E

36. i. Let A be the event of committing an error and E_1, E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form.

$P(E_1) = 0.5, P(E_2) = 0.2, P(E_3) = 0.3$

$P(\frac{A}{E_1}) = 0.06, P(\frac{A}{E_2}) = 0.04, P(\frac{A}{E_3}) = 0.03$

Using Bayes' theorem, we have

$P(\frac{E_1}{A}) = \frac{P(E_1)\cdot P\left(\frac{A}{E_1}\right)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right)+P(E_2)\cdot P\left(\frac{A}{E_2}\right)+P(E_3)\cdot P\left(\frac{A}{E_3}\right)}$

$= \frac{0.5\times0.06}{0.5\times0.06+0.2\times0.04+0.3\times0.03} = \frac{30}{47}$

$\therefore$ Required probability = $P\left(\frac{\bar{E}_1}{A}\right)$

$= 1 - P\left(\frac{E_1}{A}\right) = 1 - \frac{30}{47} = \frac{17}{47}$

ii. Let A be the event of committing an error and E_1, E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form.

$P(E_1) = 0.5, P(E_2) = 0.2, P(E_3) = 0.3$

$P(\frac{A}{E_1}) = 0.06, P(\frac{A}{E_2}) = 0.04, P(\frac{A}{E_3}) = 0.03$

$P(A \cap E_2) = P\left(\frac{A}{E_2}\right).P(E_2)$

$\Rightarrow 0.04 \times 0.2 = 0.008$

iii. Let A be the event of committing an error and E_1, E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form.

$P(E_1) = 0.5, P(E_2) = 0.2, P(E_3) = 0.3$

$P(\frac{A}{E_1}) = 0.06, P(\frac{A}{E_2}) = 0.04, P(\frac{A}{E_3}) = 0.03$

$P(A) = P\left(\frac{A}{E_1}\right).P(E_1) + P\left(\frac{A}{E_2}\right).P(E_2) + P\left(\frac{A}{E_3}\right).P(E_3)$

$= 0.5 \times 0.06 + 0.2 \times 0.04 + 0.3 \times 0.03 = 0.047$

OR

Let A be the event of committing an error and E_1, E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form.

$P(E_1) = 0.5, P(E_2) = 0.2, P(E_3) = 0.3$

$P(\frac{A}{E_1}) = 0.06, P(\frac{A}{E_2}) = 0.04, P(\frac{A}{E_3}) = 0.03$

$\sum_{i=1}^{3} P\left(\frac{E_i}{A}\right) = P\left(\frac{E_1}{A}\right) + P\left(\frac{E_2}{A}\right) + P\left(\frac{E_3}{A}\right)$

= 1 [$\because$ Sum of posterior probabilities is 1]

37. i. If a set P has m elements and set Q has n elements then the number of functions possible from P to Q is n^m.

So, number of functions from A to B = 6^2

ii. As the total number of Relations that can be defined from a set P to Q is the number of possible subsets of P $\times$ Q.

If n(P) = m and n(Q) = n then n(P $\times$ Q) = mn and the number of subsets of P $\times$ Q = 2^{mn}.

So number of relations possible from A to B = $2^{2\times6} = 2^{12}$

If n(A) = P and n(B) = q then n(A $\times$ B) = pq and the number of subsets of A $\times$ B = 2^{pq}.

iii. R = {(1, 2), (2, 2),(1, 3), (3, 4), (3, 1), (4, 3), (5, 5),}

R is not reflexive. (3, 3) $\notin$ R

R is not symmetric.

Because for (1, 2) $\in$ R there

(2, 1) $\notin$ R.

R is not transitive.

Because for all element of B there does not exist,

(a, b) (b, c) $\in$ R and (a, c) $\in$ R.

OR

R is reflexive, since every element of B i.e,

B = {1, 2, 3, 4, 5, 6} is divisible by itself.

i.e, (1, 1), (2, 2), (3, 3), (4, 4,), (5, 5), (6, 6) $\in$ R

Further, (1, 2) $\in$ R

But (2, 1) $\notin$ R

Moreover,

(1, 2), (2, 4) $\in$ R

$\Rightarrow$ (1, 4) $\in$ R

$\Rightarrow$ R is transitive.

Therefore, R is reflexive and transitive but not symmetric.

38. i. R.H.D. of f(x) at x = 1 = $\lim\limits_{h\to 0} \frac{f(1+h)-f(1)}{h}$

$= \lim\limits_{h\to 0} \frac{|1+h-3|-|-2|}{h} = \lim\limits_{h\to 0} \frac{2-h-2}{h} = -1$

ii. L.H.D. of f(x) at x = 1 = $\lim\limits_{h\to 0} \frac{f(1-h)-f(1)}{-h}$

$= \lim\limits_{h\to 0} \frac{\left[\frac{(1-h)^2}{4} - \frac{3(1-h)}{2} + \frac{13}{4} - 2\right]}{-h}$

$= \lim\limits_{h\to 0} \left[\frac{h^2-2h+1-6+6h+13-8}{-4h}\right]$

$= \lim\limits_{h\to 0} \left[\frac{h^2+4h}{-4h}\right] = -1.$

iii. Since L.H.D. of f(x) at x = 1

is same as R.H.D. of f(x) at x = 1,

f(x) is differentiable at x = 1.

OR

$f(x) = \begin{cases} x - 3, x \geq 3 \\ 3 - x, 1 \leq x < 3 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, x < 1 \end{cases}$

$[f'(x)]_{x=2} = 0 - 1 = -1$

$[f'(x)]_{x=-1} = \frac{2(-1)}{4} - \frac{3}{2} = -2$