Class X Session 2025-26
Subject - Mathematics (Standard)
Sample Question Paper - 06

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

Read the following instructions carefully and follow them:

1. This question paper contains 38 questions.
2. This Question Paper is divided into 5 Sections A, B, C, D and E.
3. In Section A, Questions no. 1-18 are multiple choice questions (MCQs) and questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21-25 are very short answer (VSA) type questions, carrying 02 marks each.
5. In Section C, Questions no. 26-31 are short answer (SA) type questions, carrying 03 marks each.
6. In Section D, Questions no. 32-35 are long answer (LA) type questions, carrying 05 marks each.
7. In Section E, Questions no. 36-38 are case study-based questions carrying 4 marks each with sub-parts of the values of 1,1 and 2 marks each respectively.
8. All Questions are compulsory. However, an internal choice in 2 Questions of Section B, 2 Questions of Section C and 2 Questions of Section D has been provided. An internal choice has been provided in all the 2 marks questions of Section E.
9. Draw neat and clean figures wherever required.
10. Take $\pi = 22/7$ wherever required if not stated.
11. Use of calculators is not allowed.

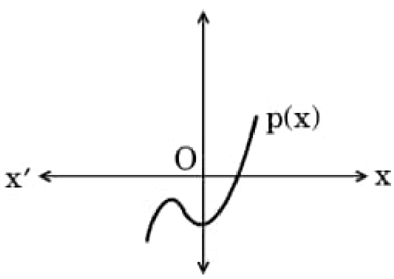
Section A

1. The prime factorisation of 432 is: **[1]**

a) $2^4 \times 3^3$ b) $2^3 \times 3^4$

c) $2^3 \times 3^3$ d) $2^4 \times 3^4$

2. Number of zeroes of the polynomial p(x) shown in the Figure, are: **[1]**



a) 2 b) 3

c) 0 d) 1

3. The pair of linear equations y = 0 and y = - 6 has: **[1]**

a) no solution b) only solution (0, 0)

c) a unique solution d) infinitely many solutions

4. The roots of the quadratic equation x^2 - 4 = 0 is/are: **[1]**

a) 2 only b) -2, 2

c) -4, 4 d) 4 only

5. The nth term of an AP is 7 – 4n, then its common difference is **[1]**

a) – 3 b) 4

c) – 4 d) 3

6. The distance between the points P $\left(-\frac{11}{3}, 5\right)$ and Q $\left(-\frac{2}{3}, 5\right)$ is: **[1]**

a) 3 units b) 4 units

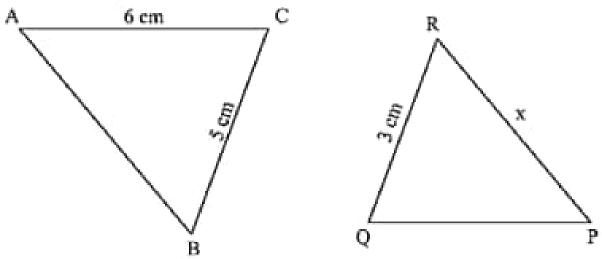
c) 2 units d) 6 units

7. The distance between the points (0, 0) and (a - b, a + b) is **[1]**

a) $2\sqrt{ab}$ b) $\sqrt{2a^2 + ab}$

c) $2\sqrt{a^2 + b^2}$ d) $\sqrt{2a^2 + 2b^2}$

8. **[1]**

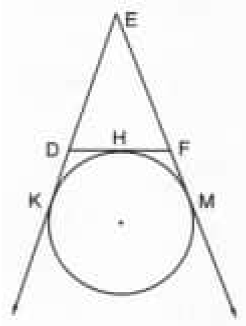


In the given figure, $\triangle ABC \sim \triangle QPR$. If AC = 6 cm, BC = 5 cm, QR = 3 cm and PR = x; then the value of x is:

a) 3.6 cm b) 2.5 cm

c) 3.2 cm d) 10 cm

9. In Figure, a circle touches the side DF of $\triangle EDF$ at H and touches ED and EF produced at K and M respectively. If EK = 9 cm, then the perimeter of $\triangle EDF$ is **[1]**



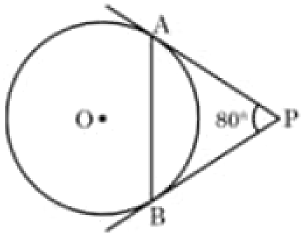
a) 12 cm

b) 13.5 cm

c) 9 cm

d) 18 cm

10. In the given figure, tangents PA and PB drawn from P to circle are inclined to each other at an angle of 80^o. The measure of $\angle$PAB is **[1]**



a) 80^o

b) 40^o

c) 50^o

d) 60^o

11. $(\sec^2\theta - 1)(1 - \operatorname{cosec}^2\theta)$ is equal to: **[1]**

a) 1

b) 2

c) -2

d) -1

12. sec θ when expressed in terms of cot θ, is equal to: **[1]**

a) $\frac{\sqrt{1-\cot^2\theta}}{\cot\theta}$

b) $\sqrt{1+\cot^2\theta}$

c) $\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$

d) $\frac{1+\cot^2\theta}{\cot\theta}$

13. The height of a tower is 20 m. The length of its shadow made on the level ground when the Sun's altitude is 60^o, is: **[1]**

a) 20 m

b) $\frac{20}{3}$ m

c) $20\sqrt{3}$ m

d) $\frac{20}{\sqrt{3}}$ m

14. If the area of a sector of a circle is $\frac{1}{8}$ of the area of the circle, then the central angle of the sector is: **[1]**

a) 60^o

b) 90^o

c) 45^o

d) 30^o

15. The area of the sector of a circle of radius 10.5 cm is 69.3 cm^2. Find the central angle of the sector. **[1]**

a) 85^o

b) 70^o

c) 72^o

d) 26^o

16. If P (A) denotes the probability of an event A, then **[1]**

a) $P(A) < 0$ b) $0 \leq P(A) \leq 1$

c) $-1 \leq P(A) \leq 1$ d) $P(A) > 1$

17. A dice is thrown once. The probability of getting an odd number is **[1]**

a) $\frac{2}{6}$ b) $\frac{4}{6}$

c) 1 d) $\frac{1}{2}$

18. For the following distribution: **[1]**

Marks Below	10	20	30	40	50	60
Number of Students	3	12	27	57	75	80

the modal class is:

a) 20 – 30 b) 40 – 50

c) 50 – 60 d) 30 – 40

19. **Assertion (A):** A sphere of radius 7 cm is mounted on the solid cone of radius 6 cm and height 8 cm. The volume of the combined solid is 1737.97 cm^3. **[1]**

Reason (R): Volume of sphere is $\frac{4}{3}\pi r^3$.

a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

20. **Assertion (A):** The constant difference between any two terms of an AP is commonly known as common difference. **[1]**

Reason (R): The common difference of 2, 4, 6, 8 this A.P. is 2.

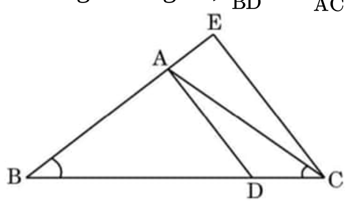
a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

Section B

21. Prove that $5 + 2\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number. **[2]**

22. In the given figure, $\frac{BC}{BD} = \frac{BE}{AC}$ and $\angle ABD = \angle ACD$. Show that $\triangle ABD \sim \Delta EBC$. **[2]**



23. Prove that the tangents drawn at the ends of a diameter of a circle are parallel. **[2]**

24. Evaluate: $2(\sin^2 45^o + \cot^2 30^o) - 6(\cos^2 45^o - \tan^2 30^o)$ **[2]**

OR

Prove that $\frac{\cot A - \cos A}{\cot A + \cos A} = \sec^2 A + \tan^2 A - 2 \sec A \tan A$.

25. Find the area of the minor and the major sectors of a circle with radius 6 cm, if the angle subtended by the minor arc at the centre is 60^o. (Use $\pi = 3.14$) **[2]**

OR

An arc of a circle of radius 10 cm subtends a right angle at the centre of the circle. Find the area of the corresponding major sector. (Use $\pi = 3.14$)

Section C

26. 144 cartons of Coke cans and 90 cartons of Pepsi cans are to be stacked in a canteen. If each stack is of the same height and if it equal contain cartons of the same drink, what would be the greatest number of cartons each stack would have? **[3]**

27. If α and β are zeroes of the quadratic polynomial $4x^2 + 4x + 1$, then form a quadratic polynomial whose zeroes are 2α and 2β. **[3]**

28. Find the sum of the first 15 multiples of 8. **[3]**

OR

If S_n denotes, the sum of the first n terms of an A.P. prove that $S_{12} = 3(S_8 - S_4)$.

29. Two concentric circles are of radii 7 cm and r cm respectively, where r >7. A chord of the larger circle of length 46 cm, touches the smaller circle. Find the value of r. **[3]**

OR

Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.

30. Prove that $\frac{\tan\theta}{1-\tan\theta} - \frac{\cot\theta}{1-\cot\theta} = \frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$. **[3]**

31. Find the mean of the following data, using step-deviation method: **[3]**

Class	5 - 15	15 - 25	25 - 35	35 - 45	45 - 55	55 - 65	65 - 75
Frequency	6	10	16	15	24	8	7

Section D

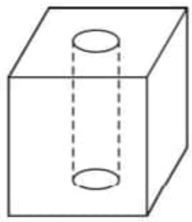
32. A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m. Find the length and breadth of the park. **[5]**

OR

If the roots of the quadratic equation (x - a) (x - b) + (x - b)(x - c) + (x - c)(x - a) = 0 are equal. Then show that a = b = c

33. From the top of an 8 m high building, the angle of elevation of the top of a cable tower is 60^o and the angle of depression of its foot is 45^o. Determine the height of the tower. (Take $\sqrt{3} = 1.732$). **[5]**

34. In Figure, from a solid cube of side 7 cm, a cylinder of radius 2.1 cm and height 7 cm is scooped out. Find the total surface area of the remaining solid. **[5]**



OR

A tent is in the shape of a right circular cylinder up to a height of 3 m and then a right circular cone, with a maximum height of 13.5 m above the ground. Calculate the cost of painting the inner side of the tent at the rate of ₹ 2 per square metre, if the radius of the base is 14 m.

35. Find the mode, median and mean for the following data: **[5]**

Marks Obtained	25 - 35	35 - 45	45 - 55	55 - 65	65 - 75	75 - 85
Number of students	7	31	33	17	11	1

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

Lokesh, a production manager in Mumbai, hires a taxi everyday to go to his office. The taxi charges in Mumbai consists of a fixed charges together with the charges for the distance covered. His office is at a distance of 10 km from his home. For a distance of 10 km to his office, Lokesh paid ₹ 105. While coming back home, he took another route. He covered a distance of 15 km and the charges paid by him were ₹ 155.

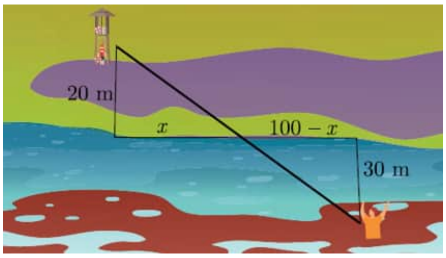
i. What are the fixed charges? (1)
ii. What are the charges per km? (1)
iii. If fixed charges are ₹ 20 and charges per km are ₹ 10, then how much Lokesh have to pay for travelling a distance of 10 km? (2)

OR

Find the total amount paid by Lokesh for travelling 10 km from home to office and 25 km from office to home. [Fixed charges and charges per km are as in (i) & (ii). (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Swimmer in Distress: A lifeguard located 20 metre from the water spots a swimmer in distress. The swimmer is 30 metre from shore and 100 metre east of the lifeguard. Suppose the lifeguard runs and then swims to the swimmer in a direct line, as shown in the figure.



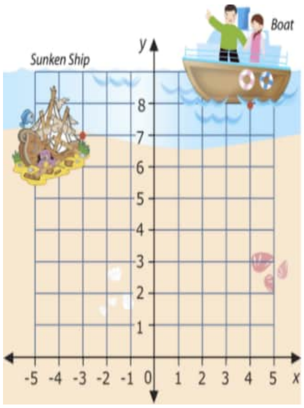
i. How far east from his original position will he enter the water? (Hint: Find the value of x in the sketch.) (1)
ii. Which similarity criterion of triangle is used? (1)
iii. What is the distance of swimmer from the shore? (2)

OR

What is the length of AD? (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Mary and John are very excited because they are going to go on a dive to see a sunken ship. The dive is quite shallow which is unusual because most sunken ship dives are found at depths that are too deep for two junior divers. However, this one is at 40 feet, so the two divers can go to see it.



They have the following map to chart their course. John wants to figure out exactly how far the boat will be from the sunken ship. Use the information in this lesson to help John figure out the following.

i. What are the coordinates of the boat and the sunken ship respectively? (1)

ii. How much distance will Mary and John swim through the water from the boat to the sunken ship? (1)

iii. If each square represents 160 cubic feet of water, how many cubic feet of water will Mary and John swim through from the boat to the sunken ship? (2)

OR

If the distance between the points (x, -1) and (3, 2) is 5, then what is the value of x? (2)

Solution

Section A

1. **(a)** $2^4 \times 3^3$
   Explanation:
   $2^4 \times 3^3$

2. **(d)** 1
   Explanation:
   We see that the graph cuts the x-axis at 1 point which implies p(x) is zero at this 1 point only.

3. **(a)** no solution
   Explanation:
   Since, we have y = 0 and y = -6 are two parallel lines.
   therefore, no solution exists.

4. **(b)** -2, 2
   Explanation:
   $x^2 - 4 = 0$
   $x^2 = 4$
   $x = \pm 2$
   roots are +2, -2

5. **(c)** – 4
   Explanation:
   Given: $a_n = 7 - 4n$
   $\therefore a_1 = 7 - 4 \times 1 = 7 - 4 = 3$
   $a_2 = 7 - 4 \times 2 = 7 - 8 = -1$
   $\therefore d = -1 - 3 = -4$

6. **(a)** 3 units
   Explanation:
   PQ = $\sqrt{\left(\frac{-2}{3} + \frac{11}{3}\right)^2 + (5-5)^2}$
   = $\sqrt{9+0}$
   = 3 units

7. **(d)** $\sqrt{2a^2 + 2b^2}$
   Explanation:
   distance between the point. (0, 0) and (a - b, a + b) is
   = $\sqrt{(a-b-0)^2 + (a+b-0)^2}$
   = $\sqrt{(a-b)^2 + (a+b)^2}$
   = $\sqrt{a^2 + b^2 - 2ab + a^2 + b^2 + 2ab}$
   = $\sqrt{2\left(a^2 + b^2\right)}$ = $\sqrt{2a^2 + 2b^2}$ units.

8.

(b) 2.5 cm

Explanation:

Given $\triangle$ABC $\sim$ $\triangle$QPR

Ac = 6 cm, BC = 5 cm, QR = 3 cm and PR = x

Since, triangles are similar

$\therefore \frac{AC}{QR} = \frac{BC}{PR}$ ...(By proportionality theorem)

$\Rightarrow \frac{6}{3} = \frac{5}{x}$

$\Rightarrow x = \frac{5\times3}{6}$

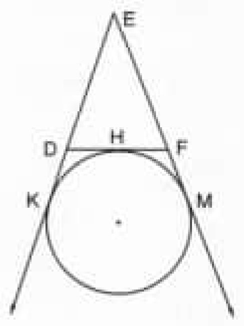
$= \frac{5}{2}$

= 2.5 cm

9.

(d) 18 cm

Explanation:



In $\triangle$DEF

DF touches the circle at H

and circle touches ED and EF Produced at K and M respectively

EK = 9 cm

EK and EM are the tangents to the circle

EM = EK = 9 cm

Similarly DH and DK are the tangent

DH = DK and FH and FM are tangents

FH = FM

Now, perimeter of ΔDEF

= ED + DF + EF

= ED + DH + FH + EF

= ED + DK + FM + EF

= EK + EM

= 9 + 9

= 18 cm

10.

(c) 50^o

Explanation:

In $\triangle$APB,

AP = BP [$\because$ tangents are equal from an external point to the circle]

$\therefore \angle PAB = \angle PBA$ [$\because$ Angles opp. to equal sides of a triangle are equal]

And

$\angle A + \angle PAB + \angle PBA = 180^\circ$

$80^\circ + \angle PBA + \angle PBA = 180^\circ$

$2. \angle PBA = 180^\circ - 80^\circ$

$\angle PBA = \frac{100}{2}$

$\angle PBA = 50^{\circ}$
$\therefore \angle PAB = 50^{\circ}$

11.

(d) -1

Explanation:

-1

12.

(c) $\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$

Explanation:

As we know that,

$\sec^2\theta = 1 + \tan^2\theta$

and $\cot\theta = \frac{1}{\tan\theta}$

$\Rightarrow \tan\theta = \frac{1}{\cot\theta}$

$\therefore \sec^2\theta = 1 + \left(\frac{1}{\cot\theta}\right)^2$

$= 1 + \frac{1}{\cot^2\theta}$

$\Rightarrow \sec^2\theta = \frac{\cot^2\theta+1}{\cot^2\theta}$

$\Rightarrow \sec\theta = \frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$

13.

(d) $\frac{20}{\sqrt{3}}$ m

Explanation:

$\frac{20}{\sqrt{3}}$ m

14.

(c) 45^{o}

Explanation:

Given

$\frac{\text{area of sector}}{\text{area of circle}} = \frac{1}{8}$

$\frac{\frac{\theta}{360^{\circ}} \times \pi r^2}{\pi r^2} = \frac{1}{8}$

$\frac{\theta}{360^{\circ}} = \frac{1}{8}$

$\theta = \frac{360^{\circ}}{8}$

$\theta = 45^{\circ}$

15.

(c) 72^{o}

Explanation:

It is given that area of the sector = 69.3 cm^2

and Radius = 10.5 cm

Now, Area of the sector $= \frac{\pi r^2\theta}{360}$

$\Rightarrow \frac{\pi \times (10.5)^2 \times \theta}{360} = 69.3$

$\Rightarrow \theta = \frac{69.3 \times 360 \times 7}{10.5 \times 10.5 \times 22} = 72^{o}$

Therefore, Central angle of the sector = 72^{o}

16.

(b) $0 \leq$ P(A) ≤ 1

Explanation:

The probability of any event is always positive. It could be at the least equal to zero but not less than that. The probability of sure event at the maximum could be equal to 1 so probability lies between 0 and 1 both included.

17.

(d) $\frac{1}{2}$

Explanation:

$\frac{1}{2}$

18.

(d) 30 – 40

Explanation:

According to the question,

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Freq	3	9	15	30	18	5

Here Maximum frequency is 30.

Therefore, the modal class is 30 – 40.

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

Section B

21. Given

$\sqrt{3}$ is an irrational number

Let 5 + 2$\sqrt{3}$ is a rational number

$\therefore$ we can write 5 + 2$\sqrt{3}$ = $\frac{p}{q}$, where p and q are integers

$\Rightarrow$ 2$\sqrt{3}$ = $\frac{p}{q} - 5 = \frac{p-5q}{q}$

$\sqrt{3}$ = $\frac{p-5q}{2q}$

Here, $\frac{p-5q}{2q}$ is a rational number

So, $\sqrt{3}$ is also a rational number.

But it is given that $\sqrt{3}$ is irrational number.

$\Rightarrow$ our assumption was wrong

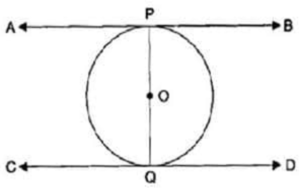
$\Rightarrow$ 5 + 2$\sqrt{3}$ is an irrational number.

22. $\angle$ABD $=$ $\angle$ACD $\Rightarrow$ AB = AC

$\therefore \frac{BC}{BD} = \frac{BE}{AC} \Rightarrow \frac{BC}{BD} = \frac{BE}{BA}$

$\angle B = \angle B$ (common)

$\therefore \triangle\mathrm{ABD} \sim \triangle\mathrm{EBC}$ [SAS similarity criterion]

23. 

Given: PQ is a diameter of a circle with centre O.

The lines AB and CD are the tangents at P and Q respectively.

To Prove: AB || CD

Proof: Since AB is a tangent to the circle at P and OP is the radius through the point of contact.

$\therefore \angle$OPA = 90^o .... (i)

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

$\because$ CD is a tangent to the circle at Q and OQ is the radius through the point of contact.

$\therefore \angle$OQD = 90^o .... (ii)

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

From eq. (i) and (ii), $\angle$OPA = $\angle$OQD

But these form a pair of equal alternate angles also,

$\therefore$ AB $\parallel$ CD

24. $2(\sin^2 45^o + \cot^2 30^o) - 6(\cos^2 45^o - \tan^2 30^o)$

$\left[\left(\frac{1}{\sqrt{2}}\right)^2+(\sqrt{3})^2\right]-6\left[\left(\frac{1}{\sqrt{2}}\right)^2-\left(\frac{1}{\sqrt{3}}\right)^2\right]$

= 7 - 1 = 6

OR

LHS = $\frac{\cot A-\cos A}{\cot A+\cos A}$

$=\frac{\frac{1}{\tan A}-\frac{1}{\sec A}}{\frac{1}{\tan A}+\frac{1}{\sec A}}$

$=\frac{\sec A-\tan A}{\sec A+\tan A}\times\frac{\sec A-\tan A}{\sec A-\tan A}$

$=\frac{\left(\sec^2-\tan A\right)^2}{\sec^2 A-\tan^2 A}$

= $\sec^2$ A + $\tan^2$ A - 2 sec A tan A = RHS

25. Area of minor sector = $\frac{3.14\times(6)^2\times60^\circ}{360^\circ}$

= 18.84

Hence, area of minor sector is 18.84 cm^2

Area of major sector = Area of circle - Area of minor sector

= 3.14 $\times$ $(6)^2$ - 18.84

= 94.2

Hence, area of major sector is 94.2 cm^2

OR

Area of circle = 3.14 $\times$ 10 $\times$ 10 = 314 cm^2

Area of minor sector = $\frac{3.14\times10\times10\times90}{360}=\frac{157}{2}$ cm^2 or 78.5 cm^2

Area of major sector = 314 - 78.5 = 235.5 cm^2

Section C

26. The greatest number of cartons is the HCF of 144 and 90

Now the prime factorization of 144 and 90 are

$144=16\times9=2^4\ \times3^{2.}$

$90=2\times3\times3\times5=2\times3^2\times5$

HCF (144,90)= 2 $\times$ 3^2 = 18

$\therefore$ The greatest number of cartons each stack would have= 18.

27. Let the given polynomial is p(x) = $4x^2$ + 4x + 1

Since, α,β are zeroes of p(x),

$\therefore\alpha+\beta$= sum of zeroes = $\frac{-4}{4}$

Also, $\alpha.\beta$= Product of zeroes = $\alpha.\beta=\frac{1}{4}$

Now a quadratic polynomial whose zeroes are 2α and 2β

x^2 - (sum of zeroes)x + Product of zeroes

$=x^2-(2\alpha+2\beta)x+2\alpha\times2\beta$

$=x^2-2(\alpha+\beta)x+4(\alpha\beta)$

$=x^2-2\times(-1)x+4\times\frac{1}{4}$

$= x^2 + 2x + 1$

The quadratic polynomial whose zeroes are 2α and 2β is $x^2 + 2x + 1$

28. The first 15 multiples of 8 are 8, 16, 24, 32,....

Here, $a_2 - a_1 = 16 - 8 = 8$

$a_3 - a_2 = 24 - 16 = 8$

$a_4 - a_3 = 32 - 24 = 8$

i.e. $a_{k-1} - a_k$ is the same everytime.

So, the above list of numbers forms an AP.

Here, a = 8

d = 8

n = 15

$\therefore$ Sum of first 15 multiples of 8 $= S_{15}$

$= \frac{15}{2}[2a + (15 - 1)d]$ .... $\because S_n = \frac{n}{2}[2a + (n - 1)d]$

$= \frac{15}{2}[2a + 14d]$

= 15 (a + 7d)

$= (15)(8 + 7 \times 8)$

= (15) (8 + 56)

= (15) (64)

= 960

OR

Let a be the first term and the common difference be d.

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{12} = \frac{12}{2}[2a + (12 - 1)d]$

= 6[2a + 11d]

= 12a + 66d

$S_8 = \frac{8}{2}[2a + (8 - 1)d]$

= 4[2a + 7d]

= 8a + 28d

$S_4 = \frac{4}{2}[2a + (4 - 1)d]$

= 2[2a + 3d]

= 4a + 6d

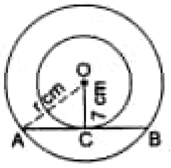
$3(S_8 - S_4) = 3[(8a + 28d) - (4a + 6d)]$

= 3[8a + 28d - 4a - 6d]

= 3[4a + 22d]

= 12a + 66d

$= S_{12}$

29. 

Let O be the common centre of the two circles and AB be the chord of the larger circle which touches the smaller circle at C.

Then, AB = 46 cm.

Join OA and OC

Then,OA = r cm and OC = 7cm.

Now, $OC \perp AB$ and OC bisects AB. [as perpendicular to a chord through the chord bisects it]

In right,$\triangle ACO$, we have,

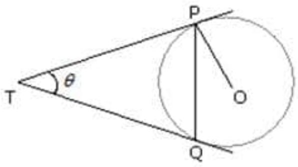
$OA^2 = OC^2 + AC^2$ [by Pythagoras' theorem]

$\Rightarrow$ OA $= \sqrt{OC^2 + AC^2}$

$= \sqrt{OC^2 + \left(\frac{1}{2}AB\right)^2}$ [$\because C$ is the midpoint of AB]

$\Rightarrow r\text{cm} = \sqrt{7^2 + 23^2}\text{cm}$
$= \sqrt{578}\text{cm}$
$= 17\sqrt{2}$
$\Rightarrow r = 17\sqrt{2}\text{cm}$.

OR



Given A circle with centre O and an external point T and two tangents TP and TQ to the circle, where P, Q are the points of contact.

To Prove: ∠PTQ = 2∠OPQ

Proof: Let ∠PTQ = θ

Since TP, TQ are tangents drawn from point T to the circle.

TP = TQ

∴ TPQ is an isoscles triangle

∴ ∠TPQ = ∠TQP = $\frac{1}{2}$ (180^o - θ) = 90^o - $\frac{\theta}{2}$

Since, TP is a tangent to the circle at point of contact P

∴ ∠OPT = 90^o

∴ ∠OPQ = ∠OPT - ∠TPQ = 90^o - (90^o - $\frac{1}{2}$ θ) = $\frac{\theta}{2}$= $\frac{1}{2}$∠PTQ

Thus, ∠PTQ = 2∠OPQ

30. To prove : $\frac{\tan\theta}{1-\tan\theta} - \frac{\cot\theta}{1-\cot\theta} = \frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$

L.H.S = $\frac{\tan\theta}{1-\tan\theta} - \frac{\cot\theta}{1-\cot\theta}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\sin\theta}{\cos\theta}} - \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\cos\theta}{\sin\theta}}$ ...[∵ tanθ = $\frac{\sin\theta}{cos\theta}$]

$= \frac{\sin\theta}{\cos\theta-\sin\theta} + \frac{\cos\theta}{\cos\theta-\sin\theta}$

$= \frac{\sin\theta+\cos\theta}{\cos\theta-\sin\theta}$

= R.H.S.

31.

Class Interval	Frequency(f_i)	Mid value x_i	$u_i = \frac{x_i-A}{h} = \frac{x_i-40}{10}$	$(f_i \times u_i)$
5 - 15	6	10	-3	-18
15 - 25	10	20	-2	-20
25 - 35	16	30	-1	-16
35 - 45	15	40 = A	0	0
45 - 55	24	50	1	24
55 - 65	8	60	2	16
65 - 75	7	70	3	21
	$\Sigma f_i = 86$			$\Sigma(f_i \times u_i)$= 7

Thus, A = 40, h = 10, $\sum f_i = 86$ and $\sum f_i u_i = 7$

Mean = $A + \left\{h \times \frac{\sum f_i u_i}{\sum f_i}\right\}$

$= 40 + \left\{10 \times \frac{7}{86}\right\}$

= 40 + 0.81

= 40.81

Section D

32. Let breadth of the rectangular park = x m

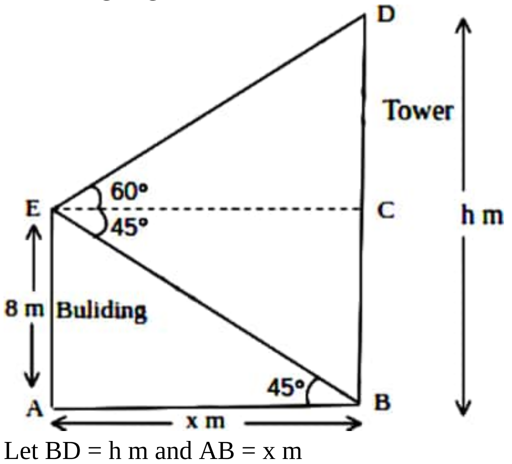
Then, length of the rectangular park = (x + 3) m

Now, area of the rectangular park is = x(x + 3) = $(x^2 + 3x)m^2$ [∵ area = length × breadth]
Given, base of the triangular park = Breadth of the rectangular park
Therefore, base of triangular park is = x m
altitude of triangular park is = 12 m
Therefore, area of the triangular park will be = $\frac{1}{2} \times x \times 12$ = 6x m^2 [∵ area = $\frac{1}{2}\times$ base × height]
As per the question area of rectangular park is = 4 + Area of triangular park
⇒ $x^2 + 3x = 4 + 6x$
⇒ $x^2 + 3x - 6x - 4 = 0$
⇒ $x^2 - 3x - 4 = 0$
⇒ $x^2 - 4x + x - 4 = 0$[by factorization]
⇒ x(x - 4) + 1(x - 4) = 0
⇒ (x - 4)(x + 1) = 0
⇒ x - 4 = 0 or x + 1 = 0
⇒ x = 4 or x = - 1
Since, breadth cannot be negative , so we will neglect x = -1 and choose x = 4
Hence, breadth of the rectangular park will be = 4 m
and length of the rectangular park will be = x + 3 = 4 + 3 = 7m

OR

Given,
(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0
$\Rightarrow x^2 - ax - bx + ab + x^2 - bx - cx + bc + x^2 - cx - ax + ac = 0$
$\Rightarrow 3x^2 - 2ax - 2bx - 2cx + ab + bc + ca = 0$
For equal roots $B^2 - 4AC = 0$
or, $\{-2(a+b+c)\}^2 = 4 \times 3(ab+bc+ca)$
or, $4(a+b+c)^2 - 12(ab+bc+ca) = 0$
or, $a^2 + b^2 + c^2 + 2ab + 2bc + 2ac - 3ab - 3bc - 3ac = 0$
or, $\frac{1}{2}\left[2a^2 + 2b^2 + 2c^2 - 2ab - 2ac - 2bc\right] = 0$
or, $\frac{1}{2}\left[\left(a^2 + b^2 - 2ab\right) + \left(b^2 + c^2 - 2bc\right) + \left(c^2 + a^2 - 2ac\right)\right] = 0$
or, $\frac{1}{2}\left[\left(a^2 + b^2 - 2ab\right) + \left(b^2 + c^2 - 2bc\right) + \left(c^2 + a^2 - 2ac\right)\right] = 0$
or, $(a-b)^2 + (b-c)^2 + (c-a)^2 = 0$ if $a \neq b \neq c$
Since $(a-b)^2 > 0, (b-c)^2 > 0 (c-a)^2 > 0$
Hence, $(a-b)^2 = 0 \Rightarrow a = b$
$(a-c)^2 = 0 \Rightarrow b = c$
$(c-a)^2 = 0 \Rightarrow c = a$
$\therefore a = b = c$ Hence Proved.

33. According to given information, we have following figure



Let BD = h m and AB = x m
Then, CD = h - 8 ...(∵ BC = 8m)
Also, In △EAB, we have
tan 45^o = $\frac{8}{x}$

$\Rightarrow 1 = \frac{8}{x}$

$\Rightarrow$ x = 8 m

Similarly, In $\triangle$DCE, we have

tan60^o = $\frac{CD}{EC}$

$\Rightarrow \sqrt{3} = \frac{h-8}{x}$ ...($\because$ EC = AB = x m)

$\Rightarrow \sqrt{3}x$ = h - 8

$\Rightarrow 8\sqrt{3}$ = h - 8 ...($\because$ x = 8 m)

$\Rightarrow$ h = 8($\sqrt{3}$ + 1)

= 8 $\times$ 2.732

= 21.856 m

34. We have;

A Cube,

Cube's $\frac{length}{Edge}$, a = 7 cm

A Cylinder:

Cylinder's Radius, r = 2.1 cm or r = $\frac{21}{10}$cm

Cylinder's Height, h = 7 cm

$\because$ A cylinder is scooped out from a cube,

$\therefore$ TSA of the resulting cuboid:

= TSA of whole Cube - 2 $\times$ (Area of upper circle or Area of lower circle) + CSA of the scooped out Cylinder

= 6a^2 + 2πrh - 2 $\times$ (πr^2)

= 6 $\times$ (7)^2 + 2 $\times$ (22 $\div$ 7 $\times$ 2.1 $\times$ 7) - 2 $\times$ [22 $\div$ 7 $\times$ (2.1)^2]

= 6 $\times$ 49 + (44 $\div$ 7 $\times$ 14.7) - (44 $\div$ 7 $\times$ 4.41)

= 294 + 92.4 - 27.72

= 294 + 64.68

= 358.68 cm^2

Hence, the total surface area of the remaining solid is 358.68 cm^2

OR

Height of the cylinder = 3 m.

Total height of the tent above the ground = 13.5 m

height of the cone = (13.5 - 3)m = 10.5 m

Radius of the cylinder = radius of cone = 14 m

Curved surface area of the cylinder = $2\pi rh\mathrm{m}^2 = \left(2 \times \frac{22}{7} \times 14 \times 3\right)\mathrm{m}^2 = 264\mathrm{m}^2$

$\therefore \quad l = \sqrt{r^2 + h^2} = \sqrt{14^2 + (10.5)^2} = \sqrt{196 + 110.25} = \sqrt{306.25} = 17.5$

$\therefore$ Cured surface area of the cone = $\pi rl = \left(\frac{22}{7} \times 14 \times 17.5\right)\mathrm{m}^2 = 770\mathrm{m}^2$

Let S be the total area which is to be painted. Then,

S = Curved surface area of the cylinder + Curved surface area of the cone

$\Rightarrow$ S = (264 + 770) m^2 = 1034 m^2

Hence, Cost of painting = S $\times$ Rate = ₹ (1034×2) = ₹ 2068

35. Table:

Class	**Frequency**	**Mid value x_i**	**f_ix_i**	**Cumulative frequency**
25 - 35	7	30	210	7
35 - 45	31	40	1240	38
45 - 55	33	50	1650	71
55 - 65	17	60	1020	88
65 - 75	11	70	770	99
75 - 85	1	80	80	100
	N = 100		$\sum f_ix_i = 4970$	

i. Mean
$\frac{\sum f_i x_i}{\sum f_i} = \frac{4970}{100} = 49.70$

ii. N = 100, $\frac{N}{2}$= 50
Median Class is 45 - 55
$l = 45, h = 10, N = 100, c = 38, f = 33$
$\therefore$Median = $l + h\left(\frac{\frac{N}{2}-c}{f}\right)$
$= 45 + \left\{10 \times \frac{50-38}{33}\right\}$
$= 45 + 3.64 = 48.64$

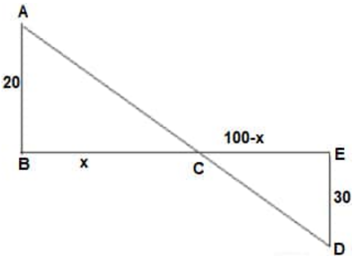
iii. we know that, Mode = $3 \times$ median $- 2 \times$ mean
$= 3 \times 48.64 - 2 \times 49.70$
$= 145.92 - 99.4 = 46.52$

Section E

36. i. Let the fixed charge be ₹ x and per kilometer charge be ₹ y
$\therefore$ x + 10y = 105 ...(i)
x + 15y = 155 ...(ii)
From (i) and (ii)
5y = 50
y = $\frac{50}{5}$ = 10
From equation (i)
x + 100 = 105
x = 105 - 100 = 5
Fixed charges = ₹ 5

ii. Let the fixed charge be ₹ x and per kilometer charge be ₹ y
$\therefore$ x + 10y = 105 ...(1)
x + 15y = 155 ...(2)
From (1) and (2)
5y = 50
y = $\frac{50}{5}$ = 10
From equation (1)
x + 100 = 105
x = 105 - 100 = 5
Per km charges = ₹ 10

iii. Let the fixed charge be ₹ a and per kilometer charge be ₹ b
a + 10b
20 + 10 $\times$ 10 = ₹ 120
OR
Total amount = x + 10y + x + 25y
= 2x + 35y
= 2 $\times$ 5 + 35 $\times$ 10
= 10 + 350
= ₹ 360

37. i.


$\triangle$ABC $\sim$ $\triangle$DEC
$\frac{20}{30} = \frac{x}{100-x}$
2000 - 20x = 30x

2000 = 50x

x = 40 m

ii. AA

iii. 60 metres

OR

AD = AC + CD

$= \sqrt{20^2 + 40^2} + \sqrt{60^2 + 30}^2$

$= \sqrt{400 + 1600} + \sqrt{3600 + 900}$

$= \sqrt{2000} + \sqrt{4500}$

$\Rightarrow 20\sqrt{5}$ + $30\sqrt{5}$

$\Rightarrow 50\sqrt{5}$ m

38. i. (4, 8) and (-3, 7)

ii. 8 units

iii. 1280 cubic feet

OR

7 or -1