

Class XII Session 2025-26

Subject - Mathematics

Sample Question Paper - 6

Time Allowed: 3 hours

Maximum Marks: 80

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If $B \begin{vmatrix} 1 & -2 \\ 1 & 4 \end{vmatrix} = \begin{vmatrix} 6 & 0 \\ 0 & 6 \end{vmatrix}$ then matrix B is [1]
a) I
b) $\begin{vmatrix} 4 & 2 \\ -1 & 1 \end{vmatrix}$
c) $\begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix}$
d) $\begin{vmatrix} 4 & 2 \\ 1 & 1 \end{vmatrix}$
2. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $A^{-1} = kA$, then k equals [1]
a) 19
b) -1/19
c) 1/19
d) -19
3. A square matrix A is called singular if det. A is [1]
a) 0
b) Negative
c) Positive
d) Non-zero
4. If $y = \frac{\log x}{x}$, then $\frac{d^2y}{dx^2} =$ [1]
a) $\frac{2 \log x - 3}{x^4}$
b) $\frac{2 \log x + 3}{x^3}$
c) $\frac{3 - 2 \log x}{x^3}$
d) $\frac{2 \log x - 3}{x^3}$
5. If lines $\frac{2x-2}{2k} = \frac{4-y}{3} = \frac{z+2}{-1}$ and $\frac{x-5}{1} = \frac{y}{k} = \frac{z+6}{4}$ are at right angles, then the value of k is [1]
a) -2
b) 4
c) 0
d) 2
6. The general solution of the differential equation $\frac{dy}{dx} + y \cot x = \operatorname{cosec} x$, is [1]

a) $y \sin x = x + C$

b) $y + x (\sin x + \cos x) = C$

c) $x + y \sin x = C$

d) $x + y \cos x = C$

7. Minimize $Z = 5x + 10y$ subject to $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x, y \geq 0$ [1]

 - Minimum $Z = 300$ at $(60, 0)$
 - Minimum $Z = 330$ at $(60, 0)$
 - Minimum $Z = 310$ at $(60, 0)$
 - Minimum $Z = 320$ at $(60, 0)$

8. The value of $\cos^{-1} \left(\cos \frac{3\pi}{2} \right)$ is equal to [1]

 - $\frac{\pi}{2}$
 - $\frac{7\pi}{2}$
 - $\frac{5\pi}{2}$
 - $\frac{3\pi}{2}$

9. $\int \frac{3x^2}{\sqrt{9-16x^6}} dx = ?$ [1]

 - $2 \sin^{-1} \left(\frac{3x^3}{4} \right) + C$
 - $4 \sin^{-1} \left(\frac{x^3}{4} \right) + C$
 - $\frac{1}{4} \sin^{-1} \left(\frac{4x^3}{3} \right) + C$
 - $\frac{1}{4} \sin^{-1} \left(\frac{x^3}{3} \right) + C$

10. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 & 2 \\ 4 & 3 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $D = \begin{bmatrix} 4 & 6 & 8 \\ 5 & 7 & 9 \end{bmatrix}$, then which of the following is defined? [1]

 - $A + B$
 - $C + D$
 - $B + D$
 - $B + C$

11. A feasible region of a system of linear inequalities is said to be ..., if it can be enclosed within a circle. [1]

 - In squared form
 - bounded
 - unbounded
 - in circled form

12. What is the angle which the vector $(\hat{i} + \hat{j} + \sqrt{2}\hat{k})$ makes with the z-axis? [1]

 - $\frac{\pi}{3}$
 - $\frac{\pi}{4}$
 - $\frac{2\pi}{3}$
 - $\frac{\pi}{6}$

13. Let A and B be two invertible matrices of order 3×3 . If $\det(AA'B') = 8$ and $\det(AB^{-1}) = 8$, then $\det(BA^{-1}B')$ is equal to [1]

 - 1
 - $\frac{1}{16}$
 - 16
 - $\frac{1}{4}$

14. Three dice are thrown simultaneously. The probability of obtaining a total score of 5 is [1]

 - $\frac{1}{36}$
 - $\frac{5}{216}$
 - $\frac{1}{6}$
 - $\frac{1}{49}$

15. The solution of the differential equation $x \frac{dy}{dx} = y + x \tan \frac{y}{x}$, is [1]

 - $\sin \frac{y}{x} = cx$
 - $\sin \frac{x}{y} = x + C$
 - $\sin \frac{x}{y} = Cy$
 - $\sin \frac{y}{x} = Cy$

16. The value of $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j})$ is [1]

 - 1
 - 3

- c) 0 d) -1
17. Let $f(x) = \begin{cases} e^{1/x}, & x < 0 \\ x, & x \geq 0 \end{cases}$, then $\lim_{x \rightarrow 0} f(x)$ [1]
- a) does exist b) is equal to non – zero real number
c) is equal to 0 d) does not exist
18. If (a_1, b_1, c_1) and (a_2, b_2, c_2) be the direction ratios of two parallel lines then [1]
- a) $a_1 = a_2, b_1 = b_2, c_1 = c_2$ b) $a_1^2 + b_1^2 + c_1^2 = a_2^2 + b_2^2 + c_2^2$
c) $a_1a_2 + b_1b_2 + c_1c_2 = 0$ d) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
19. **Assertion (A):** The function $f(x) = x^2 - 4x + 6$ is strictly increasing in the interval $(2, \infty)$. [1]
Reason (R): The function $f(x) = x^2 - 4x + 6$ is strictly decreasing in the interval $(-\infty, 2)$.
- a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false. d) A is false but R is true.
20. **Assertion (A):** The function $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x^3$ is injective. [1]
Reason (R): The function $f : X \rightarrow Y$ is injective, if $f(x) = f(y) \Rightarrow x = y$ for all $x, y \in X$.
- a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false. d) A is false but R is true.

Section B

21. Find the domain of $f(x) = \sin^{-1}(-x^2)$. [2]

OR

Which is greater, $\tan 1$ or $\tan^{-1} 1$?

22. A stone is dropped into a quiet lake and waves move in circles at the speed of 5 cm/s. At the instant when the radius of the circular wave is 8 cm, how fast is the enclosed area increasing? [2]
23. Water is running into an inverted cone at the rate of π cubic metres per minute. The height of the cone is 10 metres, and the radius of its base is 5 m. How fast the water level is rising when the water stands 7.5 m below the base. [2]

OR

Prove that the function f given by $f(x) = x^2 - x + 1$ is neither strictly increasing nor strictly decreasing on $(-1, 1)$.

24. Evaluate: $\int \tan^3 x \sec^2 x \, dx$ [2]
25. A matrix A of order 3×3 is such that $|A| = 4$. Find the value of $|2A|$. [2]

Section C

26. Find $\int \frac{x^2+1}{x^2-5x+6} \, dx$ [3]
27. An urn contains 10 white and 3 black balls. Another urn contains 3 white and 5 black balls. Two are drawn from first urn and put into the second urn and then a ball is drawn from the latter. Find the probability that it is a white ball. [3]
28. Prove that [3]

$$\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx = \frac{1}{\sqrt{2}} \log(\sqrt{2} + 1).$$

OR

Evaluate: $\int \frac{(x-1)^2}{x^2+2x+2} dx$

29. Find a particular solution of the differential equation $(x - y)(dx + dy) = dx - dy$, given that $y = -1$, when $x = 0$. [3]

OR

Solve the differential equation $x \log |x| \frac{dy}{dx} + y = \frac{2}{x} \log |x|$.

30. If $\vec{a} = 3\hat{i} - \hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$, then express \vec{b} in the form $\vec{b} = \vec{b}_1 + \vec{b}_2$, where $\vec{b}_1 \parallel \vec{a}$ and $\vec{b}_2 \perp \vec{a}$. [3]

OR

If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}, \vec{a} \times \vec{b} = \vec{a} \times \vec{c}, \vec{a} \neq \vec{0}$, then show that $\vec{b} = \vec{c}$.

31. If $x = \sin t, y = \sin pt$, prove that $(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + p^2 y = 0$. [3]

Section D

32. Find the area bounded by the curves $y = x$ and $y = x^3$ [5]

33. Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b| \text{ is divisible by } 2\}$ is an equivalence relation. Write all the equivalence classes of R . [5]

OR

Let $A = [-1, 1]$. Then, discuss whether the following functions defined on A are one-one, onto or bijective:

i. $f(x) = \frac{x}{2}$

ii. $g(x) = |x|$

iii. $h(x) = x|x|$

iv. $k(x) = x^2$

34. Find X and Y , if $2x + 3y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and $3x + 2y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$ [5]

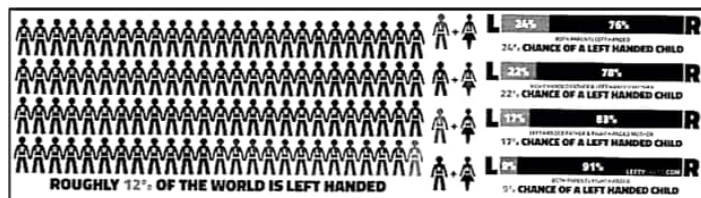
35. A wire of length 28m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined areas of the square and the circle is minimum? [5]

OR

If length of three sides of a trapezium other than base are equal to 10cm, then find the area of the trapezium when it is maximum.

Section E

36. Recent studies suggest that roughly 12% of the world population is left handed. [4]



Depending upon the parents, the chances of having a left handed child are as follows:

A. When both father and mother are left handed:

Chances of left handed child is 24%.

B. When father is right handed and mother is left handed:

Chances of left handed child is 22%.

C. When father is left handed and mother is right handed:

Chances of left handed child is 17%.

D. When both father and mother are right handed:

Chances of left handed child is 9%.

Assuming that $P(A) = P(B) = P(C) = P(D) = \frac{1}{4}$ and L denotes the event that child is left handed.

i. Find $P\left(\frac{L}{C}\right)$. (1)

ii. Find $P\left(\frac{L}{A}\right)$. (1)

iii. Find $P\left(\frac{A}{L}\right)$. (2)

OR

Find the probability that a randomly selected child is left handed given that exactly one of the parents is left handed. (2)

37. **Read the following text carefully and answer the questions that follow:**

[4]

Two motorcycles A and B are running at the speed more than allowed speed on the road along the lines

$\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = 3\hat{i} + 3\hat{j} + \mu(2\hat{i} + \hat{j} + \hat{k})$, respectively.



i. Find the cartesian equation of the line along which motorcycle A is running. (1)

ii. Find the direction cosines of line along which motorcycle A is running. (1)

iii. Find the direction ratios of line along which motorcycle B is running. (2)

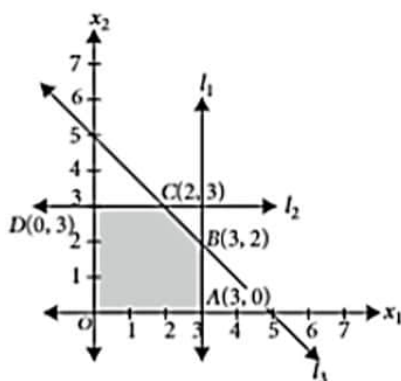
OR

Find the shortest distance between the given lines. (2)

38. **Read the following text carefully and answer the questions that follow:**

[4]

Corner points of the feasible region for an LPP are (0, 3), (5, 0), (6, 8), (0, 8). Let $Z = 4x - 6y$ be the objective function.



i. At which corner point the minimum value of Z occurs? (1)

ii. At which corner point the maximum value of Z occurs? (1)

iii. What is the value of (maximum of Z - minimum of Z)? (2)

OR

The corner points of the feasible region determined by the system of linear inequalities are (2)

Solution

Section A

1.

(b) $\begin{vmatrix} 4 & 2 \\ -1 & 1 \end{vmatrix}$

Explanation:

Let $B = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} \begin{vmatrix} 1 & -2 \\ 1 & -4 \end{vmatrix} = \begin{vmatrix} 6 & 0 \\ 0 & 6 \end{vmatrix}$$

$$\begin{vmatrix} a+b & -2a+4b \\ c+d & -2c+4d \end{vmatrix} = \begin{vmatrix} 6 & 0 \\ 0 & 6 \end{vmatrix}$$

Now, comparing the corresponding element

$$a + b = 6 \dots (i)$$

$$-2a + 4b = 0$$

$$-2a = -4b$$

$$a = 2b \dots (ii)$$

$$c + d = 0 \dots (iii)$$

$$-2c + 4d = 6$$

putting (ii) in (i)

$$3b = 6$$

$$b = 2$$

$$\therefore a = 4$$

Now, from equation (iii) and (iv)

$$-2c + 4(-c) = 6$$

$$-6c = 6$$

$$c = -1$$

$$\therefore d = 1$$

$$\text{matrix } B = \begin{vmatrix} 4 & 2 \\ -1 & 1 \end{vmatrix}$$

2.

(c) $\frac{1}{19}$

Explanation:

$$A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$$

Using adjoint matrix

$$A^{-1} = \frac{-1}{19} \begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{19} \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix} = \frac{1}{19} A$$

$$k = \frac{1}{19}$$

3. (a) 0

Explanation:

For a singular matrix, $|A| = 0$.

4.

(d) $\frac{2 \log x - 3}{x^3}$

Explanation:

$$\begin{aligned}\frac{d}{dx} \left(\frac{\log x}{x} \right) &= \frac{x \cdot \frac{1}{x} - \log x \cdot 1}{x^2} = \frac{1 - \log x}{x^2} \\ \Rightarrow \frac{d}{dx} \left(\frac{1 - \log x}{x^2} \right) \\ &= \frac{x^2 \cdot (-\frac{1}{x}) - (1 - \log x) \cdot 2x}{x^4} = \frac{-x - 2x + 2x \log x}{x^4} = \frac{x(2 \log x - 3)}{x^4} = \frac{2 \log x - 3}{x^3}\end{aligned}$$

5. (a) -2

Explanation:

Given lines are $\frac{2x-2}{2k} = \frac{4-y}{3} = \frac{z+2}{-1}$ and $\frac{x-5}{1} = \frac{y}{k} = \frac{z+6}{4}$

Writing the above equation in standard form, we get

$$\Rightarrow \frac{2(x-1)}{2k} = \frac{-(y-4)}{3} = \frac{z+2}{-1}$$

$$\Leftrightarrow \frac{(x-1)}{k} = \frac{y-4}{-3} = \frac{z+2}{-1}$$

Now, the direction ratio of the first line is (k, -3, -1) and the direction ratio of second line is (1, k, 4)

Since, lines are perpendicular,

$$\therefore (k \times 1) + (-3 \times k) + (-1 \times 4) = 0$$

$$\Rightarrow k - 3k - 4 = 0$$

$$\Rightarrow -2k - 4 = 0$$

$$\therefore k = -2$$

6. (a) $y \sin x = x + C$

Explanation:

We have,

$$\frac{dy}{dx} + y \cot x = \operatorname{cosec} x$$

Comparing with $\frac{dy}{dx} + Py = Q$ of the above equation then, we get

$$\Rightarrow P = \cot x, Q = \operatorname{cosec} x$$

$$\text{I.F.} = e^{\int P dx} = e^{\int \cot x dx} = e^{\log \sin x} = \sin x$$

Multiplying on both sides by sin x

$$\sin x \frac{dy}{dx} + y \cos x = 1$$

$$\Rightarrow \frac{d}{dx} (y \sin x) = 1$$

$$\Rightarrow y \sin x = \int 1 dx$$

$$\Rightarrow y \sin x = x + C$$

7. (a) Minimum $Z = 300$ at (60, 0)

Explanation:

Objective function is $Z = 5x + 10y$ (1).

The given constraints are : $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x, y \geq 0$.

The corner points are obtained by drawing the lines $x+2y=120$, $x+y=60$ and $x-2y=0$. The points so obtained are (60,30), (120,0), (60,0) and (40,20)

Corner points	$Z = 5x + 10y$
D(60 ,30)	600
A(120,0)	600
B(60,0)	300.....(Min.)
C(40,20)	400

Here , $Z = 300$ is minimum at (60, 0).

8. (a) $\frac{\pi}{2}$

Explanation:

$$\text{We have, } \cos^{-1} \cos \frac{3\pi}{2}$$

We know that,

$$\cos \frac{3\pi}{2} = 0$$

$$\text{So, } \cos^{-1} \cos \frac{3\pi}{2} = \cos^{-1} 0$$

Let, $\cos^{-1} 0 = \theta$

$$\Rightarrow \cos \theta = 0$$

Principal value of $\cos^{-1} x$ is $[0, \pi]$

$$\text{For, } \cos \theta = 0$$

$$\text{So, } \theta = \frac{\pi}{2}$$

9.

$$(c) \frac{1}{4} \sin^{-1} \left(\frac{4x^3}{3} \right) + C$$

Explanation:

$$\text{Put } x^3 = t \text{ and } 3x^2 dx = dt$$

$$\begin{aligned} \therefore I &= \int \frac{dt}{\sqrt{9-16t^2}} = \frac{1}{4} \int \frac{dt}{\sqrt{\frac{9}{16}-t^2}} = \frac{1}{4} \cdot \int \frac{dt}{\sqrt{\left(\frac{3}{4}\right)^2-t^2}} \\ &= \frac{1}{4} \sin^{-1} \frac{t}{(3/4)} + C = \frac{1}{4} \sin^{-1} \left(\frac{4x^3}{3} \right) + C \end{aligned}$$

10.

$$(c) B + D$$

Explanation:

Only $B + D$ is defined because matrices of the same order can only be added.

11.

$$(b) \text{ bounded}$$

Explanation:

A feasible region of a system of linear inequalities is said to be bounded, if it can be enclosed within a circle.

12.

$$(b) \frac{\pi}{4}$$

Explanation:

$$\frac{\pi}{4}$$

Hint

$$\vec{a} = (\hat{i} + \hat{j} + \sqrt{2}\hat{k}) \Rightarrow |\vec{a}| = |\vec{a}| = \sqrt{1^2 + 1^2 + (\sqrt{2})^2} = \sqrt{4} = 2$$

Direction ratios of \vec{a} are $(1, 1, \sqrt{2})$

Direction cosines of \vec{a} are $(\frac{1}{2}, \frac{1}{2}, \frac{\sqrt{2}}{2})$, i.e., $(\frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}})$

Direction cosines along to z-axis $(0, 0, 1)$

$$\therefore \cos \gamma = \frac{1}{\sqrt{2}} \Rightarrow \gamma = \frac{\pi}{4}$$

13.

$$(b) \frac{1}{16}$$

Explanation:

$$\frac{1}{16}$$

14.

$$(a) \frac{1}{36}$$

Explanation:

$$\frac{1}{36}$$

15.

$$(a) \sin \frac{y}{x} = cx$$

Explanation:

We have ,

$$x \frac{dy}{dx} = y + x \tan \left(\frac{y}{x} \right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} + \tan \left(\frac{y}{x} \right) \dots (I)$$

$$\text{Put } \frac{y}{x} = v$$

$$y = vx$$

Differentiating on both sides,

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$v + \tan v = v + x \frac{dv}{dx} \text{ ...from (i)}$$

$$\tan v = x \frac{dv}{dx}$$

$$\frac{dx}{x} = \cot v dv$$

$$\int \frac{dx}{x} = \int \cot v dv$$

$$\log |x| + \log C = \log |\sin v|$$

$$Cx = \sin v$$

$$\sin\left(\frac{y}{x}\right) = cx$$

16. (a) 1

Explanation:

$$\text{Given: } (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k}(\hat{i} \times \hat{j})$$

$$\text{i. } (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j}) = \hat{i} \cdot \hat{i} + \hat{j} \cdot (-\hat{j}) + \hat{k} \cdot \hat{k}$$

$$= 1 - \hat{j} \cdot \hat{j} + 1$$

$$= 1 - 1 + 1$$

$$= 1$$

17.

(c) is equal to 0

Explanation:

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} e^{\frac{1}{x}} = 0, \quad \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} x = 0 \therefore \lim_{x \rightarrow 0} f(x) = 0$$

18.

$$(d) \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Explanation:

We know that if there are two parallel lines then their direction ratios must have a relation

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

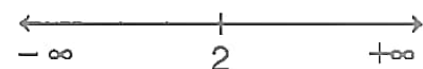
19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation:

$$\text{We have, } f(x) = x^2 - 4x + 6$$

$$\text{or } f'(x) = 2x - 4 = 2(x - 2)$$



Therefore, $f'(x) = 0$ gives $x = 2$.

Now, the point $x = 2$ divides the real line into two disjoint intervals namely, $(-\infty, 2)$ and $(2, \infty)$.

In the interval $(-\infty, 2)$, $f'(x) = 2x - 4 < 0$.

Therefore, f is strictly decreasing in this interval.

Also, in the interval $(2, \infty)$, $f'(x) > 0$ and so the function f is strictly increasing in this interval.

Hence, both the statements are true but Reason is not the correct explanation of Assertion.

20. (a) Both A and R are true and R is the correct explanation of A.

Explanation:

Assertion: Here, $f: \mathbb{R} \rightarrow \mathbb{R}$ is given as

$$f(x) = x^3.$$

Suppose $f(x) = f(y)$

where $x, y \in \mathbb{R}$

$$\Rightarrow x^3 = y^3 \text{ ... (i)}$$

Now, we try to show that $x = y$

Suppose $x \neq y$, their cubes will also not be equal.

$$x^3 \neq y^3$$

However, this will be a contradiction to Eq. (i).

Therefore, $x = y$. Hence, f is injective. Hence, both Assertion and Reason are true and Reason is the correct explanation of Assertion.

Section B

21. The domain of $\sin^{-1} x$ is $[-1, 1]$. Therefore, $f(x) = \sin^{-1}(-x^2)$ is defined for all x satisfying $-1 \leq -x^2 \leq 1$

$$\Rightarrow 1 \geq x^2 \geq -1$$

$$\Rightarrow 0 \leq x^2 \leq 1$$

$$\Rightarrow x^2 \leq 1$$

$$\Rightarrow x^2 - 1 \leq 0$$

$$\Rightarrow (x - 1)(x + 1) \leq 0$$

$$\Rightarrow -1 \leq x \leq 1$$

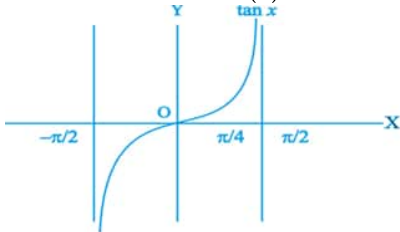
Hence, the domain of $f(x) = \sin^{-1}(-x^2)$ is $[-1, 1]$.

OR

From Fig. we note that $\tan x$ is an increasing function in the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$, since $1 > \frac{\pi}{4} \Rightarrow \tan 1 > \tan \frac{\pi}{4}$. This gives $\tan 1 > 1$

$$\Rightarrow \tan 1 > 1 > \frac{\pi}{4}$$

$$\Rightarrow \tan 1 > 1 > \tan^{-1}(1)$$



22. Let x cm be the radius and y be the enclosed area of the circular wave at any time t .

Rate of increase of radius of circular wave = 5 cm/sec

$$\Rightarrow \frac{dx}{dt} \text{ is positive and } = 5 \text{ cm/sec}$$

$$\Rightarrow \frac{dx}{dt} = 5 \text{ cm/sec} \dots (i)$$

$$y = \pi x^2$$

$$\therefore \text{Rate of change of area} = \frac{dy}{dt} = \pi \frac{d}{dt} x^2$$

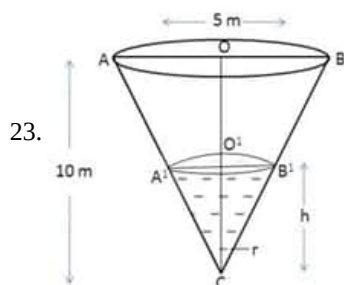
$$= \pi \cdot 2x \frac{dx}{dt} = 2\pi x (5) \text{ (from (i))}$$

$$= 10\pi x \text{ cm}^2 / \text{sec}$$

Putting $x = 8 \text{ cm}$ (given),

$$\frac{dy}{dt} = 10\pi (8) = 80\pi \text{ cm}^2 / \text{sec}$$

Since $\frac{dy}{dt}$ is positive, therefore area of circular wave is increasing at the rate of $80\pi \text{ cm}^2 / \text{sec}$.



23.

Let α be the semi vertical angle of the cone whose height $CO = 10 \text{ m}$ and radius $OB = 5 \text{ m}$.

$$\text{Now, } \tan \alpha = \frac{OB}{CO} = \frac{5}{10}$$

$$\tan \alpha = \frac{1}{2}$$

Let V be the volume of water in the cone, then

$$v = \frac{1}{3} \pi (O'B')^2 (CO')$$

$$v = \frac{1}{3} \pi h^3 \tan^2 \alpha$$

$$\begin{aligned}
 v &= \frac{\pi}{12} h^3 \\
 \frac{dv}{dt} &= \frac{3\pi}{12} h^2 \frac{dh}{dt} \\
 \pi &= \frac{\pi}{4} h^2 \frac{dh}{dt} \\
 \frac{dh}{dt} &= \frac{4}{h^2} \\
 \left(\frac{dh}{dt} \right)_{h=2.5} &= \frac{4}{(2.5)^2} \\
 &= \frac{4}{6.25} \\
 &= 0.64 \text{ m/min.}
 \end{aligned}$$

So, the water level is rising at the rate of 0.64 m/min.

OR

$$\text{Given: } f(x) = x^2 - x + 1 \quad f'(x) = 2x - 1$$

$$\Rightarrow f'(x) = 2x - 1$$

$f(x)$ is strictly increasing if $f'(x) > 0$

$$\Rightarrow 2x - 1 > 0$$

$$\Rightarrow x > \frac{1}{2}$$

i.e., increasing on the interval $\left(\frac{1}{2}, 1\right)$

$f(x)$ is strictly decreasing if $f'(x) < 0$

$$\Rightarrow 2x - 1 < 0$$

$$\Rightarrow x < \frac{1}{2}$$

i.e., decreasing on the interval $\left(-1, \frac{1}{2}\right)$

hence, $f(x)$ is neither strictly increasing nor decreasing on the interval $(-1, 1)$.

$$24. \text{ Let } I = \int \tan^3 x \sec^2 x \, dx$$

$$\text{Now let } \tan x = t. \text{ Then, } d(\tan x) = dt \Rightarrow \sec^2 x \, dx = dt \Rightarrow dx = \frac{dt}{\sec^2 x}$$

Put $\tan x = t$ and $dx = \frac{dt}{\sec^2 x}$, we get

$$I = \int \tan^3 x \sec^2 x \, dx$$

$$= \int t^3 \sec^2 x \times \frac{dt}{\sec^2 x}$$

$$= \int t^3 \, dt$$

$$= \frac{t^4}{4} + C$$

$$= \frac{\tan^4 x}{4} + C$$

25. We are given that,

Order of matrix $A = 3$

$$|A| = 4$$

We need to find the value of $|2A|$.

By the property of determinant of a matrix,

$$|KA| = K^n |A|$$

Where the order of the matrix A is n .

Similarly,

$$|2A| = 2^3 |A| \quad \dots [\because \text{Order of matrix } A = 3]$$

$$\Rightarrow |2A| = 8 |A|$$

Substituting the value of $|A|$ in the above equation,

$$\Rightarrow |2A| = 8 \times 4$$

$$\Rightarrow |2A| = 32$$

Thus, the value of $|2A|$ is 32.

Section C

26. Here the integrand $\frac{x^2+1}{x^2-5x+6}$ is not proper rational function, so we divide $x^2 + 1$ by $x^2 - 5x + 6$ and find that

$$\frac{x^2+1}{x^2-5x+6} = 1 + \frac{5x-5}{x^2-5x+6} = 1 + \frac{5x-5}{(x-2)(x-3)}$$

$$\text{Let } \frac{5x-5}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$$

$$\text{So that } 5x - 5 = A(x - 3) + B(x - 2)$$

Equating the coefficients of x and constant terms on both sides,

we get $A + B = 5$ and $3A + 2B = 5$.

Solving these equations, we get $A = -5$ and $B = 10$

$$\text{Thus, } \frac{x^2+1}{x^2-5x+6} = 1 - \frac{5}{x-2} + \frac{10}{x-3}$$

$$\begin{aligned} \text{Therefore, } \int \frac{x^2+1}{x^2-5x+6} dx &= \int dx - 5 \int \frac{1}{x-2} dx + 10 \int \frac{dx}{x-3} \\ &= x - 5 \log |x-2| + 10 \log |x-3| + C. \end{aligned}$$

27. A white ball can be drawn in three mutually exclusive ways:

- i. By transferring two black balls from first to second urn, then drawing a white ball
- ii. By transferring two white balls from first to second urn, then drawing a white ball
- iii. By transferring a white and a black ball from first to second urn, then drawing a white ball

Consider the following events:

E_1 = Two black balls are transferred from first to second bag

E_2 = Two white balls are transferred from first to second bag

E_3 = A white and a black ball is transferred from first to second bag

A = A white ball is drawn

Therefore, we have,

$$P(E_1) = \frac{{}^3C_2}{{}^{13}C_2} = \frac{3}{78}$$

$$P(E_2) = \frac{{}^{10}C_2}{{}^{13}C_2} = \frac{45}{78}$$

$$P(E_3) = \frac{{}^{10}C_1 \times {}^3C_1}{{}^{13}C_2} = \frac{30}{78}$$

Now,

$$P\left(\frac{A}{E_1}\right) = \frac{3}{10}$$

$$P\left(\frac{A}{E_2}\right) = \frac{5}{10}$$

$$P\left(\frac{A}{E_3}\right) = \frac{4}{10}$$

Using the law of total probability, we get

$$\text{Required probability} = P(A) = P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)$$

$$= \frac{3}{78} \times \frac{3}{10} + \frac{45}{78} \times \frac{5}{10} + \frac{30}{78} \times \frac{4}{10}$$

$$= \frac{9}{780} + \frac{225}{780} + \frac{120}{780}$$

$$= \frac{354}{780} = \frac{59}{130}$$

28. According to the question, $I = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx \dots(i)$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^2\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right) + \cos\left(\frac{\pi}{2}-x\right)} dx$$

$$\left[\because \int_0^a f(x) dx = \int_0^a f(a-x) dx \right]$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos x + \sin x} dx \dots(ii)$$

Adding Equations (i) and (ii),

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x + \cos^2 x}{\sin x + \cos x} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \frac{1}{\sin x + \cos x} dx$$

$$[\because \sin^2 x + \cos^2 x = 1]$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \frac{1}{\frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} + \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}} dx$$

$$\left[\because \sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \text{ and } \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right]$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sec^2 \frac{x}{2}}{2 \tan \frac{x}{2} + 1 - \tan^2 \frac{x}{2}} dx$$

$$\text{Put } \tan \frac{x}{2} = t \Rightarrow \sec^2 \frac{x}{2} \cdot \frac{1}{2} dx = dt \Rightarrow \sec^2 \frac{x}{2} dx = 2dt$$

Lower limit when $x = 0$, then $t = \tan 0 = 0$

Upper limit when $x = \frac{\pi}{2}$, then $t = \tan \frac{\pi}{4} = 1$.

$$\therefore 2I = \int_0^1 \frac{2dt}{2t+1-t^2} = 2 \int_0^1 \frac{dt}{-[t^2-2t-1]}$$

$$= 2 \int_0^1 \frac{dt}{-[(t-1)^2-1-1]} = 2 \int_0^1 \frac{dt}{(\sqrt{2})^2-(t-1)^2}$$

$$= \left[\frac{2}{2\sqrt{2}} \log \left| \frac{\sqrt{2}+t-1}{\sqrt{2}-t+1} \right| \right]_0^1$$

$$\left[\therefore \int \frac{dx}{a^2-x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C \right]$$

$$= \frac{1}{\sqrt{2}} \left[\log \frac{\sqrt{2}+1-1}{\sqrt{2}-1+1} - \log \frac{\sqrt{2}+0-1}{\sqrt{2}-0+1} \right]$$

$$= \frac{1}{\sqrt{2}} \left[\log 1 - \log \frac{\sqrt{2}-1}{\sqrt{2}+1} \right]$$

$$= -\frac{1}{\sqrt{2}} \log \left[\frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} \right] [\therefore \log 1 = 0]$$

$$= \frac{-1}{\sqrt{2}} \log \frac{2-1}{(\sqrt{2}+1)^2} [\therefore (a-b)(a+b) = a^2 - b^2]$$

$$= \frac{-1}{\sqrt{2}} \log \frac{1}{(\sqrt{2}+1)^2}$$

$$\Rightarrow 2I = \frac{2}{\sqrt{2}} \log(\sqrt{2}+1) \Rightarrow I = \frac{1}{\sqrt{2}} \log(\sqrt{2}+1)$$

OR

$$\text{Let } I = \int \frac{(x-1)^2}{x^2+2x+2} dx$$

$$= \int \left(\frac{x^2-2x+1}{x^2+2x+2} \right) dx$$

Therefore by long division we have,

$$\begin{array}{r} x^2 + 2x + 2 \overline{) x^2 - 2x + 1} \\ \underline{x^2 + 2x + 2} \\ -4x - 1 \end{array}$$

Therefore,

$$\frac{x^2-2x+1}{x^2+2x+2} = 1 - \frac{(4x+1)}{x^2+2x+2} \dots(i)$$

$$\text{Let } 4x+1 = A \frac{d}{dx}(x^2+2x+2) + B$$

$$4x+1 = A(2x+2) + B$$

$$4x+1 = (2A)x + 2A+B$$

Equating Coefficients of like terms

$$2A = 4$$

$$A = 2$$

$$2A + B = 1$$

$$2 \times 2 + B = 1$$

$$B = -3$$

$$\int \left(\frac{x^2-2x+1}{x^2+2x+2} \right) dx$$

$$= \int dx - 2 \int \frac{(2x+2)}{x^2+2x+2} dx + 3 \int \frac{dx}{x^2+2x+2}$$

$$= \int dx - 2 \int \frac{(2x+2)}{x^2+2x+2} dx + 3 \int \frac{dx}{(x+1)^2+1^2}$$

$$= x - 2 \log |x+2x+2| + \frac{3}{1} \tan^{-1} \left(\frac{x+1}{1} \right) + C$$

$$= x - 2 \log |x+2x+2| + 3 \tan^{-1} (x+1) + C$$

29. It is given that $(x-y)(dx+dy) = dx-dy$

$$\Rightarrow (x-y+1)dy = (1-x+y)dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{1-x+y}{x-y+1} \dots\dots(i)$$

Let $x - y = t$

$$\Rightarrow \frac{d}{dx}(x - y) = \frac{dt}{dx}$$

$$\Rightarrow 1 - \frac{dy}{dx} = \frac{dt}{dx}$$

Now, let us substitute the value of $x-y$ and $\frac{dy}{dx}$ in equation (i), we get,

$$1 - \frac{dt}{dx} = \frac{1-t}{1+t}$$

$$\Rightarrow \frac{dt}{dx} = 1 - \left(\frac{1-t}{1+t} \right)$$

$$\Rightarrow \frac{dt}{dx} = \frac{(1+t)-(1-t)}{1+t}$$

$$\Rightarrow \frac{dt}{dx} = \frac{2t}{1+t}$$

$$\Rightarrow \left(\frac{1+t}{t} \right) dt = 2dx$$

$$\Rightarrow \left(1 + \frac{1}{t} \right) dt = 2dx \dots\dots(ii)$$

On integrating both side, we get,

$$t + \log|t| = 2x + C$$

$$\Rightarrow (x - y) + \log|x - y| = 2x + C$$

$$\Rightarrow \log|x - y| = x + y + C \dots\dots(iii)$$

Now, $y = -1$ at $x = 0$

Then, equation (iii), we get,

$$\log 1 = 0 - 1 + C$$

$$\Rightarrow C = 1$$

Substituting $C = 1$ in equation (iii), we get,

$$\log|x - y| = x + y + 1$$

Therefore, a particular solution of the given differential equation is $\log|x - y| = x + y + 1$.

OR

We have to solve,

$$x \log|x| \frac{dy}{dx} + y = \frac{2}{x} \log|x|$$

On dividing both sides by $x \log x$, we get

$$\frac{dy}{dx} + \frac{y}{x \log|x|} = \frac{2}{x^2} \frac{\log|x|}{\log|x|} = \frac{2}{x^2}$$

which is a linear differential equation of first order, which is of the form of $\frac{dy}{dx} + Py = Q$,

$$\text{Here, } P = \frac{1}{x \log|x|} \text{ and } Q = \frac{2}{x^2}$$

We know that ,

$$IF = e^{\int P dx} = e^{\int \frac{1}{x \log|x|} dx}$$

$$\text{put } \log|x| = t \Rightarrow \frac{1}{x} dx = dt$$

$$\therefore IF = \int \frac{1}{t} dt = \log|t| = \log|\log|x||$$

$$\Rightarrow IF = \log|x| \quad [\because e^{\log x} = x]$$

Now, solution of above equation is given by

$$y \times IF = \int (Q \times IF) dx + C$$

$$y \log|x| = \int \frac{2}{x^2} \log|x| dx + C$$

$$\Rightarrow y \log|x| = 2 \left[\log|x| \int \frac{1}{x^2} dx - \int \left(\frac{d}{dx}(\log|x|) \cdot \int \frac{1}{x^2} dx \right) dx \right] + C \text{ [using integration by parts]}$$

$$\Rightarrow y \log|x| = 2 \left[\log|x| \cdot \left(-\frac{1}{x} \right) - \int \frac{1}{x} \cdot \left(-\frac{1}{x} \right) dx \right] + C$$

$$\Rightarrow y \log|x| = 2 \left[-\frac{1}{x} \log|x| + \int \frac{1}{x^2} dx \right] + C$$

$$\therefore y \log|x| = -\frac{2}{x} \log|x| - \frac{2}{x} + C$$

30. According to the question,

$$\vec{a} = 3\hat{i} - \hat{j} \text{ and}$$

$$\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$$

$$\vec{b}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k} \text{ and}$$

$$\vec{b}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$$

$$\vec{b}_1 + \vec{b}_2 = \vec{b}, \vec{b}_1 \parallel \vec{a} \text{ and } \vec{b}_2 \perp \vec{a}.$$

$$\text{Consider, } \vec{b}_1 + \vec{b}_2 = \vec{b}$$

$$\Rightarrow (x_1 + x_2)\hat{i} + (y_1 + y_2)\hat{j} + (z_1 + z_2)\hat{k} = 2\hat{i} + \hat{j} - 3\hat{k}$$

On comparing the coefficient of \hat{i} , \hat{j} and \hat{k} both sides; we get

$$\Rightarrow x_1 + x_2 = 2 \dots(i)$$

$$y_1 + y_2 = 1 \dots(ii)$$

$$\text{and } z_1 + z_2 = -3 \dots(iii)$$

$$\text{Now, consider } \vec{b}_1 \parallel \vec{a}$$

$$\Rightarrow \frac{x_1}{3} = \frac{y_1}{-1} = \frac{z_1}{0} = \lambda(\text{say})$$

$$\Rightarrow x_1 = 3\lambda, y_1 = -\lambda \text{ and } z_1 = 0 \dots(iv)$$

On substituting the values of x,y and z, form Eq. (iv) to Eq. (i), (ii) and (iii), respectively, we get

$$x_2 = 2 - 3\lambda, y_2 = 1 + \lambda \text{ and } z_2 = -3 \dots(v)$$

$$\text{Since, } \vec{b}_2 \perp \vec{a}, \text{ therefore } \vec{b}_2 \cdot \vec{a} = 0$$

$$\Rightarrow 3x_2 - y_2 = 0$$

$$\Rightarrow 3(2 - 3\lambda) - (1 + \lambda) = 0$$

$$\Rightarrow 6 - 9\lambda - 1 - \lambda = 0$$

$$\Rightarrow 5 - 10\lambda = 0 \Rightarrow \lambda = \frac{1}{2}$$

On substituting $\lambda = \frac{1}{2}$ in Eqs. (iv) and Eqs. (iv) and (v), we get

$$x_1 = \frac{3}{2}, y_1 = \frac{-1}{2}, z_1 = 0$$

$$\text{and } x_2 = \frac{1}{2}, y_2 = \frac{3}{2} \text{ and } z_2 = -3$$

$$\text{Hence, } \vec{b} = \vec{b}_1 + \vec{b}_2 = \left(\frac{3}{2}\hat{i} - \frac{1}{2}\hat{j}\right) + \left(\frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}\right)$$

$$= 2\hat{i} + \hat{j} - 3\hat{k}$$

OR

Given ,

$$\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} \text{ and } \vec{a} \neq \vec{0}$$

$$\Rightarrow \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c} = 0 \text{ and } \vec{a} \neq \vec{0}$$

$$\Rightarrow \vec{a} \cdot (\vec{b} - \vec{c}) = 0 \text{ and } \vec{a} \neq \vec{0}$$

$$\Rightarrow \vec{b} - \vec{c} = \vec{0} \text{ or } \vec{a} \perp (\vec{b} - \vec{c}) [\because \vec{a} \neq \vec{0}]$$

$$\Rightarrow \vec{b} = \vec{c} \text{ or, } \vec{a} \perp (\vec{b} - \vec{c}) \dots(i)$$

Again given,

$$\vec{a} \times \vec{b} = \vec{a} \times \vec{c} \text{ and } \vec{a} \neq \vec{0}$$

$$\Rightarrow \vec{a} \times \vec{b} - \vec{a} \times \vec{c} = \vec{0} \text{ and } \vec{a} \neq \vec{0}$$

$$\Rightarrow \vec{a} \times (\vec{b} - \vec{c}) = \vec{0} \text{ and } \vec{a} \neq \vec{0}$$

$$\Rightarrow \vec{b} - \vec{c} = \vec{0} \text{ or } \vec{a} \parallel (\vec{b} - \vec{c}) [\because \vec{a} \neq \vec{0}]$$

$$\Rightarrow \vec{b} = \vec{c} \text{ or } \vec{a} \parallel (\vec{b} - \vec{c}) \dots(ii)$$

From (i) and (ii), it follows that $\vec{b} = \vec{c}$, because \vec{a} cannot be both parallel and perpendicular to vectors $(\vec{b} - \vec{c})$

31. We have, $x = \sin t$ and $y = \sin pt$,

$$\therefore \frac{dx}{dt} = \cos t \text{ and } \frac{dy}{dt} = \cos pt \cdot p$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{p \cdot \cos pt}{\cos t} \dots(i)$$

Again, differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{\cos t \cdot \frac{d}{dt}(p \cdot \cos pt) \frac{dt}{dx} - p \cos pt \cdot \frac{d}{dt} \cos t \cdot \frac{dt}{dx}}{\cos^2 t}$$

$$= \frac{[\cos t \cdot p \cdot (-\sin pt) \cdot p - p \cos pt \cdot (-\sin t)] \frac{dt}{dx}}{\cos^2 t}$$

$$= \frac{[-p^2 \sin pt \cdot \cos t + p \sin t \cdot \cos pt] \cdot \frac{1}{\cos t}}{\cos^2 t}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-p^2 \sin pt \cdot \cos t + p \cos pt \cdot \sin t}{\cos^3 t} \dots(ii)$$

Since, we have to prove

$$(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + p^2 y = 0$$

$$\therefore LHS = (1 - \sin^2 t) \frac{[-p^2 \sin pt \cdot \cos t + p \cos pt \cdot \sin t]}{\cos^3 t}$$

$$- \sin t \cdot \frac{p \cos pt}{\cos t} + p^2 \sin pt$$

$$= \frac{1}{\cos^3 t} \left[(1 - \sin^2 t) (-p^2 \sin pt \cdot \cos t + p \cos pt \cdot \sin t) - p \cos pt \cdot \sin t \cdot \cos^2 t + p^2 \sin pt \cdot \cos^3 t \right]$$

$$= \frac{1}{\cos^3 t} \left[-p^2 \sin pt \cdot \cos^3 t + p \cos pt \cdot \sin t \cdot \cos^2 t - p \cos pt \cdot \sin t \cdot \cos^2 t + p^2 \sin pt \cdot \cos^3 t \right] [\because 1 - \sin^2 t = \cos^2 t]$$

$$= \frac{1}{\cos^3 t} \cdot 0$$

= 0 Hence proved.

Section D

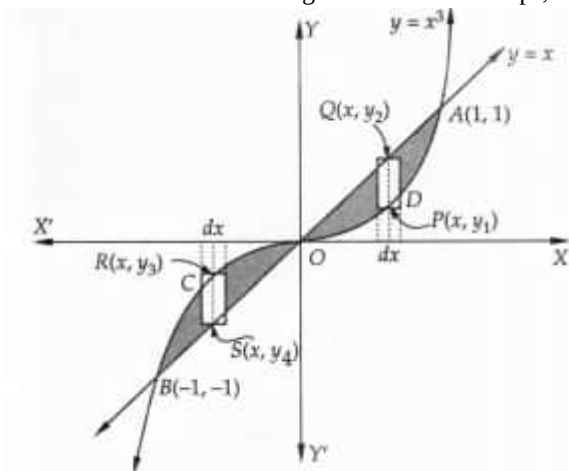
32. The given curves are ,

$$y = x \dots (i)$$

$$\text{and } y = x^3 \dots (ii)$$

The sketch of the curve $y = x^3$ is shown in Fig. Clearly, $y = x$ is a line passing through the origin and making an angle of 45° with x -axis. The shaded portion shown in Fig. is the region bounded by the curves $y = x$ and $y = x^3$. Solving $y = x$ and $y = x^3$ simultaneously, we find that the two curves intersect at $O(0, 0)$, $A(1, 1)$ and $B(-1, -1)$.

When we slice the shaded region into vertical strips, we observe that the vertical strips change their character at O .



Therefore, the required area is given by,

$$\text{Required area} = \text{Area BCOB} + \text{Area ODAO}$$

Area BCOB: Each vertical strip in this region has its lower end on $y = x$ and the upper end on $y = x^3$. Therefore, the approximating rectangle shown in this region has length = $|y_4 - y_3|$, width = dx and area = $|y_4 - y_3| dx$. Since the approximating rectangle can move from $x = -1$ to $x = 0$.

$$\therefore \text{Area BCOB} = \int_0^{-1} |y_4 - y_3| dx = \int_0^{-1} -(y_4 - y_3) dx \quad [\because y_4 < y_3 \therefore y_4 - y_3 < 0]$$

$$= \int_0^{-1} -(x - x^3) dx \quad [\because R(x, y_3) \text{ and } S(x, y_4) \text{ lie on (ii) and (i) respectively } \therefore y_3 = x_3 \text{ and } y_4 = x]]$$

$$= \int_0^{-1} (x^3 - x) dx = \left[\frac{x^4}{4} - \frac{x^2}{2} \right]_0^{-1} = 0 - \left(\frac{1}{4} - \frac{1}{2} \right) = \frac{1}{4} \text{ sq. units}$$

Area ODAO: Each vertical strip in this region has its two ends on (ii) and (i) respectively. So, the approximating rectangle shown in this region has length = $|y_2 - y_1|$, width = dx and therefore, we have,

$$\text{Area ODAO} = \int_0^1 |y_2 - y_1| dx = \int_0^1 (y_2 - y_1) dx \quad [\because y_2 > y_1 \therefore y_2 - y_1 > 0]$$

$$= \int_0^1 (x - x^3) dx \quad [\because P(x, y_1) \text{ and } Q(x, y_2) \text{ lie on (ii) and (i) respectively } \therefore y_1 = x^3 \text{ and } y_2 = x]$$

$$= \left[\frac{x^2}{2} - \frac{x^4}{4} \right]_0^1 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4} \text{ sq. units}$$

$$\therefore \text{Required area} = \text{Area BCOB} + \text{Area ODAO} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \text{ sq. units}$$

33. $R = \{(a, b) = |a \cdot b| \text{ is divisible by } 2\}$

where $a, b \in A = \{1, 2, 3, 4, 5\}$

reflexivity

For any $a \in A, |a - a| = 0$ Which is divisible by 2.

$\therefore (a, a) \in r$ for all $a \in A$

So, R is Reflexive

Symmetric :

Let $(a, b) \in R$ for all $a, b \in R$

$|a-b|$ is divisible by 2

$|b-a|$ is divisible by 2

$(a, b) \in R \Rightarrow (b, a) \in R$

So, R is symmetric.

Transitive :

Let $(a, b) \in R$ and $(b, c) \in R$ then

$(a, b) \in R$ and $(b, c) \in R$

$|a-b|$ is divisible by 2

$|b-c|$ is divisible by 2

Two cases :

Case 1:

When b is even

$(a, b) \in R$ and $(b, c) \in R$

$|a-c|$ is divisible by 2

$|b-c|$ is divisible by 2

$|a-c|$ is divisible by 2

$\therefore (a, c) \in R$

Case 2:

When b is odd

$(a, b) \in R$ and $(b, c) \in R$

$|a-c|$ is divisible by 2

$|b-c|$ is divisible by 2

$|a-c|$ is divisible by 2

Thus, $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$

So R is transitive.

Hence, R is an equivalence relation

OR

Given that $A = [-1, 1]$

i. $f(x) = \frac{x}{2}$

Let $f(x_1) = f(x_2)$

$$\Rightarrow \frac{x_1}{2} = \frac{x_2}{2} \Rightarrow x_1 = x_2$$

So, $f(x)$ is one-one.

Now, let $y = \frac{x}{2}$

$$\Rightarrow x = 2y \notin A, \forall y \in A$$

As for $y = 1 \in A$, $x = 2 \notin A$

So, $f(x)$ is not onto.

Also, $f(x)$ is not bijective as it is not onto.

ii. $g(x) = |x|$

Let $g(x_1) = g(x_2)$

$$\Rightarrow |x_1| = |x_2| \Rightarrow x_1 = \pm x_2$$

So, $g(x)$ is not one-one.

Now, $x = \pm y \notin A$ for all $y \in R$

So, $g(x)$ is not onto, also, $g(x)$ is not bijective.

iii. $h(x) = x|x|$

$$\Rightarrow x_1|x_1| = x_2|x_2| \Rightarrow x_1 = x_2$$

So, $h(x)$ is one-one

Now, let $y = x|x|$

$$\Rightarrow y = x^2 \in A, \forall x \in A$$

So, $h(x)$ is onto also, $h(x)$ is a bijective.

iv. $k(x) = x^2$

Let $k(x_1) = k(x_2)$

$\Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$

Thus, $k(x)$ is not one-one.

Now, let $y = x^2$

$\Rightarrow x\sqrt{y} \notin A, \forall y \in A \quad x = \sqrt{y} \notin A, \forall y \in A$

As for $y = -1, x = \sqrt{-1} \notin A$

Hence, $k(x)$ is neither one-one nor onto.

34. On adding

$$5x + 5y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$$

$$5(x + y) = \begin{bmatrix} 4 & 1 \\ 3 & 5 \end{bmatrix}$$

$$(x + y) = \frac{1}{5} \begin{bmatrix} 4 & 1 \\ 3 & 5 \end{bmatrix} \text{-----(i)}$$

On subtracting

$$x - y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \text{-----(ii)}$$

$$x - y = \begin{bmatrix} 0 & -5 \\ -5 & 5 \end{bmatrix}$$

Adding (i) and (ii) gives,

$$2x = \begin{bmatrix} \frac{4}{5} & \frac{-24}{5} \\ \frac{-22}{5} & 6 \end{bmatrix}$$

$$x = \begin{bmatrix} \frac{2}{5} & \frac{-12}{5} \\ \frac{-11}{5} & 3 \end{bmatrix}$$

$$x + y = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} \\ \frac{3}{5} & 1 \end{bmatrix}$$

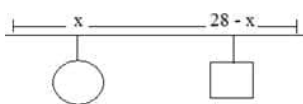
$$\begin{bmatrix} \frac{2}{5} & \frac{-12}{5} \\ \frac{-11}{5} & 3 \end{bmatrix} + y = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} \\ \frac{3}{5} & \frac{1}{5} \end{bmatrix}$$

$$y = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} \\ \frac{3}{5} & 1 \end{bmatrix} + \begin{bmatrix} \frac{-2}{5} & \frac{12}{5} \\ \frac{11}{5} & -3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{5} - \frac{2}{5} & \frac{1}{5} + \frac{12}{5} \\ \frac{3}{5} + \frac{11}{5} & 1 - 3 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$$

35. Let 1st length = x

2nd length = 28 - x



Now circumference of circle is $2\pi r$

$\therefore 2\pi r = x$

$\Rightarrow r = \frac{x}{2\pi}$

Now perimeter of rectangle = $4a$

$\therefore 4a = 28 - x$

$\Rightarrow a = 7 - \frac{x}{4}$

ATQ

A = area of circle + area of square

$$\pi \left(\frac{x}{2\pi} \right)^2 + \left(7 - \frac{x}{4} \right)^2$$

Now, $A = \pi \cdot \frac{x^2}{4\pi^2} + \left(7 - \frac{x}{4} \right)^2$

So, $\frac{dA}{dx} = \frac{2x}{4\pi} + 2 \left(7 - \frac{x}{4} \right) \left(-\frac{1}{4} \right)$

$$\begin{aligned}
\frac{dA}{dx} &= 0 \\
\Rightarrow \frac{2x}{4\pi} + 2 \left(7 - \frac{x}{4} \right) \left(-\frac{1}{4} \right) &= 0 \\
\Rightarrow \frac{1}{2} \left(7 - \frac{x}{4} \right) &= \frac{x}{2\pi} \\
\Rightarrow 7 - \frac{x}{4} &= \frac{x}{\pi} \\
\Rightarrow 7 &= \frac{x}{\pi} + \frac{x}{4} \\
\Rightarrow 7 &= x \left(\frac{4+\pi}{4\pi} \right) \\
\Rightarrow \frac{28\pi}{4+\pi} &= x
\end{aligned}$$

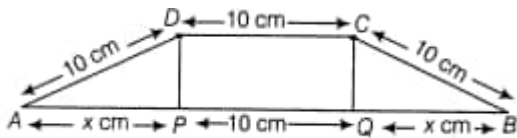
$$\begin{aligned}
\text{Now, } \frac{d^2y}{dx^2} &= \frac{1}{2\pi} - \frac{1}{2} \left(\frac{-1}{4} \right) \\
&= \frac{1}{2\pi} + \frac{1}{8}
\end{aligned}$$

positive, hence minimum

$$\text{Therefore, 1}^{\text{st}} \text{ length} = \frac{28\pi}{4+\pi}$$

$$\begin{aligned}
2^{\text{nd}} \text{ length} &= \frac{28}{1} - \frac{28\pi}{4+\pi} \\
&= 28 \left[\frac{4+\pi-\pi}{4+\pi} \right] \\
&= \frac{112}{4+\pi}
\end{aligned}$$

OR



Let ABCD be the given trapezium in which $AD = BC = CD = 10$ cm.

Draw a perpendicular DP and CQ on AB. Let $AP = x$ cm

In $\triangle APD$ & $\triangle BQC$,

$$\angle APD = \angle BQC \text{ [each } = 90^\circ]$$

$$AD = BC \text{ [Both } 10 \text{ cm]}$$

$$DP = CQ \text{ [Perpendicular between parallel lines are equal in length]}$$

$$\therefore \triangle APD \cong \triangle BQC \text{ [RHS Congruency]}$$

$$\therefore QB = AP \text{ [CPCT]}$$

$$\Rightarrow QB = x \text{ cm}$$

$$DP = \sqrt{10^2 - x^2} \text{ [by Pythagoras theorem]}$$

Now, area of trapezium,

$$A = \frac{1}{2} \times (\text{sum of parallel sides}) \times \text{height}$$

$$= \frac{1}{2} \times (2x + 10 + 10) \times \sqrt{100 - x^2}$$

$$= (x + 10)\sqrt{100 - x^2} \dots (i)$$

We need to find the area of trapezium when it is maximum i.e. we need to maximize area.

On differentiating both sides of eq(i) w.r.t.x, we get

$$\frac{dA}{dx} = (x + 10) \frac{(-2x)}{2\sqrt{100-x^2}} + \sqrt{100-x^2}$$

$$= \frac{-x^2 - 10x + 100 - x^2}{\sqrt{100-x^2}}$$

$$= \frac{-2x^2 - 10x + 100}{\sqrt{100-x^2}} \dots (ii)$$

$$\text{For maxima or minima, put } \frac{dA}{dx} = 0$$

$$\Rightarrow \frac{-2x^2 - 10x + 100}{\sqrt{100-x^2}} = 0$$

$$= -2(x^2 + 5x - 50) = 0$$

$$= -2(x+10)(x-5) = 0$$

$$x = 5 \text{ or } -10$$

Since, x represents distance, so it cannot be negative.

Therefore, we take $x = 5$.

On differentiating both sides of eq.(ii) w.r.t.x, we get

$$\begin{aligned}\frac{d^2 A}{dx^2} &= \frac{\sqrt{100-x^2} \cdot \frac{d}{dx}(-2x^2-10x+100) - (-2x^2-10x+100) \frac{d}{dx}(\sqrt{100-x^2})}{(\sqrt{100-x^2})^2} \quad [\text{by using the quotient rule of derivative}] \\ &= \frac{\sqrt{100-x^2} \cdot (-4x-10) - (-2x^2-10x+100) \left(\frac{-2x}{2\sqrt{100-x^2}}\right)}{(\sqrt{100-x^2})^2} \\ &= \frac{\sqrt{100-x^2} \cdot (-4x-10) + \frac{x(-2x^2-10x+100)}{\sqrt{100-x^2}}}{100-x^2} \\ &= \frac{(100-x^2) \cdot (-4x-10) + x(-2x^2-10x+100)}{(100-x^2)^{\frac{3}{2}}} \\ &= \frac{-400x+4x^3-1000+10x^2-2x^3-10x^2+100x}{(100-x^2)^{\frac{3}{2}}}\end{aligned}$$

$$\therefore \frac{d^2 A}{dx^2} = \frac{2x^3-300x-1000}{(100-x^2)^{3/2}}$$

When $x = 5$,

$$\begin{aligned}\frac{d^2 A}{dx^2} &= \frac{2(5)^3-300(5)-1000}{[100-(5)^2]^{3/2}} \\ &= \frac{250-1500-1000}{(100-25)^{3/2}} = \frac{-2250}{75\sqrt{75}} < 0\end{aligned}$$

\therefore It is maximum when $x = 5$

Thus, area of trapezium is maximum at $x = 5$ and maximum area is

$$\begin{aligned}A_{\max} &= (5+10)\sqrt{100-(5)^2} \quad [\text{put } x = 5 \text{ in Eq. (i)}] \\ &= 15\sqrt{100-25} = 15\sqrt{75} = 75\sqrt{3} \text{ cm}^2\end{aligned}$$

Section E

36. i. $P\left(\frac{L}{C}\right) = \frac{17}{100}$
 ii. $P\left(\frac{L}{A}\right) = 1 - P\left(\frac{L}{B}\right) = 1 - \frac{24}{100} = \frac{76}{100}$ or $\frac{19}{25}$
 iii. $P\left(\frac{A}{L}\right) = \frac{\frac{1}{4} \times \frac{24}{100}}{\frac{1}{4} \times \frac{24}{100} + \frac{1}{4} \times \frac{22}{100} + \frac{1}{4} \times \frac{17}{100} + \frac{1}{4} \times \frac{9}{100}} = \frac{24}{72} = \frac{1}{3}$

Probability that a randomly selected child is left-handed given that exactly one of the parents is left-handed.

$$= P\left(\frac{L}{B \cup C}\right) = \frac{22}{100} + \frac{17}{100} = \frac{39}{100}$$

37. i. The line along which motorcycle A is running, $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$, which can be rewritten as

$$(x\hat{i} + y\hat{j} + z\hat{k}) = \lambda\hat{i} + 2\lambda\hat{j} - \lambda\hat{k}$$

$$\Rightarrow x = \lambda, y = 2\lambda, z = -\lambda \Rightarrow \frac{x}{1} = \lambda, \frac{y}{2} = \lambda, \frac{z}{-1} = \lambda$$

Thus, the required cartesian equation is $\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$

- ii. Clearly, D.R.'s of the required line are $< 1, 2, -1 >$

\therefore D.C.'s are

$$\left(\frac{1}{\sqrt{1^2+2^2+(-1)^2}}, \frac{2}{\sqrt{1^2+2^2+(-1)^2}}, \frac{-1}{\sqrt{1^2+2^2+(-1)^2}} \right)$$

i.e., $\left(\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}} \right)$

- iii. The line along which motorcycle B is running, is $\vec{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k})$, which is parallel to the vector $2\hat{i} + \hat{j} + \hat{k}$.

\therefore D.R.'s of the required line are $(2, 1, 1)$.

OR

Here, $\vec{a}_1 = 0\hat{i} + 0\hat{j} + 0\hat{k}$, $\vec{a}_2 = 3\hat{i} + 3\hat{j}$, $\vec{b}_1 = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{b}_2 = 2\hat{i} + \hat{j} + \hat{k}$

$$\therefore \vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j}$$

$$\text{and } \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & 1 & 1 \end{vmatrix} = 3\hat{i} - 3\hat{j} - 3\hat{k}$$

$$\text{Now, } (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (3\hat{i} + 3\hat{j}) \cdot (3\hat{i} - 3\hat{j} - 3\hat{k})$$

$$= 9 - 9 = 0$$

Hence, shortest distance between the given lines is 0.

38. i.

Corner Points	Value of $Z = 4x - 6y$
(0, 3)	$4 \times 0 - 6 \times 3 = -18$
(5, 0)	$4 \times 5 - 6 \times 0 = 20$
(6, 8)	$4 \times 6 - 6 \times 8 = -24$
(0, 8)	$4 \times 0 - 6 \times 8 = -48$

Minimum value of Z is -48 which occurs at (0, 8).

ii.

Corner Points	Value of $Z = 4x - 6y$
(0, 3)	$4 \times 0 - 6 \times 3 = -18$
(5, 0)	$4 \times 5 - 6 \times 0 = 20$
(6, 8)	$4 \times 6 - 6 \times 8 = -24$
(0, 8)	$4 \times 0 - 6 \times 8 = -48$

Maximum value of Z is 20, which occurs at (5, 0).

iii.

Corner Points	Value of $Z = 4x - 6y$
(0, 3)	$4 \times 0 - 6 \times 3 = -18$
(5, 0)	$4 \times 5 - 6 \times 0 = 20$
(6, 8)	$4 \times 6 - 6 \times 8 = -24$
(0, 8)	$4 \times 0 - 6 \times 8 = -48$

Maximum of Z - Minimum of Z = $20 - (-48) = 20 + 48 = 68$

OR

The corner points of the feasible region are O(0, 0), A(3, 0), B(3, 2), C(2, 3), D(0, 3).