Class XII Session 2025-26
Subject - Mathematics
Sample Question Paper - 7

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculators is not allowed.

Section A

1. Let A be a skew-symmetric matrix of order 3. If |A| = x, then $(2023)^x$ is equal to: **[1]**

a) $(2023)^2$ b) $\frac{1}{2023}$

c) 2023 d) 1

2. If A is a square matrix of order 3 and |A| = 6, then the value of |adj A| is: **[1]**

a) 6 b) 216

c) 27 d) 36

3. The value of $\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$ is **[1]**

a) 1 b) x + y + z

c) 0 d) 2(x + y + z)

4. If A is a square matrix such that $A^2 = I$, then A^{-1} is equal to **[1]**

a) 0 b) A + I

c) 2A d) A

5. A line making angles $45^0 and 60^0$ with the positive directions of the axis of x and y makes with the positive **[1]**

direction of Z axis , an angle of

a) 45^0

b) 60^0

c) $60^0 or 120^0$

d) 120^0

6. The Integrating Factor of the differential equation $\left(1-y^2\right)\frac{dx}{dy}+yx=ay\,(-1<y<1)$ is **[1]**

a) $\frac{1}{1-y^2}$

b) $\frac{1}{y^2-1}$

c) $\frac{1}{\sqrt{1-y^2}}$

d) $\frac{1}{\sqrt{y^2-1}}$

7. In an LPP, if the objective function z = ax + by has the same maximum value on two corner points of the feasible region, then the number of points at which z_{max} occurs is: **[1]**

a) 0

b) 2

c) infinite

d) finite

8. If $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along with three mutually perpendicular directions, then: **[1]**

a) $\hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = 1$

b) $\hat{i} \cdot \hat{k} = 0$

c) $\hat{\mathrm{i}} \times \hat{\mathrm{k}} = 0$

d) $\hat{\mathrm{i}} \times \hat{\mathrm{j}} = 1$

9. $\int 2^{x+2}$ dx is equal to: **[1]**

a) $\frac{2^{x+2}}{\log 2} + C$

b) $2^{x+2} + C$

c) $2 \cdot \frac{2^x}{\log 2} + C$

d) $2^{x+2} \log 2 + C$

10. If $\left|\frac{A^{-1}}{2}\right| = \frac{1}{k|A|}$, where A is a 3 $\times$ 3 matrix, then the value of k is: **[1]**

a) $\frac{1}{8}$

b) 2

c) 8

d) $\frac{1}{2}$

11. The region represented by the inequation system x, y $\geq$ 0, y $\leq$ 6, x + y $\leq$ 3 is **[1]**

a) bounded in second quadrant

b) unbounded in first and second quadrants

c) unbounded in first quadrant

d) bounded in first quadrant

12. The position vectors of three consecutive vertices of a parallelogram ABCD are $A(4\hat{i}+2\hat{j}-6\hat{k})$, B $(5\hat{i}-3\hat{j}+\hat{k})$ and $C(12\hat{i}+4\hat{j}+5\hat{k})$. The position vector of D is given by **[1]**

a) $-11\hat{i}-9\hat{j}+2\hat{k}$

b) $-3\hat{i}-5\hat{j}-10\hat{k}$

c) $11\hat{i}+9\hat{j}-2\hat{k}$

d) $21\hat{i}+3\hat{j}$

13. If d is the determinant of a square matrix A of order n, then the determinant of its adjoint is **[1]**

a) d^{n+1}

b) d^{n-1}

c) d^n

d) d

14. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accidents are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver? **[1]**

a) $\frac{1}{52}$

b) $\frac{3}{52}$

c) $\frac{5}{52}$

d) $\frac{1}{4}$

15. The number of arbitrary constants in the general solution of a differential equation of fourth order are: **[1]**

a) 4
b) 3
c) 1
d) 2

16. In △ABC, $\overrightarrow{AB} = \hat{i} + \hat{j} + 2\hat{k}$ and $\overrightarrow{AC} = 3\hat{i} - \hat{j} + 4\hat{k}$. If D is mid-point of BC, then vector $\overrightarrow{AD}$ is equal to: **[1]**

a) $2\hat{i} - 2\hat{j} + 2\hat{k}$
b) $\hat{i} - \hat{j} + \hat{k}$
c) $4\hat{i} + 6\hat{k}$
d) $2\hat{i} + 3\hat{k}$

17. If x = log (1 + t^2) and y = t - $\tan^{-1}$ t, then $\frac{dy}{dx}$ is equal to **[1]**

a) e^x - y
b) e^x - 1
c) $\frac{\sqrt{e^x-1}}{2}$
d) t^2 - 1

18. The cartesian equation of a line is given by $\frac{2x-1}{\sqrt{3}} = \frac{y+2}{2} = \frac{z-3}{3}$ **[1]**

The direction cosines of the line is

a) $\frac{\sqrt{3}}{\sqrt{55}}, \frac{-4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$
b) $\frac{\sqrt{3}}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$
c) $\frac{3}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$
d) $\frac{-3}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$

19. **Assertion (A):** If manufacturer can sell x items at a price of ₹$\left(5 - \frac{x}{100}\right)$ each. The cost price of x items is ₹ $\left(\frac{x}{5} + 500\right)$. Then, the number of items he should sell to earn maximum profit is 240 items. **[1]**

Reason (R): The profit for selling x items is given by $\frac{24}{5}x - \frac{x^2}{100}$ - 300.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** A function f: Z → Z defined as f(x) = x^3 is injective. **[1]**

Reason (R): A function f: A → B is said to be injective if every element of B has a pre-Image in A.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Simplify $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$, 0 < x < $\frac{1}{\sqrt{2}}$. **[2]**

OR

Write the value of $\sin^{-1}\left(\frac{1}{3}\right) - \cos^{-1}\left(-\frac{1}{3}\right)$

22. The volume of a cube is increasing at the rate of 7 cm^3 /sec. How fast is its surface area increasing at the instant when the length of an edge of the cube is 12 cm? **[2]**

23. Find the value of a for which f(x) = a (x + sin x) + a is increasing on R. **[2]**

OR

The money to be spent for the welfare of the employees of a firm is proportional to the rate of change of its total revenue (marginal revenue).

If the total revenue (in Rs.) received from the sale of x units of a product is given by R(x) = $3x^2$ + 36x + 5, then find the marginal revenue, when x = 5 and write which value does the question indicate?

24. Evaluate: $\int \frac{x\cos^{-1}x}{\sqrt{1-x^2}}dx$ **[2]**

25. Show that $f(x) = \cos(2x + \frac{\pi}{4})$ is an increasing function on $(\frac{3\pi}{8}, \frac{7\pi}{8})$ **[2]**

Section C

26. Evaluate: $\int \frac{(3\sin\theta - 2)\cos\theta}{(5-\cos^2\theta - 4\sin\theta)}d\theta$ **[3]**

27. In a set of 10 coins, 2 coins are with heads on both the sides. A coin is selected at random from this set and tossed five times. If all the five times, the result was heads, find the probability that the selected coin had heads on both the sides. **[3]**

28. Evaluate $\int_{-1}^{2} \left|x^3 - x\right| dx$. **[3]**

OR

Evaluate: $\int \frac{\sin 2x}{(1-\cos 2x)(2-\cos 2x)}dx$.

29. Solve the differential equation: y (1 - x^2) $\frac{dy}{dx}$ = x (1 + y^2) **[3]**

OR

Find the particular solution of the differential equation (1 + e^{2x})dy + (1 + y^2)e^x dx = 0, given that y = 1, when x = 0.

30. Minimise Z = 13x - 15y subject to the constraints: $x + y \leqslant 7, 2x - 3y + 6 \geqslant 0, x \geqslant 0\ y \geqslant 0$. **[3]**

OR

Solve graphically the following linear programming problem:

Maximise z = 6x + 3y,

Subject to the constraints:

4x + y $\geq$ 80,

3x + 2y $\leq$ 150,

x + 5y $\geq$115,

x > 0, y $\geq$ 0.

31. Find $\frac{dy}{dx}$ when y = $x^{\log x}$ + $(\log x)^x$ **[3]**

Section D

32. Using integration, find the area of the region bounded by the line x - y + 2 = 0, the curve x = $\sqrt{y}$ and Y-axis. **[5]**

33. Each of the following defines a relation on N: **[5]**

i. x is greater than y, x, y $\in$ N
ii. x + y = 10, x, y $\in$ N
iii. xy is square of an integer x, y $\in$ N
iv. x + 4y = 10x, y $\in$ N.

Determine which of the above relations are reflexive, symmetric and transitive.

OR

Show that the function f : R - {3} $\rightarrow$ R - {1} given by $f(x) = \frac{x-2}{x-3}$ is a bijection.

34. If A = $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ find A^{-1}, using A^{-1} solve the system of equations **[5]**

2x - 3y + 5z = 11

3x + 2y - 4z = -5

x + y - 2z = -3

35. Find the shortest distance between the lines l_1 and l_2 whose vector equations are **[5]**

$\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$ ...(1)

and $\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$ ...(2)

OR

Find the image of the point with position vector $3\hat{i}+\hat{j}+2\hat{k}$ in the plan $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})=4$ Also, find the position vectors of the foot of the perpendicular and the equation of the perpendicular line through $3\hat{i}+\hat{j}+2\hat{k}$.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

In a bilateral cricket series between India and South Africa, the probability that India wins the first match is 0.6. If India wins any match, then the probability that it wins the next match is 0.4, otherwise, the probability is 0.3. Also, it is given that there is no tie in any match.

i. Find the probability that India won the second match, if India has already loose the first match. (1)

ii. Find the probability that India losing the third match, if India has already lost the first two matches. (1)

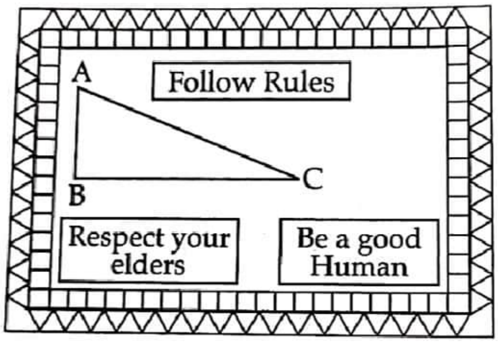
iii. Find the probability that India losing the first two matches. (2)

OR

Find the probability that India winning the first three matches. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

The slogans on chart papers are to be placed on a school bulletin board at the points A, B and C displaying A (follow Rules), B (Respect your elders) and C (Be a good human). The coordinates of these points are (1, 4, 2), (3, -3, -2) and (-2, 2, 6), respectively.



i. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ be the position vectors of points A, B, C, respectively, then find $|\vec{a}+\vec{b}+\vec{c}|$. (1)

ii. If $\vec{a}=4\hat{i}+6\hat{j}+12\hat{k}$, then find the unit vector in direction of $\vec{a}$. (1)

iii. Find area of $\triangle$ABC. (2)

OR

Write the triangle law of addition for $\triangle$ABC. Suppose, if the given slogans are to be placed on a straight line, then the value of $|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|$. (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

The temperature of a person during an intestinal illness is given by $f(x) = -0.1x^2 + mx + 98.6$, $0 \leq x < 12$, m being a constant, where f(x) is the temperature in ^{o}F at x days.

i. Is the function differentiable in the interval (0, 12)? Justify your answer. (1)

ii. If 6 is the critical point of the function, then find the value of the constant m. (1)

iii. Find the intervals in which the function is strictly increasing/strictly decreasing. (2)

OR

Find the points of local maximum/local minimum, if any, in the interval (0, 12) as well as the points of absolute maximum/absolute minimum in the interval [0, 12]. Also, find the corresponding local minimum and the absolute maximum/absolute minimum values of the function. (2)

Solution

Section A

1.

(d) 1

Explanation:

1

2.

(d) 36

Explanation:

36

|adj A|= $|A|^{n-1}$

$=(6)^{3-1}$

= 36

3.

(c) 0

Explanation:

0

4.

(d) A

Explanation:

$A^2 = I$

$A^{-1}A^2 = A^{-1}I$

$A = A^{-1}$

5.

(c) $60^0 or 120^0$

Explanation:

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\alpha = 45, \beta = 60$

put the values in above equation

$(1/\sqrt{2})^2 + (1/2)^2 + \cos^2\gamma = 1$

$\cos\gamma = \pm 1/2$

6.

(c) $\frac{1}{\sqrt{1-y^2}}$

Explanation:

It is given that $\left(1-y^2\right)\frac{dx}{dy} + yx = ay$

$\Rightarrow \frac{dx}{dy} + \frac{yx}{1-y^2} = \frac{ay}{1-y^2}$

This is equation in the form of $\frac{dx}{dy} + px = Q$ (where $p = \frac{y}{1-y^2}$ and Q = $\frac{ay}{1-y^2}$)

Now, I.F. = $e^{\int p dy} = e^{\int \frac{y}{1-y^2} dy} = e^{-\frac{1}{2}\log(1-y^2)} = e^{\log\left[\frac{1}{\sqrt{(1-y^2)}}\right]}$

$= \frac{1}{\sqrt{(1-y^2)}}$

7.

(c) infinite

Explanation:

In a LPP, if the objective f^n Z = ax + by has the maximum value on two corner point of the feasible region then every point on the line segment joining these two points gives the same maximum value.

hence, Z_{max} occurs at infinite no of times.

8.

(b) $\hat{i} \cdot \hat{k} = 0$

Explanation:

Dot product of the mutually perpendicular vector is always zero.

here $\hat{i}$ and $\hat{k}$ are perpendicular to each other

hence, $\hat{i} \cdot \hat{k} = 0$

9. **(a)** $\frac{2^{x+2}}{\log 2} + C$

Explanation:

$\frac{2^{x+2}}{\log 2} + C$

10.

(c) 8

Explanation:

8

11.

(d) bounded in first quadrant

Explanation:

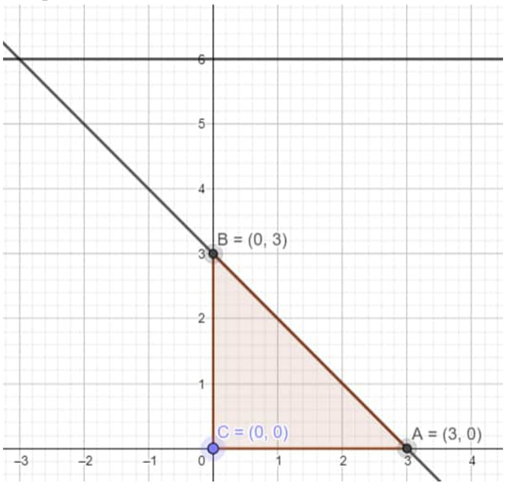
Converting the given inequations into equations, we obtain

y = 6, x + y = 3, x = 0 and y = 0, y = 6 is the line passing through (0, 6) and parallel to the X axis.The region below the line y = 6 will satisfy the given inequation.

The line x + y = 3 meets the coordinate axis at A(3, 0) and B(0, 3). Join these points to obtain the line x + y = 3 Clearly, (0, 0) satisfies the inequation x + y ≤ 3 . So, the region in x y -plane that contains the origin represents the solution set of the given equation.

The region represented by x $\geq$ 0 and y $\geq$ 0 :

Since every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations.



12.

(c) $11\hat{i} + 9\hat{j} - 2\hat{k}$

Explanation:

$11\hat{i} + 9\hat{j} - 2\hat{k}$

13.

(b) d^{n-1}

Explanation:

$|A| = d$

$|adjA| = |A|^{n - 1}$

$|adjA| = d^{n - 1}$

14. **(a)** $\frac{1}{52}$

Explanation:

Let, E_1, E_2 and E_3 are events of selection of a scooter driver, car driver and truck driver respectively.

$\therefore P(E_1) = \frac{2000}{12000} = \frac{1}{6}$, $P(E_2) = \frac{4000}{12000} = \frac{1}{3}$,$P(E_3) = \frac{6000}{12000} = \frac{1}{2}$.

Let A = event that the insured person meet with the accident.

$\therefore$ $P(A|E_1) = 0.01$, $P(A|E_2) = 0.03$, $P(A|E_3) = 0.15$

$\Rightarrow P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)+P(E_3)P(A|E_3)}$

$= \frac{\frac{1}{6}\times 0.01}{\frac{1}{6}\times 0.01+\frac{1}{3}\times 0.03+\frac{1}{2}\times 0.15} = \frac{1}{52}$

15. **(a)** 4

Explanation:

4, because the no. of arbitrary constants is equal to order of the differential equation.

16.

(d) $2\hat{i} + 3\hat{k}$

Explanation:

$2\hat{i} + 3\hat{k}$

17.

(c) $\frac{\sqrt{e^x - 1}}{2}$

Explanation:

Given, $x = \log (1 + t^2)$ and $y = t - \tan^{-1} t$

On differentiating both sides w.r.t.x, we get

$\frac{dx}{dt} = \frac{1}{1+t^2}(2t)$ and $\frac{dy}{dt}$ = 1 - $\frac{1}{1+t^2} = \frac{t^2}{1+t^2}$

$\therefore \frac{dy}{dx} = \frac{\frac{t^2}{\left(1+t^2\right)}}{\frac{2t}{\left(1+t^2\right)}} = \frac{t}{2}$ ...(i)

Also, x = log (1 + t^2)

$\Rightarrow$ t^2 = e^x-1 ...(ii)

From Eqs. (i) and (ii), we get

$\frac{dy}{dx} = \frac{\sqrt{e^x-1}}{2}$

18.

(b) $\frac{\sqrt{3}}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$

Explanation:

Rewrite the given line as

$r\frac{2\left(x-\frac{1}{2}\right)}{\sqrt{3}} = \frac{y+2}{2} = \frac{z-3}{3}$

or $\frac{x-\frac{1}{2}}{\sqrt{3}} = \frac{y+2}{4} = \frac{z-3}{6}$

$\therefore$ DR's of line are $\sqrt{3}$, 4 and 6

Therefore, direction cosines are:

$\frac{\sqrt{3}}{\sqrt{(\sqrt{3})^2+4^2+6^2}}, \frac{4}{\sqrt{(\sqrt{3})^2+4^2+6^2}}, \frac{6}{\sqrt{(\sqrt{3})^2+4^2+6^2}}$ or $\frac{\sqrt{3}}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$

19.

(c) A is true but R is false.

Explanation:

Let S(x) be the selling price of x items and let C(x) be the cost price of x items.

Then, we have

S(x) = (5 - $\frac{x}{100}$)x = 5x - $\frac{x^2}{100}$

and C(x) = $\frac{x}{5}$ + 500

Thus, the profit function P(x) is given by

P(x) = S(x) - C(x) = 5x - $\frac{x^2}{100} - \frac{x}{5}$ - 500

i.e. P(x) = $\frac{24}{5}x - \frac{x^2}{100}$ - 500

On differentiating both sides w.r.t. x, we get

P'(x) = $\frac{24}{5} - \frac{x}{50}$

Now, P'(x) = 0 gives x = 240.

Also, P'(x) = $\frac{-1}{50}$.

So, P'(240) = $\frac{-1}{50}$ < 0

Thus, x = 240 is a point of maxima.

Hence, the manufacturer can earn maximum profit, if he sells 240 items.

20.

(c) A is true but R is false.

Explanation:

Assertion is true because distinct elements in Z (domain) has distinct images in Z (codomain).

Reason is false because of: A $\rightarrow$ B is said to be surjective if every element of B has at least one pre-Image in A.

Section B

21. Let x = cos θ

$\therefore \theta = \cos^{-1}$ x

$\sec^{-1}\left(\frac{1}{2x^2-1}\right) = \sec^{-1}\left(\frac{1}{2\cos^2\theta-1}\right) = \sec^{-1}\left(\frac{1}{\cos 2\theta}\right) = \sec^{-1}(\sec 2\theta)$

$= 2\theta$

= 2 $\cos^{-1}$ x

OR

Given $\sin^{-1}\left(\frac{1}{3}\right) - \cos^{-1}\left(-\frac{1}{3}\right)$

We know that $\cos^{-1}(-\theta) = \pi - \cos^{-1}\theta$

$= \sin^{-1}\left(\frac{1}{3}\right) - \left[\pi - \cos^{-1}\left(\frac{1}{3}\right)\right]$

$= \sin^{-1}\left(\frac{1}{3}\right) - \pi + \cos^{-1}\left(\frac{1}{3}\right)$

$= \sin^{-1}\left(\frac{1}{3}\right) + \cos^{-1}\left(\frac{1}{3}\right) - \pi$

$= \frac{\pi}{2} - \pi$

$= -\frac{\pi}{2}$

Therefore we have,

$\sin^{-1}\left(\frac{1}{3}\right) - \cos^{-1}\left(-\frac{1}{3}\right) = -\frac{\pi}{2}$

22. At any instant t, let the length of each edge of the cube be x, V be its volume and S be its surface area. Then,

$\frac{dV}{dt}$ = 7 cm^3 / sec ... (given) ... (i)

Now, V = x^3 $\Rightarrow \frac{dV}{dt} = \frac{dV}{dx} \cdot \frac{dx}{dt}$

$\Rightarrow 7 = \frac{d}{dx}\left(x^3\right) \cdot \frac{dx}{dt}$ .... [$\because V = x^3$]

$\Rightarrow 3x^2 \cdot \frac{dx}{dt} = 7$

$\Rightarrow \frac{dx}{dt} = \frac{7}{3x^2}$

$\therefore S = 6x^2 \Rightarrow \frac{dS}{dt} = \frac{dS}{dx} \cdot \frac{dx}{dt}$

$= \frac{d}{dx}\left(6x^2\right) \cdot \frac{7}{3x^2}$

$= \left(12x \times \frac{7}{3x^2}\right) = \frac{28}{x}$

$\Rightarrow \left[\frac{dS}{dt}\right]_{x=12} = \left(\frac{28}{12}\right)$ cm^2 /sec = $2\frac{1}{3}\ cm^2/sec$

Hence, the surface area of the cube is increasing at the rate of $2\frac{1}{3}\ cm^2/sec$ at the instant when its edge is 12 cm.

23. Given: $f(x) = a(x + \sin x) + a$

$f'(x) = a(1 + \cos x)$

for (x) to be increasing we must have

$f'(x) > 0$

$\Rightarrow a(1 + \cos x) > 0 \ldots (1)$

We know

$-1 \leq \cos x \leq 1,\ \forall\ x \in R$

$\Rightarrow 0 \leq (1 + \cos x) \leq 2, \forall\ x \in R$

$\Rightarrow a \in (0, \infty) \Rightarrow a \geq 0$ f(x) is increasing on R.

OR

By definition of marginal revenue we have,

marginal revenue $(MR) = \frac{dR}{dx}$

$= \frac{d}{dx}\left(3x^2 + 36x + 5\right) = 6x + 36$

When x = 5, then

MR = 6(5) + 36 = 30 + 36 = 66

Hence, the required marginal revenue is Rs. 66.

More amount of money spent for the welfare of the employees with the increase in marginal revenue show affinity and care for employees.

24. Let I = $\int \frac{x \cos^{-1} x}{\sqrt{1-x^2}} dx$

Put, t = $\cos^{-1}$ x

$\Rightarrow dt = \frac{-1}{\sqrt{1-x^2}} dx$

Also, cos t = x

Thus, we have

I = - $\int$ t cot dt

Now let us solve this by the 'by parts' method.

I = - [t sint - $\int$ sin t dt]

$\Rightarrow$ I = -[t sin t - cos t] + C

Substituting the value t = $\cos^{-1}$ x, we get

I = -[$\cos^{-1}$ x sin t + x] + c

$\Rightarrow$ I = -[$\cos^{-1}$ x $\sqrt{1-x^2}$ + x] + c

25. Given: $f(x) = \cos(2x + \frac{\pi}{4})$

$f'(x) = -2\sin(2x + \frac{\pi}{4})$

Now,

$x \in (\frac{3\pi}{8}, \frac{7\pi}{8})$

$\Rightarrow \quad \frac{3\pi}{8} < x < \frac{7\pi}{8}$

$\Rightarrow \quad \frac{3\pi}{4} < 2x < \frac{7\pi}{4}$

$\Rightarrow \quad \frac{\pi}{4} + \frac{3\pi}{4} < 2x + \frac{\pi}{4} < \frac{7\pi}{4} + \frac{\pi}{4}$

$\Rightarrow \quad \pi < 2x + \frac{\pi}{4} < 2\pi$

$\Rightarrow \quad \sin(2x + \frac{\pi}{4}) < 0$

$\Rightarrow \quad -2\sin(2x + \frac{\pi}{4}) > 0$

$\Rightarrow \quad f'(x) > 0$

Hence, f(x) is increasing on $(\frac{3\pi}{8}, \frac{7\pi}{8})$

Section C

26. According to the question, $I = \int \frac{(3\sin\theta - 2)\cos\theta}{5 - \cos^2\theta - 4\sin\theta} d\theta$

$= \int \frac{(3\sin\theta - 2)\cos\theta}{5 - \left(1 - \sin^{2\theta}\right) - 4\sin\theta} d\theta$

Put sin$\theta = t \Rightarrow \cos\theta d\theta = dt$

$\therefore \quad I = \int \frac{3t-2}{5-(1-t^2)-4t} dt$

$= \int \frac{3t-2}{4+t^2-4t} dt = \int \frac{3t-2}{(t-2)^2} dt$

$= \int \frac{3t-6+4}{(t-2)^2} dt = \int \frac{3(t-2)+4}{(t-2)^2} dt$

$= \int \frac{3}{(t-2)} dt + \int \frac{4}{(t-2)^2} dt$

$= 3\log|t-2| + \frac{4(t-2)^{-2+1}}{-2+1} + C$

$\left[\because \int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1\right]$

$= 3\log|t-2| - \frac{4}{(t-2)} + C$

$= 3\log|\sin\theta - 2| - \frac{4}{(\sin\theta - 2)} + C$

$[Put\ t = \sin\theta]$

27. Let E_1 and E_2 be the events that two headed coin and unbiased coin is chosen respectively. Also let E be the event that all 5 tosses are heads.

Therefore,we have,

$P(E_1) = \frac{2}{10} = \frac{1}{5}$, $P(E_2) = \frac{4}{5}$

$P(E|E_1) = 1$, $P(E|E_2) = {}^5C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^{5-5} = \frac{1}{32}$

By using Bayes' theorem

$P(E_1|E) = \frac{P(E|E_1)P(E_1)}{P(E|E_1)P(E_1)+P(E|E_2)P(E_2)}$

$\Rightarrow P(E_1|E) = \frac{1\times\frac{1}{5}}{1\times\frac{1}{5}+\frac{1}{32}\times\frac{4}{5}} = \frac{32}{36} = \frac{8}{9}$

This is the required probability.

28. According to the question , $I = \int_{-1}^{2}\left|x^3 - x\right| dx$

We can observe that,

$$\left|x^3 - x\right| = \begin{cases} \left(x^3 - x\right), \text{ when } -1 < x < 0 \\ -\left(x^3 - x\right), \text{ when } 0 \leq x < 1 \\ \left(x^3 - x\right), \text{ when } 1 \leq x < 2 \end{cases}$$

By Splitting the intervals , we get

$I = \int_{-1}^{0} |x^3 - x|\, dx + \int_{0}^{1} |x^3 - x|\, dx + \int_{1}^{2} |x^3 - x|\, dx$

$\therefore I = \int_{-1}^{0} \left(x^3 - x\right) dx + \int_{0}^{1} - \left(x^3 - x\right) dx + \int_{1}^{2} \left(x^3 - x\right) dx$

$= \int_{-1}^{0} \left(x^3 - x\right) dx - \int_{0}^{1} \left(x^3 - x\right) dx + \int_{1}^{2} \left(x^3 - x\right) dx$

$= \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} - \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{0}^{1} + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{1}^{2}$

$= \left[0 - \left(\frac{1}{4} - \frac{1}{2}\right)\right] - \left[\left(\frac{1}{4} - \frac{1}{2}\right) - 0\right] + \left[\left(\frac{16}{4} - \frac{4}{2}\right) - \left(\frac{1}{4} - \frac{1}{2}\right)\right]$

$= -\frac{1}{4} + \frac{1}{2} - \frac{1}{4} + \frac{1}{2} + 4 - 2 - \frac{1}{4} + \frac{1}{2}$

$= -\frac{3}{4} + \frac{3}{2} + 2$

$= \frac{-3+6+8}{4}$

$= \frac{11}{4}$

$\therefore I = \frac{11}{4}$ sq units.

OR

Let, $I = \int \frac{\sin 2x}{(1-\cos 2x)(2-\cos 2x)} dx$

Put t = cos2x

dt = -2sin2xdx

$I = \int \frac{-dt/2}{(1-t)(2-t)} = \frac{1}{2} \int \frac{dt}{(t-2)(1-t)}$

Using partial fractions,

$\frac{1}{(t-2)(1-t)} = \frac{A}{t-2} + \frac{B}{1-t}$ .... (1)

A(1 - t) + B(t - 2) = 1

Putting 1 - t = 0

t = 1

A(0) + B(1 - 2) = 1

B = -1

Putting t - 2 = 0

t = 2

A(1 - 2) + B(0) = 1

A = -1

From equation (1), we get,

$\frac{1}{(t-2)(1-t)} = \frac{-1}{t-2} + \frac{-1}{1-t}$

$\int \frac{1}{(t-2)(1-t)} dt = \int \frac{1}{2-t} dt + \int \frac{1}{t-1} dt$

$= -\log|2 - t| + \log|t - 1| + c$

$= \log|t - 1| - \log|2 - t| + c$

$= \log|\cos 2x - 1| - \log|2 - \cos 2x| + c$

29. The given differential equation is,

y (1 - x^2) $\frac{dy}{dx}$ = x (1 + y^2)

$\Rightarrow \frac{y}{1+y^2} dy = \frac{x}{1-x^2} dx$

Integrating both sides,

$\int \frac{y}{1+y^2} dy = \int \frac{x}{1-x^2} dx$

Substituting 1 + y^2 = t and 1 - x^2 = u

2y dy = dt and -2x dx = du

$\therefore \frac{1}{2} \int \frac{1}{t} dt = \frac{-1}{2} \int \frac{1}{u} du$

$\Rightarrow \frac{1}{2}$ log |t| = $-\frac{1}{2}$ log |u| + log C

$\Rightarrow \frac{1}{2}$ log |1 + y^2| = $-\frac{1}{2}$ log |1 - x^2| + log C

$\Rightarrow \frac{1}{2}$ log (|1 + y^2| |1 - x^2|) = 2 log C

$\Rightarrow$ (1 + y^2) (1 - x^2) = C^2

$\Rightarrow$ (1 + y^2) (1 - x^2) = C_1 ...(where $C_1 = C^2$)

Hence, (1 + y^2) (1 - x^2) = C_1 is the required solution.

OR

Given differential equation is,

$(1 + e^{2x})dy + (1 + y^2)e^x\, dx = 0$

Above equation may be written as

$\frac{dy}{1+y^2} = \frac{-e^x}{1+e^{2x}} dx$

On integrating both sides, we get

$\int \frac{dy}{1+y^2} = -\int \frac{e^x}{1+e^{2x}} dx$

On putting $e^x = t \Rightarrow e^x\, dx = dt$ in RHS, we get

$\tan^{-1} y = -\int \frac{1}{1+t^2} dt$

$\Rightarrow \quad \tan^{-1} y = -\tan^{-1} t + C$

$\Rightarrow \quad \tan^{-1} y = -\tan^{-1}(e^x) + C$ ...(i) [put $t = e^x$]

Also, given that y = 1, when x = 0.

On putting above values in Eq. (i), we get

$\tan^{-1}1 = -\tan^{-1}(e^0) + C$

$\Rightarrow \quad \tan^{-1} 1 = -\tan^{-1} 1 + C \quad [\because e^0 = 1]$

$\Rightarrow \quad 2\tan^{-1} 1 = C$

$\Rightarrow \quad 2\tan^{-1}\left(\tan \frac{\pi}{4}\right) = C$

$\Rightarrow \quad C = 2 \times \frac{\pi}{4} = \frac{\pi}{2}$

On putting $C = \frac{\pi}{2}$ in Eq. (i), we get

$\tan^{-1} y = -\tan^{-1} e^x + \frac{\pi}{2}$

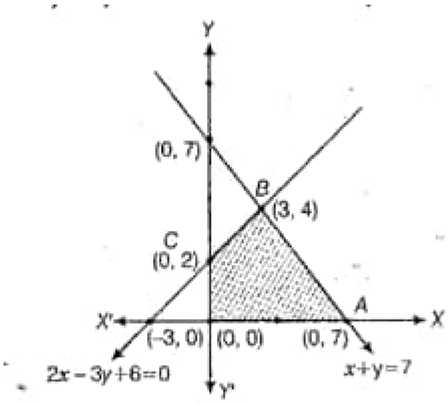
$\Rightarrow \quad y = \tan\left[\frac{\pi}{2} - \tan^{-1}(e^x)\right] = \cot\left[\tan^{-1}(e^x)\right]$

$= \cot\left[\cot^{-1}\left(\frac{1}{e^x}\right)\right] \left[\because \tan^{-1} x = \cot^{-1} \frac{1}{x}\right]$

$\therefore \quad y = \frac{1}{e^x}$

which is the required solution.

30. Minimise Z = 13x - 15y subject to the constraints: $x + y \leqslant 7, 2x - 3y + 6 \geqslant 0, x \geqslant 0, y \geqslant 0$.



replace (0,7) by (7,0) in horizontal line

Shaded region shown as OABC is bounded and coordinates of its corner points are (0, 0), (7, 0), (3, 4) and (0, 2), respectively.

Corner Points	**Corresponding value of Z**
(0, 0)	0
(7, 0)	91
(3, 4)	-21
(0, 2)	-30 (Minimum)

Hence, the minimum value of Z is -30 at (0,2).

OR

Subject to the constraints are

$4x + y \geq 80$

$x + 5y \geq 115$

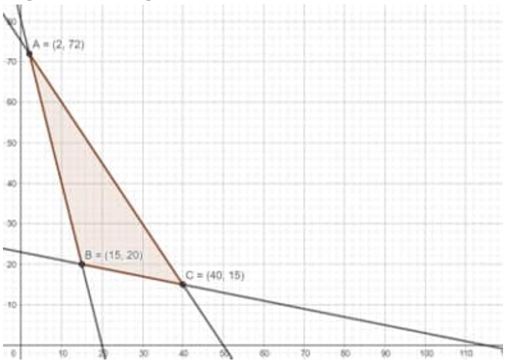
$3x + 2y \leq 150$

and the non negative constraint x,y ≥ 0

Converting the given inequations into equations, we get 4x + y = 80, x + 5y = 115, 3x + 2y = 150, x = 0 and y = 0

These lines are drawn on the graph and the shaded region ABC represents the feasible

region of the given LPP.



It can be observed that the feasible region is bounded. The coordinates of the corner

points of the feasible region are A(2, 72), B(15, 20) and C(40, 15) The values of the objective function, Z at these corner points are given in the following table:

Corner Point Value of the Objective Function Z = 6x + 3y

A(2, 72) : $Z = 6 \times 2 + 3 \times 72 = 228$

B(15, 20) : $Z = 6 \times 15 + 3 \times 20 = 150$

C(40, 15) : $Z = 6 \times 40 + 3 \times 15 = 285$

From the table, Z is minimum at x = 15 and y = 20 and the minimum value of Z is 150. Thus, the minimum value of Z is 150.

31. Let y = $x^{\log x} + (\log x)^x$

Also, let u = $(\log x)^x$ and v = $x^{\log x}$

$\therefore$ y = v + u

$\Rightarrow \frac{dy}{dx} = \frac{dv}{dx} + \frac{du}{dx}$ ...(i)

Now, u = $(\log x)^x$

$\Rightarrow$ log u = $\log[(\log x)^x]$

$\Rightarrow$ log u = x log(log x)

Differentiating both sides with respect to x,

$\frac{1}{u}\frac{du}{dx} = \log(\log x)\frac{d}{dx}(x) + x\frac{d}{dx}[\log(\log x)]$

$\Rightarrow \frac{du}{dx} = u\left[\log(\log x) + x\frac{1}{\log x}\frac{d}{dx}(\log x)\right]$

$\Rightarrow \frac{du}{dx} = (\log x)^x\left[\log(\log x) + \frac{x}{\log x} \times \frac{1}{x}\right]$

$\Rightarrow \frac{du}{dx} = (\log x)^x\left[\log(\log x) + \frac{1}{\log x}\right]$ ...(ii)

Also, v = $x^{\log x}$

$\Rightarrow$ log v = log $x^{\log x}$

$\Rightarrow$ log v = log x log x = $(\log x)^2$

Differentiating both sides with respect to x,

$\frac{1}{v}\frac{dv}{dx} = \frac{d}{dx}\left[(\log x)^2\right]$

$\Rightarrow \frac{1}{v}\frac{dv}{dx} = 2(\log x)\frac{d}{dx}(\log x)$

$\Rightarrow \frac{dv}{dx} = 2v(\log x)\frac{1}{x}$

$\Rightarrow \frac{dv}{dx} = 2x^{\log x}\frac{\log x}{x}$

$\Rightarrow \frac{dv}{dx} = 2x^{\log x}\frac{\log x}{x}$ ....(iii)

From (i), (ii) and (iii), we obtain

$\frac{dy}{dx} = 2x^{\log x}\frac{\log x}{x} + (\log x)^x\left[\log(\log x) + \frac{1}{\log x}\right]$

Section D

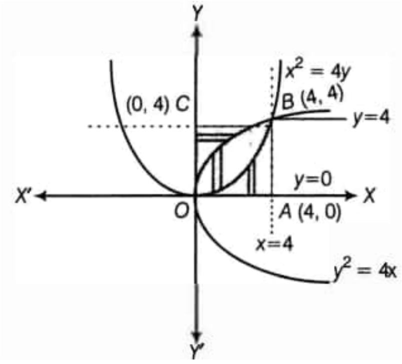
32. According to the question ,

Given curves are

x - y + 2 = 0....(i)

x = $\sqrt{y}$...(ii)

Consider x = $\sqrt{y}$ ⇒x^2 = y, which represents the parabola
vertex of parabola is (0, 0)
axis of parabola is Y-axis.
Now, the point of intersection of Eqs.(i) and (ii) is given by
x = $\sqrt{x+2}$
Squaring on both sides ,
⇒x^2 = x + 2
⇒x^2 - x - 2 = 0
⇒(x - 2)(x + 1) = 0
⇒x = -1, 2
When x = -1 , does not satisfy the Eq. (ii).
When x = 2 , then 2 = $\sqrt{y} \Rightarrow$ y = 4
Hence, the point of intersection is (2, 4).
But actual equation of given parabola is x = $\sqrt{y}$, it means a semi-parabola which is on right side of Y -axis.
The graph of given curves are shown below:



Clearly, area of bounded region = Area of region OABO
$= \int_0^2 \left[y_{(\text{ line })} - y_{(\text{ parabola })}\right] dx$
$= \int_0^2 (x+2)dx - \int_0^2 x^2 dx$
$= \left[\frac{x^2}{2} + 2x\right]_0^2 - \left[\frac{x^3}{3}\right]_0^2$
= $\left[\frac{4}{2} + 4 - 0\right] - \left[\frac{8}{3} - 0\right]$
$= 6 - \frac{8}{3}$
$= \frac{18-8}{3}$
$= \frac{10}{3}$ sq.units.

33. i. x is greater than y, x, y ∈ N
For xRx x > x is not true for any x ∈ N.
Therefore, R is not reflexive.
Let (x, y) ∈ R ⇒ xRy
x > y
but y > x is not true for any x, y ∈ N
Thus, R is not symmetric.
Let xRy and yRz
x > y and y > z ⇒ x > z
⇒ xRz
So, R is transitive.

ii. x + y = 10, x, y ∈ N
R={(x,y) : x+y = 10 ,x,y ∈ R}
R = {(1, 9), (2, 8), (3, 7), (4, 6), (5, 5), (6, 4), (7, 3), (8, 2), (9, 1)} (1, 1) ∉ R
So, R is not reflexive.
(x, y) ∈ R ⇒ (y, x) ∈ R
Therefore, R is symmetric.

(1, 9) ∈ R, (9, 1) ∈ R ⇒ (1, 1) ∉ R
Hence, R is not transitive.

iii. Given xy, is square of an integer x, y ∈ N
⇒ R = {(x, y) : xy is a square of an integer x, y ∈ N}
(x, x) ∈ R, ∀x ∈ N
As x^2 is square of an integer for any x ∈ N
Hence, R is reflexive.
If (x, y) ∈ R ⇒ (y, x) ∈ R
Therefore, R is symmetric.
If (x, y) ∈ R (y, z) ∈ R
So, xy is square of an integer and yz is square of an integer.
Let xy = m^2 and yz = n^2 for some m, n ∈ Z
$x = \frac{m^2}{y}$ and $z = \frac{x^2}{y}$
$xz = \frac{m^2n^2}{y^2}$, Which is square of an integer.
So, R is transitive.

iv. x + 4y = 10, x, y ∈ N
R = {(x, y) : x + 4y = 10, x, y ∈ N}
R{(2, 2), (6, 1)}
(1, 1), (3, 3) ..... ∉ R
Thus, R is not reflexive.
(6, 1) ∈ R but (1, 6) ∉ R
Hence, R is not symmetric.
(x, y) ∈ R ⇒ x + 4y = 10 but (y, z) ∈ R
y + 4z = 10 ⇒ (x, z) ∈ R
So, R is transitive.

OR

A = R - {3}, B = R - {1}
f : A → B is defined as $f(x) = \left(\frac{x-2}{x-3}\right)$.
Let x, y ∈ A such that f(x) = f(y).
⇒ $\frac{x-2}{x-3} = \frac{y-2}{y-3}$
⇒ (x - 2) (y - 3) = (y - 2) (x - 3)
⇒ xy - 3x - 2y + 6 = xy - 3y - 2x + 6
⇒ -3x - 2y = -3y - 2x
⇒ 3x - 2x = 3y - 2y
⇒ x = y
Therefore, f is one-one.
Let y ∈ B = R - {1}
Then, y ≠ 1.
The function f is onto if there exists x ∈ A such that f(x) = y.
Now, f (x) = y
⇒ $\frac{x-2}{x-3} = y$
⇒ x - 2 = xy - 3y
⇒ x(1 - y) = -3y + 2
⇒ $x = \frac{2-3y}{1-y} \in \mathbf{A} \qquad [y \neq 1]$
Thus, for any y ∈ B, there exists $\frac{2-3y}{1-y} \in \mathrm{A}$ such that
$$f\left(\frac{2-3y}{1-y}\right) = \frac{\left(\frac{2-3y}{1-y}\right)-2}{\left(\frac{2-3y}{1-y}\right)-3} = \frac{2-3y-2+2y}{2-3y-3+3y} = \frac{-y}{-1} = y$$
∴ f is onto.
Hence, function f is one-one and onto.

34. $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

$\therefore$ |A| = 2(-4 + 4) + 3(-6 +4)+ 5(3 - 2) = 0 -6 +5 = -1

Now, A_{11} = 0, A_{12}= 2, A_{13}= 1

A_{21}= -1, A_{22} = -9, A_{23}= -5

A_{31}= 2, A_{32}= 23,A_{33} = 13

$\therefore A^{-1} = \frac{1}{|A|}(adjA) = -\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$ ....(1)

Now, the given system of equations can be written in the form of AX = B, where

$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

The solution of the system of equations is given by X = A^{-1}B

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$ [Using (1)]

$= \begin{bmatrix} 0-5+6 \\ -22-45+69 \\ -11-25+39 \end{bmatrix}$

$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Hence, x = 1, y = 2 and z = 3.

35. $\vec{a}_1 = \hat{i} + \hat{j}, \vec{b}_1 = 2\hat{i} - \hat{j} + \hat{k}$

$\vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k}, \vec{b}_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$

$\vec{a}_2 - \vec{a}_1 = \hat{i} - \hat{k}$

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$

$= \hat{i}(-2+5) - \hat{j}(4-3) + \hat{k}(-10+3)$

$= 3\hat{i} - \hat{j} - 7\hat{k}$

$\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{9+1+49} = \sqrt{59}$

Also, $(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1) = (3\hat{i} - \hat{j} - 7\hat{k})(\hat{i} - \hat{k}) = 3 + 7 + 0 = 10$

$d = \left|\frac{(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|}\right| = \frac{10}{\sqrt{59}}$

OR

Let P be the point with position vector $\vec{p} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$ and M be the image of P in the plane $\vec{r} \cdot (2\hat{\imath} - \hat{\jmath} + \hat{k}) = 4$.

In addition, let Q be the foot of the perpendicular from P on to the given plane. So, Q is the midpoint of PM.

Direction ratios of PM are proportional to 2, –1, 1 as PM is normal to the plane and parallel to $2\hat{\imath} - \hat{\jmath} + \hat{k}$.

Recall the vector equation of the line passing through the point with position vector $\vec{r}$ and parallel to vector $\vec{b}$ is given by $\vec{r} = \vec{a} + \lambda\vec{b}$

Here, $\vec{a} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$

Hence, the equation of PM is

$\vec{r} = (3\hat{i} + \hat{j} + 2\hat{k}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$

$\therefore \vec{r} = (3+2\lambda)\hat{i} + (1-\lambda)\hat{j} + (2+\lambda)\hat{k}$

Let the position vector of M be $\vec{m}$. As M is a point on this line, for some scalar α, we have

$\Rightarrow \vec{m} = (3+2\alpha)\hat{i} + (1-\alpha)\hat{j} + (2+\alpha)\hat{k}$

Now, let us find the position vector of Q, the midpoint of PM.

Let this be $\vec{q}$.

Using the midpoint formula, we have

$\overrightarrow{q}=\frac{\overrightarrow{p}+\overrightarrow{m}}{2}$

$\Rightarrow \overrightarrow{q}=\frac{[3\hat{i}+\hat{j}+2\hat{k}]+[(3+2\alpha)\hat{i}+(1-\alpha)\hat{j}+(2+\alpha)\hat{k}]}{2}$

$\Rightarrow \overrightarrow{q}=\frac{(3+(3+2\alpha))\hat{i}+(1+(1-\alpha))\hat{j}+(2+(2+\alpha))\hat{k}}{2}$

$\Rightarrow \overrightarrow{q}=\frac{(6+2\alpha)\hat{i}+(2-\alpha)\hat{j}+(4+\alpha)\hat{k}}{2}$

$\therefore \overrightarrow{q}=(3+\alpha)\hat{i}+\frac{(2-\alpha)}{2}\hat{j}+\frac{(4+\alpha)}{2}\hat{k}$

This point lies on the given plane, which means this point satisfies the plane equation $\overrightarrow{r}\cdot(2\hat{\imath}-\hat{\jmath}+\hat{k})=4$.

$\Rightarrow \left[(3+\alpha)\hat{i}+\frac{(2-\alpha)}{2}\hat{j}+\frac{(4+\alpha)}{2}\hat{k}\right]\cdot(2\hat{\imath}-\hat{j}+\hat{k})=4$

$\Rightarrow 2(3+\alpha)-\left(\frac{2-\alpha}{2}\right)(1)+\left(\frac{4+\alpha}{2}\right)(1)=4$

$\Rightarrow 6+2\alpha+\frac{4+\alpha-(2-\alpha)}{2}=4$

$\Rightarrow 2\alpha+(1+\alpha)=-2$

$\Rightarrow 3\alpha=-3$

$\therefore \alpha=-1$

We have the image $\overrightarrow{m}=(3+2\alpha)\hat{i}+(1-\alpha)\hat{j}+(2+\alpha)\hat{k}$

$\Rightarrow \overrightarrow{m}=[3+2(-1)]\hat{i}+[1-(-1)]\hat{j}+[2+(-1)]\hat{k}$

$\therefore \overrightarrow{m}=\hat{\imath}+2\hat{\jmath}+\hat{k}$

Therefore, the image is (1, 2, 1)

The foot of the perpendicular $\overrightarrow{q}=(3+\alpha)\hat{i}+\frac{(2-\alpha)}{2}\hat{j}+\frac{(4+\alpha)}{2}\hat{k}$

$\Rightarrow \overrightarrow{q}=[3+(-1)]\hat{i}+\left[\frac{2-(-1)}{2}\right]\hat{j}+\left[\frac{4+(-1)}{2}\right]\hat{k}$

$\therefore \overrightarrow{q}=2\hat{i}+\frac{3}{2}\hat{j}+\frac{3}{2}\hat{k}$

Thus, the position vector of the image is $\hat{i}+2\hat{j}+\hat{k}$ and that of the foot of the perpendicular is $2\hat{\imath}+\frac{3}{2}\hat{\jmath}+\frac{3}{2}\hat{k}$

Section E

36. i. It is given that if India loose any match, then the probability that it wins the next match is 0.3.

$\therefore$ Required probability = 0.3

ii. It is given that, if India loose any match, then the probability that it wins the next match is 0.3.

$\therefore$ Required probability = 1 - 0.3 = 0.7

iii. Required probability = P(India losing first match) $\cdot$ P(India losing second match when India has already lost first match) = 0.4 $\times$ 0.7 = 0.28

OR

Required probability = P(India winning first match) $\cdot$ P(India winning second match if India has already won first match) $\cdot$ P(India winning third match if India has already won first two matches) = 0.6 $\times$ 0.4 $\times$ 0.4 = 0.096

37. i. Here,

Position vector of A is $\vec{a}=\hat{i}+4\hat{j}+2\hat{k}$

Position vector of B is $\vec{b}=3\hat{i}-3\hat{j}-2\hat{k}$

Position vector of C is $\vec{c}=-2\hat{i}+2\hat{j}+6\hat{k}$

$\therefore \vec{a}+\vec{b}+\vec{c}$ = (1 + 3 - 2)$\hat{i}$ + (4 - 3 + 2)$\hat{j}$ + (2 - 2 + 6)$\hat{k}$

= 2$\hat{i}$ + 3$\hat{j}$ + 6$\hat{k}$

Thus, $|\vec{a}+\vec{b}+\vec{c}|$ = $\left|\sqrt{(2)^2+(3)^2+(6)^2}\right|$

= $|\sqrt{4+9+16}|$

= $\sqrt{29}$

ii. Given, $\vec{a}=4\hat{i}+6\hat{j}+12\hat{k}$,

$|\vec{a}|=\sqrt{4^2+6^2+12^2}$ = 14

Therefore, the unit vector in direction of $\vec{a}$ is given by

$\hat{a}=\frac{\vec{a}}{|\vec{a}|}=\frac{4\hat{i}+6\hat{j}+12\hat{k}}{14}$

= $\frac{4}{14}\hat{i}+\frac{6}{14}\hat{j}+\frac{12}{14}\hat{k}$

= $\frac{2}{7}\hat{i}+\frac{3}{7}\hat{j}+\frac{6}{7}\hat{k}$

iii. We have, A(1, 4, 2), B(3, -3, -2) and C(-2, 2, 6)

Now, $\overrightarrow{AB} = \vec{b} - \vec{a} = 2\hat{i} - 7\hat{j} - 4\hat{k}$

and $\overrightarrow{AC} = \vec{c} - \vec{a} = -3\hat{i} - 2\hat{j} + 4\hat{k}$

$\therefore \overrightarrow{AB} \times \overrightarrow{AC}$ = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & -4 \\ -3 & -2 & 4 \end{vmatrix}$

= $\hat{i}(-28 - 8) - \hat{j}(8 - 12) + \hat{k}(-4 - 21)$

= - $36\hat{i} + 4\hat{j} - 25\hat{k}$

Now, $|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(-36)^2 + 4^2 + (-25)^2}$

= $|\sqrt{1296 + 16 + 625}| = \sqrt{1937}$

$\therefore$ Area of $\triangle$ABC = $\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

= $\frac{1}{2}\sqrt{1937}$ sq. units

OR

Triangle law of addition for $\triangle$ABC is given by

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$ = $\vec{0}$

If the given points lie on the straight line, then the points will be collinear and so area of $\triangle$ABC = 0

Then, $|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$ = 0.

Also, if a, b, c are the position vector of the three vertices A, B and C of $\triangle$ABC, then area of triangle is $\frac{1}{2}|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$.

38. i. f(x) = -0.1x^2 + mx + 98.6, being a polynomial function, is differentiable everywhere, hence, differentiable in (0, 12).

ii. f(x) = -0.2x + m

At Critical point

0 = -0.2 $\times$ 6 + m

m = 1.2

iii. f(x) = -0.1x^2 + 1.2x + 98.6

f'(x) = -0.2x + 1.2 = -0.2(x - 6)

In the Interval	f'(x)	Conclusion
(0, 6)	+Ve	f is strictly increasing in [0, 6]
(6, 12)	-Ve	f is strictly decreasing in [6, 12]

OR

f(x) = -0.1x^2 + 1.2x + 98.6,

f'(x) = -0.2x + 1.2, f'(6) = 0,

f''(x) = -0.2

f''(6) = -0.2 < 0

Hence, by second derivative test 6 is a point of local maximum. The local maximum value = f(6) = -0.1 $\times$ 6^2 + 1.2 $\times$ 6 + 98.6 = 102.2

We have f(0) = 98.6, f(6) = 102.2, f(12) = 98.6

6 is the point of absolute maximum and the absolute maximum value of the function = 102.2.

0 and 12 both are the points of absolute minimum and the absolute minimum value of the function = 98.6.