



Series : 1PQRS

SET-1



प्रश्न-पत्र कोड
Q.P. Code 65/1/1

रोल नं.

Roll No.

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परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।

Candidates must write the Q.P. Code on the title page of the answer-book.



गणित

MATHEMATICS

निर्धारित समय : 3 घण्टे

Time allowed : 3 hours

अधिकतम अंक : 80

Maximum Marks : 80

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं।
- कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में यथा स्थान पर प्रश्न का क्रमांक अवश्य लिखें।
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।
- Please check that this question paper contains 23 printed pages.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 38 questions.
- Please write down the serial number of the question in the answer-book at the given place before attempting it.
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period. {}

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P.T.O.



सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख्ती से पालन कीजिए :

- (i) इस प्रश्न-पत्र में कुल 38 प्रश्न हैं। सभी प्रश्न अनिवार्य हैं।
- (ii) यह प्रश्न-पत्र पाँच खण्डों में विभाजित है – खण्ड-क, ख, ग, घ एवं ङ।
- (iii) खण्ड – क में प्रश्न संख्या 1 से 18 तक बहुविकल्पीय तथा प्रश्न संख्या 19 एवं 20 अभिकथन एवं तर्क आधारित 1 अंक के प्रश्न हैं।
- (iv) खण्ड – ख में प्रश्न संख्या 21 से 25 तक अति लघु-उत्तरीय (VSA) प्रकार के 2 अंकों के प्रश्न हैं।
- (v) खण्ड – ग में प्रश्न संख्या 26 से 31 तक लघु-उत्तरीय (SA) प्रकार के 3 अंकों के प्रश्न हैं।
- (vi) खण्ड – घ में प्रश्न संख्या 32 से 35 तक दीर्घ-उत्तरीय (LA) प्रकार के 5 अंकों के प्रश्न हैं।
- (vii) खण्ड – ङ में प्रश्न संख्या 36 से 38 प्रकरण अध्ययन आधारित 4 अंकों के प्रश्न हैं।
- (viii) प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड – क के अतिरिक्त अन्य खण्डों के कुछ प्रश्नों में आंतरिक विकल्प दिया गया है।
- (ix) कैल्कुलेटर का उपयोग वर्जित है।

खण्ड – क

प्रश्न 1 से 20 तक बहुविकल्पीय प्रश्न हैं तथा प्रत्येक प्रश्न 1 अंक का है।

1. यदि $2 \cos^{-1}x = y$ है, तो 1
 - (A) $0 \leq y \leq \pi$
 - (B) $-\pi \leq y \leq \pi$
 - (C) $0 \leq y \leq 2\pi$
 - (D) $-\pi \leq y \leq 0$
2. निम्न में से कौन सी एक पंक्ति-आव्यूह की कोटि नहीं हो सकती ? 1
 - (A) 2×1
 - (B) 1×2
 - (C) 1×1
 - (D) $1 \times n$



General Instructions :

Read the following instructions very carefully and strictly follow them :

- (i) This Question paper contains **38** questions. **All** questions are **compulsory**.
- (ii) Question paper is divided into **FIVE** Sections – Section **A, B, C, D** and **E**.
- (iii) In Section **A** – Question Number **1** to **18** are Multiple Choice Questions (MCQs) and Question Number **19 & 20** are Assertion-Reason based questions of **1** mark each.
- (iv) In Section **B** – Question Number **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.
- (v) In Section **C** – Question Number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.
- (vi) In Section **D** – Question Number **32** to **35** are Long Answer (LA) type questions, carrying **5** marks each.
- (vii) In Section **E** – Question Number **36** to **38** are case study based questions, carrying **4** marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in few questions in all the Sections except Section – **A**.
- (ix) Use of calculator is **NOT** allowed.

SECTION – A

Question **1** to **20** are multiple choice questions of **1** mark each.

- 1. If $2 \cos^{-1}x = y$, then 1
 - (A) $0 \leq y \leq \pi$
 - (B) $-\pi \leq y \leq \pi$
 - (C) $0 \leq y \leq 2\pi$
 - (D) $-\pi \leq y \leq 0$

- 2. Which of the following cannot be the order of a row-matrix ? 1
 - (A) 2×1
 - (B) 1×2
 - (C) 1×1
 - (D) $1 \times n$



3. उपयुक्त कोटि के दो आव्यूहों के लिए निम्नलिखित में से कौन सा/से गुण सत्य है/हैं ? 1
- (i) $(A + B)' = A' + B'$ (ii) $(A - B)' = B' - A'$
(iii) $(AB)' = A'B'$ (iv) $(kAB)' = kB'A'$ (k एक अदिश है)
(A) केवल (i) (B) (i), (ii) तथा (iii)
(C) (i) तथा (ii) (D) (i) तथा (iv)
4. यदि $\Delta_1 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{vmatrix}$ तथा $\Delta_2 = \begin{vmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 6 \end{vmatrix}$ है, तो 1
- (A) $\Delta_1 = 2\Delta_2$ (B) $\Delta_2 = -2\Delta_1$
(C) $\Delta_1 = \Delta_2$ (D) $\Delta_2 = -\Delta_1$
5. x का एक मान जिसके लिए $\begin{vmatrix} \cos x & \sin x \\ -\cos x & \sin x \end{vmatrix} = 1$ है 1
- (A) 0 (B) $\frac{\pi}{4}$
(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$
6. यदि आव्यूह A तथा B समान कोटि के विषम सममित आव्यूह हैं, तो निम्न में से कौन सा आव्यूह विषम सममित है ? 1
- (A) AB (B) AB + BA
(C) $(A + B)^2$ (D) A - B
7. अंतराल $[0, 3]$ में $f(x) = x^3 - 12x$ का न्यूनतम मान है 1
- (A) -16 (B) -9
(C) 0 (D) 16



3. Which of the following properties is/are true for two matrices of suitable orders ?

1

(i) $(A + B)' = A' + B'$

(ii) $(A - B)' = B' - A'$

(iii) $(AB)' = A'B'$

(iv) $(kAB)' = kB'A'$ (k is a scalar)

(A) (i) only

(B) (i), (ii) and (iii)

(C) (i) and (ii)

(D) (i) and (iv)

4. If $\Delta_1 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 6 \end{vmatrix}$, then

1

(A) $\Delta_1 = 2\Delta_2$

(B) $\Delta_2 = -2\Delta_1$

(C) $\Delta_1 = \Delta_2$

(D) $\Delta_2 = -\Delta_1$

5. One of the values of x for which $\begin{vmatrix} \cos x & \sin x \\ -\cos x & \sin x \end{vmatrix} = 1$ is

1

(A) 0

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

6. If A and B are skew symmetric matrices of same order, then which of the following matrices is also skew symmetric ?

1

(A) AB

(B) AB + BA

(C) $(A + B)^2$

(D) A - B

7. The least value of $f(x) = x^3 - 12x$, $x \in [0, 3]$ is

1

(A) -16

(B) -9

(C) 0

(D) 16



8. यदि $\int \frac{3ax}{b^2 + c^2x^2} dx = A \log |b^2 + c^2x^2| + K$ है, तो A का मान है : 1

(A) $3a$ (B) $\frac{3a}{2b^2}$

(C) $\frac{3a}{b^2c^2}$ (D) $\frac{3a}{2c^2}$

9. $\int_{-1}^1 \frac{x^3}{x^2 + 2|x| + 1} dx$ का मान है : 1

(A) 0 (B) $\log 2$

(C) $2 \log 2$ (D) $\frac{1}{2} \log 2$

10. वक्र $y = x|x|$, x -अक्ष एवं कोटियों $x = -1$ तथा $x = 1$ से घिरे क्षेत्र का क्षेत्रफल है 1

(A) 0 (B) $\frac{1}{3}$

(C) $\frac{2}{3}$ (D) $\frac{4}{3}$

11. अवकल समीकरण $R \frac{dx}{dy} + Px = Q$, जहाँ P, Q तथा R, y के फलन हैं, का समाकलन गुणक है 1

(A) $e^{\int \frac{P}{Q} dy}$ (B) $e^{\int P dy}$

(C) $e^{\int \frac{P}{R} dy}$ (D) $e^{\int \frac{P}{R} dx}$



8. If $\int \frac{3ax}{b^2 + c^2x^2} dx = A \log |b^2 + c^2x^2| + K$, then the value of A is 1

(A) $3a$ (B) $\frac{3a}{2b^2}$

(C) $\frac{3a}{b^2c^2}$ (D) $\frac{3a}{2c^2}$

9. The value of $\int_{-1}^1 \frac{x^3}{x^2 + 2|x| + 1} dx$ is 1

(A) 0 (B) $\log 2$

(C) $2 \log 2$ (D) $\frac{1}{2} \log 2$

10. The area bounded by the curve $y = x|x|$, x -axis and the ordinates $x = -1$ and $x = 1$ is given by 1

(A) 0 (B) $\frac{1}{3}$

(C) $\frac{2}{3}$ (D) $\frac{4}{3}$

11. The integrating factor of differential equation $R \frac{dx}{dy} + Px = Q$ where P, Q, R are functions of y is 1

(A) $e^{\int \frac{P}{Q} dy}$ (B) $e^{\int P dy}$

(C) $e^{\int \frac{P}{R} dy}$ (D) $e^{\int \frac{P}{R} dx}$



12. अवकल समीकरण $\frac{d}{dx}(e^y) = 0$ की कोटि तथा घात, क्रमशः हैं 1
- (A) 0, 1 (B) 1, 1
(C) 2, 1 (D) 1, परिभाषित नहीं है।
13. p का वह मान जिसके लिए सदिश $\hat{i} + 2\hat{j} + 3\hat{k}$ तथा $2\hat{i} - p\hat{j} + \hat{k}$ परस्पर लंबवत हैं, है 1
- (A) 0 (B) 1
(C) $\frac{5}{2}$ (D) $-\frac{5}{2}$
14. m का वह मान जिसके लिए बिंदु जिनके स्थिति सदिश $-\hat{i} - \hat{j} + 2\hat{k}$, $2\hat{i} + m\hat{j} + 5\hat{k}$ तथा $3\hat{i} + 11\hat{j} + 6\hat{k}$ हैं, संरेख हैं, है 1
- (A) 8 (B) -8
(C) 2 (D) $\frac{5}{2}$
15. यदि $|\vec{a}| = 8$, $|\vec{b}| = 3$ तथा $|\vec{a} \times \vec{b}| = 12$ है, तो $|\vec{a} \cdot \vec{b}|$ का मान है 1
- (A) $6\sqrt{3}$ (B) $8\sqrt{3}$
(C) $12\sqrt{3}$ (D) $3\sqrt{12}$
16. बिंदु $(2, 5, 7)$ से रेखा $\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$ पर डाले गए लंब की लंबाई है 1
- (A) 2 (B) 5
(C) $\sqrt{74}$ (D) $\sqrt{78}$



12. The order and degree of the differential equation $\frac{d}{dx}(e^y) = 0$ respectively are 1

- (A) 0, 1 (B) 1, 1
(C) 2, 1 (D) 1, not defined

13. The value of p for which vectors $\hat{i} + 2\hat{j} + 3\hat{k}$ and $2\hat{i} - p\hat{j} + \hat{k}$ are perpendicular to each other is 1

- (A) 0 (B) 1
(C) $\frac{5}{2}$ (D) $-\frac{5}{2}$

14. The value of m for which the points with position vectors $-\hat{i} - \hat{j} + 2\hat{k}$, $2\hat{i} + m\hat{j} + 5\hat{k}$ and $3\hat{i} + 11\hat{j} + 6\hat{k}$ are collinear, is 1

- (A) 8 (B) -8
(C) 2 (D) $\frac{5}{2}$

15. If $|\vec{a}| = 8$, $|\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 12$, then the value of $|\vec{a} \cdot \vec{b}|$ 1

- (A) $6\sqrt{3}$ (B) $8\sqrt{3}$
(C) $12\sqrt{3}$ (D) $3\sqrt{12}$

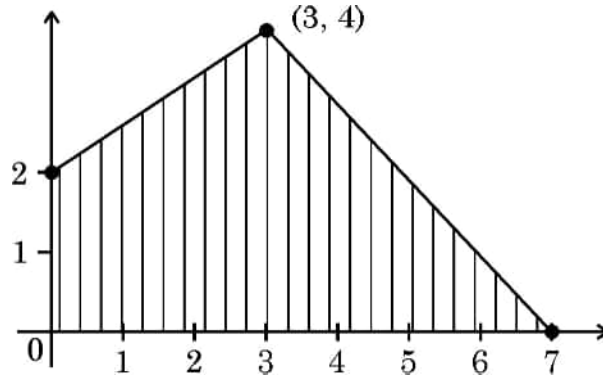
16. The length of perpendicular drawn from point (2, 5, 7) on line $\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$ is 1

- (A) 2 (B) 5
(C) $\sqrt{74}$ (D) $\sqrt{78}$



17. एक LPP जिसका उद्देश्य फलन $Z = 5x + 7y$ है, का सुसंगत क्षेत्र नीचे दर्शाया गया है :

1



Z का अधिकतम मान – Z का न्यूनतम मान है :

- (A) 8 (B) 29
(C) 35 (D) 43
18. एक LPP के उद्देश्य फलन की घात है
- (A) 0 (B) 1
(C) 2 (D) कोई प्राकृत संख्या

1

अभिकथन – तर्क आधारित प्रश्न

निर्देश : प्रश्न संख्या 19 और 20 अभिकथन (A) एवं तर्क (R) पर आधारित प्रश्न हैं। दो कथन दिए गए हैं जिनमें एक को अभिकथन (A) तथा दूसरे को तर्क (R) द्वारा अंकित किया गया है। इन प्रश्नों के सही उत्तर नीचे दिए गए कोडो (A), (B), (C) और (D) में से चुनकर दीजिए।

- (A) अभिकथन (A) तथा तर्क (R) दोनों सही हैं और तर्क (R), अभिकथन (A) की सही व्याख्या करता है।
(B) अभिकथन (A) तथा तर्क (R) दोनों सही हैं, परन्तु तर्क (R), अभिकथन (A) की सही व्याख्या नहीं करता है।
(C) अभिकथन (A) सही है, परन्तु तर्क (R) गलत है।
(D) अभिकथन (A) गलत है, परन्तु तर्क (R) सही है।

19. **अभिकथन (A) :** एक निष्पक्ष पासे को उछालने के प्रयोग में, एक अभाज्य संख्या के आने की प्रायिकता, जबकि दिया है कि पासे पर एक विषम संख्या आई है, $\frac{2}{3}$ है।

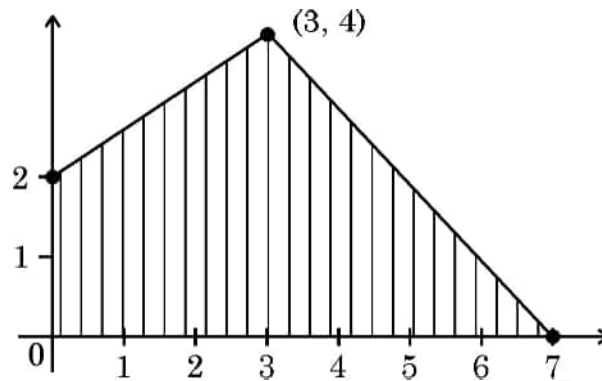
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तर्क (R) : किन्हीं दो घटनाओं A तथा B के लिए $P(A|B) = \frac{P(A \cup B)}{P(B)}$



17. The feasible region of a linear programming problem with objective function $Z = 5x + 7y$ is shown below :

1



The maximum value of Z – minimum value of Z is

- (A) 8 (B) 29
(C) 35 (D) 43
18. The degree of an objective function of a linear programming problem is
- (A) 0 (B) 1
(C) 2 (D) Any natural number

Assertion – Reason Based Questions

Direction : Questions number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R).

Select the correct answer from the codes (A), (B), (C) and (D) as given below.

- (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true and Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is true.
19. **Assertion (A) :** In an experiment of throwing an unbiased die, the probability of getting a prime number given that number appearing on the die being odd is $\frac{2}{3}$.

1

Reason (R) : For any two events A and B, $P(A|B) = \frac{P(A \cap B)}{P(B)}$



20. अभिकथन (A) : $x = py + q$, $z = ry + s$ तथा $x = p'y + q'$, $z = r'y + s'$ द्वारा दी गई रेखाएँ परस्पर लंबवत हैं यदि $pp' + rr' = 1$ है ।

1

तर्क (R) : दो रेखाएँ $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ तथा $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ परस्पर लंबवत हैं यदि $\vec{b}_1 \cdot \vec{b}_2 = 0$ है।

खण्ड – ख

इस खण्ड में अति लघु-उत्तरीय प्रकार के प्रश्न हैं। प्रत्येक प्रश्न के 2 अंक हैं।

21. (a) जाँच कीजिए कि क्या $f(x)$ द्वारा दिया गया फलन

2

$$f(x) = \begin{cases} \frac{|x-3|}{2(x-3)}, & x < 3 \\ \frac{x-6}{6}, & x \geq 3 \end{cases} \quad x = 3 \text{ पर संतत है या नहीं।}$$

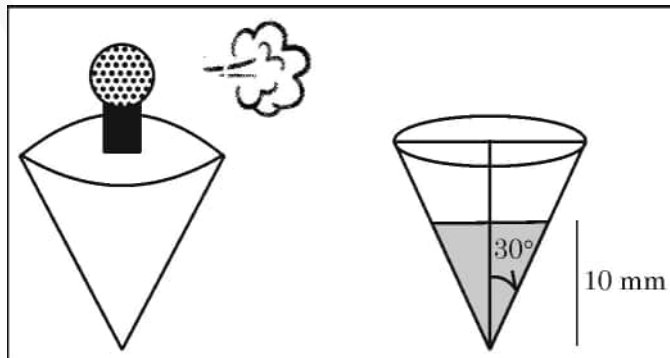
अथवा

(b) यदि $\sqrt{3}(x^2 + y^2) = 4xy$ है, तो $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ पर $\frac{dy}{dx}$ ज्ञात कीजिए।

2

22. एक उलटे शंकु के आकार की रूम फ्रेशनर बोतल नियमित अंतरालों में स्प्रे करती है, जिससे बोतल में परफ्यूम के आयतन के घटने की दर $1 \text{ mm}^3/\text{min.}$ है। यदि शंकुवाकार बोतल का सेमी वर्टिकल कोण $\frac{\pi}{6}$ है तो बोतल में परफ्यूम के तल के घटने की दर ज्ञात कीजिए, जब बोतल में परफ्यूम का लेवल 10 mm हो।

2





20. **Assertion (A)** : Lines given by $x = py + q$, $z = ry + s$ and $x = p'y + q'$, $z = r'y + s'$ are perpendicular to each other when $pp' + rr' = 1$. 1

Reason (R) : Two lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ are perpendicular to each other if $\vec{b}_1 \cdot \vec{b}_2 = 0$.

SECTION – B

This section comprises Very Short Answer (VSA) type questions of 2 marks each.

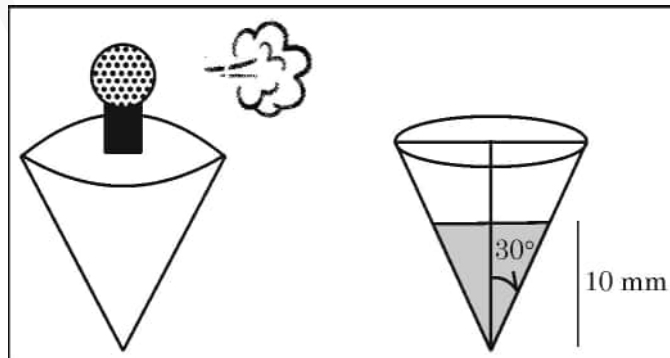
21. (a) Check whether function $f(x)$ defined as 2

$$f(x) = \begin{cases} \frac{|x-3|}{2(x-3)}, & x < 3 \\ \frac{x-6}{6}, & x \geq 3 \end{cases} \text{ is continuous at } x = 3 \text{ or not?}$$

OR

- (b) If $\sqrt{3}(x^2 + y^2) = 4xy$, then find $\frac{dy}{dx}$ at $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. 2

22. A room freshner bottle in the shape of an inverted cone sprays the perfume at regular intervals such that volume of the perfume in the bottle decreases at the steady rate of $1 \text{ mm}^3/\text{min}$. Find the rate at which level of perfume is dropping at an instant when level of perfume in the bottle is 10 mm, if the semi-vertical angle of conical bottle is $\frac{\pi}{6}$. 2





23. बिंदुओं P(1, 3, 2) तथा Q(-1, 0, 8) के लिए $\overrightarrow{QP}$ की दिशा में परिमाण 14 का एक सदिश ज्ञात कीजिए।

2

24. सदिश $\vec{a} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ तथा $\vec{b} = \hat{i} + 2\hat{k}$ एक समांतर चतुर्भुज की दो संलग्न भुजाओं को निरूपित करते हैं। इस चतुर्भुज के विकर्णों को निरूपित करने वाले सदिश ज्ञात कीजिए। अतः इन विकर्णों की लम्बाइयाँ भी ज्ञात कीजिए।

2

25. (a) सरल कीजिए : $\tan^{-1} \left(\frac{\cos 2x - \sin 2x}{\cos 2x + \sin 2x} \right), 0 < x < \frac{\pi}{4}$.

2

अथवा

(b) मान ज्ञात कीजिए : $\tan \left(\sin^{-1} 1 - \cos^{-1} \left(-\frac{1}{2} \right) \right)$

2

खण्ड - ग

इस खण्ड में लघु-उत्तरीय प्रकार के प्रश्न हैं। प्रत्येक प्रश्न के 3 अंक हैं।

26. मान ज्ञात कीजिए : $\int_0^1 x \tan^{-1} x \, dx$.

3

27. (a) ज्ञात कीजिए : $\int \sqrt{\frac{x+2}{x-2}} \, dx$

3

अथवा

(b) ज्ञात कीजिए : $\int \frac{x^2}{(x^2 + 9)(x^2 + 16)} \, dx$

3

28. यदि $I_1 = \int_{-\pi/4}^{\pi/4} \frac{dx}{1 + \cos 2x}$ तथा $I_2 = \int_{-1/2}^{1/2} |x| \, dx$ है, तो दर्शाइए कि $I_1 - 4I_2 = 0$.

3

29. (a) अवकल समीकरण $x^2 \frac{dy}{dx} = x^2 + xy + y^2$ का व्यापक हल ज्ञात कीजिए।

3

अथवा

(b) अवकल समीकरण $xy \frac{dy}{dx} = (x + 2)(y + 2)$, का विशिष्ट हल ज्ञात कीजिए, दिया है कि $y(1) = -1$.

3



23. Find the vector of magnitude 14 in the direction of $\overrightarrow{QP}$, where P and Q are the points (1, 3, 2) and (-1, 0, 8) respectively. 2

24. Vectors $\vec{a} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{k}$ represent the two adjacent sides of a parallelogram. Find the vectors representing its diagonals and hence find their lengths. 2

25. (a) Simplify : $\tan^{-1} \left(\frac{\cos 2x - \sin 2x}{\cos 2x + \sin 2x} \right)$, $0 < x < \frac{\pi}{4}$. 2

OR

- (b) Evaluate : $\tan \left(\sin^{-1} 1 - \cos^{-1} \left(-\frac{1}{2} \right) \right)$ 2

SECTION – C

This section comprises Short Answer (SA) type questions of 3 marks each.

26. Evaluate : $\int_0^1 x \tan^{-1} x \, dx$. 3

27. (a) Find $\int \sqrt{\frac{x+2}{x-2}} \, dx$ 3

OR

- (b) Find : $\int \frac{x^2}{(x^2 + 9)(x^2 + 16)} \, dx$ 3

28. If $I_1 = \int_{-\pi/4}^{\pi/4} \frac{dx}{1 + \cos 2x}$ and $I_2 = \int_{-1/2}^{1/2} |x| \, dx$, then show that $I_1 - 4I_2 = 0$. 3

29. (a) Find the general solution of the following differential equation : 3

$$x^2 \frac{dy}{dx} = x^2 + xy + y^2$$

OR

- (b) Find the particular solution of the differential equation 3

$$xy \frac{dy}{dx} = (x + 2)(y + 2), \text{ given that } y(1) = -1.$$



30. निम्न रैखिक प्रोग्रामन समस्या (LLP) को ग्राफ द्वारा हल कीजिए :

3

अवरोधों $x + y \leq 7,$

$$2x - 3y + 6 \geq 0,$$

$$x \geq 0, y \geq 0$$

के अंतर्गत $Z = 13x - 15y$ का न्यूनतमीकरण कीजिए ।

31. (a) दो थैलों में से थैला I में 3 लाल तथा 4 सफेद गेंदें हैं, जबकि थैला II में 8 लाल तथा 6 सफेद गेंदें हैं । एक पासा उछाला गया । यदि 3 से कम संख्या आई तो थैला I में से एक गेंद को यादृच्छया निकाला गया अन्यथा थैला II में से यादृच्छया एक गेंद को निकाला गया । दोनों थैलों में से एक में से निकाली गई गेंद के लाल रंग की होने की प्रायिकता ज्ञात कीजिए ।

3

अथवा

- (b) दो घटनाओं X तथा Y में से कम से कम एक के एक साथ होने की प्रायिकता a है । यदि घटनाओं X, Y में ठीक एक के होने की संभावना b है, तो सिद्ध कीजिए कि $P(X') + P(Y') = 2 - 2a + b$.

3

खण्ड – घ

इस खण्ड में दीर्घ-उत्तरीय प्रकार के प्रश्न हैं । प्रत्येक प्रश्न के 5 अंक हैं ।

32. (a) एक संबंध R, सभी पूर्णाकों के समुच्चय Z में इस प्रकार परिभाषित है कि $R = \{(x, y) : |x - y| \text{ एक अभाज्य संख्या 'p' से भाज्य है, } x, y \in Z\}$ है ।
जाँच कीजिए कि R एक तुल्यता संबंध है या नहीं ।

5

अथवा

- (b) फलन $f : R - \left\{ \frac{3}{5} \right\} \longrightarrow R - \left\{ \frac{3}{5} \right\}$, इस प्रकार परिभाषित है $f(x) = \frac{3x + 2}{5x - 3}$. दर्शाइए कि f एक एकैकी एवं आच्छादक फलन है ।

5



30. Solve the following linear programming problem graphically : 3

Minimize $Z = 13x - 15y$

Subject to constraints

$$x + y \leq 7,$$

$$2x - 3y + 6 \geq 0,$$

$$x \geq 0, y \geq 0$$

31. (a) Out of two bags, bag I contains 3 red and 4 white balls and bag II contains 8 red and 6 white balls. A die is thrown. If it shows a number less than 3 then a ball is drawn at random from bag I, otherwise a ball is drawn at random from bag II. Find the probability that the ball drawn from one of the bags is a red ball. 3

OR

- (b) The probability of simultaneous occurrence of atleast one of the two events X and Y is a. If the probability that exactly one of the events X, Y occurs is b, prove that $P(X') + P(Y') = 2 - 2a + b$. 3

SECTION – D

This section comprises Long Answer (LA) type questions of 5 marks each.

32. (a) A relation R is defined on Z, the set of integers, as 5

$$R = \{(x, y) : |x - y| \text{ is divisible by a prime number 'p', } x, y \in \mathbb{Z}\}$$

check whether R is an equivalence relation or not.

OR

- (b) A function $f : \mathbb{R} - \left\{ \frac{3}{5} \right\} \longrightarrow \mathbb{R} - \left\{ \frac{3}{5} \right\}$ is defined as $f(x) = \frac{3x+2}{5x-3}$. Show that f is one-one and onto. 5



33. (a) यदि $A = \begin{bmatrix} 0 & 2 & 1 \\ -2 & -1 & -2 \\ 1 & -1 & 0 \end{bmatrix}$ है, तो A^{-1} ज्ञात कीजिए।

A^{-1} के प्रयोग से निम्न समीकरण निकाय को हल कीजिए :

$$-2y + z = 7, 2x - y - z = 8, x - 2y = 10$$

5

अथवा

(b) यदि $\begin{bmatrix} 3 & -1 & \sin 3x \\ -7 & 4 & \cos 2x \\ -11 & 7 & 2 \end{bmatrix}$ एक अव्युत्क्रमणीय (singular) आव्यूह है, तो x के सभी

मान ज्ञात कीजिए जहाँ $x \in \left[0, \frac{\pi}{2}\right]$ है।

5

34. यदि $x = \cos t$, $y = \cos mt$ है, तो सिद्ध कीजिए कि $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + m^2y = 0$.

5

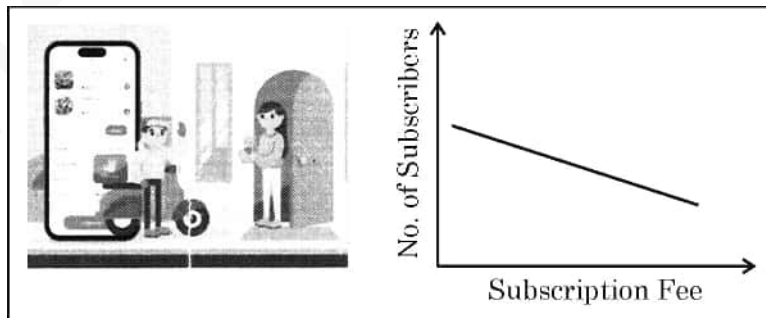
35. जाँच कीजिए कि रेखाएँ $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ तथा $\frac{x-4}{5} = \frac{y-1}{2} = z$, परस्पर समांतर हैं या नहीं। यदि समांतर हैं, तो उनके बीच की दूरी ज्ञात कीजिए, अन्यथा यदि रेखाएँ प्रतिच्छेदी हैं, तो उनका प्रतिच्छेदन बिंदु ज्ञात कीजिए।

5

खण्ड - ड

इस खण्ड में 3 प्रकरण अध्ययन आधारित प्रश्न हैं। प्रत्येक प्रश्न के 4 अंक हैं।

36. एक ऑन-लाइन डिलिवरी कम्पनी के एक शहर में 5000 ग्राहक हैं तथा वह प्रत्येक ग्राहक से असीमित मुफ्त डिलिवरी के लिए ₹ 300 प्रतिवर्ष लेती है।



कम्पनी अपना वार्षिक सदस्यता शुल्क बढ़ाना चाहती है। यह अनुमान है कि प्रत्येक ₹ 1 की बढ़ोतरी से 10 सदस्य छोड़ जाएँगे। माना कि कम्पनी ने वार्षिक शुल्क में ₹ x की बढ़ोतरी की।



33. (a) If $A = \begin{bmatrix} 0 & 2 & 1 \\ -2 & -1 & -2 \\ 1 & -1 & 0 \end{bmatrix}$, find A^{-1} and use it to solve the following

system of equations :

$$-2y + z = 7, 2x - y - z = 8, x - 2y = 10$$

5

OR

- (b) If $\begin{bmatrix} 3 & -1 & \sin 3x \\ -7 & 4 & \cos 2x \\ -11 & 7 & 2 \end{bmatrix}$ is a singular matrix, then find all values of x

where $x \in \left[0, \frac{\pi}{2}\right]$.

5

34. If $x = \cos t$, $y = \cos mt$, prove that $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + m^2y = 0$.

5

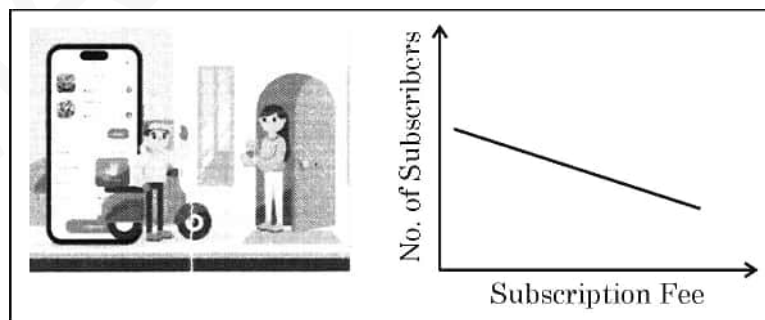
35. Check whether the lines given by $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ are parallel or not. If parallel, find the distance between them, otherwise find their point of intersection, if the lines are intersecting.

5

SECTION – E

This section comprises of 3 case study based questions of 4 marks each.

36. An online delivery company in a city has 5000 subscribers and collects annual subscription fees of ₹ 300 per subscriber for unlimited free deliveries.



The company wishes to increase the annual subscription fee. It is predicted that, for every increase of ₹ 1, ten subscribers will discontinue. Assume that the company increased the annual fee by ₹ x .



उपरोक्त सूचनाओं के आधार पर निम्न प्रश्नों के उत्तर दीजिए :

- (i) वार्षिक शुल्क में ₹ x की बढ़ोतरी पर कितने सदस्य छोड़ जाएँगे ? 1
- (ii) यदि $R(x)$ वार्षिक शुल्क में ₹ x की बढ़ोतरी के बाद इकट्ठा हुए कुल राजस्व को दर्शाता है, तो $R(x)$ को x के फलन के रूप में व्यक्त कीजिए । 1
- (iii) (a) x का वह मान ज्ञात कीजिए, जिसके लिए $R(x)$ अधिकतम हो । 2

अथवा

- (iii) (b) वह उपअन्तराल ज्ञात कीजिए जहाँ $(0, 5000)$ में $R(x)$ वर्धमान अथवा हासमान है । 2

37. एक ऑन-लाइन जैकपॉट में ₹ 3,00,000 का पहला इनाम, ₹ 2,00,000 के दो दूसरे इनाम और ₹ 50,000 के तीन तीसरे इनाम हैं । जैकपॉट के लिए प्रत्येक ₹ 100 मूल्य के 1,00,000 टिकटों की बिक्री से ₹ 1,00,00,000 एकत्र हुए ।



रोहन ने एक टिकट खरीदा ।

उपरोक्त सूचनाओं के आधार पर निम्न प्रश्नों के उत्तर दीजिए :

- (i) एक व्यक्ति कौन-कौन सी संभावित राशियाँ जीत सकता है ? 1
- (ii) (a) एक व्यक्ति द्वारा कम से कम ₹ 2,00,000 जीतने की क्या प्रायिकता है ? 2

अथवा

- (ii) (b) एक व्यक्ति के कुछ भी न जीतने की क्या प्रायिकता है ? 2
- (iii) एक अन्य जैकपॉट जिसमें इनाम की राशि ₹ 5,00,000 है, का एक टिकट भी रोहन ने लिया । इस जैकपॉट में जीतने की संभावना 1,00,000 में 1 है । प्रायिकता ज्ञात कीजिए कि रोहन किसी एक ही जैकपॉट की टिकट पर इनाम जीतता है । 1



Based on the given information, answer the following questions :

- (i) How many subscribers will discontinue after an increase of ₹ x in annual fee ? 1
- (ii) If $R(x)$ denotes the total revenue collected after the increase of ₹ x in subscription fee, express $R(x)$ as a function of x . 1
- (iii) (a) Find the value of x for which $R(x)$ is maximum. 2

OR

- (iii) (b) Find the sub-intervals of $(0, 5000)$ in which $R(x)$ is increasing and decreasing. 2

37. In an online jackpot, there is one first prize of ₹ 3,00,000, two second prizes of ₹ 2,00,000 each and three third prizes of ₹ 50,000 each.



A total of 1,00,000 jackpot tickets each costing ₹ 100 were sold there by raising a fund of ₹ 1,00,00,000.

Rohan bought one ticket.

Based on given information, answer the following questions :

- (i) What are the possible amounts, the person can win ? 1
- (ii) (a) What is the probability that the person wins atleast ₹ 2,00,000 ? 2

OR

- (ii) (b) What is the probability that the person does not win any amount ? 2
- (iii) In another jackpot, Rohan also bought a ticket having a prize money of ₹ 5,00,000. The chances of winning the jackpot are 1 in 1,00,000. Find the probability that on exactly one of tickets he wins the jackpot. 1



38. ट्रैफिक कम करने और रेडलाइट से बचने के लिए अक्सर व्यस्त सड़कों पर गोल चक्कर बनाए जाते हैं।



एक ऐसा ही गोल चक्कर इस प्रकार बनाया गया कि उसकी सीमा $C_1 : x^2 + y^2 = 64$ द्वारा प्रदत्त है। इस गोल चक्कर के मध्य में एक वृत्ताकार तालाब है जिसमें एक फव्वारा लगा हुआ है, जिसकी समीकरण $C_2 : x^2 + y^2 = 4$ द्वारा प्रदत्त है।

उपरोक्त सूचनाओं के आधार पर निम्न प्रश्नों के उत्तर दीजिए :

- | | | |
|-------|--|---|
| (i) | समीकरण C_1 तथा C_2 को आकृति से दर्शाइए। | 1 |
| (ii) | दोनों C_1 तथा C_2 के लिए y को x के फलन के रूप में दर्शाइए। ($y = f(x)$) | 1 |
| (iii) | (a) पूरे गोल चक्कर द्वारा घेरे गए क्षेत्र का क्षेत्रफल समाकलन विधि से ज्ञात कीजिए। | 2 |

अथवा

- | | | |
|-------|---|---|
| (iii) | (b) वृत्ताकार तालाब द्वारा घेरे गए क्षेत्र का क्षेत्रफल समाकलन विधि द्वारा ज्ञात कीजिए। | 2 |
|-------|---|---|



38. Roundabouts are often made on busy roads to ease the traffic and avoid red lights.



One such round-about is made such that equation representing its boundary is given by $C_1 : x^2 + y^2 = 64$.

There is a circular pond with a fountain in the middle of the roundabout whose equation is given by $C_2 : x^2 + y^2 = 4$.

Based on the given information, answer the following questions :

- (i) Represent the given equations C_1 and C_2 with the help of a diagram. 1
- (ii) Express y as a function of x , ($y = f(x)$), for both C_1 and C_2 . 1
- (iii) (a) Using integration find the area of region covered by the roundabout. 2

OR

- (iii) (b) Using integration, find the area of region covered by circular pond. 2



MATHEMATICS

Marking Scheme Strictly Confidential
(For Internal and Restricted use only) Senior Secondary Examination, 2026
SUBJECT NAME MATHEMATICS (Q.P. CODE – 65/1/1)

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	“Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its leakage to the public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in Newspaper/Website, etc. may invite action under various rules of the Board and BNS.”
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one’s own interpretation or any other consideration. The Marking Scheme should be strictly adhered to and religiously followed. However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-XII, while evaluating the competency-based questions, please try to understand the given answer and even if reply is not from a marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.
4	The Marking Scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (✓) wherever answer is correct. For wrong answer CROSS ‘X’ be marked. Evaluators will not put right (✓) while evaluating which gives the impression that the answer is correct, and no marks are awarded. This is the most common mistake which evaluators are committing.
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left- hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	If a student has attempted an extra question, answer to the question deserving more marks should be retained and the other answer scored out with a note “Extra Question” .

10	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
11	A full scale of marks (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
12	Every examiner must necessarily do evaluation work for full working hours, i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
13	Ensure that you do not make the following common types of errors committed by the Examiner in the past: • Leaving answer or part thereof unassessed in an answer book. • Giving more marks for an answer than assigned to it. • Wrong totalling of marks awarded on an answer. • Wrong transfer of marks from the inside pages of the answer book to the title page. • Wrong question wise totalling on the title page. • Wrong totalling of marks of the two columns on the title page. • Wrong grand total. • Marks in words and figures not tallying/not same. • Wrong transfer of marks from the answer book to online award list. • Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) • Half or a part of the answer marked correct and the rest as wrong, but no marks awarded.
14	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
15	Any unassessed portion, non-carrying over of marks to the title page, or total error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
16	The Examiners should acquaint themselves with the guidelines given in the “Guidelines for Spot Evaluation” before starting the actual evaluation.
17	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totalled and written in figures and words.
18	The candidates are entitled to obtain a photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

MARKING SCHEME
MATHEMATICS (Subject Code-041)
(PAPER CODE: 65/1/1)

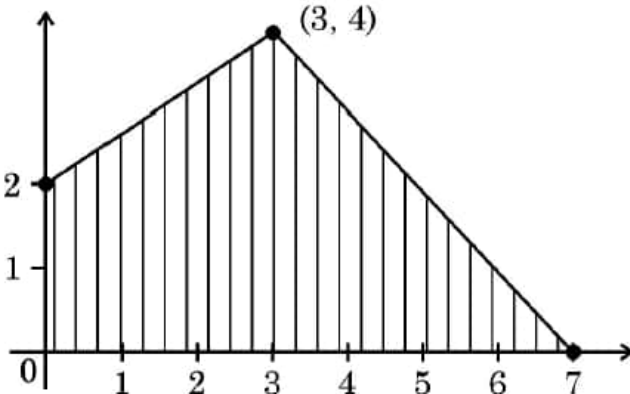
Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Steps	Marks
	SECTION A		
	Q. Number 1 to 20 are multiple choice questions of 1 mark each.		
1.	If $2 \cos^{-1}x = y$, then (A) $0 \leq y \leq \pi$ (B) $-\pi \leq y \leq \pi$ (C) $0 \leq y \leq 2\pi$ (D) $-\pi \leq y \leq 0$		
Sol.	(C) $0 \leq y \leq 2\pi$		1
2.	Which of the following cannot be the order of a row-matrix ? (A) 2×1 (B) 1×2 (C) 1×1 (D) $1 \times n$		
Sol.	(A) 2×1		1
3.	Which of the following properties is/are true for two matrices of suitable orders ? (i) $(A + B)' = A' + B'$ (ii) $(A - B)' = B' - A'$ (iii) $(AB)' = A'B'$ (iv) $(kAB)' = kB'A'$ (k is a scalar) (A) (i) only (B) (i), (ii) and (iii) (C) (i) and (ii) (D) (i) and (iv)		
Sol.	(D) (i) and (iv)		1

4.	If $\Delta_1 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 6 \end{vmatrix}$, then (A) $\Delta_1 = 2\Delta_2$ (B) $\Delta_2 = -2\Delta_1$ (C) $\Delta_1 = \Delta_2$ (D) $\Delta_2 = -\Delta_1$		
Sol.	(B) $\Delta_2 = -2\Delta_1$		1
5.	One of the values of x for which $\begin{vmatrix} \cos x & \sin x \\ -\cos x & \sin x \end{vmatrix} = 1$ is (A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$		
Sol.	(B) $\frac{\pi}{4}$		1
6.	If A and B are skew symmetric matrices of same order, then which of the following matrices is also skew symmetric ? (A) AB (B) AB + BA (C) $(A + B)^2$ (D) A – B		
Sol.	(D) A – B		1

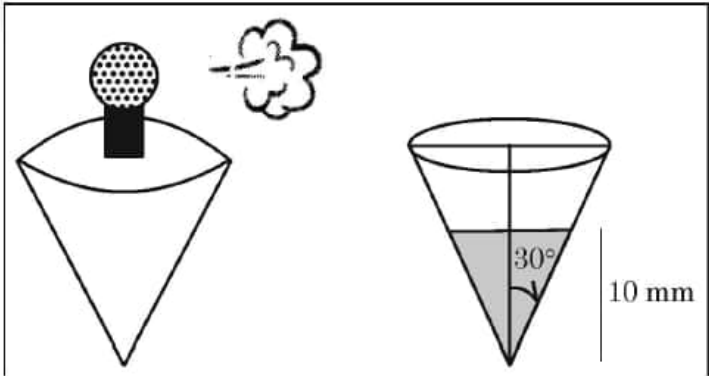
7.	The least value of $f(x) = x^3 - 12x$, $x \in [0, 3]$ is (A) -16 (B) -9 (C) 0 (D) 16		
Sol.	(A) -16		1
8.	If $\int \frac{3ax}{b^2 + c^2x^2} dx = A \log b^2 + c^2x^2 + K$, then the value of A is (A) $3a$ (B) $\frac{3a}{2b^2}$ (C) $\frac{3a}{b^2c^2}$ (D) $\frac{3a}{2c^2}$		
Sol.	(D) $\frac{3a}{2c^2}$		1
9.	The value of $\int_{-1}^1 \frac{x^3}{x^2 + 2 x + 1} dx$ is (A) 0 (B) $\log 2$ (C) $2 \log 2$ (D) $\frac{1}{2} \log 2$		
Sol.	(A) 0		1

10.	The area bounded by the curve $y = x x$, x-axis and the ordinates $x = -1$ and $x = 1$ is given by (A) 0 (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{4}{3}$		
Sol.	(C) $\frac{2}{3}$		1
11.	The integrating factor of differential equation $R \frac{dx}{dy} + Px = Q$ where P, Q, R are functions of y is (A) $e^{\int \frac{P}{Q} dy}$ (B) $e^{\int P dy}$ (C) $e^{\int \frac{P}{R} dy}$ (D) $e^{\int \frac{P}{R} dx}$		
Sol.	(C) $e^{\int \frac{P}{R} dy}$		1
12.	The order and degree of the differential equation $\frac{d}{dx}(e^y) = 0$ respectively are (A) 0, 1 (B) 1, 1 (C) 2, 1 (D) 1, not defined		
Sol.	(B) 1, 1		1

13.	The value of p for which vectors $\hat{i} + 2\hat{j} + 3\hat{k}$ and $2\hat{i} - p\hat{j} + \hat{k}$ are perpendicular to each other is (A) 0 (B) 1 (C) $\frac{5}{2}$ (D) $-\frac{5}{2}$		
Sol.	(C) $\frac{5}{2}$		1
14.	The value of m for which the points with position vectors $-\hat{i} - \hat{j} + 2\hat{k}$, $2\hat{i} + m\hat{j} + 5\hat{k}$ and $3\hat{i} + 11\hat{j} + 6\hat{k}$ are collinear, is (A) 8 (B) -8 (C) 2 (D) $\frac{5}{2}$		
Sol.	(A) 8		1
15.	If $\vec{a} = 8$, $\vec{b} = 3$ and $\vec{a} \times \vec{b} = 12$, then the value of $\vec{a} \cdot \vec{b}$ (A) $6\sqrt{3}$ (B) $8\sqrt{3}$ (C) $12\sqrt{3}$ (D) $3\sqrt{12}$		
Sol.	(C) $12\sqrt{3}$		1
16.	The length of perpendicular drawn from point (2, 5, 7) on line $\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$ is (A) 2 (B) 5 (C) $\sqrt{74}$ (D) $\sqrt{78}$		
Sol.	(C) $\sqrt{74}$		1

17.	The feasible region of a linear programming problem with objective function $Z = 5x + 7y$ is shown below :  The maximum value of Z – minimum value of Z is (A) 8 (B) 29 (C) 35 (D) 43		
Sol.	(D) 43		1
18.	The degree of an objective function of a linear programming problem is (A) 0 (B) 1 (C) 2 (D) Any natural number		
Sol.	(B) 1		1
	Assertion – Reason Based Questions Direction : Questions number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below. (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A). (C) Assertion (A) is true and Reason (R) is false. (D) Assertion (A) is false and Reason (R) is true.		

19.	Assertion (A) : In an experiment of throwing an unbiased die, the probability of getting a prime number given that number appearing on the die being odd is $\frac{2}{3}$. Reason (R) : For any two events A and B, $P(A B) = \frac{P(A \cap B)}{P(B)}$		
Sol.	(C) Assertion (A) is true and Reason (R) is false.		1
20.	Assertion (A) : Lines given by $x = py + q$, $z = ry + s$ and $x = p'y + q'$, $z = r'y + s'$ are perpendicular to each other when $pp' + rr' = 1$. Reason (R) : Two lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ are perpendicular to each other if $\vec{b}_1 \cdot \vec{b}_2 = 0$.		
Sol.	(D) Assertion (A) is false and Reason (R) is true.		1
SECTION B			
Q. Numbers 21 to 25 are very short answer questions of 2 marks each.			
21.(a)	Check whether function $f(x)$ defined as $f(x) = \begin{cases} \frac{ x-3 }{2(x-3)}, & x < 3 \\ \frac{x-6}{6}, & x \geq 3 \end{cases}$ is continuous at $x = 3$ or not ?		
Sol.	$f(x) = \begin{cases} \frac{-(x-3)}{2(x-3)}, & x < 3 \\ \frac{x-6}{6}, & x \geq 3 \end{cases} \Rightarrow f(x) = \begin{cases} \frac{-1}{2}, & x < 3 \\ \frac{x-6}{6}, & x \geq 3 \end{cases}$ $f(3) = \frac{-1}{2}$ $\text{LHL} = \lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} \frac{-1}{2} = \frac{-1}{2}$ $\text{RHL} = \lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} \frac{x-6}{6} = \frac{-1}{2}$	I II	1 $\frac{1}{2}$

	LHL = RHL = $f(3)$, hence the function $f(x)$ is continuous at $x = 3$.	III	$\frac{1}{2}$
	OR		
21.(b)	(b) If $\sqrt{3}(x^2 + y^2) = 4xy$, then find $\frac{dy}{dx}$ at $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.		
Sol.	Differentiating both sides w.r.to x , we get $\sqrt{3}\left(2x + 2y\frac{dy}{dx}\right) = 4\left(x\frac{dy}{dx} + y\right)$	I	1
	$\Rightarrow \frac{dy}{dx} = \frac{2y - \sqrt{3}x}{\sqrt{3}y - 2x}$	II	$\frac{1}{2}$
	$\Rightarrow \left(\frac{dy}{dx}\right)_{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)} = \sqrt{3}$	III	$\frac{1}{2}$
22.	A room freshner bottle in the shape of an inverted cone sprays the perfume at regular intervals such that volume of the perfume in the bottle decreases at the steady rate of $1 \text{ mm}^3/\text{min}$. Find the rate at which level of perfume is dropping at an instant when level of perfume in the bottle is 10 mm, if the semi-vertical angle of conical bottle is $\frac{\pi}{6}$. 		
Sol.	Let v, r, h , respectively, be the volume, radius and the height of perfume at any time t .		
	$\frac{dv}{dt} = -1 \text{ mm}^3/\text{min}$		
	$\frac{r}{h} = \tan \frac{\pi}{6} \Rightarrow r = \frac{h}{\sqrt{3}}$	I	$\frac{1}{2}$
	$v = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \frac{h^3}{3}$	II	$\frac{1}{2}$
	$\Rightarrow \frac{dv}{dt} = \frac{\pi}{9} 3h^2 \frac{dh}{dt}$	III	$\frac{1}{2}$

	$\Rightarrow \frac{dh}{dt} = -\frac{3}{\pi h^2}$ $\Rightarrow \left(\frac{dh}{dt}\right)_{h=10} = -\frac{3}{100\pi}$ The level of perfume is dropping at the rate of $\frac{3}{100\pi} \text{ mm/min}$	IV	$\frac{1}{2}$
23.	Find the vector of magnitude 14 in the direction of $\overrightarrow{QP}$, where P and Q are the points (1, 3, 2) and (-1, 0, 8) respectively.		
Sol.	$\overrightarrow{QP} = 2\hat{i} + 3\hat{j} - 6\hat{k}$ $ \overrightarrow{QP} = \sqrt{4 + 9 + 36} = 7$ The unit vector in the direction of $\overrightarrow{QP} = \frac{\overrightarrow{QP}}{ \overrightarrow{QP} } = \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}$ The required vector = $14\left(\frac{\overrightarrow{QP}}{ \overrightarrow{QP} }\right) = 4\hat{i} + 6\hat{j} - 12\hat{k}$	I II III	$\frac{1}{2}$ $\frac{1}{2}$ 1
24.	Vectors $\vec{a} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{k}$ represent the two adjacent sides of a parallelogram. Find the vectors representing its diagonals and hence find their lengths.		
Sol.	$\vec{a} + \vec{b} = 4\hat{i} - 2\hat{j} + 4\hat{k}, \vec{a} - \vec{b} = 2\hat{i} - 2\hat{j}$ The vector representing one of its diagonals is $4\hat{i} - 2\hat{j} + 4\hat{k}$ or $-(4\hat{i} - 2\hat{j} + 4\hat{k})$ The vector representing the other diagonal is $2\hat{i} - 2\hat{j}$ or $-(2\hat{i} - 2\hat{j})$ The lengths of the two diagonals are $\sqrt{16 + 4 + 16} = 6$ and $\sqrt{4 + 4} = 2\sqrt{2}$	I II III	$\frac{1}{2}$ $\frac{1}{2}$ 1
25. (a)	Simplify : $\tan^{-1} \left(\frac{\cos 2x - \sin 2x}{\cos 2x + \sin 2x} \right), 0 < x < \frac{\pi}{4}$.		
Sol.	Given expression = $\tan^{-1} \left(\frac{1 - \tan 2x}{1 + \tan 2x} \right)$ = $\tan^{-1} \left[\tan \left(\frac{\pi}{4} - 2x \right) \right] = \frac{\pi}{4} - 2x$	I II	1 1
	OR		

25. (b)	Evaluate : $\tan\left(\sin^{-1}1 - \cos^{-1}\left(-\frac{1}{2}\right)\right)$		
Sol.	Given expression = $\tan\left(\frac{\pi}{2} - \frac{2\pi}{3}\right)$ = $\tan\left(-\frac{\pi}{6}\right) = -\frac{1}{\sqrt{3}}$	I II	1 1
SECTION C Q. Numbers 26 to 31 are short answer questions of 3 marks each.			
26.	Evaluate : $\int_0^1 x \tan^{-1}x \, dx.$		
Sol.	$\int_0^1 x \tan^{-1}x \, dx = \left[\tan^{-1}x \times \frac{x^2}{2} \right]_0^1 - \int_0^1 \frac{1}{1+x^2} \times \frac{x^2}{2} \, dx$ = $\frac{\pi}{8} - \int_0^1 \frac{1+x^2-1}{1+x^2} \times \frac{1}{2} \, dx$ = $\frac{\pi}{8} - \frac{1}{2} \left[\int_0^1 dx - \int_0^1 \frac{1}{1+x^2} \, dx \right]$ = $\frac{\pi}{8} - \frac{1}{2} [x - \tan^{-1}x]_0^1$ = $\frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{8} = \frac{\pi}{4} - \frac{1}{2}$	I II III	1 1 1
27. (a)	Find $\int \sqrt{\frac{x+2}{x-2}} \, dx$		
Sol.	$I = \int \sqrt{\frac{x+2}{x-2}} \, dx = \int \sqrt{\frac{x+2}{x-2} \times \frac{x+2}{x+2}} \, dx = \int \frac{x+2}{\sqrt{x^2-4}} \, dx$ = $\frac{1}{2} \int \frac{2x}{\sqrt{x^2-4}} \, dx + 2 \int \frac{1}{\sqrt{x^2-4}} \, dx$	I II	1 1

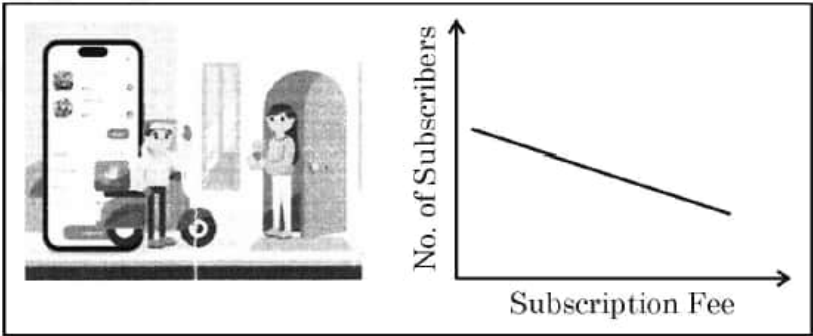
	$I_1 = \frac{1}{2} \int \frac{2x}{\sqrt{x^2-4}} dx = \frac{1}{2} \int \frac{dt}{\sqrt{t}} \quad [(x^2-4) = t \Rightarrow 2x dx = dt]$ $= \sqrt{t} = \sqrt{x^2-4}$	III	1/2
	$I_2 = 2 \int \frac{1}{\sqrt{x^2-4}} dx = 2 \log x + \sqrt{x^2-4} $ $I = I_1 + I_2 = \sqrt{x^2-4} + 2 \log x + \sqrt{x^2-4} + C$	IV	1/2
	OR		
27. (b)	Find : $\int \frac{x^2}{(x^2+9)(x^2+16)} dx$		
Sol.	Put $x^2 = t$ to get $\frac{x^2}{(x^2+9)(x^2+16)} = \frac{t}{(t+9)(t+16)} = \frac{A}{t+9} + \frac{B}{t+16}$ Getting $A = \frac{-9}{7}, B = \frac{16}{7}$ Given integral $= \frac{-9}{7} \int \frac{1}{x^2+9} dx + \frac{16}{7} \int \frac{1}{x^2+16} dx$ $= \frac{-3}{7} \tan^{-1}\left(\frac{x}{3}\right) + \frac{4}{7} \tan^{-1}\left(\frac{x}{4}\right) + C$	I II III IV	1/2 1 1/2 1
28.	If $I_1 = \int_{-\pi/4}^{\pi/4} \frac{dx}{1+\cos 2x}$ and $I_2 = \int_{-1/2}^{1/2} x dx$, then show that $I_1 - 4I_2 = 0$.		
Sol.	$I_1 = 2 \int_0^{\pi/4} \frac{dx}{1+\cos 2x}, \quad \left(\frac{1}{1+\cos 2x} \text{ is an even function}\right)$ $= 2 \int_0^{\pi/4} \frac{dx}{2\cos^2 x} = \int_0^{\pi/4} \sec^2 x dx$ $= [\tan x]_0^{\pi/4} = 1$ Note: Students attempting I_1 without the application of any property should be marked proportionately. $I_2 = 2 \int_0^{1/2} x dx, \quad (x \text{ is an even function})$ $= 2 \int_0^{1/2} x dx$	I II	1 1/2 1/2

	$= [x^2]_0^{\frac{1}{2}} = \frac{1}{4}$ $I_1 - 4I_2 = 0$	III	$\frac{1}{2}$
		IV	$\frac{1}{2}$
29. (a)	Find the general solution of the following differential equation : $x^2 \frac{dy}{dx} = x^2 + xy + y^2$		
Sol.	Given differential equation is $\frac{dy}{dx} = 1 + \frac{y}{x} + \left(\frac{y}{x}\right)^2$ Put $y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$ The given differential equation becomes $x \frac{dv}{dx} = 1 + v^2$ $\Rightarrow \frac{dv}{1+v^2} = \frac{dx}{x}$ $\Rightarrow \int \frac{dv}{1+v^2} = \int \frac{dx}{x}$ $\Rightarrow \tan^{-1}v = \log x + C$ $\Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = \log x + C$	I	$\frac{1}{2}$
		II	$\frac{1}{2}$
		III	$\frac{1}{2}$
		IV	1
		V	$\frac{1}{2}$
	OR		
29. (b)	Find the particular solution of the differential equation $xy \frac{dy}{dx} = (x+2)(y+2), \text{ given that } y(1) = -1.$		
Sol.	Given differential equation is $\frac{y}{y+2} dy = \frac{(x+2)}{x} dx$ $\Rightarrow \int \left(1 - \frac{2}{y+2}\right) dy = \int \left(1 + \frac{2}{x}\right) dx$ $\Rightarrow y - 2\log y+2 = x + 2\log x + C \text{ or } y = x + 2\log x(y+2) + C.$ When $x = 1, y = -1 \Rightarrow C = -2$ The required particular solution is	I	1
		II	$1\frac{1}{2}$

31.(a)	Out of two bags, bag I contains 3 red and 4 white balls and bag II contains 8 red and 6 white balls. A die is thrown. If it shows a number less than 3 then a ball is drawn at random from bag I, otherwise a ball is drawn at random from bag II. Find the probability that the ball drawn from one of the bags is a red ball.		
Sol.	Let E_1: The number appearing on the die < 3 E_2: The number appearing on the die ≥ 3 A: Red ball is drawn $P(E_1) = \frac{2}{6}, P(A/E_1) = \frac{3}{7}, P(E_2) = \frac{4}{6}, P(A/E_2) = \frac{8}{14}$ The required probability $= P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2)$ $= \frac{2}{6} \times \frac{3}{7} + \frac{4}{6} \times \frac{8}{14}$ $= \frac{11}{21}$	I 1/2 II 1 III 1 IV 1/2	
	OR		
31.(b)	The probability of simultaneous occurrence of atleast one of the two events X and Y is a. If the probability that exactly one of the events X, Y occurs is b, prove that $P(X') + P(Y') = 2 - 2a + b$.		
Sol.	$P(X \cup Y) = a$ $P(X \cup Y) - P(X \cap Y) = b$ Hence, $P(X \cap Y) = a - b$ $P(X') + P(Y') = [1 - P(X)] + [1 - P(Y)]$ $= 2 - [P(X) + P(Y)]$ $= 2 - [P(X \cup Y) + P(X \cap Y)]$ $= 2 - [a + (a - b)]$	I 1/2 II 1 III 1	

	$\Rightarrow \frac{3x_1 + 2}{5x_1 - 3} = \frac{3x_2 + 2}{5x_2 - 3}$ $\Rightarrow 15x_1x_2 - 9x_1 + 10x_2 - 6 = 15x_1x_2 + 10x_1 - 9x_2 - 6$ $\Rightarrow x_1 = x_2$ Hence, f is one-one. Let $y \in R - \left\{\frac{3}{5}\right\}$ (Codomain). Then $f(x) = y$ or, $\frac{3x+2}{5x-3} = y \Rightarrow x = \frac{3y+2}{5y-3} \in \text{Domain}.$ Hence, for every $y \in R - \left\{\frac{3}{5}\right\}$ (codomain), there exists $x = \frac{3y+2}{5y-3} \in R - \left\{\frac{3}{5}\right\}$ (domain) such that $f\left(\frac{3y+2}{5y-3}\right) = y$ Thus, f is onto.	I	2½
		II	2½
33. (a)	If $A = \begin{bmatrix} 0 & 2 & 1 \\ -2 & -1 & -2 \\ 1 & -1 & 0 \end{bmatrix}$, find A^{-1} and use it to solve the following system of equations : $-2y + z = 7, 2x - y - z = 8, x - 2y = 10$		
Sol.	$ A = -2(2) + 1(3) = -1 \neq 0$ $\text{adj}A = \begin{bmatrix} -2 & -1 & -3 \\ -2 & -1 & -2 \\ 3 & 2 & 4 \end{bmatrix}$ $A^{-1} = \frac{1}{ A } \text{adj}A = \begin{bmatrix} 2 & 1 & 3 \\ 2 & 1 & 2 \\ -3 & -2 & -4 \end{bmatrix}$ Given system of equations is equivalent to $A^T X = B$, where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 7 \\ 8 \\ 10 \end{bmatrix}$ $X = (A^T)^{-1} B = (A^{-1})^T B$ $\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & 2 & -3 \\ 1 & 1 & -2 \\ 3 & 2 & -4 \end{bmatrix} \begin{bmatrix} 7 \\ 8 \\ 10 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$ $\therefore x = 0, y = -5, z = -3$	I II III IV V	1 1½ ½ ½ 1½
	OR		

33. (b)	If $\begin{bmatrix} 3 & -1 & \sin 3x \\ -7 & 4 & \cos 2x \\ -11 & 7 & 2 \end{bmatrix}$ is a singular matrix, then find all values of x where $x \in \left[0, \frac{\pi}{2}\right]$.		
Sol.	Given matrix is singular, hence, its determinant = 0, i. e., $\begin{vmatrix} 3 & -1 & \sin 3x \\ -7 & 4 & \cos 2x \\ -11 & 7 & 2 \end{vmatrix} = 0$ $\Rightarrow 2\cos 2x + \sin 3x = 2$ $\Rightarrow 4\sin^3 x + 4\sin^2 x - 3\sin x = 0$ $\Rightarrow \sin x(4\sin^2 x + 4\sin x - 3) = 0$ $\Rightarrow \sin x(2\sin x - 1)(2\sin x + 3) = 0$ Solving to get $x = 0, \frac{\pi}{6}$	I II III IV V	1 1 1 1 1
34.	If $x = \cos t$, $y = \cos mt$, prove that $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + m^2y = 0$.		
Sol.	$\frac{dx}{dt} = -\sin t, \frac{dy}{dt} = -m \times \sin(mt)$ $\frac{dy}{dx} = \frac{m \times \sin(mt)}{\sin t}$ or $\sin t \frac{dy}{dx} = m \times \sin(mt)$ $\Rightarrow \sin t \frac{d^2y}{dx^2} + \frac{dy}{dx} \cos t \times \frac{dt}{dx} = m^2 \cos(mt) \times \frac{dt}{dx}$ $\Rightarrow (1 - \cos^2 t) \frac{d^2y}{dx^2} - x \frac{dy}{dx} = -m^2 \cos(mt)$ $\Rightarrow (1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + m^2y = 0$	I II III IV	1 1 2 1
35.	Check whether the lines given by $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ are parallel or not. If parallel, find the distance between them, otherwise find their point of intersection, if the lines are intersecting.		
Sol.	Drs of first line are $\langle 2, 3, 4 \rangle$		

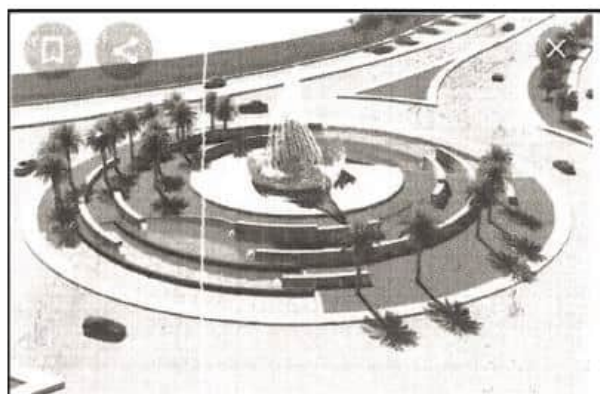
	Drs of second line are $\langle 5, 2, 1 \rangle$ Drs of the two lines are not proportional. Hence, the lines are not parallel. Any point on the first line is $(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$ Any point on the second line is $(5\mu + 4, 2\mu + 1, \mu)$ For the lines to intersect, we must have some λ and μ, for which these coordinates must coincide, i.e., we must have $2\lambda + 1 = 5\mu + 4, 3\lambda + 2 = 2\mu + 1, 4\lambda + 3 = \mu$, for some λ and μ The first two equations, when solved, give us $\lambda = -1$ and $\mu = -1$. These values satisfy the third equation. Hence, the lines intersect. The point of intersection is $(-1, -1, -1)$	I II III IV V	1 1 1 1 1
	SECTION E This section (Q. 36 to 38) has 3 case study-based questions of 4 marks each.		
36.	An online delivery company in a city has 5000 subscribers and collects annual subscription fees of ₹ 300 per subscriber for unlimited free deliveries.  The company wishes to increase the annual subscription fee. It is predicted that, for every increase of ₹ 1, ten subscribers will discontinue. Assume that the company increased the annual fee by ₹ x.		

	Based on the given information, answer the following questions : (i) How many subscribers will discontinue after an increase of ₹ x in annual fee ? (ii) If $R(x)$ denotes the total revenue collected after the increase of ₹ x in subscription fee, express $R(x)$ as a function of x. (iii) (a) Find the value of x for which $R(x)$ is maximum. OR (iii) (b) Find the sub-intervals of $(0, 5000)$ in which $R(x)$ is increasing and decreasing.		
Sol.(i)	$10x$	I	1
Sol.(ii)	$R(x) = (5000 - 10x)(300 + x)$	I	1
Sol.(iii)	$R'(x) = 2000 - 20x$	I	$\frac{1}{2}$
(a)	$R'(x) = 0 \Rightarrow x = 100$	II	$\frac{1}{2}$
	$R''(x) = -20 < 0$ at $x = 100$	III	$\frac{1}{2}$
	At $x = 100$, $R(x)$ is maximum	IV	$\frac{1}{2}$
	OR		
Sol.(iii)	$R'(x) = 2000 - 20x$	I	$\frac{1}{2}$
(b)	$R'(x) = 0 \Rightarrow x = 100$	II	$\frac{1}{2}$
	In $(0, 100)$, $R'(x) > 0$. Therefore, $R(x)$ is increasing in $(0, 100)$ or $(0, 100]$	III	$\frac{1}{2}$
	In $(100, 5000)$, $R'(x) < 0$. Therefore, $R(x)$ is decreasing in $(100, 5000)$ or $[100, 5000)$	IV	$\frac{1}{2}$

37.	In an online jackpot, there is one first prize of ₹ 3,00,000, two second prizes of ₹ 2,00,000 each and three third prizes of ₹ 50,000 each.  A total of 1,00,000 jackpot tickets each costing ₹ 100 were sold there by raising a fund of ₹ 1,00,00,000. Rohan bought one ticket. Based on given information, answer the following questions : (i) What are the possible amounts, the person can win ? (ii) (a) What is the probability that the person wins atleast ₹ 2,00,000 ? OR (ii) (b) What is the probability that the person does not win any amount ? (iii) In another jackpot, Rohan also bought a ticket having a prize money of ₹ 5,00,000. The chances of winning the jackpot are 1 in 1,00,000. Find the probability that on exactly one of tickets he wins the jackpot.		
Sol.(i)	₹ 300000, ₹ 200000, ₹ 50000	I	1
Sol.(ii) (a)	Required probability = $\frac{3}{100000}$	I	2
	OR		
Sol.(ii) (b)	Required probability = $1 - \frac{6}{100000} = \frac{99994}{100000}$ or $\frac{49997}{50000}$	I	2
Sol.(iii)	P (Exactly on one of the tickets Rohan wins the jackpot) = P [(He wins first and he loses second) or (He loses first and wins second)] $= \frac{6}{100000} \times \frac{99999}{100000} + \frac{99994}{100000} \times \frac{1}{100000} = \frac{699988}{10^{10}} \text{ or } \frac{174997}{25000000000}$	I	1

38.

Roundabouts are often made on busy roads to ease the traffic and avoid red lights.



One such round-about is made such that equation representing its boundary is given by $C_1 : x^2 + y^2 = 64$.

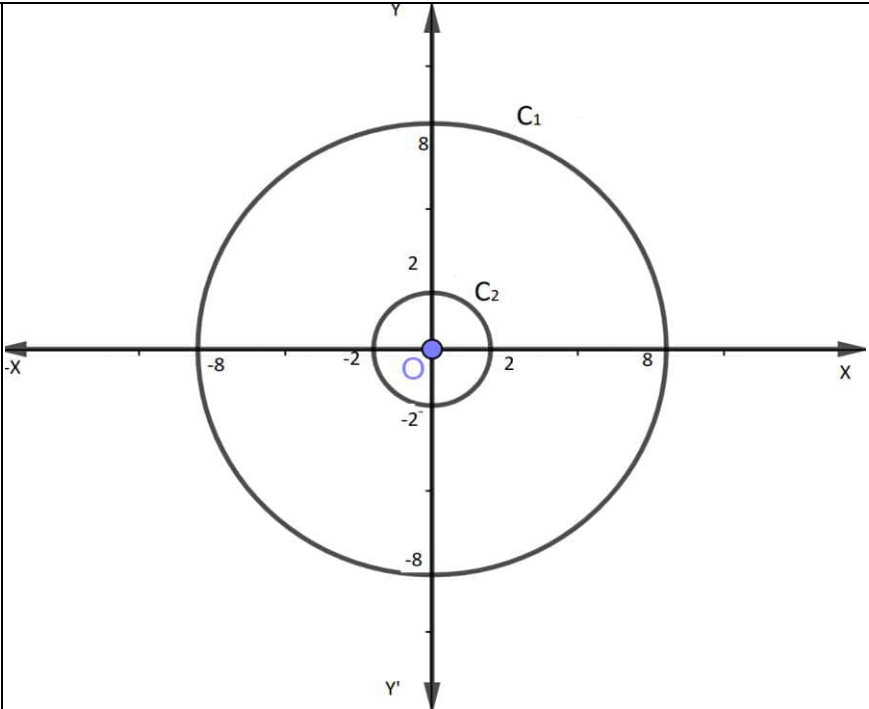
There is a circular pond with a fountain in the middle of the roundabout whose equation is given by $C_2 : x^2 + y^2 = 4$.

Based on the given information, answer the following questions :

- (i) Represent the given equations C_1 and C_2 with the help of a diagram.
- (ii) Express y as a function of x , ($y = f(x)$), for both C_1 and C_2 .
- (iii) (a) Using integration find the area of region covered by the roundabout.

OR

- (iii) (b) Using integration, find the area of region covered by circular pond.

Sol.(i)		I	1
Sol.(ii)	Circle C_1: $y = \sqrt{64 - x^2}$ or $y = -\sqrt{64 - x^2}$ Circle C_2: $y = \sqrt{4 - x^2}$ or $y = -\sqrt{4 - x^2}$	I II	$\frac{1}{2}$ $\frac{1}{2}$
Sol.(iii) (a)	Required area = $4 \times \int_0^8 \sqrt{64 - x^2} dx$ $= 2[x\sqrt{64 - x^2} + 64\sin^{-1}\frac{x}{8}]_0^8 = 64\pi$	I II	1 1
	OR		
Sol.(iii) (b)	Required area = $4 \times \int_0^2 \sqrt{4 - x^2} dx$ $= 2[x\sqrt{4 - x^2} + 4\sin^{-1}\frac{x}{2}]_0^2 = 4\pi$	I II	1 1