

The number of zeroes of $f(x)$ is

a) 2

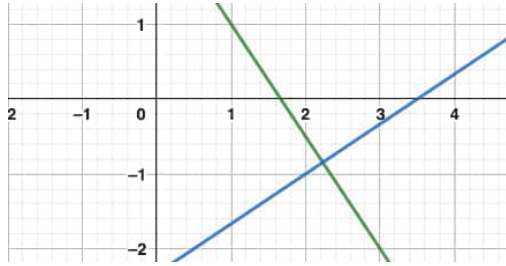
b) 0

c) 3

d) 4

3. The pair of linear equations $3x + 2y = 5$ and $2x - 3y = 7$ are

[1]



a) inconsistent

b) consistent

c) dependent

d) independent

4. Which of the following is a quadratic equation?

[1]

a) $x^2 + 2x + 1 = (4 - x)^2 + 3$

b) $x^3 - x^2 = (x - 1)^3$

c) $(k + 1)x^2 + \frac{3}{2}x - 5 = 0$, where $k = -1$

d) $-2x^2 = (5 - x)\left(2x - \frac{2}{5}\right)$

5. If the sum of first n terms of an A.P. is $3n^2 + 4n$ and its common difference is 6, then its first term is:

[1]

a) 7

b) 4

c) 6

d) 3

6. The abscissa of any point on the y -axis is

[1]

a) 0

b) 1

c) y

d) -1

7. The coordinates of the mid-point of the line segment joining the points $(-2, 3)$ and $(4, -5)$ are

[1]

a) $(-1, 1)$

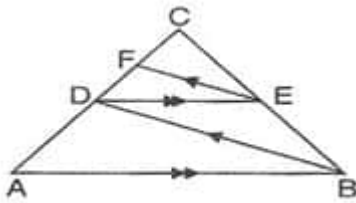
b) $(1, -1)$

c) $(-2, 4)$

d) $(0, 0)$

8. We have, $AB \parallel DE$ and $BD \parallel EF$. Then,

[1]



a) $AC^2 = BC \cdot DC$

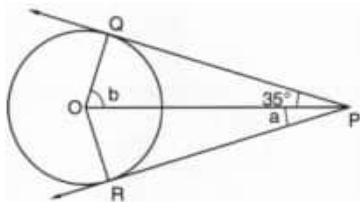
b) $AB^2 = AC \cdot DE$

c) $BC^2 = AB \cdot CE$

d) $DC^2 = CF \times AC$

9. In Figure, PQ and PR are tangents drawn from P to a circle with centre O. If $\angle OPQ = 35^\circ$, then

[1]



a) $a = 40^\circ$, $b = 50^\circ$

b) $a = 30^\circ$, $b = 60^\circ$

c) $a = 45^\circ$, $b = 45^\circ$

d) $a = 35^\circ$, $b = 55^\circ$

10. Quadrilateral ABCD is circumscribed to a circle. If $AB = 6$ cm, $BC = 7$ cm and $CD = 4$ cm then the length of AD is [1]

a) 7 cm

b) 4 cm

c) 6 cm

d) 3 cm

11. If $\sin\theta + \cos\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$, then $q(p^2 - 1) =$ [1]

a) $2p$

b) 2

c) $\frac{2q}{p^2}$

d) $\frac{q}{p^2}$

12. If $\sin\theta = \frac{a}{b}$ then $\cos\theta = ?$ [1]

a) $\frac{b}{a}$

b) $\frac{a}{\sqrt{b^2 - a^2}}$

c) $\frac{\sqrt{b^2 - a^2}}{b}$

d) $\frac{b}{\sqrt{b^2 - a^2}}$

13. Two men are on opposite sides of a tower. They observe the angles of elevation of the top of the tower as 30° and 45° respectively. If the height of the tower is 100m, then the distance between them is [1]

a) $100(\sqrt{3} - 1)m$

b) $100(1 - \sqrt{3})m$

c) $100(2 - \sqrt{3})m$

d) $100(\sqrt{3} + 1)m$

14. Area of a sector of a circle of radius 36 cm is 54π cm². The length of the corresponding arc of the sector is: [1]

a) 5π cm

b) 4π cm

c) 3π cm

d) 2π cm

15. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. The length of the arc is [1]

a) 21 cm

b) 22 cm

c) 23.5 cm

d) 18.16 cm

16. A card is drawn at random from a well-shuffled deck of 52 playing cards. The probability of getting an ace of spade is: [1]

a) $\frac{1}{52}$

b) $\frac{3}{52}$

c) $\frac{1}{26}$

d) $\frac{1}{13}$

17. If three coins are tossed simultaneously, then the probability of getting at least two heads, is [1]

a) $\frac{1}{4}$

b) $\frac{7}{4}$

c) $\frac{3}{8}$

d) $\frac{1}{2}$

18. Consider the following frequency distribution of the heights of 60 students of a class : [1]

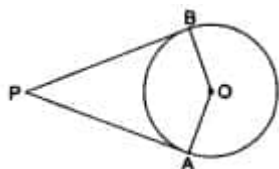
Height (in cm)	Number of students
150-155	15
155-160	13
160-165	10

27. Find the zeroes of the polynomial $2s^2 + (1 + 2\sqrt{2})s + \sqrt{2}$ by factorisation method and verify the relationship between the zeroes and coefficient of the polynomial. [3]
28. A lady has only 25-paisa and 50-paisa coins in her purse. If she has 50 coins in all totalling ₹19.50, how many coins of each kind does she have? [3]

OR

The sum of two numbers is 8. If their sum is four times their difference, find the numbers.

29. In the given figure, PA and PB are the tangent segments to a circle with centre O. Show that the points A, O, B and P are concyclic. [3]



OR

Two tangent segments PA and PB are drawn to a circle with centre O such that $\angle APB = 120^\circ$. Prove that $OP = 2AP$.

30. If $\tan \theta + \frac{1}{\tan \theta} = 2$, find the value of $\tan^2 \theta + \frac{1}{\tan^2 \theta}$ [3]
31. To find out the concentration of SO_2 in the air (in parts per million, i.e. ppm), the data was collected for 30 localities in a certain city and is presented below: [3]

Concentration of SO_2 (in ppm)	Frequency
0.00-0.04	4
0.04-0.08	9
0.08-0.12	9
0.12-0.16	2
0.16-0.20	4
0.20-0.24	2

Find the mean concentration of SO_2 in the air.

Section D

32. Solve for x: $\left(\frac{2x}{x-5}\right)^2 + 5\left(\frac{2x}{x-5}\right) - 24 = 0, x \neq 5$ [5]

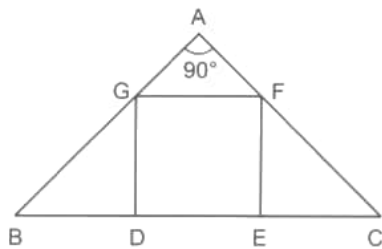
OR

The sum of two numbers is 34. If 3 is subtracted from one number and 2 is added to another, the product of these two numbers becomes 260. Find the numbers.

33. In the given figure, DEFG is a square and $\angle BAC = 90^\circ$. [5]

Prove that

- $\triangle AGF \sim \triangle DBG$
- $\triangle AGF \sim \triangle EFC$
- $\triangle DBG \sim \triangle EFC$
- $DE^2 = BD \times EC$



34. A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of cone is 4 cm and the diameter of the base is 8 cm. Determine the volume of the toy. If a cube circumscribes the toy, then find the difference of the volumes of cube and the toy. Also, find the total surface area of the toy. [5]

OR

An iron pillar has some part in the form of a right circular cylinder and remaining in the form of a right circular cone. The radius of base of each of cone and cylinder is 8 cm. The cylindrical part is 240 cm high and the conical part is 36 cm high. Find the weight of the pillar, if one cubic cm of iron weighs 10 g.

35. To find out the concentration of SO_2 in the air (in parts per million, i.e., ppm), the data was collected for 30 localities in a certain city and is presented below: [5]

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0.00 - 0.04	4
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0.12 - 0.16	2
0.16 - 0.20	4
0.20 - 0.24	2

find the mean concentration of SO_2 in the air.

Section E

36. Read the following text carefully and answer the questions that follow: [4]

Students of a school thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of class I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII.

There are three sections of each class.



- Find total number of trees planted by primary 1 to 5 class students? (1)
- Find the total number of trees planted by the students of the school. (1)
- Find the total number of trees planted by class 10th student. (2)

OR

Find the total no of trees planted by class 12th students. (2)

37. **Read the following text carefully and answer the questions that follow:**

[4]

A satellite image of a colony is shown below. In this view, a particular house is pointed out by a flag, which is situated at the point of intersection of x and y-axes. If we go 2 cm east and 3 cm north from the house, then we reach to a Grocery store. If we go 4 cm west and 6 cm south from the house, then we reach to an Electricians's shop. If we go 6 cm east and 8 cm south from the house, then we reach to a food cart. If we go 6 cm west and 8 cm north from the house, then we reach a bus stand.

Scale:

x-axis : 1 cm = 1 unit

y-axis : 1 cm = 1 unit



- What is the distance between the grocery store and food cart? (1)
- What is the distance of the bus stand from the house? (1)
- If the grocery store and electricians shop lie on a line, then what will be the ratio of distance of house from grocery store to that from electrician's shop? (2)

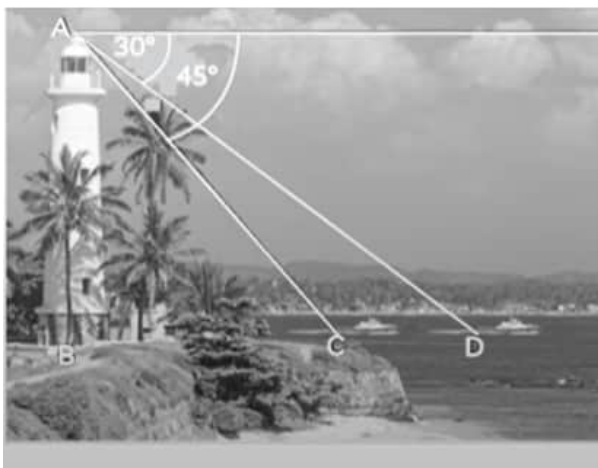
OR

What are the ratio of distances of the house from bus stand to food cart? (2)

38. **Read the following text carefully and answer the questions that follow:**

[4]

An observer on the top of a 40m tall light house (including height of the observer) observes a ship at an angle of depression 30° coming towards the base of the light house along straight line joining the ship and the base of the light house. The angle of depression of ship changes to 45° after 6 seconds.



- Find the distance of ship from the base of the light house after 6 seconds from the initial position when angle of depression is 45° . (1)
- Find the distance between two positions of ship after 6 seconds? (1)
- Find the speed of the ship? (2)

OR

Find the distance of ship from the base of the light house when angle of depression is 30° . (2)

Solution

Section A

1.

(c) $2^3 \times 3^3$

Explanation:

L.C.M. of $2^3 \times 3^2$ and $2^2 \times 3^3$ is the product of all prime numbers with the greatest power of every given number, hence it will be $2^3 \times 3^3$

2.

(b) 0

Explanation:

Here $y = f(x)$ is not intersecting or touching the X-axis.

\therefore Number of zeroes of $f(x) = 0$

3.

(b) consistent

Explanation:

Given: $a_1 = 3, a_2 = 2, b_1 = 2, b_2 = -3, c_1 = 5$ and $c_2 = 7$

Here, $\frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{2}{-3}$

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefore, the pair of given linear equations is consistent.

4.

(b) $x^3 - x^2 = (x - 1)^3$

Explanation:

In equation $x^3 - x^2 = (x - 1)^3$

$\Rightarrow x^3 - x^2 = x^3 - 1 - 3x^2 + 3x$

$\Rightarrow -x^2 + 3x^2 - 3x + 1 = 0$

$\Rightarrow 2x^2 - 3x + 1 = 0$

It is a quadratic equation as its degree is 2.

5. (a) 7

Explanation:

$S_n = 3n^2 + 4n$

for $n = 1$

$S_1 = a_1$

$S_1 = 3(1) + 4(1)$

$= 3 + 4$

$a_1 = 7$

6. (a) 0

Explanation:

Since coordinates of any point on y-axis is $(0, y)$

Therefore, the abscissa is 0.

7.

(b) $(1, -1)$

Explanation:

Let the coordinates of midpoint C(x, y) of the line segment joining the points A(-2, 3) and B(4, -5)

$$\therefore x = \frac{x_1 + x_2}{2} = \frac{-2 + 4}{2} = \frac{2}{2} = 1$$

$$\text{And } y = \frac{y_1 + y_2}{2} = \frac{3 - 5}{2} = \frac{-2}{2} = -1$$

Therefore, the coordinates of mid-point C are (1, -1)

8.

(d) $DC^2 = CF \times AC$

Explanation:

In $\triangle ABC$, using Thales theorem,

$$\frac{DC}{AC} = \frac{CE}{BC} \quad [AB \parallel DE] \dots\dots\dots(i)$$

And in triangle BCD, using Thales theorem,

$$\frac{CF}{DC} = \frac{CE}{BC} \quad [BD \parallel EF] \dots\dots\dots(ii)$$

From eq. (i) and (ii), we have

$$\frac{DC}{AC} = \frac{CF}{DC}$$

$$\Rightarrow DC^2 = CF \times AC$$

9.

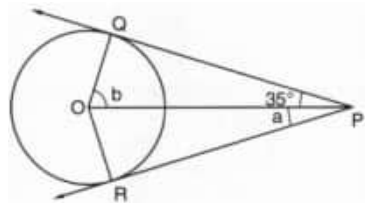
(d) $a = 35^\circ$, $b = 55^\circ$

Explanation:

In the figure, PQ and PR are the tangents drawn from P to the circle with centre O

$$\angle OPQ = 35^\circ$$

PO is joined



$PQ = PR$ (tangents from P to the circle)

$$\angle OPQ = \angle OPR$$

$$\Rightarrow 35^\circ = a$$

$$\Rightarrow a = 35^\circ$$

OQ is radius and PQ is tangent $OQ \perp PQ$

$$\Rightarrow \angle OQP = 90^\circ$$

In $\triangle OQP$

$$\angle POQ + \angle QPO = 90^\circ$$

$$\Rightarrow b + 35^\circ = 90^\circ$$

$$\Rightarrow b = 90^\circ - 35^\circ = 55^\circ$$

$$a = 35^\circ, b = 55^\circ$$

10.

(d) 3 cm

Explanation:

A quadrilateral ABCD is circumscribed to a circle with centre O.

$AB = 6$ cm, $BC = 7$ cm, $CD = 4$ cm, $AD = 7$ cm

ABCD circumscribed to a circle.

$$AB + CD = BC + AD$$

$$\Rightarrow 6 + 4 = 7 + AD$$

$$\Rightarrow 10 = 7 + AD$$

$$AD = 10 - 7 = 3 \text{ cm}$$

11. (a) 2p

Explanation:

Given: $\sin\theta + \cos\theta = p$

squaring both sides we get

$$\sin^2\theta + \cos^2\theta + \cos^2\theta + 2\sin\theta \cos\theta = p^2$$

$$1 + 2\sin\theta \cos\theta = p^2 (\sin^2\theta + \cos^2\theta = 1)$$

$$2\sin\theta \cos\theta = p^2 - 1 \dots (i)$$

and also $\sec\theta + \operatorname{cosec}\theta = q$ (given)

$$\frac{1}{\cos\theta} + \frac{1}{\sin\theta} = q$$

$$\frac{\sin\theta + \cos\theta}{\sin\theta \cos\theta} = q$$

but $\sin\theta + \cos\theta = p \dots$ (given)

$$\frac{p}{\sin\theta \cos\theta} = q \dots (ii)$$

from (i) and (ii) we get

$$q(p^2 - 1) = 2p$$

12.

(c) $\frac{\sqrt{b^2 - a^2}}{b}$

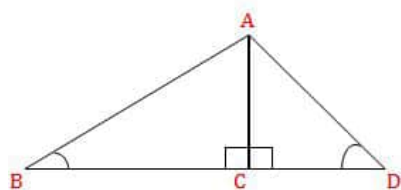
Explanation:

$$\cos^2\theta = (1 - \sin^2\theta) = \left(1 - \frac{a^2}{b^2}\right) = \frac{b^2 - a^2}{b^2} \Rightarrow \cos\theta = \frac{\sqrt{b^2 - a^2}}{b}$$

13.

(d) $100(\sqrt{3} + 1)m$

Explanation:



Let the height of the tower $AC = 100$ m

Now, in triangle ABC,

$$\tan 30^\circ = \frac{AC}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{BC}$$

$$\Rightarrow BC = 100\sqrt{3} \text{ m}$$

Now, in triangle ACD,

$$\tan 45^\circ = \frac{AC}{CD}$$

$$\Rightarrow 1 = \frac{100}{CD}$$

$$\Rightarrow CD = 100 \text{ m}$$

Therefore, the required distance $= BC + CD = 100\sqrt{3} + 100 = 100(\sqrt{3} + 1) \text{ m}$

14.

(c) 3π cm

Explanation:

We know that:

$$\text{Area of a sector} = \frac{\pi r^2 \theta}{360^\circ}$$

$$\frac{\pi \times 36 \times 36 \times \theta}{360} = 54$$

$$\Rightarrow \theta = \frac{54 \times 360}{36 \times 36} = 15^\circ$$

$$\Rightarrow \text{Length of arc} = \frac{\theta}{360^\circ} \times 2\pi r$$

$$= \frac{15}{360} \times 2\pi \times 36$$

$$= 3\pi \text{ cm}$$

15.

(b) 22 cm

Explanation:

$$\text{Arc length} = \frac{2\pi r\theta}{360} = \left(2 \times \frac{22}{7} \times 21 \times \frac{60}{360}\right) \text{ cm} = 22 \text{ cm}$$

16. (a) $\frac{1}{52}$

Explanation:

A card is drawn at random from a pack of well shuffled 52 playing cards. 'S' is the sample space.

$$\therefore n(S) = 52$$

Event A: The card drawn is an ace of spade

$$\therefore n(A) = 1$$

$$\therefore P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{1}{52}$$

17.

(d) $\frac{1}{2}$

Explanation:

Possible outcomes of tossing three coins are:

(HHH), (HHT), (HTH), (THH), (TTT), (TTH), (THT), (HTT)

here H and T are denoted for Head and Tail.

Total outcomes = 8

no. of outcomes with at least two heads = 4

$$\therefore \text{required probability} = \frac{4}{8} = \frac{1}{2}$$

18.

(d) 315

Explanation:

150-155 is the modal class

Height in cm	Number of students (f)	Cumulative Frequency (CF)
150-155	15	15
155-160	13	28
160-165	10	38
165-170	8	46
170-175	9	55
175-180	5	60
Total	60	

Here, $\frac{N}{2} = 30$, the cumulative frequency just above 30 is 38 and the corresponding class is 160-165 which is the median class.

hence the required sum = 115 + 165 = 315

19. (a) Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

20.

(c) A is true but R is false.

Explanation:

$$a_{10} = a + 9d$$

$$= 5 + 9(3) = 5 + 27 = 32$$

Section B

21. Given number,

$$7 \times 9 \times 13 \times 15 + 15 \times 14$$

$$= 15(7 \times 9 \times 13 + 14)$$

Clearly, this number is a product of two numbers other than 1 and has factors other than 1, and itself.

Therefore, it is a composite number.

22. In $\triangle ABC$, it is given that

$$DE \parallel BC$$

$$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE} \dots\dots\dots(i)$$

In $\triangle ADC$, it is given that

$$FE \parallel DC$$

$$\Rightarrow \frac{AD}{AF} = \frac{AC}{AE} \dots\dots\dots(ii)$$

From (i) and (ii), we get

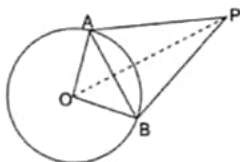
$$\frac{AB}{AD} = \frac{AD}{AF}$$

$$\Rightarrow AD^2 = AB \times AF$$

23. Draw a circle with centre O and take an external point P.

PA and PB are the tangents.

Join OP.



Now in $\triangle OAP$ and $\triangle OBP$,

OA = OB ...(Radius of circle)

OP = OP ...(Common)

PA = PB ...(Tangents are equal)

So, by S.S.S criteria,

$$\triangle OAP \cong \triangle OBP$$

So, $\angle APO = \angle BPO$

Hence, OP bisects $\angle APB$.

24. According to the question,

$$\sec \theta = \operatorname{cosec} 60^\circ$$

$$\Rightarrow \sec \theta = \operatorname{cosec} (90^\circ - 30^\circ)$$

$$\Rightarrow \sec \theta = \sec 30^\circ [\operatorname{cosec} (90^\circ - \theta) = \sec \theta]$$

$$\Rightarrow \theta = 30^\circ$$

$$\therefore 2\cos^2 \theta - 1$$

$$= 2\cos^2 30^\circ - 1$$

$$= 2\left(\frac{\sqrt{3}}{2}\right)^2 - 1$$

$$= 2 \times \frac{3}{4} - 1$$

$$= \frac{3}{2} - 1$$

$$= \frac{3-2}{2}$$

$$= \frac{1}{2}$$

OR

We have,

$$\text{LHS} = \sqrt{\sec^2 \theta + \operatorname{cosec}^2 \theta}$$

$$= \sqrt{(1 + \tan^2 \theta) + (1 + \cot^2 \theta)}$$

$$= \sqrt{\tan^2 \theta + \cot^2 \theta + 2} = \sqrt{\tan^2 \theta + \cot^2 \theta + 2 \tan \theta \cot \theta} \quad [\because \tan \theta \cot \theta = 1]$$

$$= \sqrt{(\tan \theta + \cot \theta)^2} = \tan \theta + \cot \theta = \text{RHS}$$

25. We have

$$r = 5 \text{ cm}$$

$$\theta = 36^\circ$$

We have to find the length of the arc.

$$\text{Length of the arc} = \frac{\theta}{360} \times 2\pi r$$

Substituting the values we get,

$$\text{Length of the arc} = \frac{36}{360} \times 2\pi \times 5 \dots (1)$$

Now we will simplify the equation (1) as below,

$$= \frac{1}{10} \times 2\pi \times 5$$

$$= \frac{1}{2} \times 2\pi$$

$$= \pi$$

Therefore, the length of the arc is π cm.

OR

$$\text{Area of minor segment} = \frac{3.14 \times (10)^2 \times 60^\circ}{360^\circ} - \frac{1}{2} \times (10)^2 \times \frac{\sqrt{3}}{2}$$

$$= \frac{314}{6} - \frac{173}{4}$$

$$= 9\frac{1}{12} \text{ or } 9.08$$

Hence, area of minor segment is 9.08 cm^2 .

Section C

26. For maximum number of students to put into each group

Mr patil sir should have to take H.C.F of 28, 42 and 56

so

maximum number of students Mr. Patil can put into each group is 14.

$$27. 2s^2 + (1 + 2\sqrt{2})s + \sqrt{2}$$

$$= 2s^2 + s + 2\sqrt{2}s + \sqrt{2}$$

$$= s(2s + 1) + \sqrt{2}(2s + 1)$$

$$= (2s + 1)(s + \sqrt{2})$$

$$\Rightarrow s = -\frac{1}{2}, -\sqrt{2} \text{ are zeroes of the polynomial.}$$

$$\text{Sum of zeroes} = -\left[\frac{1}{2} + \sqrt{2}\right] = -\frac{1+2\sqrt{2}}{2}$$

$$\text{Also, } \frac{-b}{a} = -\frac{1+2\sqrt{2}}{2}$$

$$\Rightarrow \text{Sum of zeroes} = \frac{-b}{a}$$

$$\text{Product of zeroes} = \frac{-1}{2} \times -\sqrt{2} = \frac{\sqrt{2}}{2}$$

$$\text{and } \frac{c}{a} = \frac{\sqrt{2}}{2}$$

$$\Rightarrow \text{Product of zeroes} = \frac{c}{a}$$

28. Suppose the number of 25 - paisa coins be x and the number of 50 - paisa coins be y.

Then,

$$x + y = 50 \dots\dots\dots(i)$$

She has a total of ₹19.50,

$$25x + 50y = 19.50 \times (100)$$

$$\Rightarrow 25x + 50y = 1950$$

$$\Rightarrow x + 2y = 78 \dots\dots\dots(ii)$$

Subtracting equation (i) from (ii),

$$\Rightarrow y = 28$$

Substituting y = 28 in (i), we get x = 22.

\therefore the number of 25 - paisa coins = 22

the number of 50 - paisa coins = 28

OR

Let the numbers be 'a' and 'b'.

Given that, sum of two numbers is 8.

$$\Rightarrow a + b = 8 \dots(1)$$

Also, their sum is four times their difference

$$\Rightarrow a + b = 4(a - b)$$

$$\Rightarrow a + b = 4a - 4b$$

$$\Rightarrow 3a - 5b = 0 \dots (2)$$

Substituting value of 'a' from equation (2) in equation (1)

$$\Rightarrow \frac{5}{3}b + b = 8$$

$$\Rightarrow \frac{5b+3b}{3} = 8$$

$$\Rightarrow 8b = 24$$

$$\Rightarrow b = 3$$

Using this value of 'b' in (1), gives

$$a = 8 - 3 = 5$$

Thus, a = 5 and b = 3.

29. Here, OA = OB

And OA \perp AP, OB \perp BP (Since tangent is perpendicular to the radius at the point of contact)

$$\therefore \angle OAP = 90^\circ,$$

$$\angle OBP = 90^\circ$$

$$\therefore \angle OAP + \angle OBP = 90^\circ + 90^\circ = 180^\circ$$

$$\therefore \angle AOB + \angle APB = 180^\circ$$

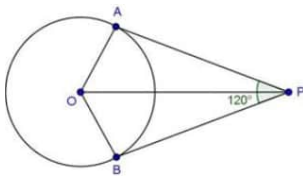
(Since, $\angle AOB + \angle OAP + \angle OBP + \angle APB = 360^\circ$)

Thus, sum of opposite angle of a quadrilateral is 180° .

Hence, A, O, B and P are concyclic.

OR

In $\triangle AOP$ and $\triangle BOP$, we have,



$$\angle OAP = \angle OBP = 90^\circ$$

$$OP = OP \text{ [Common]}$$

PA = PB [\because Tangents from an external point are equal in length]

So, by RHS congruence criterion, we have

$$\triangle AOP \cong \triangle BOP$$

$$\therefore \angle APO = \angle BPO \text{ [By C.P.C.T.]}$$

$$\Rightarrow \angle APO = \angle BPO = \frac{1}{2} \angle APB$$

$$= \frac{1}{2} \times 120^\circ$$

$$= 60^\circ$$

$$\Rightarrow \angle APO = \angle BPO = 60^\circ$$

In $\triangle OAP$, we have

$$\cos 60^\circ = \frac{AP}{OP}$$

$$\Rightarrow \frac{1}{2} = \frac{AP}{OP}$$

$$\Rightarrow OP = 2AP$$

Hence proved.

30. We have,

$$\tan \theta + \frac{1}{\tan \theta} = 2$$

Squaring both sides, we get

$$\Rightarrow \left(\tan \theta + \frac{1}{\tan \theta} \right)^2 = 2^2$$

$$\Rightarrow \tan^2 \theta + \frac{1}{\tan^2 \theta} + 2 \times \tan \theta \times \frac{1}{\tan \theta} = 4$$

$$\Rightarrow \tan^2 \theta + \frac{1}{\tan^2 \theta} + 2 = 4$$

$$\Rightarrow \tan^2 \theta + \frac{1}{\tan^2 \theta} = 2$$

Alternate method, We have

$$\tan \theta + \frac{1}{\tan \theta} = 2$$

$$\Rightarrow \tan^2 \theta + 1 = 2 \tan \theta$$

$$\Rightarrow \tan^2 \theta - 2 \tan \theta + 1 = 0$$

$$\Rightarrow (\tan \theta - 1)^2 = 0$$

$$\Rightarrow \tan \theta = 1$$

$$\therefore \tan^2 \theta + \frac{1}{\tan^2 \theta} = 1 + 1 = 2$$

31. Take $a = 0.14$, $h = 0.04$

Concentration of SO ₂ (in ppm)	Frequency (f _i)	Class Mark (x _i)	d _i = x _i - 0.14	u _i = $\frac{x_i - 0.14}{0.04}$	f _i u _i
0.00-0.04	4	0.02	-0.12	-3	-12
0.04-0.08	9	0.06	-0.08	-2	-18
0.08-0.12	9	0.10	0.04	-1	-9
0.12-0.16	2	0.14	0	0	0
0.16-0.20	4	0.18	0.04	1	4
0.20-0.24	2	0.22	0.08	2	4
Total	$\sum f_i = 30$				$\sum f_i u_i = -31$

Using the step-deviation method,

$$\bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) \times h = 0.14 + \left(\frac{-31}{30} \right) \times (0.04)$$

$$= 0.14 - 0.041 = 0.0999 \text{ ppm.}$$

Therefore, the mean concentration of SO₂ in the air is 0.099

Section D

32. We have given,

$$\left(\frac{2x}{x-5} \right)^2 + 5 \left(\frac{2x}{x-5} \right) - 24 = 0$$

Let $\frac{2x}{(x-5)}$ be y

$$\therefore y^2 + 5y - 24 = 0$$

Now factorise,

$$y^2 + 8y - 3y - 24 = 0$$

$$y(y + 8) - 3(y + 8) = 0$$

$$(y + 8)(y - 3) = 0$$

$$y = 3, -8$$

Putting $y=3$

$$\frac{2x}{x-5} = 3$$

$$2x = 3x - 15$$

$$x = 15$$

Putting $y = -8$

$$\frac{2x}{x-5} = -8$$

$$2x = -8x + 40$$

$$10x = 40$$

$$x = 4$$

Hence, x is 15, 4

OR

Then, according to given condition, we have

$$x + y = 34 \dots (i)$$

$$\text{and } (x - 3)(y + 2) = 260 \dots (ii)$$

On substituting the value of y from equation (i) in equation (ii), we get

$$(x - 3)(34 - x + 2) = 260 \dots [\because y = 34 - x]$$

$$\Rightarrow (x - 3)(36 - x) = 260$$

$$\Rightarrow 36x - x^2 - 108 + 3x = 260$$

$$\Rightarrow x^2 - 39x + 368 = 0$$

$$\Rightarrow x^2 - (16x + 23x) + 368 = 0$$

$$\Rightarrow x^2 - 16x - 23x + 368 = 0$$

$$\Rightarrow x(x - 16) - 23(x - 16) = 0$$

$$\Rightarrow (x - 16)(x - 23) = 0$$

$$\Rightarrow x = 16 \text{ or } x = 23$$

$$\text{When } x = 16, \text{ then } y = 34 - 16 = 18$$

$$\text{When } x = 23, \text{ then } y = 34 - 23 = 11$$

So, the numbers are 16, 18 or 23, 11

33. Given $\triangle ABC$ in which $\angle BAC = 90^\circ$ and DEFG is a square.

Proof

i. In $\triangle AGF$ and $\triangle DBG$, we have

$$\angle GAF = \angle BDG = 90^\circ$$

$$\angle AGF = \angle DBG \text{ [corresponding angles]}$$

$$[\because GF \parallel BC \text{ and AB is the transversal}]$$

$$\therefore \triangle AGF \sim \triangle DBG$$

ii. In $\triangle AGF$ and $\triangle EFC$, we have

$$\angle FAG = \angle CEF = 90^\circ$$

$$\angle GFA = \angle FCE \text{ [corresponding angles]}$$

$$[\because GF \parallel BC \text{ and AC is the transversal}]$$

$$\therefore \triangle AGF \sim \triangle EFC$$

iii. $\triangle DBG \sim \triangle AGF$ and $\triangle AGF \sim \triangle EFC$

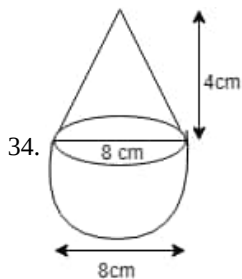
$$\Rightarrow \triangle DBG \sim \triangle EFC$$

iv. $\triangle DBG \sim \triangle EFC$

$$\Rightarrow \frac{BD}{FE} = \frac{DG}{EC}$$

$$\Rightarrow \frac{BD}{DE} = \frac{DE}{EC} \text{ } [\because DG = DE \text{ and } FE = DE].$$

$$\text{Hence, } DE^2 = BD \times EC.$$



Volume of toy = volume of cone + volume of hemisphere

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi r^2 (h + 2r)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 4 \times 4(4 + 2 \times 4)$$

$$= 201.14 \text{ cm}^3$$

If a cube circumscribes the toy then,

$$\text{Volume of cubi} = (\text{side})^3$$

$$\text{Volume} = 512 \text{ cm}^3$$

Difference of the volume of cube and toy

$$= 512 - 201.14$$

$$= 310.86 \text{ cm}^3$$

Total surface Area of toy = Curved surface area of cone + Curved surface area of hemisphere

$$l = \sqrt{h^2 + r^2}$$

$$l = \sqrt{4^2 + 4^2}$$

$$l = \sqrt{32}$$

$$l = 4\sqrt{2}$$

$$l = 5.64 \text{ cm}$$

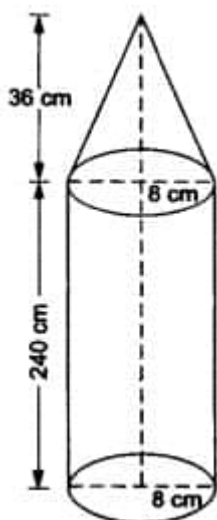
$$\text{Total surface area of toy} = \pi r l + 2\pi r^2$$

$$= \pi r(l + 2r)$$

$$= \frac{22}{7} \times 4(5.64 + 2 \times 4)$$

Total surface area of toy = 171.47 cm²

OR



Let us suppose that r denotes the radius of the cylinder = 8 cm.

Suppose R denotes the radius of the cone = 8 cm.

Let h be the height of the cylinder = 240cm.

Suppose H is the height of the cone = 36 cm.

Total volume of the iron = volume of the cylinder + volume of the cone

$$= \pi r^2 h + \frac{1}{3} \pi R^2 H = \pi r^2 \left(h + \frac{1}{3} H \right) \quad [\text{as } r=R= 8\text{cm each}]$$

$$= \left[\frac{22}{7} \times 8 \times 8 \times \left(240 + \frac{1}{3} \times 36 \right) \right] \text{cm}^3$$

$$= 50688 \text{ cm}^3$$

\therefore Weight of the pillar = volume in cm³ \times weight per cm³

$$= \left(\frac{50688 \times 10}{1000} \right) \text{kg} = 506.88 \text{kg}$$

Therefore, the weight of the pillar is 506.88 kg.

35. We may find class marks for each interval by using the relation

$$x = \frac{\text{upperlimit} + \text{lowerclasslimit}}{2}$$

Class size of this data = 0.04

Concentration of SO ₂	Frequency f_i	Class interval x_i	$d_i = x_i - 0.14$	u_i	$f_i u_i$
0.00 – 0.04	4	0.02	-0.12	-3	-12
0.04 – 0.08	9	0.06	-0.08	-2	-18
0.08 – 0.12	9	0.10	-0.04	-1	-9
0.12 – 0.16	2	0.14	0	0	0
0.16 – 0.20	4	0.18	0.04	1	4
0.20 – 0.24	2	0.22	0.08	2	4
Total	$\sum f_i = 30$				$\sum f_i u_i = -31$

let $a = 0.14$

$$\text{Mean } \bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) \times h$$

$$= 0.14 + (0.04) \left(\frac{-31}{30} \right)$$

$$= 0.099 \text{ ppm}$$

Section E

36. i. Each class has 3 section

class 1 plants = 3 trees

class 2 plants = 6 trees

class 3 plants = 9 trees

$\therefore 3, 6, 9, \dots$

The no of trees planted by each class is in AP.

$$S_n = \frac{n}{2} \{2a + (n - 1)d\}$$

$$S_5 = \frac{5}{2} \{2 \times 3 + (5 - 1)3\}$$

$$S_5 = \frac{5}{2} \{6 + 12\}$$

$$S_5 = \frac{5}{2} \times 18$$

$$S_5 = 45$$

\therefore class 1 to 5 students plant 45 trees.

ii. $S_n = \frac{n}{2} \{2a + (n - 1)d\}$

$$S_{12} = \frac{12}{2} \{2 \times 3 + (12 - 1)3\}$$

$$S_{12} = 6 \{6 + 33\}$$

$$S_{12} = 6 \times 39$$

$$S_{12} = 234$$

\therefore total no of trees planted by school = 234

- iii. 30

OR

\therefore Class 12th has 3 sections and each section plants 12 trees.

\therefore total no of trees = 12×3

= 36 trees.

37. i. Consider the house is at origin (0, 0), then coordinates of grocery store, electrician's shop, food cart and bus stand are respectively (2, 3), (-4, -6), (6, -8) and (-6, 8).

Since, grocery store is at (2, 3) and food cart is at (6, -8)

$$\therefore \text{Required distance} = \sqrt{(6 - 2)^2 + (-8 - 3)^2}$$

$$= \sqrt{4^2 + 11^2} = \sqrt{16 + 121} = \sqrt{137} \text{ cm}$$

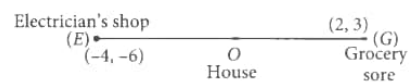
- ii. Consider the house is at origin (0, 0), then coordinates of the grocery store, electrician's shop, food cart and bus stand are respectively (2, 3), (-4, -6), (6, -8) and (-6, 8).

Required distance

$$= \sqrt{(-6)^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10 \text{ cm}$$

- iii. Consider the house is at origin (0, 0), then coordinates of grocery store, electrician's shop, food cart and bus stand are respectively (2, 3), (-4, -6), (6, -8) and (-6, 8).

Let O divides EG in the ratio k: 1, then



$$0 = \frac{2k-4}{k+1}$$

$$\Rightarrow 2k = 4$$

$$\Rightarrow k = 2$$

Thus, O divides EG in the ratio 2 : 1

Hence, required ratio = OG : OE i.e., 1 : 2.

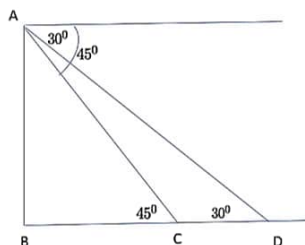
OR

Consider the house is at origin (0, 0), then coordinates of grocery store, electrician's shop, food cart and bus stand are respectively (2, 3), (-4, -6), (6, -8) and (-6, 8).

Since, (0, 0) is the mid-point of (-6, 8) and (6, -8), therefore both bus stand and food cart are at equal distances from the house.

Hence, required ratio is 1 : 1.

38. i.



The distance of ship from the base of the light house after 6 seconds from the initial position when angle of depression is 45°.

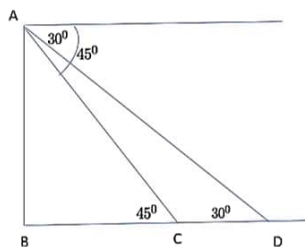
In $\triangle ABC$

$$\tan 45^\circ = \frac{AB}{BC}$$

$$\Rightarrow 1 = \frac{40}{BC}$$

$$\Rightarrow BC = 40 \text{ m}$$

ii.



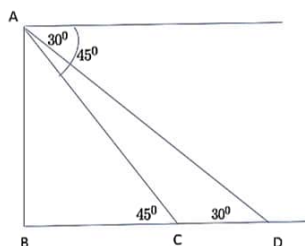
The distance between two positions of ship after 6 seconds

$$CD = BD - BC$$

$$\Rightarrow CD = 40\sqrt{3} - 40 = 40(\sqrt{3} - 1)$$

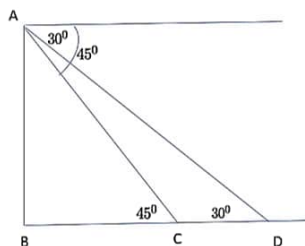
$$\Rightarrow CD = 29.28 \text{ m}$$

iii.



$$\text{Speed of ship} = \frac{\text{Distance}}{\text{Time}} = \frac{29.28}{6} = 4.88 \text{ m/sec}$$

OR



The distance of ship from the base of the light house when angle of depression is 30°.

In $\triangle ABD$

$$\tan 30^\circ = \frac{AB}{BD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{40}{BD}$$

$$\Rightarrow BD = 40\sqrt{3} \text{ m}$$