Class X Session 2025-26
Subject - Mathematics (Standard)
Sample Question Paper - 03

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

Read the following instructions carefully and follow them:

1. This question paper contains 38 questions.
2. This Question Paper is divided into 5 Sections A, B, C, D and E.
3. In Section A, Questions no. 1-18 are multiple choice questions (MCQs) and questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21-25 are very short answer (VSA) type questions, carrying 02 marks each.
5. In Section C, Questions no. 26-31 are short answer (SA) type questions, carrying 03 marks each.
6. In Section D, Questions no. 32-35 are long answer (LA) type questions, carrying 05 marks each.
7. In Section E, Questions no. 36-38 are case study-based questions carrying 4 marks each with sub-parts of the values of 1,1 and 2 marks each respectively.
8. All Questions are compulsory. However, an internal choice in 2 Questions of Section B, 2 Questions of Section C and 2 Questions of Section D has been provided. An internal choice has been provided in all the 2 marks questions of Section E.
9. Draw neat and clean figures wherever required.
10. Take $\pi = 22/7$ wherever required if not stated.
11. Use of calculators is not allowed.

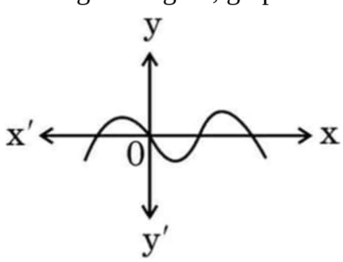
Section A

1. (HCF $\times$ LCM) for the numbers 70 and 40 is: **[1]**

   a) 280 b) 70

   c) 10 d) 2800

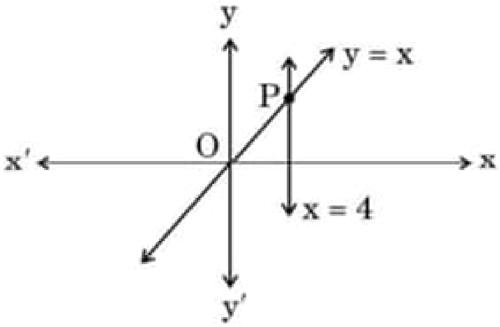
2. In the given figure, graph of a polynomial p(x) is given. Number of zeroes of p(x) is: **[1]**



   a) 5 b) 4

   c) 3 d) 2

3. The lines represented by the linear equations y = x and x = 4 intersect at P. The coordinates of the point P are: **[1]**



a) (4, 4)

b) (-4, 4)

c) (4, 0)

d) (0, 4)

4. If the roots of equation $ax^2 + bx + c = 0, a \neq 0$ are real and equal, then which of the following relation is true? **[1]**

a) $c = \frac{b^2}{a}$

b) $ac = \frac{b^2}{4}$

c) $a = \frac{b^2}{c}$

d) $b^2 = ac$

5. The sum of the first 100 even natural numbers is: **[1]**

a) 2550

b) 10100

c) 5050

d) 10010

6. The coordinates of the point P dividing the line segment joining the points A (1, 3) and B(4, 6) in the ratio 2: 1 are **[1]**

a) (4, 2)

b) (2, 4)

c) (5, 3)

d) (3, 5)

7. The mid-point of the line segment joining the points (-1, 3) and (8, $\frac{3}{2}$) is: **[1]**

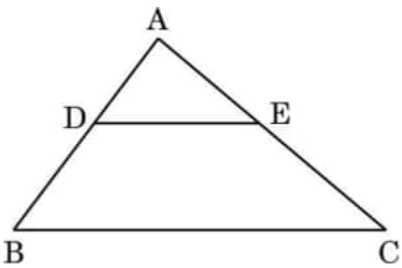
a) $\left(\frac{7}{2}, -\frac{3}{4}\right)$

b) $\left(\frac{7}{2}, \frac{9}{4}\right)$

c) $\left(\frac{9}{2}, -\frac{3}{4}\right)$

d) $\left(\frac{7}{2}, \frac{9}{2}\right)$

8. In $\Delta\mathrm{ABC}, \mathrm{DE} || \mathrm{BC}$ (as shown in the figure). If AD = 4 cm, AB = 9 cm and AC = 13.5 cm, then the length of EC is: **[1]**



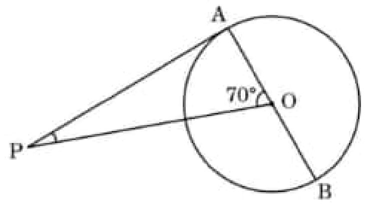
a) 5.7 cm

b) 7.5 cm

c) 9 cm

d) 6 cm

9. In the given figure, PA is a tangent from an external point P to a circle with centre O. If $\angle$AOP = 70°, then the measure of $\angle$APO is: **[1]**



a) 90^o

b) 110^o

c) 20^o

d) 70^o

10. In a right triangle ABC, right angled at B, BC = 12 cm and AB = 5 cm. The radius of the circle inscribed in the triangle (in cm) is **[1]**

a) 1

b) 3

c) 4

d) 2

11. If cos $\theta = \frac{1}{\sqrt{2}}$ then tan θ is equal to **[1]**

a) 0

b) $\sqrt{2} + 1$

c) 1

d) $\frac{1}{\sqrt{2}}$

12. For $\theta = 30^o$, the value of (2 sin θ cos θ) is: **[1]**

a) 1

b) $\frac{3}{2}$

c) $\frac{\sqrt{3}}{2}$

d) $\frac{\sqrt{3}}{4}$

13. The string of a kite in air is 50 m long and it makes an angle of 60^o with the horizontal. Assuming the string to be straight, the height of the kite from the ground is: **[1]**

a) $50\sqrt{3}$ m

b) $25\sqrt{3}$ m

c) $\frac{100}{\sqrt{3}}$ m

d) $\frac{50}{\sqrt{3}}$ m

14. Area of a segment of a circle of radius r and central angle 90^o is: **[1]**

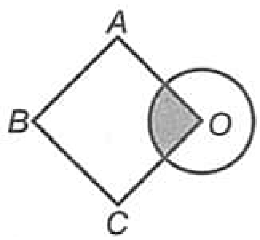
a) $\frac{2\pi r}{4} - \frac{1}{2}r^2$

b) $\frac{\pi r^2}{2} - \frac{1}{2}r^2$

c) $\frac{\pi r^2}{4} - \frac{1}{2}r^2$

d) $\frac{2\pi r}{4} - r^2 \sin 90^o$

15. O is the centre of a circle of diameter 4 cm and OABC is a square, if the shaded area is $\frac{1}{3}$ area of the square, then the side of the square is _______. **[1]**



a) $\pi\sqrt{3}$ cm

b) $\sqrt{3\pi}$ cm

c) 3π cm

d) $3\sqrt{\pi}$ cm

16. If a digit is chosen at random from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9; then the probability that this digit is an odd prime number is: **[1]**

a) $\frac{4}{9}$

b) $\frac{1}{3}$

c) $\frac{5}{9}$

d) $\frac{2}{3}$

17. A die is rolled once. The probability that a composite number comes up, is: **[1]**

a) $\frac{1}{3}$

b) $\frac{1}{2}$

c) $\frac{2}{3}$

d) 0

18. The time, in seconds, taken by 150 athletes to run a 100 m hurdle race are tabulated below: **[1]**

Time (sec.)	13 - 14	14 - 15	15 - 16	16 - 17	17 - 18	18 - 19
Number of Athletes	2	4	5	71	48	20

The number of athletes who completed the race in less than 17 seconds is

a) 11

b) 82

c) 71

d) 68

19. **Assertion (A):** In a solid hemisphere of radius 10 cm, a right cone of same radius is removed out. The volume of the remaining solid is 523.33 cm^3 [Take π = 3.14 and $\sqrt{2}$ = 1.4] **[1]**

Reason (R): Expression used here to calculate volume of remaining solid = Volume of hemisphere - Volume of cone

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Common difference of the A.P. 5, 1, -3, -7 ... is 4. **[1]**

Reason (R): Common difference of the A.P. a_1, a_2, a_3 ... an is obtained by d = a_n - a_{n-1}.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

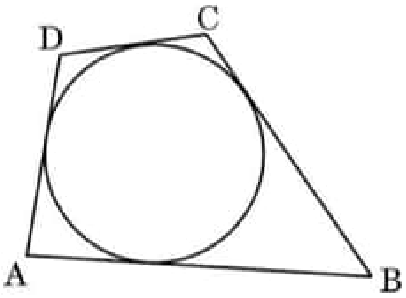
d) A is false but R is true.

Section B

21. Define a prime number and a composite number. Hence explain why $7 \times 11 \times 13$ + 13 is a composite number. **[2]**

22. ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$. **[2]**

23. A quadrilateral ABCD is drawn to circumscribe a circle, as shown in the figure. Prove that AB + CD = AD + BC. **[2]**



24. Prove that: $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$ **[2]**

OR

Prove that: $\frac{\cot\theta + cosec\theta - 1}{\cot\theta - cosec\theta + 1} = \frac{1+\cos\theta}{\sin\theta}$

25. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes. **[2]**

OR

A sector is cut from a circle of radius 21 cm. The central angle of the sector is 150^o. Find the length of the arc of this sector and the area of the sector.

Section C

26. Prove that $\sqrt{2}$ is an irrational number. **[3]**

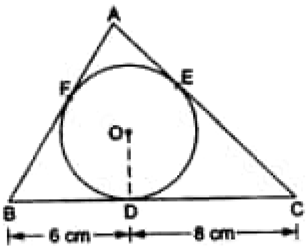
27. One zero of the polynomial $x^2 - 2x - (7p + 3)$ is -1, find the value of p and the other zero. **[3]**

28. Find the sum of all two-digit natural numbers which are divisible by 4. **[3]**

OR

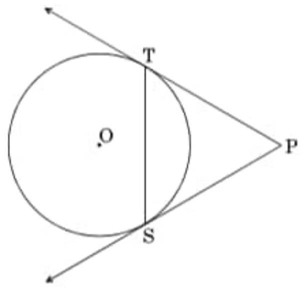
If the ratio of the sums of first n terms of two A.P.'s is (7n + 1) : (4n + 27), find the ratio of their m^{th} terms.

29. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 6 cm and 8cm respectively. Find the lengths of the sides AB and AC. **[3]**



OR

In the given figure, PT and PS are tangents to a circle with centre O, from a point P, such that PT = 4 cm and $\angle TPS = 60^o$. Find the length of the chord TS. Also, find the radius of the circle.



30. If $1 + \sin^2\theta = 3 \sin\theta \cos\theta$, then prove that $\tan\theta = 1$, or $\frac{1}{2}$. **[3]**

31. Find the mean of the following frequency distribution: **[3]**

Class	Frequency
0-10	12
10-20	18
20-30	27
30-40	20
40-50	17
50-60	6

Section D

32. A girl is twice as old as her sister. Four years hence, the product of their ages (in years) will be 160. Find their present ages **[5]**

OR

The sum of the ages of a father and his son is 45 years. Five years ago, the product of their ages was four times the father's age at that time. Find their present ages.

33. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30^{o}, which is approaching the foot of the tower with a uniform speed. Ten seconds later, the angle of depression of the car is found to be 60^{o}. Find the time taken by the car to reach the foot of the tower from this point. **[5]**

34. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area and the volume of the vessel. **[5]**

OR

A hemispherical depression is cut out from one face of a cubical block of side 7 cm, such that the diameter of the hemisphere is equal to the edge of the cube. Find the surface area of the remaining solid.

35. The marks obtained by 45 students of a class in a test are given below: **[5]**

Marks	40 - 45	45 - 50	50 - 55	55 - 60	60 - 65	65 - 70
No. of Students	8	9	10	9	5	4

Find the mean and median marks.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

TOWER OF PISA : To prove that objects of different weights fall at the same rate, Galileo dropped two objects with different weights from the Leaning Tower of Pisa in Italy. The objects hit the ground at the same time. An object dropped off the top of Leaning Tower of Pisa falls vertically with constant acceleration. If s is the distance of the object above the ground (in feet) t seconds after its release, then s and t are related by an equation of the form $s = a + bt^2$ where a and b are constants. Suppose the object is 180 feet above the ground 1 second after its release and 132 feet above the ground 2 seconds after its release.

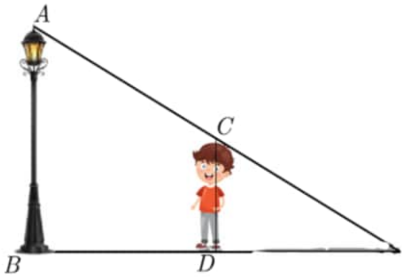
i. Find the constants a and b. (1)
ii. How high is the Leaning Tower of Pisa? (1)
iii. How long does the object fall? (2)

OR

At t = 2 sec, the object is at what height? (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Priyanshu is very intelligent in maths. He always try to relate the concept of maths in daily life. One day he is walking away from the base of a lamp post at a speed of 1 m/s. Lamp is 4.5 m above the ground.



i. If after 2 second, length of shadow is 1 meter, what is the height of Priyanshu? (1)

ii. What is the minimum time after which his shadow will become larger than his original height? (1)

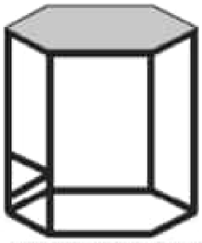
iii. What is the distance of Priyanshu from pole at this point? (2)

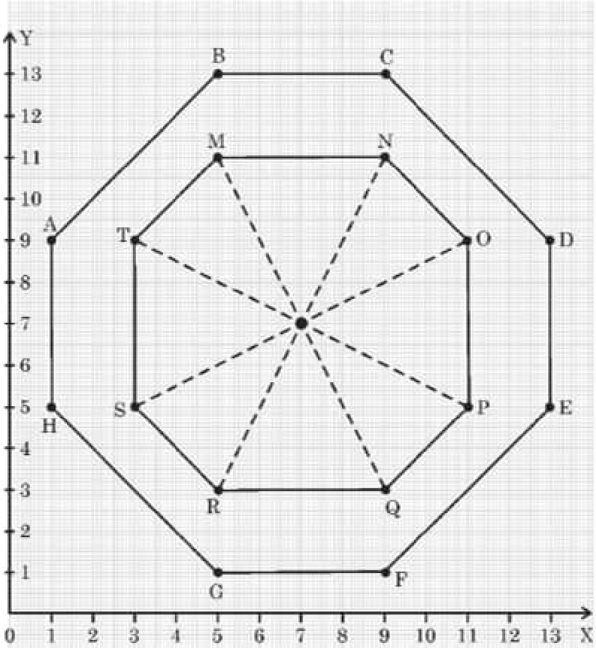
OR

What will be the length of his shadow after 4 seconds? (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

The top of a table is hexagonal in shape.





On the basis of the information given above, answer the following questions:

i. Write the coordinates of A and B.

ii. Write the coordinates of the mid-point of line segment joining C and D.

iii. a. Find the distance between M and Q.

OR

b. Find the coordinates of the point which divides the line segment joining M and N in the ratio 1:3 internally.

Solution

Section A

1.

(d) 2800

Explanation:

Given

Numbers are 70 and 40

We know that, HCF $\times$ LCM = Product of numbers

So, HCF $\times$ LCM = 70 $\times$ 40 = 2800.

2.

(b) 4

Explanation:

Number of zeros= number of times the graph touches x-axis.

Here the graph touches x-axis 4 times.

3. **(a)** (4, 4)

Explanation:

(4, 4)

4.

(b) $ac = \frac{b^2}{4}$

Explanation:

If root are real and equal, then D = 0

b^2 - 4ac = 0

b^2 = 4ac

$\frac{b^2}{4}$ = ac

5.

(b) 10100

Explanation:

The number series 2, 4, 6, 8, 10, 12, ..., 200.

a = 2, d = 2, n = 100

$S_n = \frac{n}{2} \times (a + t_n)$

$= \frac{100}{2} \times (2 + 200)$

$= \frac{(100 \times 202)}{2}$

$= \frac{20200}{2}$

2 + 4 + 6 + 8 + 10 + 12 + ...+ 200 = 10100

6.

(d) (3, 5)

Explanation:

Point P divides the line segment joining the points A(1, 3) and B(4, 6) in the ratio 2: 1

Let coordinates of P be (x, y), then

$x = \frac{m_1x_2+m_2x_1}{m_1+m_2} = \frac{2\times4+1\times1}{2+1} = \frac{8+1}{3} = \frac{9}{3} = 3$

$y = \frac{m_1y_2+m_2y_1}{m_1+m_2} = \frac{2\times6+1\times3}{2+1} = \frac{12+3}{3} = \frac{15}{3} = 5$

$\therefore$ Coordinates of P are (3, 5)

7.

(b) $\left(\frac{7}{2}, \frac{9}{4}\right)$

Explanation:

Cordinates of mid points are $\left[\frac{(8-1)}{2}, \left(\frac{3+\left(\frac{3}{2}\right)}{2}\right)\right]$

$= (\frac{7}{2}, \frac{9}{4})$

8.

(b) 7.5 cm

Explanation:

By BPT

$\frac{AD}{AB} = \frac{AE}{AC}$

$\frac{4}{9} = \frac{AE}{13.5}$

$\frac{13.5 \times 4}{9} = AE$

AE = 6 cm

EC = AC - AE

= 13.5 - 6

EC = 7.5 cm

9.

(c) 20^{o}

Explanation:

We know that,

Radius is $\perp$ to the tangent at point of contact, So

$\angle PAO = 90^{\circ}$

And $\angle AOP = 70^{\circ}$ (given)

Now,

In $\triangle APO$,

$\angle PAO + \angle AOP + \angle APO = 180^{\circ}$

$\Rightarrow 90 + 70 + \angle APO = 180^{\circ}$

$\Rightarrow 160^{\circ} + \angle APO = 180^{\circ}$

$\Rightarrow \angle APO = 180 - 160^{o}$

$\Rightarrow \angle APO = 20^{o}$

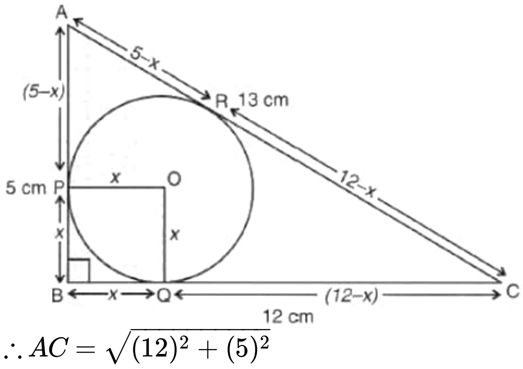
10.

(d) 2

Explanation:

Here, AB = 5cm, BC = 12 and $\angle B = 90^{\circ}$

Let the radius of circle be x cm



$\therefore AC = \sqrt{(12)^2 + (5)^2}$

$= \sqrt{144 + 25}$

$= \sqrt{169} = 13\text{cm}$

$\because$ AC = AR + RC

$\therefore$ AC = (5 - x) + 12 - x

$\Rightarrow$ 13 = 5 - x + 12 - x

$\Rightarrow$ 2x = 17 - 13 = 4

$\Rightarrow \quad x = \frac{4}{2} = 2\text{cm}$

Hence, radius of the circle = 2cm.

11.

(c) 1

Explanation:

$\cos\theta = \frac{1}{\sqrt{2}}$

$\cos\theta = \cos 45^\circ$

$\theta = 45^\circ$

$\text{Tan}\,\theta = \text{Tan}\,45^\circ$

= 1

12.

(c) $\frac{\sqrt{3}}{2}$

Explanation:

$2\sin 30^\circ \cos 30^\circ$

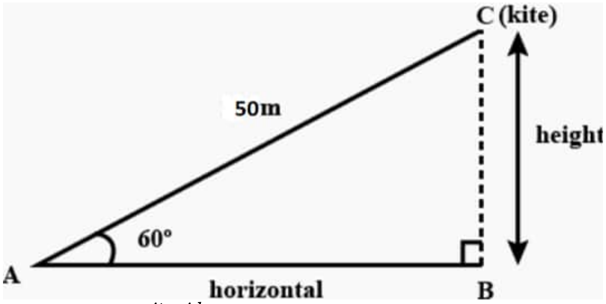
$= 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{2}$

13.

(b) $25\sqrt{3}$ m

Explanation:



Sin(θ) = $\frac{opposite side}{hypotenuse}$

son60$^{\text{o}}$ = $\frac{\text{BC}}{\text{AC}}$ = $\frac{h}{50}$

$\frac{\sqrt{3}}{2}$ = $\frac{h}{50}$ ($\because$ sin60$^{\text{o}}$ = $\frac{\sqrt{3}}{2}$)

h = $25\sqrt{3}$ m

14.

(c) $\frac{\pi r^2}{4} - \frac{1}{2}r^2$

Explanation:

$\frac{\pi r^2}{4} - \frac{1}{2}r^2$

15.

(b) $\sqrt{3\pi}$ cm

Explanation:

Let the length of side of square be x cm

Then area of square = x^2 cm^2

Area of sector of circle

= $\frac{\theta}{360^{\circ}} \times \pi r^2$

= $\frac{90^{\circ}}{360^{\circ}} \times \pi r^2$ [$\because$ angle of square = θ = 90^{o}]

$\therefore$ Shaded area = $\frac{\pi \times 4}{4}$ = π

According to question, Area of square = 3 $\times$ shaded area

$\Rightarrow$ 3π = x^2 $\therefore$ x = $\sqrt{3\pi}$ cm

16.

(b) $\frac{1}{3}$

Explanation:

p(odd prime no.)

fav case $\rightarrow$ 3, 5, 7

p(odd prime no.) $= \frac{3}{9} = \frac{1}{3}$

17. **(a)** $\frac{1}{3}$

Explanation:

The composite numbers among the numbers on a die are 4 and 6.

Thus, we have 2 favourable outcomes out of a total of 6 outcomes.

Hence, the required probability is $\frac{2}{6}$ = $\frac{1}{3}$.

18.

(b) 82

Explanation:

Required no. of athletes = sum of all frequencies upto 16-17

= 2 + 4 + 5 + 71

= 82

19.

(d) A is false but R is true.

Explanation:

A is false but R is true.

20.

(d) A is false but R is true.

Explanation:

A is false but R is true.

Section B

21. **Prime Number:** A number which have exactly two factors 1 and the number itself.

Composite Number: A number having more than two factors.

7 $\times$ 11 $\times$ 13 + 13 = 13(7 $\times$ 11 + 1)

= 13 $\times$ 78

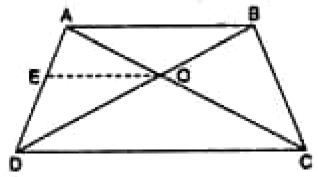
The resulting number have more then 2 factors.

Hence, it is composite.

22. Given: ABCD is a trapezium in which AB||DC.

ITs diagonals intersect each other at the point O.

To prove: $\frac{AO}{BO} = \frac{CO}{DO}$



Construction:

Through O, draw a line OE parallel to AB or DC intersecting AD at E.

Proof: In $\triangle ADC$

$\because OE||DC$

$\therefore \frac{AO}{CO} = \frac{AE}{DE}$ (1)......[By basic proportionality theorem]

In $\triangle DBA$, $\because OE||AB$

$\therefore \frac{DE}{AE} = \frac{DO}{BO}$ .....[By basic proportionality theorem]

$\Rightarrow \frac{DE}{AE} = \frac{DO}{BO}\ldots\ldots$ [By basic proportionality theorem]

From (1) and (2) $\frac{AO}{CO} = \frac{BO}{DO} \Rightarrow \frac{AO}{BO} = \frac{CO}{DO}$

23. We know that the tangents from an external point to a circle are equal.

$\therefore$ AP = AS .... (i)

BP = BQ .... (ii)

CR = CQ .... (iii)

DR = DS .... (iv)

On adding eq. (i), (ii), (iii) and (iv), we get

(AP + BP) + (CR + DR) = (AS + BQ) + (CQ + DS)

$\Rightarrow$ AB + CD = (AS + DS) + (BQ + CQ)

$\Rightarrow$ AB + CD = AD + BC

24. $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

L.H.S. $= \sin^6\theta + \cos^6\theta$

$= \left(\sin^2\theta\right)^3 + \left(\cos^2\theta\right)^3$

$= \left(\sin^2\theta + \cos^2\theta\right)\left(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta\right)\ \left[\because a^3 + b^3 = (a+b)\left(a^2 + b^2 - ab\right)\right]$

$= 1\left(\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cdot\cos^2\theta - 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta\right]$ since, $\sin^2 A + \cos^2 A = 1$

$= \left(\sin^2\theta + \cos^2\theta\right)^2 - 3\sin^2\theta\cos^2\theta$

$= 1 - 3\sin^2\theta\cos^2\theta$

= R.H.S. proved.

OR

We have,

L.H.S. = $\frac{(\cot\theta + cosec\theta) - 1}{(\cot\theta - cosec\theta + 1)}$

$= \frac{(cosec\theta + \cot\theta) - \left(cosec^2\theta - \cot^2\theta\right)}{(\cot\theta - cosec\theta + 1)}$ [$\because$ 1 = cosec$^2\theta$ - cot$^2\theta$]

$= \frac{(cosec\theta + \cot\theta)[1 - (cosec\theta - \cot\theta)]}{(\cot\theta - cosec\theta + 1)}$

$= \frac{(cosec\theta + \cot\theta)(\cot\theta - cosec\theta + 1)}{(\cot\theta - cosec\theta + 1)}$

$= (cosec\theta + \cot\theta) = \left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right) = \frac{1 + \cos\theta}{\sin\theta}$ = R.H.S.

Hence, $\frac{\cot\theta + cosec\theta - 1}{\cot\theta - cosec\theta + 1} = \frac{1 + \cos\theta}{\sin\theta}$.

25. Here, r = 14 cm and $\theta = \frac{90^\circ}{3} = 30^\circ$

$\therefore$Area swept = $\frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14$

$= \frac{154}{3}$ cm^2

OR

Given:

Radius = 21 cm

Angle of sector = 150°

Now,

Length of arc $= \frac{2\pi r\theta}{360}$

$= \frac{2 \times \frac{22}{7} \times 21 \times 150}{360}$

= 55 cm

Area of the sector $= \frac{\pi r^2\theta}{360}$

$= \frac{\frac{22}{7} \times 21 \times 21 \times 150}{360}$

$= 577.5\ cm^2$

Section C

26. We have to prove that $\sqrt{2}$ is an irrational number.

Let $\sqrt{2}$ be a rational number.

$\therefore \quad \sqrt{2} = \frac{p}{q}$

where p and q are co-prime integers and $q \neq 0$

On squaring both the sides, we get,

or, $2 = \frac{p^2}{q^2}$

or, $p^2 = 2q^2$

$\therefore$ p^2 is divisible by 2.

p is divisible by 2........(i)

Let p = 2r for some integer r

or, $p^2 = 4r^2$

$2q^2 = 4r^2$ [$\because p^2 = 2q^2$]

or, $q^2 = 2r^2$

or, q^2 is divisible by 2.

$\therefore$q is divisible by 2..........(ii)

From (i) and (ii)

p and q are divisible by 2, which contradicts the fact that p and q are co-primes.

Hence, our assumption is wrong.

$\therefore$ $\sqrt{2}$ is irrational number.

27. Let $p(x) = x^2 - 2x - (7p + 3)$

Since -1 is a zero of p(x). Therefore,

p(-1) = 0

$(-1)^2 - 2(-1) - (7p + 3)=0$

1 + 2 - 7p - 3 = 0

3 - 7p - 3 = 0

7p = 0

p = 0

Thus, $p(x) = x^2 - 2x - 3$

For finding zeros of p(x), we put,

p(x) = 0

$x^2 - 2x - 3 = 0$

$x^2 - 3x - x - 3 = 0$

x(x-3) + 1(x-3) = 0

(x-3)(x+1) = 0

Put x - 3 = 0 and x + 1 = 0, we get,

Thus, x = 3, -1

Thus, the other zero is 3.

28. All the two-digit natural numbers divisible by 4 are 12, 16, 20, 24, ..., 96

Here, $a_1 = 12$

$a_2 = 16$

$a_3 = 20$

$a_4 = 24$

$\therefore$ $a_2 - a_1 = 16 - 12 = 4$

$a_3 - a_2 = 20 - 16 = 4$

$a_4 - a_3 = 24 - 20 = 4$

$\because a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = ...$ (= 4 each)

$\therefore$ This sequence is an arithmetic progression whose common difference is 4.

Here, a = 12, d = 4, l = 96

Let the number of terms be n.

Then, l = a + (n - 1)d $\Rightarrow$ 96 = 12 + (n - 1)4

$\Rightarrow$ 96 - 12 = (n - 1) 4 $\Rightarrow$ 84 = (n - 1)4

$\Rightarrow$ (n - 1) 4 = 84 $\Rightarrow$ n - 1 = $\frac{84}{4}$

$\Rightarrow$ n - 1 = 21 $\Rightarrow$ n = 21 + 1 $\Rightarrow$ n = 22

$\therefore S_n = \frac{n}{2}(a+l) = \frac{22}{2}(12+96) = (11)(108) = 1188$

OR

Let a, and A be the first terms and d and D be the common difference of two A.Ps

Then, according to the question,

$\frac{S_n}{S_n'} = \frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}[2A+(n-1)D]} = \frac{7n+1}{4n+27}$

or, $\frac{2a+(n-1)d}{2A+(n-1)D} = \frac{7n+1}{4n+27}$

or, $\frac{a+\left(\frac{n-1}{2}\right)d}{A+\left(\frac{n-1}{2}\right)D} = \frac{7n+1}{4n+27}$

Putting, $\frac{n-1}{2} = m-1$

$n-1 = 2m-2$

$n = 2m-2+1$

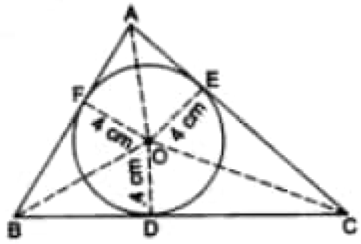
or, $n = 2m-1$

$\frac{a+(m-1)d}{A+(m-1)D} = \frac{7(2m-1)+1}{4(2m-1)+27}$

$\frac{a+(m-1)d}{A+(m-1)D} = \frac{14m-7+1}{8m-4+27}$

$\frac{a+(m-1)d}{A+(m-1)D} = \frac{14m-6}{8m+23}$

Hence, $\frac{a_m}{A_m} = \frac{14m-6}{8m+23}$

29. 

We know that the lengths of tangents drawn from an exterior point to a circle are equal.

$AE = AF = x\ cm,$

$BD = BF = 6cm, CD = CE = 8cm.$

so, AB = AF+ BF = (x + 6) cm,

BC = BD + CD = 14 cm,

AC = CE+AE = (x+8) cm.

Perimeter, 2s = AB + BC+AC

= [(x + 6) +14 + (x+8)] cm

= (2x + 28) cm

$\Rightarrow$ s = (x +14) cm.

$\therefore \quad \text{ar}(\Delta ABC) = \sqrt{s(s-AB)(s-BC)(s-AC)}$

$= \sqrt{(x+14)\{(x+14)-(x+6)\}\{(x+14)-14\}\{(x+14)-(x+8)\}}\text{cm}^2$

$= \sqrt{48x(x+14)}\text{cm}^2$ . ...(i)

Join OE and OF and also OA, OB and OC.

$\therefore \quad \text{ar}(\Delta ABC)$

$= \text{ar}(\triangle OAB) + \text{ar}(\triangle OBC) + \text{ar}(\triangle OCA)$

$= \left(\frac{1}{2} \times AB \times OF\right) + \left(\frac{1}{2} \times BC \times OD\right) + \left(\frac{1}{2} \times AC \times OE\right)$

$= \left[\frac{1}{2} \times (x+6) \times 4\right] + \left[\frac{1}{2} \times 14 \times 4\right] + \left[\frac{1}{2} \times (x+8) \times 4\right]$

$= 2[(x + 6) + 14 + (x + 8)]$
$= 4(x + 14)\text{cm}^2$. ...(ii)
From (i) and (ii), we get
$\sqrt{48x(x+14)} = 4(x + 14)$
$\Rightarrow \quad 48x(x + 14) = 16(x + 14)^2$
$\Rightarrow \quad 48x = 16(x + 14)$
$\Rightarrow x = \frac{16\times14}{32} = 7$
$\therefore \quad AB = (x + 6)\text{cm}$
$= (7 + 6)\text{cm}$
$= 13cm$
and
$AC = (x + 8)cm$
$= (7 + 8)cm$
$= 15cm$

OR

$\because$ PT = PS (tangents from an external point P)
$\therefore$ ∠PTS = ∠PST
Using Angle Sum Property in △PTS
∠PTS + ∠PST + ∠TPS = 180^o
2∠PTS = 180 - 60 = 120^o
∠PTS = 60^o
⇒ PTS Is a equilateral triangle
So, TS = 4 cm
Now, In △PTO
As PO is angle bisector of ∠TPS, ∠OTP = 90^o
tan 30^o = $\frac{OT}{TP}$
$\frac{1}{\sqrt{3}} = \frac{OT}{4}$
OT = $\frac{4}{\sqrt{3}}$
OT = $\frac{4\sqrt{3}}{3}$ cm
$\therefore$ radius of circle = $\frac{4\sqrt{3}}{3}$ cm

30. Given, 1 + $\sin^2$ θ = 3 sin θ cos θ, then we have to prove that tan θ = 1, or $\frac{1}{2}$.
Now, 1 + $\sin^2$θ = 3 sinθ cosθ
[Dividing by $\sin^2$ θ 0n both sides]
⇒ $\frac{1}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{3\sin\theta\cos\theta}{\sin^2\theta}$
⇒ cosec^2θ + 1 = 3 cotθ
⇒ 1 + $\cot^2$θ + 1 – 3 cotθ = 0
⇒ $\cot^2$ θ – 3 cot θ + 2 = 0
⇒ $\cot^2$ θ – 2 cot θ –cot θ + 2 = 0
⇒ cot θ(cot θ – 2) –1(cot θ – 2) = 0
⇒ (cotθ – 2) (cot θ – 1) = 0
⇒ cotθ – 2 = 0 or (cot θ – 1) = 0
⇒ cotθ = 2 or cot θ = 1
⇒ tanθ = $\frac{1}{2}$ or tan θ = 1
Hence, either, tan θ = $\frac{1}{2}$, or 1

31.

Class interval	Frequency	Midpoint	f_ix_i
0-10	12	5	60
10-20	18	15	270
20-30	27	25	675

30-40	20	35	700
40-50	17	45	765
50-60	6	55	330
Total	$\sum f_i$=100		$\sum f_i x_i =$ 2800

Mean= $\frac{\sum f_i x_i}{\sum f_i} = \frac{2800}{100}$ = 28

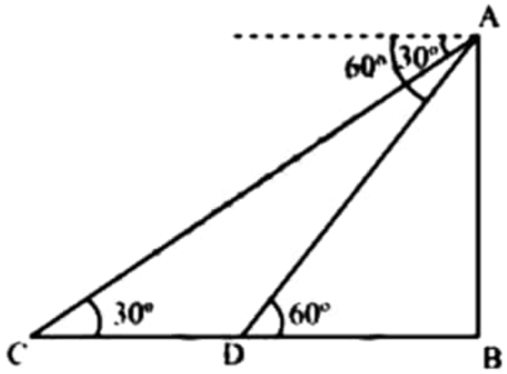
Section D

32. If the present age of sister be x,then ,by the first condition of the question,we have,
present age of the girl = 2x
By the second condition of the question,we have,
(2x + 4)(x + 4) = 160
$2x^2$ + 8x + 4x + 16 = 160
$2x^2$ + 12x -144 = 0
$2x^2$ + (24 - 12)x - 144 = 0
2x(x + 12) - 12(x + 12) = 0
(2x - 12)(x +12) = 0
$\therefore$ x = 6; x = -12
Since age can't be negative,therefore
x=6
So, Age of sister = 6 and Age of girl = 2(6)=12

OR

Let the age of father = x years
age of son = (45 - x) years
(x - 5) (45 - x - 5) = 4(x - 5)
On Solving
x = 36
Age of father = 36 years
Age of son = 9 years

33. Let AB be the tower.



Initial position of the car is C, which changes to D after ten seconds.
In $\triangle$ADB,
$\frac{AB}{DB}$ = tan 60°
$\frac{AB}{DB} = \sqrt{3}$
In $\triangle$ABC,
$\frac{AB}{BC}$ = tan 30°
$\frac{AB}{BD+DC} = \frac{1}{\sqrt{3}}$
$AB\sqrt{3}$ = BD + DC
$AB\sqrt{3} = \frac{AB}{\sqrt{3}} + DC$
DC = $AB\sqrt{3} - \frac{AB}{\sqrt{3}}$ = $AB\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$

$= \frac{2AB}{\sqrt{3}}$

Time taken by the car to travel distance DC$\left(\text{ i.e } \frac{2AB}{\sqrt{3}}\right)$ = 10 second

Time taken by the car to travel distance DB$\left(\text{ i.e } \frac{AB}{\sqrt{3}}\right) = 10 \times \frac{\sqrt{3}}{2\sqrt{3}}$

= $\frac{10}{2}$ = 5 seconds

34. Radius = 7 cm

Height of cylindrical portion = 13 - 7 = 6 cm

Inner surface area of the vessel = $2\pi r^2 + 2\pi rh$

= $2 \times \frac{22}{7} \times 7 \times 7 + 2 \times \frac{22}{7} \times 7 \times 6$

= 572 cm^2

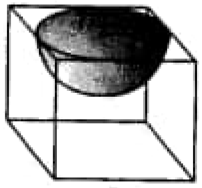
Volume of the vessel = $\frac{2}{3}\pi r^3 + \pi r^2 h$

= $\frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 + \frac{22}{7} \times 7 \times 7 \times 6$

= $\frac{4928}{3}$ or 1642.67 cm^3 approx.

Therefore, inner surface area and volume of the vessel is 572 cm^2 and 1642.67 cm^3 respectively.

OR



Edge of the cube, a = 7 cm.

Radius of the hemisphere, $r = \frac{7}{2}$cm.

Surface area of remaining solid

= total surface area of the cube - area of the top of hemispherical part + curved surface area of the hemisphere

$= 6a^2 - \pi r^2 + 2\pi r^2 = 6a^2 + \pi r^2$

$= \left(6 \times 7 \times 7 + \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right) \text{cm}^2$

= (294 + 38.5)cm^2 = 332.5 cm^2.

35.

Marks	**x**	**f**	$u = \frac{x-52\cdot5}{5}$	**fu**	**cf**
40 - 45	42.5	8	-2	-16	8
45 - 50	47.5	9	-1	-9	17
50 - 55	52.5	10	0	0	27
55 - 60	57.5	9	1	9	36
60 - 65	62.5	5	2	10	41
65 - 70	67.5	4	3	12	45
		45		6	

Mean = 52.5 + 5 $\times \frac{6}{45}$ = 53.2 (approx)

Median = 50 + $\frac{5}{10}$ (22.5 - 17)

= 52.75

Section E

36. i. S = a + bt^2

At t = 1 sec

180 = a + b ...(i)

At t = 2 sec

132 = a + 4b ...(ii)

from (i) and (ii)

180 - 132 = -3b

48 = -3b

b = -16

Put b = -16, in equation (i)

180 = a + (-16)

a = 196

ii. At t = 0

s = a + b(0)

s = a

s = 196

i.e., The height of Tower of Pisa = 196 feet

iii. $s = a + bt^2$

$0 = 196 - 16t^2$

$-196 = -16t^2$

$196 \div 16 = t$

$t = \frac{14}{4}$

t = 3.5 sec

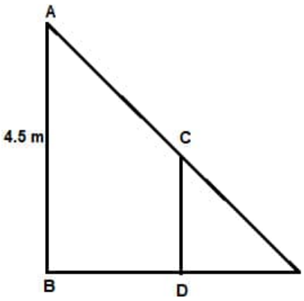
OR

$s = a + bt^2$

$s = 196 + (-16)(2)^2$

s = 196 - 64

s = 132 feet

37. i. 

Distance covered in 2 sec = 2 m

length of shadow = 1 m

Total distance from base = 2 + 1 = 3m

$\frac{1}{3} = \frac{\text{height of Rohan}}{4.5}$

height of Rohan = 1.5 m

= 150 cm

ii. When x > 1.5 m

distance walked = d m

$\Rightarrow \frac{x}{d+x} = \frac{1.5}{4.5}$

$\Rightarrow \frac{x}{d+x} = \frac{1}{3}$

2x = d

d > 3

hence, the time must be 3 sec

∴ minimum time after which his shadow become larger than his original height = 3 s

iii. 3 metres

OR

After 4 and distance = 4 m

Shadow length = y m

$\frac{y}{4+y} = \frac{1.5}{4.5}$

3y = 4 + y

y = 2 m

∴ After 4 sec, the shadow length will be 2 m

38. i. A(1, 9) and B(5, 13)

ii. C(9, 13) and D(13, 9)

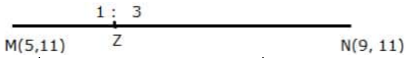
Mid-point of CD is (11, 11)

iii. a. M(5, 11) and Q(9, 3)

MQ = $\sqrt{(9-5)^2+(3-11)^2} = \sqrt{80}$ or $4\sqrt{5}$

OR

b. M(5, 11) and N(9, 11)



$Z\left(\frac{1\times9+3\times5}{1+3}, \frac{1\times11+3\times11}{1+3}\right)$

$Z(6, 11)$