

**SET-3****Series : SR3PQ**प्रश्न-पत्र कोड
Q.P. Code**65/3/3**

रोल नं.

Roll No.

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परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।

Candidates must write the Q.P. Code on the title page of the answer-book.

**गणित****MATHEMATICS**

निर्धारित समय : 3 घण्टे

Time allowed : 3 hours

अधिकतम अंक : 80

Maximum Marks : 80

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं।
- कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में यथा स्थान पर प्रश्न का क्रमांक अवश्य लिखें।
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।
- Please check that this question paper contains 23 printed pages.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 38 questions.
- Please write down the Serial Number of the question in the answer-book at the given place before attempting it.
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.

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सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख्ती से पालन कीजिए :

- इस प्रश्न-पत्र में 38 प्रश्न हैं। सभी प्रश्न अनिवार्य हैं।
- यह प्रश्न-पत्र पाँच खण्डों में विभाजित है – क, ख ग, घ एवं ङ।
- खण्ड क में प्रश्न संख्या 1 से 18 तक बहुविकल्पीय (MCQ) तथा प्रश्न संख्या 19 एवं 20 अभिकथन एवं तर्क आधारित 1 अंक के प्रश्न हैं।
- खण्ड ख में प्रश्न संख्या 21 से 25 तक अति लघु-उत्तरीय (VSA) प्रकार के 2 अंकों के प्रश्न हैं।
- खण्ड ग में प्रश्न संख्या 26 से 31 तक लघु-उत्तरीय (SA) प्रकार के 3 अंकों के प्रश्न हैं।
- खण्ड घ में प्रश्न संख्या 32 से 35 तक दीर्घ-उत्तरीय (LA) प्रकार के 5 अंकों के प्रश्न हैं।
- खण्ड ङ में प्रश्न संख्या 36 से 38 तक प्रकरण अध्ययन आधारित 4 अंकों के प्रश्न हैं।
- प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड ख के 2 प्रश्नों में, खण्ड ग के 3 प्रश्नों में, खण्ड घ के 2 प्रश्नों में तथा खण्ड ङ के 3 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है।
- कैल्कुलेटर का उपयोग वर्जित है।

खण्ड क

इस खण्ड में बहुविकल्पीय प्रश्न (MCQ) हैं, जिनमें प्रत्येक प्रश्न 1 अंक का है।

- यदि $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 198$ तथा $|\vec{a}| = 10|\vec{b}|$ है, तो :

(A) $ \vec{a} = \sqrt{2}$	(B) $ \vec{b} = \sqrt{2}$
(C) $ \vec{b} = 10\sqrt{2}$	(D) $ \vec{a} = \frac{10}{\sqrt{2}}$
- यदि l_1, m_1, n_1 तथा l_2, m_2, n_2 क्रमशः रेखाओं L_1 तथा L_2 के दिक्-कोसाइन हैं तथा θ उनके बीच का न्यून कोण है, तो :

(A) $\cos \theta = l_1 l_2 + m_1 m_2 + n_1 n_2$	(B) $\sin \theta = l_1 l_2 + m_1 m_2 + n_1 n_2$
(C) $\tan \theta = \frac{l_1}{l_2} + \frac{m_1}{m_2} + \frac{n_1}{n_2}$	(D) $\cos \theta = l_1 l_2 + m_1 m_2 + n_1 n_2 $
- रेखाओं l_1 तथा l_2 के दिक्-अनुपात क्रमशः $\langle 1, 0, 0 \rangle$ तथा $\langle 0, -1, 0 \rangle$ हैं। वह रेखा, जो l_1 तथा l_2 दोनों के लंबवत है, के दिक्-अनुपात हैं :

(A) $\langle 1, 1, 0 \rangle$	(B) $\langle 0, 0, -1 \rangle$
(C) $\langle 1, 1, 1 \rangle$	(D) $\langle 1, 0, -1 \rangle$



General Instructions :

Read the following instructions very carefully and strictly follow them :

- (i) This question paper contains **38** questions. **All** questions are **compulsory**.
- (ii) This question paper is divided into **five** Sections – **A, B, C, D** and **E**.
- (iii) In **Section A**, Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.
- (iv) In **Section B**, Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.
- (v) In **Section C**, Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.
- (vi) In **Section D**, Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.
- (vii) In **Section E**, Questions no. **36** to **38** are case study based questions carrying **4** marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 3 questions in Section E.
- (ix) Use of calculator is **not** allowed.

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 198$ and $|\vec{a}| = 10|\vec{b}|$, then :

(A) $ \vec{a} = \sqrt{2}$	(B) $ \vec{b} = \sqrt{2}$
(C) $ \vec{b} = 10\sqrt{2}$	(D) $ \vec{a} = \frac{10}{\sqrt{2}}$
2. If l_1, m_1, n_1 and l_2, m_2, n_2 are direction cosines of lines L_1 and L_2 respectively and θ is the acute angle between them, then :

(A) $\cos \theta = l_1 l_2 + m_1 m_2 + n_1 n_2$	(B) $\sin \theta = l_1 l_2 + m_1 m_2 + n_1 n_2$
(C) $\tan \theta = \frac{l_1}{l_2} + \frac{m_1}{m_2} + \frac{n_1}{n_2}$	(D) $\cos \theta = l_1 l_2 + m_1 m_2 + n_1 n_2 $
3. Direction ratios of lines l_1 and l_2 respectively are $\langle 1, 0, 0 \rangle$ and $\langle 0, -1, 0 \rangle$. The direction ratios of the line perpendicular to both l_1 and l_2 are :

(A) $\langle 1, 1, 0 \rangle$	(B) $\langle 0, 0, -1 \rangle$
(C) $\langle 1, 1, 1 \rangle$	(D) $\langle 1, 0, -1 \rangle$



4. यदि E तथा F दो स्वतंत्र घटनाएँ इस प्रकार हैं कि $P(E) = \frac{3}{10}$, $P(E \cup F) = \frac{1}{2}$ है, तो $P(E|F) - P(F|E)$ बराबर है :
- (A) $\frac{2}{7}$ (B) $\frac{3}{35}$
(C) $\frac{1}{70}$ (D) $\frac{1}{7}$
5. $\cos^{-1}(4x + 1)$ का प्रान्त है :
- (A) $[-1, 1]$ (B) $[-3, 5]$
(C) $[-4, 4]$ (D) $[-\frac{1}{2}, 0]$
6. यदि $A^2 = 4A + 3I$ है तथा $A^{-1} = xA + yI$ है, तो $(x + y)$ का मान है :
- (A) -1 (B) 1
(C) $\frac{5}{3}$ (D) 7
7. यदि A तथा B समान कोटि के विषम-सममित आव्यूह हैं, तो $AB' + BA'$ एक :
- (A) सममित आव्यूह है
(B) विषम-सममित आव्यूह है
(C) शून्य आव्यूह है
(D) तत्समक आव्यूह है
8. यदि एक आव्यूह X इस प्रकार है कि $\begin{bmatrix} 2 & 1 \end{bmatrix} X = \begin{bmatrix} 3 & 4 & 5 \end{bmatrix}$ है, तो आव्यूह X की कोटि है :
- (A) 3×1 (B) 2×3
(C) 1×2 (D) 1×3
9. यदि एक वर्ग आव्यूह A ऐसा है कि $A^2 = A$ तथा $(I - A)^3 = xA + I$ है, तो x का मान होगा :
- (A) 7 (B) 5
(C) -7 (D) -1
10. यदि $A(\text{adj } A) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ है, तो $|2A|$ का मान है :
- (A) 6 (B) 54
(C) 12 (D) 24



4. If E and F are two independent events such that $P(E) = \frac{3}{10}$, $P(E \cup F) = \frac{1}{2}$, then $P(E | F) - P(F | E)$ is equal to :
- (A) $\frac{2}{7}$ (B) $\frac{3}{35}$
(C) $\frac{1}{70}$ (D) $\frac{1}{7}$
5. The domain of $\cos^{-1}(4x + 1)$ is :
- (A) $[-1, 1]$ (B) $[-3, 5]$
(C) $[-4, 4]$ (D) $[-\frac{1}{2}, 0]$
6. If $A^2 = 4A + 3I$ and $A^{-1} = xA + yI$, then the value of $(x + y)$ is :
- (A) -1 (B) 1
(C) $\frac{5}{3}$ (D) 7
7. If A and B are skew-symmetric matrices of same order, then $AB' + BA'$ is a/an :
- (A) symmetric matrix
(B) skew-symmetric matrix
(C) null matrix
(D) identity matrix
8. If a matrix X is such that $[2 \ 1] X = [3 \ 4 \ 5]$, then the order of matrix X is :
- (A) 3×1 (B) 2×3
(C) 1×2 (D) 1×3
9. If a square matrix A is such that $A^2 = A$ and $(I - A)^3 = xA + I$, then value of x must be :
- (A) 7 (B) 5
(C) -7 (D) -1
10. If $A(\text{adj } A) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then the value of $|2A|$ is :
- (A) 6 (B) 54
(C) 12 (D) 24



11. k का वह मान जिसके लिए फलन $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ k(x+1), & x = 0 \end{cases}$ एक संतत फलन है, है :
- (A) $\frac{1}{4}$ (B) 2
(C) $\frac{1}{2}$ (D) 0
12. यदि $2 \cos^{-1} x = y$ है, तो $\frac{dy}{dx}$ है :
- (A) $-2 \sin^{-1} x$ (B) $-\frac{1}{2} \sin \frac{y}{2}$
(C) $\frac{-1}{\sqrt{1-2x^2}}$ (D) $-2 \operatorname{cosec} \frac{y}{2}$
13. एक गोले के व्यास के सापेक्ष इसके आयतन के बदलने की दर, जब इसकी त्रिज्या 5 cm है, है :
- (A) $400\pi \text{ cm}^3/\text{cm}$ (B) $100\pi \text{ cm}^3/\text{cm}$
(C) $50\pi \text{ cm}^3/\text{cm}$ (D) $25\pi \text{ cm}^3/\text{cm}$
14. $\int \frac{\cos x}{\sqrt{\sin^2 x + 1}} dx$ बराबर है :
- (A) $\tan^{-1}(\sin x) + C$
(B) $\sin^{-1}(\sin x) + C$
(C) $\log |\sin x + \sqrt{1 + \sin^2 x}| + C$
(D) $\log |\sin x - \sqrt{\sin^2 x + 1}| + C$
15. $\int_{-1}^1 (1 - |x|) dx$ बराबर है :
- (A) $2 \int_0^1 (1+x) dx$ (B) $2 \int_{-1}^0 (1+x) dx$



(C) 0

(D) $2 \int_{-1}^0 (1-x) dx$

11. The value of k for which the function $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ k(x+1), & x = 0 \end{cases}$ is a continuous function, is :

(A) $\frac{1}{4}$

(B) 2

(C) $\frac{1}{2}$

(D) 0

12. If $2 \cos^{-1} x = y$, then $\frac{dy}{dx}$ is :

(A) $-2 \sin^{-1} x$

(B) $-\frac{1}{2} \sin \frac{y}{2}$

(C) $\frac{-1}{\sqrt{1-2x^2}}$

(D) $-2 \operatorname{cosec} \frac{y}{2}$

13. The rate of change of volume of a sphere with respect to its diameter, when its radius is 5 cm, is :

(A) $400\pi \text{ cm}^3/\text{cm}$

(B) $100\pi \text{ cm}^3/\text{cm}$

(C) $50\pi \text{ cm}^3/\text{cm}$

(D) $25\pi \text{ cm}^3/\text{cm}$

14. $\int \frac{\cos x}{\sqrt{\sin^2 x + 1}} dx$ is equal to :

(A) $\tan^{-1}(\sin x) + C$

(B) $\sin^{-1}(\sin x) + C$

(C) $\log |\sin x + \sqrt{1 + \sin^2 x}| + C$

(D) $\log |\sin x - \sqrt{\sin^2 x + 1}| + C$

15. $\int_{-1}^1 (1 - |x|) dx$ is equal to :



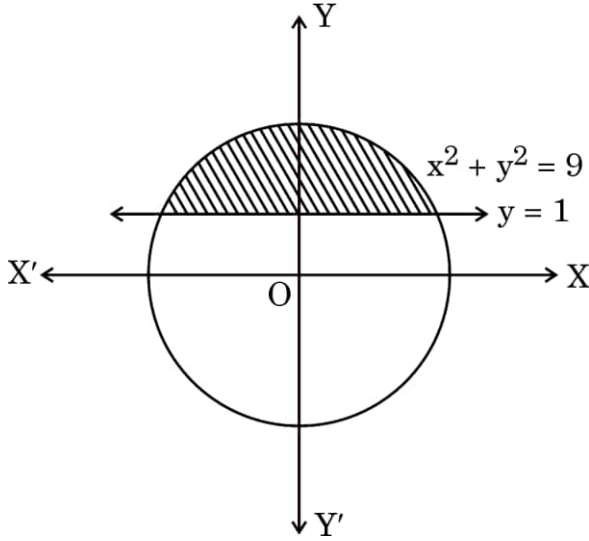
(A) $2 \int_0^1 (1+x) dx$

(B) $2 \int_{-1}^0 (1+x) dx$

(C) 0

(D) $2 \int_{-1}^0 (1-x) dx$

16. नीचे दी गई आकृति में, वृत्त के छायांकित भाग का क्षेत्रफल बराबर है :



(A) $\int_1^3 \sqrt{9-y^2} dy$

(B) $2 \int_1^3 \sqrt{9-y^2} dy$

(C) $\int_0^3 \sqrt{9-x^2} dx$

(D) $2 \int_0^3 \sqrt{9-x^2} dx$

17. निम्नलिखित में से कौन-से फलनों के लिए, $\frac{dy}{dx} = F(x, y)$ एक समघातीय अवकल समीकरण होगा ?

(i) $F(x, y) = 3x + 2y$

(ii) $F(x, y) = \sin \frac{y}{x} + \log y - \log x$

(iii) $F(x, y) = e^{y/x} + 1$

(iv) $F(x, y) = \sqrt{x^2 + y^2} - y$

(A) (i) तथा (ii)

(B) (i), (ii) तथा (iii)



(C) (ii), (iii) तथा (iv)

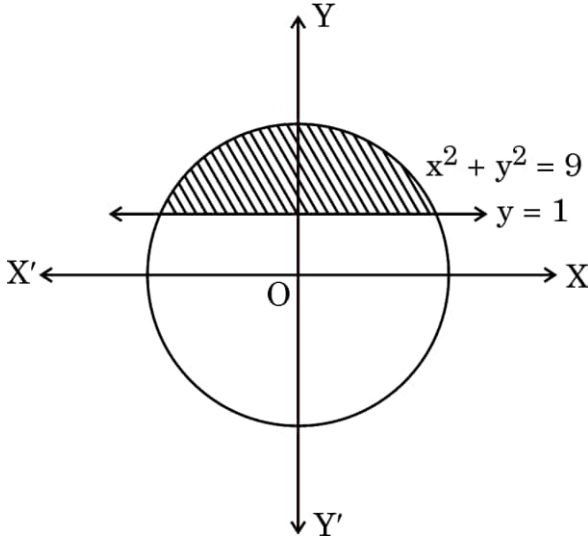
(D) (ii) तथा (iii)

18. किन्हीं दो सदिशों $\vec{a}$ तथा $\vec{b}$ के लिए, निम्नलिखित में से कौन-सा कथन सदैव सत्य है ?

(A) $\vec{a} \cdot \vec{b} \leq |\vec{a}| |\vec{b}|$ (B) $|\vec{a} + \vec{b}| \geq |\vec{a}| + |\vec{b}|$

(C) $|\vec{a} - \vec{b}| = |\vec{a}| - |\vec{b}|$ (D) $|\vec{a} \times \vec{b}| \geq |\vec{a}| |\vec{b}|$

16. The area of the shaded region of the circle given below is equal to :



(A) $\int_1^3 \sqrt{9-y^2} dy$

(B) $2 \int_1^3 \sqrt{9-y^2} dy$

(C) $\int_0^3 \sqrt{9-x^2} dx$

(D) $2 \int_0^3 \sqrt{9-x^2} dx$

17. $\frac{dy}{dx} = F(x, y)$ will be a homogeneous differential equation for which of the following functions ?

(i) $F(x, y) = 3x + 2y$

(ii) $F(x, y) = \sin \frac{y}{x} + \log y - \log x$

(iii) $F(x, y) = e^{y/x} + 1$

(iv) $F(x, y) = \sqrt{x^2 + y^2} - y$

(A) (i) and (ii)

(B) (i), (ii) and (iii)



(C) (ii), (iii) and (iv)

(D) (ii) and (iii)

18. For any two vectors $\vec{a}$ and $\vec{b}$, which of the following statements is always true ?

(A) $\vec{a} \cdot \vec{b} \leq |\vec{a}| |\vec{b}|$ (B) $|\vec{a} + \vec{b}| \geq |\vec{a}| + |\vec{b}|$

(C) $|\vec{a} - \vec{b}| = |\vec{a}| - |\vec{b}|$ (D) $|\vec{a} \times \vec{b}| \geq |\vec{a}| |\vec{b}|$

प्रश्न संख्या 19 और 20 अभिकथन एवं तर्क आधारित प्रश्न हैं। दो कथन दिए गए हैं जिनमें एक को अभिकथन (A) तथा दूसरे को तर्क (R) द्वारा अंकित किया गया है। इन प्रश्नों के सही उत्तर नीचे दिए गए कोडों (A), (B), (C) और (D) में से चुनकर दीजिए।

(A) अभिकथन (A) और तर्क (R) दोनों सही हैं और तर्क (R), अभिकथन (A) की सही व्याख्या करता है।

(B) अभिकथन (A) और तर्क (R) दोनों सही हैं, परन्तु तर्क (R), अभिकथन (A) की सही व्याख्या नहीं करता है।

(C) अभिकथन (A) सही है, परन्तु तर्क (R) गलत है।

(D) अभिकथन (A) गलत है, परन्तु तर्क (R) सही है।

19. अभिकथन (A) : अवकल समीकरण $\frac{dy}{dx} = e^{x+y}$ का एक विशिष्ट हल $e^x + e^{-y} = -2$ हो सकता है।

तर्क (R) : अवकल समीकरण $\frac{dy}{dx} = e^{x+y}$ का व्यापक हल $e^x + e^{-y} = C$ है।

20. अभिकथन (A) : सदिश $\vec{a}$ तथा $(-2\vec{a})$, जहाँ $\vec{a} \neq \vec{0}$, सरेखीय सदिश हैं।

तर्क (R) : $\vec{a} \cdot (-2\vec{a}) = 0$.

खण्ड ख

इस खण्ड में अति लघु-उत्तरीय (VSA) प्रकार के 5 प्रश्न हैं, जिनमें प्रत्येक के 2 अंक हैं।

21. (क) यदि रेखाएँ $\frac{x-3}{1} = \frac{1-y}{1} = \frac{z+2}{p}$ तथा $\frac{2-x}{3} = \frac{y+1}{5} = \frac{z+56}{2p}$ परस्पर लंबवत हैं, तो p का/के मान ज्ञात कीजिए।

अथवा



- (ख) उस रेखा का सदिश समीकरण ज्ञात कीजिए जो मूल-बिंदु से होकर जाती है तथा रेखाओं $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ तथा $\vec{r} = \mu(\hat{i} - \hat{j} + \hat{k})$ दोनों के लंबवत है।

22. यदि $x = e^{\sin^{-1} t}$ तथा $y = e^{\cos^{-1} t}$ है, तो $t = \frac{1}{\sqrt{2}}$ पर $\frac{dy}{dx}$ ज्ञात कीजिए।

23. $(0, \infty)$ का वह उप-अंतराल ज्ञात कीजिए जिसमें $f(x) = x \log x$ वर्धमान है।

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
 (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).
 (C) Assertion (A) is true, but Reason (R) is false.
 (D) Assertion (A) is false, but Reason (R) is true.

19. Assertion (A) : One of the particular solutions of the differential equation $\frac{dy}{dx} = e^{x+y}$ can be $e^x + e^{-y} = -2$.

Reason (R) : $e^x + e^{-y} = C$ is the general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$.

20. Assertion (A) : The vectors $\vec{a}$ and $(-2\vec{a})$, where $\vec{a} \neq \vec{0}$ are collinear vectors.

Reason (R) : $\vec{a} \cdot (-2\vec{a}) = 0$.

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. (a) If the lines $\frac{x-3}{1} = \frac{1-y}{1} = \frac{z+2}{p}$ and $\frac{2-x}{3} = \frac{y+1}{5} = \frac{z+56}{2p}$ are perpendicular to each other, then find the value(s) of p.



OR

- (b) Find the vector equation of a line passing through the origin and perpendicular to both the lines $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and $\vec{r} = \mu(\hat{i} - \hat{j} + \hat{k})$.

22. If $x = e^{\sin^{-1} t}$ and $y = e^{\cos^{-1} t}$, then find $\frac{dy}{dx}$ at $t = \frac{1}{\sqrt{2}}$.

23. Find the sub-interval of $(0, \infty)$ in which $f(x) = x \log x$ is increasing.

24. (क) $\sin [\cot^{-1} \sqrt{2} (\cos (\tan^{-1} 1))]$ का मान ज्ञात कीजिए।

अथवा

(ख) समुच्चय $A = \{1, 2, 3\}$ पर परिभाषित संबंध $R = \{(1, 1) (3, 3), (1, 2)\}$ है। क्या R एक सममित संबंध है? सत्यापित कीजिए। वह सबसे छोटा संबंध समुच्चय R_1 लिखिए जिससे $R \cup R_1$ समुच्चय $A = \{1, 2, 3\}$ पर एक तुल्यता संबंध बन जाए।

25. दो एकक सदिशों $\vec{a}$ तथा $\vec{b}$ के लिए, यदि $|\vec{a} + 2\vec{b}| = |2\vec{a} - \vec{b}|$ है, तो $\vec{a}$ तथा $\vec{b}$ के बीच का कोण ज्ञात कीजिए।

खण्ड ग

इस खण्ड में लघु-उत्तरीय (SA) प्रकार के 6 प्रश्न हैं, जिनमें प्रत्येक के 3 अंक हैं।

26. ऐसे मरीजों पर एक सर्वे करवाया गया जिन्होंने घुटने बदलवाने की सर्जरी करवाई है।



यह पाया गया कि रोबोटिक घुटने बदलवाने की सर्जरी 90% सफल रहती है।

एक विशेष दिन, तीन रोबोटिक सर्जरी रोगियों A, B तथा C पर एक के बाद एक की गई। यह मानते हुए कि प्रत्येक सर्जरी की सफलता और असफलता एक-दूसरे से स्वतंत्र हैं, प्रायिकता ज्ञात कीजिए कि :

- मात्र एक सर्जरी सफल रही,
- अधिक-से-अधिक दो सर्जरी सफल रहती हैं।



27. (क) ज्ञात कीजिए :

$$\int \frac{dx}{x^{1/2} + x^{1/3}}$$

अथवा

(ख) ज्ञात कीजिए :

$$\int \tan^{-1} \left(\frac{1-x}{1+x} \right) dx$$

24. (a) Find the value of $\sin [\cot^{-1} \sqrt{2} (\cos (\tan^{-1} 1))]$.

OR

(b) A relation R on $A = \{1, 2, 3\}$ is defined as $R = \{(1, 1) (3, 3), (1, 2)\}$. Is R a symmetric relation ? Justify. Write the smallest relation set R_1 such that $R \cup R_1$ becomes an equivalence relation on the set $\{1, 2, 3\}$.

25. If for two unit vectors $\vec{a}$ and $\vec{b}$, $|\vec{a} + 2\vec{b}| = |2\vec{a} - \vec{b}|$, then find the angle between $\vec{a}$ and $\vec{b}$.

SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26. A survey was conducted on the patients who have undergone knee replacement surgeries.



It was found that, Robotic Knee replacement surgeries have 90% success rate.

On a particular day, robotic surgery was performed on three patients, A, B and C, one after the other. Assuming that the success and failure of each surgery is independent of each other, find the probability that :

(i) exactly one surgery is successful,



- (ii) at most two surgeries are successful.

27. (a) Find :

$$\int \frac{dx}{x^{1/2} + x^{1/3}}$$

OR

- (b) Find :

$$\int \tan^{-1} \left(\frac{1-x}{1+x} \right) dx$$

28. मान ज्ञात कीजिए :

$$\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{1 + \sin 2x} dx$$

29. (क) ज्ञात कीजिए :

$$\int \frac{\cos x}{(2 + \sin x)(4 + \sin x)} dx$$

अथवा

- (ख) ज्ञात कीजिए :

$$\int \frac{x+3}{x^2 + 4x + 5} dx$$

30. (क) अवकल समीकरण $(x^2 - y^2) dx + 2xy dy = 0$ का व्यापक हल ज्ञात कीजिए।

अथवा

- (ख) अवकल समीकरण $\sin x \cos y dx + \cos x \sin y dy = 0$ का विशिष्ट हल ज्ञात कीजिए,

दिया गया है कि $y = \frac{\pi}{4}$ जब $x = 0$ है।

31. यदि $(\sin x)^y = y^{\cos x}$ है, तो $\frac{dy}{dx}$ ज्ञात कीजिए।

खण्ड घ



इस खण्ड में 4 दीर्घ-उत्तरीय (LA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 5 अंक हैं।

32. (क) नीचे दी गई सरल रेखाओं l_1 तथा l_2 के समीकरणों को सदिश रूप में लिखिए। अतः जाँच कीजिए कि रेखाएँ परस्पर प्रतिच्छेदी हैं या नहीं।

$$l_1: \frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$$

$$l_2: \frac{x+1}{-1} = \frac{2-y}{-2} = \frac{z-5}{5}$$

अथवा

28. Evaluate :

$$\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{1+\sin 2x} dx$$

29. (a) Find :

$$\int \frac{\cos x}{(2+\sin x)(4+\sin x)} dx$$

OR

- (b) Find :

$$\int \frac{x+3}{x^2+4x+5} dx$$

30. (a) Find the general solution of the differential equation

$$(x^2 - y^2) dx + 2xy dy = 0.$$

OR

- (b) Solve the differential equation

$$\sin x \cos y dx + \cos x \sin y dy = 0, \text{ given that } y = \frac{\pi}{4} \text{ when } x = 0.$$

31. If $(\sin x)^y = y^{\cos x}$, then find $\frac{dy}{dx}$.

SECTION D



This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32. (a) Represent the equations of lines l_1 and l_2 in vector form and check whether they are intersecting or not.

$$l_1: \frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$$

$$l_2: \frac{x+1}{-1} = \frac{2-y}{-2} = \frac{z-5}{5}$$

OR

- (ख) एक वर्ग की सम्मुख भुजाएँ निम्न रेखाओं के अनुदिश हैं :

$$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$$

यदि वर्ग की सम्मुख भुजाओं के दूसरे युग्म के दिक्-अनुपात $\langle -3, 6, p \rangle$ हैं, तो वर्ग का क्षेत्रफल ज्ञात कीजिए। p का मान भी ज्ञात कीजिए।

33. दर्शाइए कि फलन $f: R_+ \rightarrow A \subset N$, जो कि $f(x) = 4x^2 + 12x + 15$ द्वारा परिभाषित है, एक एकैकी फलन है। वह समुच्चय A ज्ञात कीजिए ताकि f एक आच्छादी फलन हो, जहाँ $R_+ = [0, \infty)$ । यह भी ज्ञात कीजिए कि क्या कोई ऐसा $a \in R_+$ है जिसके लिए $f(a) = 7$ हो। सत्यापित कीजिए।

34. निम्नलिखित रैखिक प्रोग्रामन समस्या को ग्राफीय विधि से हल कीजिए :
व्यवरोधों

$$x - 20 \leq 0$$

$$2x + 3y \leq 120$$

$$2x + y \leq 60$$

$$x \geq 0, y \geq 0$$

के अंतर्गत $Z = 8x + 8y$ का अधिकतमीकरण कीजिए।

35. (क) एक नए शोरूम के उद्घाटन वाले दिन, एक लक्की ड्रॉ रखा गया जिसमें ₹ 1,000 तथा ₹ 500 के कुछ वाउचर लक्की ड्रॉ जीतने वालों को दिए गए।



उस दिन कुल 60 वाउचर दिए गए। ₹ 1,000 वाले वाउचर की संख्या को ₹ 500 वाले वाउचरों की संख्या के 3 गुने में जोड़ने पर 100 आता है।

दी गई सूचना को दो चरों के रैखिक समीकरण निकाय के रूप में व्यक्त कीजिए। अतः आव्यूह विधि से प्रत्येक प्रकार के वाउचरों की संख्या ज्ञात कीजिए।

अथवा

- (b) Opposite sides of a square are along the lines :

$$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$$

Find the area of the square if direction ratios of other pair of opposite sides of the square are given by $\langle -3, 6, p \rangle$. Also, find the value of p .

33. Show that a function $f : \mathbb{R}_+ \rightarrow A \subset \mathbb{N}$, defined as $f(x) = 4x^2 + 12x + 15$ is one-one. Find set A so that f is onto where $\mathbb{R}_+ = [0, \infty)$. Also, find if there exists $a \in \mathbb{R}_+$ such that $f(a) = 7$. Justify.

34. Solve the following Linear Programming Problem graphically :

Maximize $Z = 8x + 8y$

subject to the constraints

$$x - 20 \leq 0$$

$$2x + 3y \leq 120$$

$$2x + y \leq 60$$

$$x \geq 0, y \geq 0$$

35. (a) On the inauguration day of a new showroom, a lucky draw was organized and some vouchers of ₹ 1,000 and ₹ 500 were given to the lucky draw winners.



A total of 60 vouchers were given on the day. The number of ₹ 1,000 vouchers added to 3 times the number of ₹ 500 vouchers, gives 100. Express the given information as a system of linear equations in two variables. Hence, find the number of vouchers of each type by matrix method.

OR

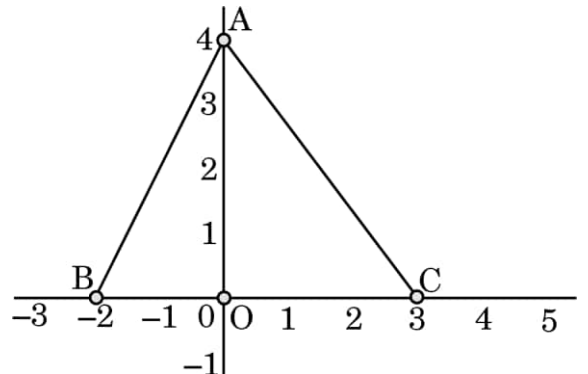
(ख) दिया गया है कि $P = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $Q = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$ तथा $R = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$, एक ऐसा आव्यूह S ज्ञात कीजिए ताकि $PQ - RS$ एक शून्य आव्यूह हो।

खण्ड ड

इस खण्ड में 3 प्रकरण-अध्ययन आधारित प्रश्न हैं, जिनमें प्रत्येक के 4 अंक हैं।

प्रकरण अध्ययन – 1

36. एक सोसाइटी में एक त्रिभुजाकार पार्क है जिसे दो भागों में बाँटा गया है, जैसा कि चित्र में दर्शाया गया है।





क्षेत्र OAC में, बच्चों को क्रिकेट, फुटबॉल खेल खेलने दिए जाते हैं जबकि क्षेत्र AOB में भागदौड़ वाले खेलों की अनुमति नहीं है।

त्रिभुजाकार पार्क ABC के शीर्ष A(0, 4), B(-2, 0) तथा C(3, 0) हैं।

उपर्युक्त सूचना पर आधारित, निम्नलिखित प्रश्नों के उत्तर दीजिए :

- (i) पार्क की सीमा रेखा AB का समीकरण लिखिए। 1
- (ii) पार्क की सीमा रेखा AC का समीकरण लिखिए। 1
- (iii) (क) समाकलन के प्रयोग से, क्षेत्र OAC, जिसमें क्रिकेट, फुटबॉल खेलने की अनुमति है, का क्षेत्रफल ज्ञात कीजिए। 2

अथवा

- (iii) (ख) समाकलन के प्रयोग से, क्षेत्र AOB का क्षेत्रफल ज्ञात कीजिए। 2

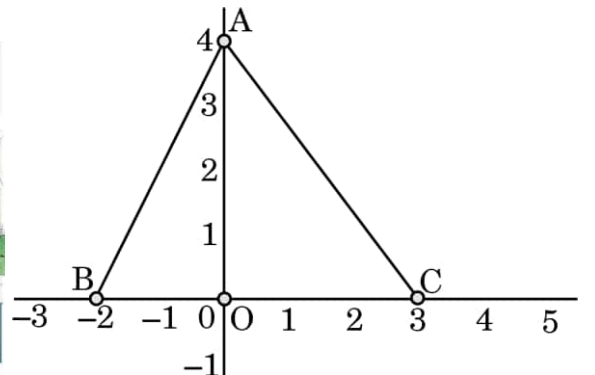
- (b) Given that $P = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $Q = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$ and $R = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$, find a matrix S such that $PQ - RS$ is a null matrix.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. There is a triangular park in the society. The park is divided into two sections as shown in the figure.



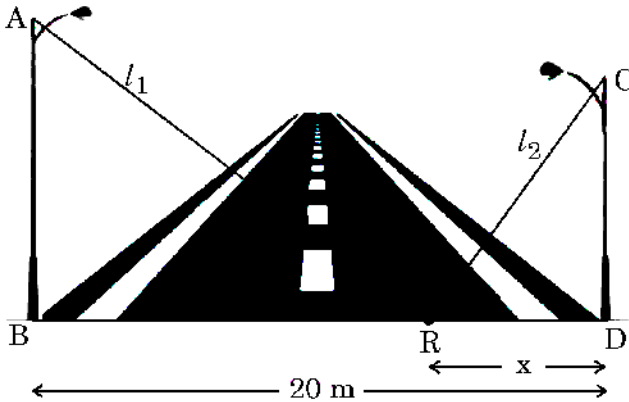


In the region OAC, children are allowed to play games like cricket, football, while in the region AOB, activities which involve running are not allowed. The vertices of the triangular park ABC are A(0, 4), B(-2, 0) and C(3, 0). Based on the above information, answer the following questions :

- (i) Write the equation of the boundary line AB of the park. 1
- (ii) Write the equation of the boundary line AC of the park. 1
- (iii) (a) Using integration, find the area of region OAC, in which children are allowed to play cricket, football. 2
- OR**
- (iii) (b) Using integration, find the area of region AOB. 2

प्रकरण अध्ययन – 2

37. नीचे चित्र में दर्शाए अनुसार, एक 20 m चौड़ी सड़क के दोनों ओर आमने सामने 22 m तथा 16 m ऊँचाई वाले दो ऊर्ध्वाधर बिजली के खम्भे खड़े हैं।



सड़क के एक बिंदु R, जो छोटे खम्भे से x m की दूरी पर है, से दोनों ओर दो सीढ़ियाँ l_1 तथा l_2 लम्बाई की हैं।

उपर्युक्त सूचना पर आधारित, निम्नलिखित प्रश्नों के उत्तर दीजिए :



- (i) $p(x) = l_1 + l_2$ को x के पदों में व्यक्त कीजिए। 1
- (ii) $p'(x)$ ज्ञात कीजिए। 1
- (iii) (क) x का वह मान ज्ञात कीजिए जिसके लिए $l_1^2 + l_2^2$ का मान न्यूनतम हो। 2

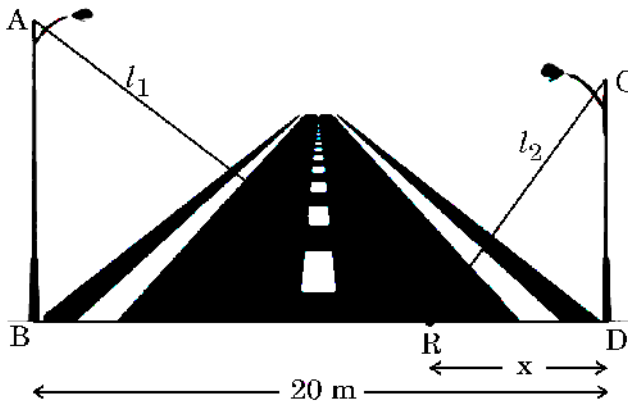
अथवा

- (iii) (ख) यदि 22 m लंबे खम्भे को हटाकर उसके स्थान पर भी एक 16 m लंबा खंभा लगाया गया हो, तो प्रत्येक खंभे से कितनी दूरी पर सीढ़ी रखी जाए ताकि दोनों ओर खम्भे के शिखर तक पहुँचने के लिए सीढ़ियों की लंबाई के वर्गों का योगफल न्यूनतम हो ? 2



Case Study – 2

37. Two vertical light poles of height 22 m and 16 m stand on the opposite sides of a 20 m wide road as shown below in the figure.



Two ladders of length l_1 and l_2 are placed from a common point R on the road at a distance of x m from the smaller pole.

Based on the above information, answer the following questions :

- (i) Express $p(x) = l_1 + l_2$ in terms of x. 1
- (ii) Find $p'(x)$. 1
- (iii) (a) Find the value of x for which $l_1^2 + l_2^2$ is minimum. 2

OR

- (iii) (b) If the 22 m long pole is also replaced by a 16 m long pole, at what distance from either pole should the ladders be kept so that the sum of squares of lengths of ladders needed to reach the top of the pole is minimum ? 2



प्रकरण अध्ययन – 3

38. बारहवीं कक्षा के बाद एक वर्ष छोड़कर बाद में एंट्रेंस परीक्षा पास करने वाले विद्यार्थियों का सक्सेस रेट ज्ञात करने के लिए एक सर्वे कराया गया।



संग्रहित आँकड़ों के अनुसार, परीक्षा में बैठने वालों में 40% विद्यार्थी एक वर्ष छोड़ने वाले थे तथा शेष विद्यार्थी बारहवीं कक्षा के नियमित विद्यार्थी थे।

एक वर्ष छोड़ने वालों में 5% परीक्षा में पास हुए जबकि नियमित विद्यार्थियों में से 10% पास हुए।

उपर्युक्त सूचना पर आधारित, निम्नलिखित प्रश्नों के उत्तर दीजिए :

- (i) यादृच्छया चुने गए एक विद्यार्थी के नियमित विद्यार्थी होने की प्रायिकता ज्ञात कीजिए। 1
- (ii) एक वर्ष छोड़ने वाले विद्यार्थियों के समूह में से यादृच्छया एक विद्यार्थी चुना गया। इस विद्यार्थी के पास न होने की प्रायिकता क्या है? 1
- (iii) (क) यादृच्छया चुना गया एक विद्यार्थी परीक्षा में पास होने वाला पाया गया। प्रायिकता ज्ञात कीजिए कि यह एक वर्ष छोड़ने वाला विद्यार्थी नहीं है। 2

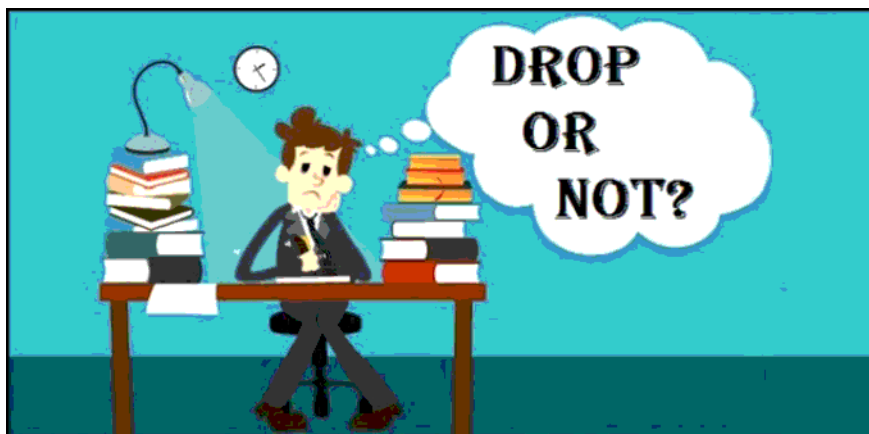
अथवा

- (iii) (ख) यादृच्छया चुना गया एक विद्यार्थी परीक्षा में पास न होने वाला पाया गया। प्रायिकता ज्ञात कीजिए कि यह एक नियमित विद्यार्थी था। 2



Case Study – 3

38. A survey was conducted to find out the success rate of students who qualified the entrance examination by dropping a year after class XII.



As per the data collected, 40% students appearing in the examination were dropouts and the remaining students were regular students of class XII.

Of the dropouts, 5% qualify the examination while 10% of the regular students qualify the examination.

Based on the above information, answer the following questions.

- (i) Find the probability that a student selected at random is a regular student. 1
- (ii) A student is selected at random from a group of dropout students. What is the probability that the student will not qualify the examination ? 1
- (iii) (a) A student selected at random qualified the examination. Find the probability that student is not a dropout. 2

OR

- (iii) (b) A student selected at random did not qualify the examination. Find the probability that the student was a regular student. 2

Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Senior Secondary School Examination 2026
MATHEMATICS (041) (PAPER CODE -65/3/3)

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	“Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its leakage to the public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in Newspaper/Website, etc. may invite action under various rules of the Board and BNS.”
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one’s own interpretation or any other consideration. The Marking Scheme should be strictly adhered to and religiously followed. However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-XII, while evaluating the competency-based questions, please try to understand the given answer and even if reply is not from a marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.
4	The Marking Scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (✓) wherever answer is correct. For wrong answer CROSS ‘X’ be marked. Evaluators will not put right (✓) while evaluating which gives the impression that the answer is correct, and no marks are awarded. This is the most common mistake which evaluators are committing.
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	If a student has attempted an extra question, answer to the question deserving more marks should be retained and the other answer scored out with a note “Extra Question”.
10	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.

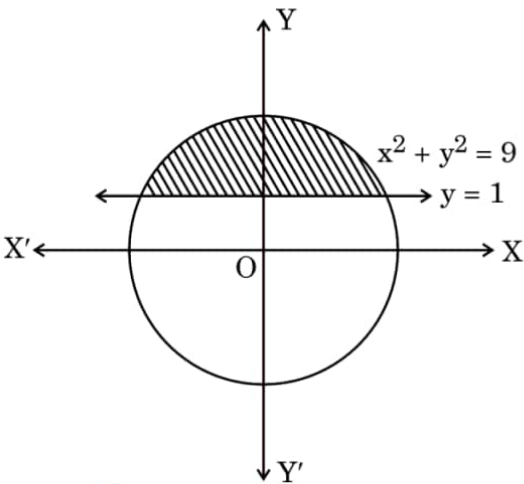
11	A full scale of marks (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
12	Every examiner must necessarily do evaluation work for full working hours, i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
13	Ensure that you do not make the following common types of errors committed by the Examiner in the past: - • Leaving answer or part thereof unassessed in an answer book. • Giving more marks for an answer than assigned to it. • Wrong totaling of marks awarded on an answer. • Wrong transfer of marks from the inside pages of the answer book to the title page. • Wrong question wise totaling on the title page. • Wrong totaling of marks of the two columns on the title page. • Wrong grand total. • Marks in words and figures not tallying/not same. • Wrong transfer of marks from the answer book to online award list. • Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) • Half or a part of the answer marked correct and the rest as wrong, but no marks awarded.
14	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
15	Any unassessed portion, non-carrying over of marks to the title page, or total error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
16	The Examiners should acquaint themselves with the guidelines given in the “Guidelines for Spot Evaluation” before starting the actual evaluation.
17	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
18	The candidates are entitled to obtain a photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

MARKING SCHEME
MATHEMATICS (Subject Code–041)
(PAPER CODE: 65/3/3)

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	STEPS	MARKS
	SECTION A This section comprises 20 multiple choice questions (MCQs) of 1 mark each.		
1.	If $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 198$ and $\vec{a} = 10 \vec{b}$, then : (A) $\vec{a} = \sqrt{2}$ (B) $\vec{b} = \sqrt{2}$ (C) $\vec{b} = 10\sqrt{2}$ (D) $\vec{a} = \frac{10}{\sqrt{2}}$		
Sol. 1	(B) $ \vec{b} = \sqrt{2}$	I	1
2.	If l_1, m_1, n_1 and l_2, m_2, n_2 are direction cosines of lines L_1 and L_2 respectively and θ is the acute angle between them, then : (A) $\cos \theta = l_1 l_2 + m_1 m_2 + n_1 n_2$ (B) $\sin \theta = l_1 l_2 + m_1 m_2 + n_1 n_2$ (C) $\tan \theta = \frac{l_1}{l_2} + \frac{m_1}{m_2} + \frac{n_1}{n_2}$ (D) $\cos \theta = l_1 l_2 + m_1 m_2 + n_1 n_2$		
Sol. 2	(D) $\cos \theta = l_1 l_2 + m_1 m_2 + n_1 n_2 $	I	1
3.	Direction ratios of lines l_1 and l_2 respectively are $\langle 1, 0, 0 \rangle$ and $\langle 0, -1, 0 \rangle$. The direction ratios of the line perpendicular to both l_1 and l_2 are : (A) $\langle 1, 1, 0 \rangle$ (B) $\langle 0, 0, -1 \rangle$ (C) $\langle 1, 1, 1 \rangle$ (D) $\langle 1, 0, -1 \rangle$		
Sol. 3	(B) $\langle 0, 0, -1 \rangle$	I	1
4.	If E and F are two independent events such that $P(E) = \frac{3}{10}$, $P(E \cup F) = \frac{1}{2}$, then $P(E F) - P(F E)$ is equal to : (A) $\frac{2}{7}$ (B) $\frac{3}{35}$ (C) $\frac{1}{70}$ (D) $\frac{1}{7}$		
Sol. 4	(C) $\frac{1}{70}$	I	1
5.	The domain of $\cos^{-1}(4x + 1)$ is : (A) $[-1, 1]$ (B) $[-3, 5]$ (C) $[-4, 4]$ (D) $[-\frac{1}{2}, 0]$		
Sol. 5	(D) $[-\frac{1}{2}, 0]$	I	1

6.	If $A^2 = 4A + 3I$ and $A^{-1} = xA + yI$, then the value of $(x + y)$ is : (A) -1 (B) 1 (C) $\frac{5}{3}$ (D) 7		
Sol. 6	(A) -1	I	1
7.	If A and B are skew-symmetric matrices of same order, then $AB' + BA'$ is a/an : (A) symmetric matrix (B) skew-symmetric matrix (C) null matrix (D) identity matrix		
Sol. 7	(A) symmetric matrix	I	1
8.	If a matrix X is such that $[2 \ 1] X = [3 \ 4 \ 5]$, then the order of matrix X is : (A) 3×1 (B) 2×3 (C) 1×2 (D) 1×3		
Sol. 8	(B) 2×3	I	1
9.	If a square matrix A is such that $A^2 = A$ and $(I - A)^3 = xA + I$, then value of x must be : (A) 7 (B) 5 (C) -7 (D) -1		
Sol. 9	(D) -1	I	1
10.	If $A(\text{adj } A) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then the value of $2A$ is : (A) 6 (B) 54 (C) 12 (D) 24		
Sol. 10	(D) 24	I	1
11.	The value of k for which the function $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ k(x+1), & x = 0 \end{cases}$ is a continuous function, is : (A) $\frac{1}{4}$ (B) 2 (C) $\frac{1}{2}$ (D) 0		
Sol. 11	(D) 0	I	1

12.	If $2 \cos^{-1} x = y$, then $\frac{dy}{dx}$ is : (A) $-2 \sin^{-1} x$ (B) $-\frac{1}{2} \sin \frac{y}{2}$ (C) $\frac{-1}{\sqrt{1-2x^2}}$ (D) $-2 \operatorname{cosec} \frac{y}{2}$		
Sol. 12	(D) $-2 \operatorname{cosec} \frac{y}{2}$	I	1
13.	The rate of change of volume of a sphere with respect to its diameter, when its radius is 5 cm, is : (A) $400\pi \text{ cm}^3/\text{cm}$ (B) $100\pi \text{ cm}^3/\text{cm}$ (C) $50\pi \text{ cm}^3/\text{cm}$ (D) $25\pi \text{ cm}^3/\text{cm}$		
Sol. 13	(C) $50\pi \text{ cm}^3/\text{cm}$	I	1
14.	$\int \frac{\cos x}{\sqrt{\sin^2 x + 1}} dx$ is equal to : (A) $\tan^{-1}(\sin x) + C$ (B) $\sin^{-1}(\sin x) + C$ (C) $\log \sin x + \sqrt{1 + \sin^2 x} + C$ (D) $\log \sin x - \sqrt{\sin^2 x + 1} + C$		
Sol. 14	(C) $\log \sin x + \sqrt{1 + \sin^2 x} + C$	I	1
15.	$\int_{-1}^1 (1 - x) dx$ is equal to : (A) $2 \int_0^1 (1+x) dx$ (B) $2 \int_{-1}^0 (1+x) dx$ (C) 0 (D) $2 \int_{-1}^0 (1-x) dx$		
Sol. 15	(B) $2 \int_{-1}^0 (1+x) dx$	I	1

16.	The area of the shaded region of the circle given below is equal to :  (A) $\int_1^3 \sqrt{9-y^2} \, dy$ (B) $2 \int_1^3 \sqrt{9-y^2} \, dy$ (C) $\int_0^3 \sqrt{9-x^2} \, dx$ (D) $2 \int_0^3 \sqrt{9-x^2} \, dx$		
Sol. 16	(B) $2 \int_1^3 \sqrt{9-y^2} \, dy$	I	1
17.	$\frac{dy}{dx} = F(x, y)$ will be a homogeneous differential equation for which of the following functions ? (i) $F(x, y) = 3x + 2y$ (ii) $F(x, y) = \sin \frac{y}{x} + \log y - \log x$ (iii) $F(x, y) = e^{y/x} + 1$ (iv) $F(x, y) = \sqrt{x^2 + y^2} - y$ (A) (i) and (ii) (B) (i), (ii) and (iii) (C) (ii), (iii) and (iv) (D) (ii) and (iii)		
Sol. 17	(D) (ii) and (iii)	I	1
18.	For any two vectors $\vec{a}$ and $\vec{b}$, which of the following statements is always true ? (A) $\vec{a} \cdot \vec{b} \leq \vec{a} \vec{b}$ (B) $\vec{a} + \vec{b} \geq \vec{a} + \vec{b}$ (C) $\vec{a} - \vec{b} = \vec{a} - \vec{b}$ (D) $\vec{a} \times \vec{b} \geq \vec{a} \vec{b}$		
Sol. 18	(A) $\vec{a} \cdot \vec{b} \leq \vec{a} \vec{b} $	I	1
Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled as Assertion (A) and the other labelled as Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.			

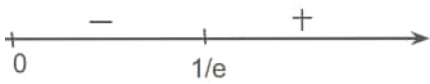
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19.	<i>Assertion (A) :</i> One of the particular solutions of the differential equation $\frac{dy}{dx} = e^{x+y}$ can be $e^x + e^{-y} = -2$. <i>Reason (R) :</i> $e^x + e^{-y} = C$ is the general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$.		
Sol. 19	(D) Assertion (A) is false, but Reason (R) is true.	I	1
20.	<i>Assertion (A) :</i> The vectors $\vec{a}$ and $(-2\vec{a})$, where $\vec{a} \neq \vec{0}$ are collinear vectors. <i>Reason (R) :</i> $\vec{a} \cdot (-2\vec{a}) = 0$.		
Sol. 20	(C) Assertion (A) is true, but Reason (R) is false.	I	1
SECTION B This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.			
21.	(a) If the lines $\frac{x-3}{1} = \frac{1-y}{1} = \frac{z+2}{p}$ and $\frac{2-x}{3} = \frac{y+1}{5} = \frac{z+56}{2p}$ are perpendicular to each other, then find the value(s) of p. OR (b) Find the vector equation of a line passing through the origin and perpendicular to both the lines $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and $\vec{r} = \mu(\hat{i} - \hat{j} + \hat{k})$.		
Sol. 21(a)	Direction Ratios of line $\frac{x-3}{1} = \frac{1-y}{1} = \frac{z+2}{p}$ are $\langle 1, -1, p \rangle$ Direction Ratios of line $\frac{2-x}{3} = \frac{y+1}{5} = \frac{z+56}{2p}$ are $\langle -3, 5, 2p \rangle$ $\therefore$ Given both lines are perpendicular to each other. $\therefore 1 \times (-3) + (-1) \times 5 + p \times 2p = 0$	I II III	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

	$\Rightarrow p = \pm 2$ OR (b) The vector parallel to required line is given by $\vec{b} = (3\hat{i} + 4\hat{j} + 2\hat{k}) \times (\hat{i} - \hat{j} + \hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 2 \\ 1 & -1 & 1 \end{vmatrix} = 6\hat{i} - \hat{j} - 7\hat{k}$ Equation of required line is given by $\vec{r} = \delta(6\hat{i} - \hat{j} - 7\hat{k})$	IV I II	$\frac{1}{2}$ $1\frac{1}{2}$ $\frac{1}{2}$
22.	If $x = e^{\sin^{-1} t}$ and $y = e^{\cos^{-1} t}$, then find $\frac{dy}{dx}$ at $t = \frac{1}{\sqrt{2}}$.		
Sol. 22	$x = e^{\sin^{-1} t} \Rightarrow \frac{dx}{dt} = \frac{1}{\sqrt{1-t^2}} e^{\sin^{-1} t}$ $y = e^{\cos^{-1} t} \Rightarrow \frac{dy}{dt} = -\frac{1}{\sqrt{1-t^2}} e^{\cos^{-1} t}$ $\frac{dy}{dx} = -\frac{e^{\cos^{-1} t}}{e^{\sin^{-1} t}}$ $\left[\frac{dy}{dx}\right]_{t=\frac{1}{\sqrt{2}}} = -1$	I II III IV	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
23.	Find the sub-interval of $(0, \infty)$ in which $f(x) = x \log x$ is increasing.		
Sol. 23	$f(x) = x \log x \Rightarrow f'(x) = 1 + \log x$ Put $1 + \log x = 0 \Rightarrow x = \frac{1}{e}$  $\therefore f'(x) > 0 \quad \forall x \in \left(\frac{1}{e}, \infty\right) \Rightarrow f(x) \text{ is increasing in } \left(\frac{1}{e}, \infty\right).$ Note: Interval $[1/e, \infty)$ can be considered.	I II III	1 $\frac{1}{2}$ $\frac{1}{2}$
24.	(a) Find the value of $\sin [\cot^{-1} \sqrt{2} (\cos (\tan^{-1} 1))]$. OR (b) A relation R on $A = \{1, 2, 3\}$ is defined as $R = \{(1, 1) (3, 3), (1, 2)\}$. Is R a symmetric relation ? Justify. Write the smallest relation set R_1 such that $R \cup R_1$ becomes an equivalence relation on the set $\{1, 2, 3\}$.		
Sol. 24(a)	$\sin[\cot^{-1} \sqrt{2} (\cos (\tan^{-1} 1))] = \sin \left[\cot^{-1} \sqrt{2} \left(\cos \frac{\pi}{4} \right) \right] = \sin(\cot^{-1} 1)$ $= \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$ OR	I II	1 1

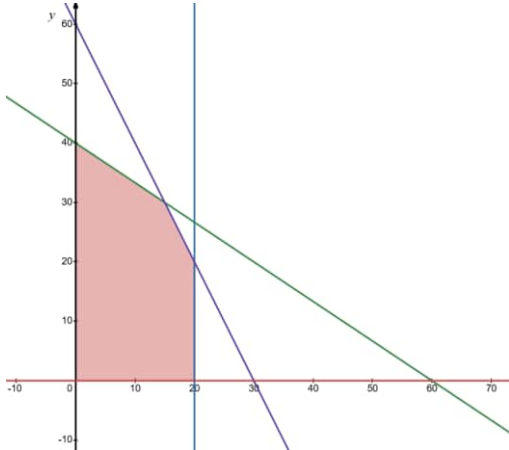
(b)	As $(1,2) \in R$, but $(2,1) \notin R \Rightarrow R$ is not symmetric relation. $R_1 = \{(2,2), (2,1)\}$	I	1
		II	1
25.	If for two unit vectors $\vec{a}$ and $\vec{b}$, $ \vec{a} + 2\vec{b} = 2\vec{a} - \vec{b} $, then find the angle between $\vec{a}$ and $\vec{b}$.		
Sol. 25	Given that $ \vec{a} = \vec{b} = 1$	I	$\frac{1}{2}$
	$ \vec{a} + 2\vec{b} = 2\vec{a} - \vec{b} \Rightarrow \vec{a} + 2\vec{b} ^2 = 2\vec{a} - \vec{b} ^2$	II	$\frac{1}{2}$
	$\Rightarrow \vec{a} ^2 + 4 \vec{b} ^2 + 4\vec{a} \cdot \vec{b} = 4 \vec{a} ^2 + \vec{b} ^2 - 4\vec{a} \cdot \vec{b} \Rightarrow \vec{a} \cdot \vec{b} = 0$	III	$\frac{1}{2}$
	$\Rightarrow$ Angle between $\vec{a}$ and $\vec{b} = \frac{\pi}{2}$	IV	$\frac{1}{2}$
SECTION C			
This section comprises 6 Short Answer (SA) type questions of 3 marks each.			
26.	A survey was conducted on the patients who have undergone knee replacement surgeries.  It was found that, Robotic Knee replacement surgeries have 90% success rate. On a particular day, robotic surgery was performed on three patients, A, B and C, one after the other. Assuming that the success and failure of each surgery is independent of each other, find the probability that : (i) exactly one surgery is successful, (ii) at most two surgeries are successful.		
Sol. 26	$P(\text{success}) = \frac{9}{10}$ and $P(\text{not success}) = \frac{1}{10}$	I	$\frac{1}{2}$
	(i) Probability that exactly one surgery is successful $= \frac{9}{10} \times \frac{1}{10} \times \frac{1}{10} \times 3 = \frac{27}{1000}$	II	1
	(ii) Probability that all three surgeries are successful $= \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10} = \frac{729}{1000}$	III	1
	Probability that at most two surgeries are successful $= 1 - \frac{729}{1000} = \frac{271}{1000}$	IV	$\frac{1}{2}$

27.	(a) Find : $\int \frac{dx}{x^{1/2} + x^{1/3}}$ OR (b) Find : $\int \tan^{-1}\left(\frac{1-x}{1+x}\right) dx$		
Sol. 27(a)	Put $x = t^6 \Rightarrow dx = 6t^5 dt$ $\int \frac{dx}{x^{1/2} + x^{1/3}} = 6 \int \frac{t^5}{t^3 + t^2} dt = 6 \int \frac{t^3}{t+1} dt$ $= 6 \int \left(t^2 - t + 1 - \frac{1}{t+1} \right) dt$ $= 6 \left(\frac{t^3}{3} - \frac{t^2}{2} + t - \log t+1 \right) + c$ $= 2x^{1/2} - 3x^{1/3} + 6x^{1/6} - 6 \log \left x^{1/6} + 1 \right + c$ OR (b) $\int \tan^{-1}\left(\frac{1-x}{1+x}\right) dx$ Getting $\int \left(\frac{\pi}{4} - \tan^{-1}x \right) dx$ $= \frac{\pi x}{4} - \left[x \tan^{-1}x - \int \frac{x}{1+x^2} dx \right]$ $= \frac{\pi x}{4} - x \tan^{-1}x + \frac{1}{2} \log(1+x^2) + c$	I II III I II III	1 1 1 1 $1\frac{1}{2}$ $\frac{1}{2}$
28.	Evaluate : $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{1 + \sin 2x} dx$		
Sol. 28	Let, $I = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{1 + \sin 2x} dx \dots \dots \dots (1)$ Getting $I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{1 + \sin 2x} dx \dots \dots \dots (2)$ On adding equations (1) and (2), we get	I	1

	$2I = \int_0^{\frac{\pi}{2}} \frac{1}{1 + \sin 2x} dx$	II	$\frac{1}{2}$
	$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \frac{1 - \sin 2x}{\cos^2 2x} dx = \int_0^{\frac{\pi}{2}} \sec^2 2x dx - \int_0^{\frac{\pi}{2}} \sec 2x \tan 2x dx$	III	$\frac{1}{2}$
	$\Rightarrow 2I = \left[\frac{\tan 2x}{2} - \frac{\sec 2x}{2} \right]_0^{\frac{\pi}{2}} \Rightarrow I = \frac{1}{2}$	IV	1
29.	(a) Find : $\int \frac{\cos x}{(2 + \sin x)(4 + \sin x)} dx$ OR (b) Find : $\int \frac{x+3}{x^2+4x+5} dx$		
Sol. 29(a)	Put $\sin x = t \Rightarrow \cos x dx = dt$ $\int \frac{\cos x}{(2 + \sin x)(4 + \sin x)} dx = \int \frac{1}{(2 + t)(4 + t)} dt$ $= \frac{1}{2} \int \left(\frac{1}{2 + t} - \frac{1}{4 + t} \right) dt = \frac{1}{2} \log \left \frac{2 + t}{4 + t} \right + c$ $= \frac{1}{2} \log \left \frac{2 + \sin x}{4 + \sin x} \right + c$ OR (b) $\int \frac{x+3}{x^2+4x+5} dx = \frac{1}{2} \int \frac{(2x+4) + 2}{x^2+4x+5} dx$ $= \frac{1}{2} \int \frac{2x+4}{x^2+4x+5} dx + \int \frac{1}{x^2+4x+5} dx$ $= \frac{1}{2} \log x^2+4x+5 + \int \frac{1}{(x+2)^2+1} dx$ $= \frac{1}{2} \log x^2+4x+5 + \tan^{-1}(x+2) + c$	I II III	1 $1\frac{1}{2}$ $\frac{1}{2}$
		I II III	1 1 1
30.	(a) Find the general solution of the differential equation $(x^2 - y^2) dx + 2xy dy = 0.$ OR (b) Solve the differential equation $\sin x \cos y dx + \cos x \sin y dy = 0, \text{ given that } y = \frac{\pi}{4} \text{ when } x = 0.$		

Sol.30 (a) (b)	$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$ Put $y = vx$ and $\frac{dy}{dx} = v + x \frac{dv}{dx}$ Thus, $v + x \frac{dv}{dx} = \frac{v^2 - 1}{2v}$ $\Rightarrow x \frac{dv}{dx} = -\frac{1 + v^2}{2v}$ $\Rightarrow \int \frac{2v}{1 + v^2} dv = -\int \frac{dx}{x}$ $\Rightarrow \log 1 + v^2 = -\log x + \log c $ $\Rightarrow 1 + v^2 = \frac{c}{x}$ $\Rightarrow x^2 + y^2 = cx$ OR $\sin x \cos y \, dx + \cos x \sin y \, dy = 0$ $\Rightarrow \frac{dy}{dx} = -\frac{\sin x \cos y}{\cos x \sin y}$ $\Rightarrow \int \tan y \, dy = -\int \tan x \, dx$ $\Rightarrow \log \sec y = -\log \sec x + \log c $ $\Rightarrow \sec x \sec y = c$ Put $x = 0, y = \frac{\pi}{4}$ we get $c = \sqrt{2}$ Hence particular solution is given by $\sec x \sec y = \sqrt{2}$	I II III IV I II III IV	1 $\frac{1}{2}$ 1 $\frac{1}{2}$ 1 $\frac{1}{2}$
31.	If $(\sin x)^y = y^{\cos x}$, then find $\frac{dy}{dx}$.		
Sol. 31	$(\sin x)^y = y^{\cos x} \Rightarrow y \log \sin x = \cos x \log y$ On diff. wrt x both sides, we get $\frac{y \cos x}{\sin x} + \log \sin x \frac{dy}{dx} = \frac{\cos x}{y} \frac{dy}{dx} - \log y \cdot \sin x$ $\Rightarrow \frac{dy}{dx} = \frac{y(y \cos x + \log y \cdot \sin^2 x)}{\sin x (\cos x - y \log \sin x)} \text{ or } \frac{(y \cot x + \sin x \cdot \log y)y}{\cos x - y \log \sin x}$	I II III	1 $1\frac{1}{2}$ $\frac{1}{2}$
SECTION D This section comprises 4 Long Answer (LA) type questions of 5 marks each.			

32.	(a) Represent the equations of lines l_1 and l_2 in vector form and check whether they are intersecting or not. $l_1: \frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ $l_2: \frac{x+1}{-1} = \frac{2-y}{-2} = \frac{z-5}{5}$ OR (b) Opposite sides of a square are along the lines : $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ $\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$ Find the area of the square if direction ratios of other pair of opposite sides of the square are given by $\langle -3, 6, p \rangle$. Also, find the value of p.		
Sol. 32(a)	$l_1: \vec{r} = (-3\hat{i} + \hat{j} + 5\hat{k}) + \lambda(-3\hat{i} + \hat{j} + 5\hat{k})$	I	1
	$l_2: \vec{r} = (-\hat{i} + 2\hat{j} + 5\hat{k}) + \mu(-\hat{i} + 2\hat{j} + 5\hat{k})$	II	1
	$\vec{a}_1 = -3\hat{i} + \hat{j} + 5\hat{k}; \quad \vec{b}_1 = -3\hat{i} + \hat{j} + 5\hat{k}$		
	$\vec{a}_2 = -\hat{i} + 2\hat{j} + 5\hat{k}; \quad \vec{b}_2 = -\hat{i} + 2\hat{j} + 5\hat{k}$		
	$\vec{a}_2 - \vec{a}_1 = 2\hat{i} + \hat{j}$	III	1
	$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix} = -5\hat{i} + 10\hat{j} - 5\hat{k}$	IV	1
	$\therefore (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 0$	V	$\frac{1}{2}$
	$\therefore$ Both lines are intersecting.	VI	$\frac{1}{2}$
	OR		
	(b) $\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}; \quad \vec{a}_2 = 3\hat{i} + 3\hat{j} - 5\hat{k}; \quad \vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$	I	1
	$\vec{a}_2 - \vec{a}_1 = 2\hat{i} + \hat{j} - \hat{k}$		
	$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix} = 9\hat{i} - 14\hat{j} + 4\hat{k}$	II	1
	$ (\vec{a}_2 - \vec{a}_1) \times \vec{b} = \sqrt{293}$	III	$\frac{1}{2}$
	Distance between given parallel lines $= \frac{ (\vec{a}_2 - \vec{a}_1) \times \vec{b} }{ \vec{b} } = \frac{\sqrt{293}}{7}$	IV	$\frac{1}{2}$
	$\therefore$ Length of side of square = distance between parallel lines $= \frac{\sqrt{293}}{7}$ or $\frac{\sqrt{293}}{7}$ units		
	Area of square $= \frac{293}{49}$ or $\frac{293}{49}$ square units	V	1
	$\therefore$ Adjacent sides of square are perpendicular to each other. $\therefore -3 \times 2 + 6 \times 3 + 6p = 0 \Rightarrow p = -2$	VI	1

33.	Show that a function $f : R_+ \rightarrow A \subset N$, defined as $f(x) = 4x^2 + 12x + 15$ is one-one. Find set A so that f is onto where $R_+ = [0, \infty)$. Also, find if there exists $a \in R_+$ such that $f(a) = 7$. Justify.														
Sol. 33	f defined in the question is not a function. So, full marks to be given for attempting the question.	I	5												
34.	Solve the following Linear Programming Problem graphically : Maximize $Z = 8x + 8y$ subject to the constraints $x - 20 \leq 0$ $2x + 3y \leq 120$ $2x + y \leq 60$ $x \geq 0, y \geq 0$														
Sol. 34	 <table border="1" data-bbox="435 1240 991 1471"><thead><tr><th>Corner Points</th><th>Value of $Z=600x+400y$</th></tr></thead><tbody><tr><td>(20,0)</td><td>160</td></tr><tr><td>(20,20)</td><td>320</td></tr><tr><td>(15,30)</td><td>360</td></tr><tr><td>(0,40)</td><td>320</td></tr><tr><td>(0,0)</td><td>0</td></tr></tbody></table> Hence, $Z_{\max.} = 360$ at point (15,30).	Corner Points	Value of $Z=600x+400y$	(20,0)	160	(20,20)	320	(15,30)	360	(0,40)	320	(0,0)	0	I II III	For correct graph and shaded region 3 $1\frac{1}{2}$ $\frac{1}{2}$
Corner Points	Value of $Z=600x+400y$														
(20,0)	160														
(20,20)	320														
(15,30)	360														
(0,40)	320														
(0,0)	0														

35.

- (a) On the inauguration day of a new showroom, a lucky draw was organized and some vouchers of ₹ 1,000 and ₹ 500 were given to the lucky draw winners.



A total of 60 vouchers were given on the day. The number of ₹ 1,000 vouchers added to 3 times the number of ₹ 500 vouchers, gives 100. Express the given information as a system of linear equations in two variables. Hence, find the number of vouchers of each type by matrix method.

OR

- (b) Given that $P = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $Q = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$ and $R = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$, find a matrix S such that $PQ - RS$ is a null matrix.

Sol. 35(a)

Let, number of ₹ 1000 vouchers = x

and number of ₹ 500 vouchers = y

Hence, $x + y = 60$ and $x + 3y = 100$

In matrix form, $\begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 60 \\ 100 \end{bmatrix}$

$\Rightarrow AX = B$ where $A = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 60 \\ 100 \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \end{bmatrix}$

$\Rightarrow X = A^{-1}B$ (1)

$|A| = 2 \quad \therefore |A| \neq 0 \Rightarrow A$ is an invertible matrix.

$\text{adj } A = \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^{-1} = \frac{1}{2} \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}$

By eq. (1), $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 60 \\ 100 \end{bmatrix} = \begin{bmatrix} 40 \\ 20 \end{bmatrix}$

On comparing, $x = 40$ and $y = 20$

Hence, number of ₹ 1000 vouchers = 40

and number of ₹ 500 vouchers = 20

OR

35(b)

Let $S = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$PQ = \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix}$

$RS = \begin{bmatrix} 2a + 5c & 2b + 5d \\ 3a + 8c & 3b + 8d \end{bmatrix}$

I

1

II

1

III $\frac{1}{2}$ **IV** $1\frac{1}{2}$ **V**

1

I $\frac{1}{2}$ **II**

1

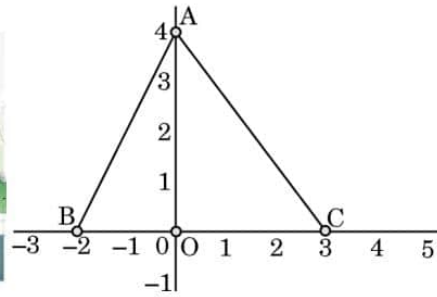
III

1

	Given that $PQ - RS = \text{Null matrix} \Rightarrow PQ = RS$ On comparing corresponding elements, $2a + 5c = 3, \quad 2b + 5d = 0, \quad 3a + 8c = 43, \quad 3b + 8d = 22$ By solving, $a = -191, \quad b = -110, \quad c = 77, \quad d = 44$ Hence, $S = \begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}$ <u>Alternative Method:</u> Given that $PQ - RS = \text{null matrix} \Rightarrow RS = PQ$ $\begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix} S = \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix}$ $\Rightarrow S = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}^{-1} \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix}$ $\Rightarrow S = \begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix}$ $\Rightarrow S = \begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}$	IV V VI I II III IV V	$\frac{1}{2}$ $1\frac{1}{2}$ $\frac{1}{2}$ 1 1 1 $1\frac{1}{2}$ $\frac{1}{2}$
SECTION E This section comprises 3 case-study based questions of 4 marks each.			
	Case Study – 1		

36.

There is a triangular park in the society. The park is divided into two sections as shown in the figure.



In the region OAC, children are allowed to play games like cricket, football, while in the region AOB, activities which involve running are not allowed.

The vertices of the triangular park ABC are A(0, 4), B(-2, 0) and C(3, 0).

Based on the above information, answer the following questions :

- (i) Write the equation of the boundary line AB of the park.
- (ii) Write the equation of the boundary line AC of the park.
- (iii) (a) Using integration, find the area of region OAC, in which children are allowed to play cricket, football.

OR

- (iii) (b) Using integration, find the area of region AOB.

Sol. 36(i)

Equation of the boundary line AB is $2x - y + 4 = 0$

I

1

(ii)

Equation of the boundary line AC is $4x + 3y = 12$

I

1

(iii)(a)

$$\begin{aligned} \text{Area of region OAC} &= \int_0^3 \frac{12 - 4x}{3} dx \\ &= \frac{1}{3} [12x - 2x^2]_0^3 = 6 \end{aligned}$$

I

1

II

1

OR

(iii)(b)

$$\begin{aligned} \text{Area of region AOB} &= \int_{-2}^0 (2x + 4) dx \\ &= [x^2 + 4x]_{-2}^0 = 4 \end{aligned}$$

I

1

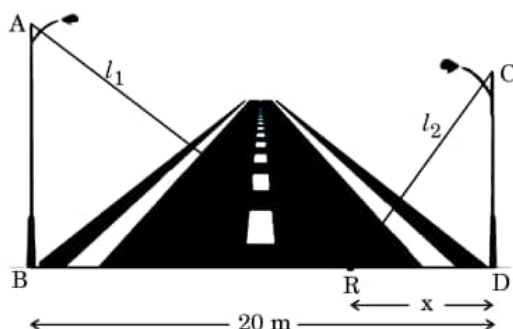
II

1

Case Study – 2

37.

Two vertical light poles of height 22 m and 16 m stand on the opposite sides of a 20 m wide road as shown below in the figure.



Two ladders of length l_1 and l_2 are placed from a common point R on the road at a distance of x m from the smaller pole.

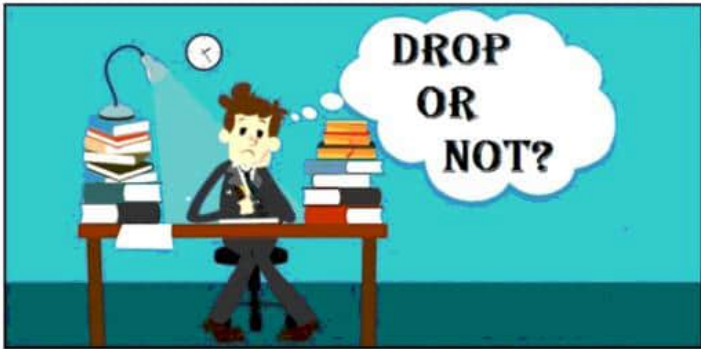
Based on the above information, answer the following questions :

- (i) Express $p(x) = l_1 + l_2$ in terms of x.
- (ii) Find $p'(x)$.
- (iii) (a) Find the value of x for which $l_1^2 + l_2^2$ is minimum.

OR

- (iii) (b) If the 22 m long pole is also replaced by a 16 m long pole, at what distance from either pole should the ladders be kept so that the sum of squares of lengths of ladders needed to reach the top of the pole is minimum ?

Sol. 37(i)	$p(x) = \sqrt{22^2 + (20 - x)^2} + \sqrt{16^2 + x^2} \quad \text{or}$ $p(x) = \sqrt{x^2 - 40x + 884} + \sqrt{x^2 + 256}$	I	1
(ii)	$p'(x) = \frac{x - 20}{\sqrt{x^2 - 40x + 884}} + \frac{x}{\sqrt{x^2 + 256}}$	I	1
(iii)(a)	Let $q(x) = l_1^2 + l_2^2 = 2x^2 - 40x + 1140$ $q'(x) = 4x - 40$ Put $q'(x) = 0 \Rightarrow x = 10$ $q''(x = 10) = 4 > 0 \Rightarrow q(x)$ is minimum at $x = 10$.	I II III IV	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
	OR		
(iii)(b)	Let, ladder is kept at y m distance from one pole. Suppose, $r(y)$ = sum of squares of lengths of ladders $\Rightarrow r(y) = (16^2 + y^2) + [16^2 + (20 - y)^2]$ $\Rightarrow r(y) = 2y^2 - 40y + 912$ On diff. wrt y, we get $r'(y) = 4y - 40$	I II	$\frac{1}{2}$ $\frac{1}{2}$

	Put $r'(y) = 0 \Rightarrow y = 10$ $r''(y) = 4 > 0 \Rightarrow r(y)$ is minimum at $y = 10$. Hence, the required distance of the ladder from each pole = 10 m	III IV	$\frac{1}{2}$ $\frac{1}{2}$
	Case Study – 3		
38.	A survey was conducted to find out the success rate of students who qualified the entrance examination by dropping a year after class XII.  As per the data collected, 40% students appearing in the examination were dropouts and the remaining students were regular students of class XII. Of the dropouts, 5% qualify the examination while 10% of the regular students qualify the examination. Based on the above information, answer the following questions. (i) Find the probability that a student selected at random is a regular student. (ii) A student is selected at random from a group of dropout students. What is the probability that the student will not qualify the examination ? (iii) (a) A student selected at random qualified the examination. Find the probability that student is not a dropout. OR (iii) (b) A student selected at random did not qualify the examination. Find the probability that the student was a regular student.		
Sol.38(i)	Probability that a student selected at random is a regular student = $\frac{3}{5}$	I	1
38(ii)	Probability that the selected student from the group of dropout will not qualify the examination = $\frac{95}{100}$ or $\frac{19}{20}$	I	1
38(iii)(a)	Let A: The student has qualified the examination. E_1: The student appearing in the examination is dropout. E_2: The student is regular. Using Bayes' Theorem,		

	$P\left(\frac{E_1'}{A}\right) = P\left(\frac{E_2}{A}\right) = \frac{\frac{60}{100} \times \frac{10}{100}}{\frac{40}{100} \times \frac{5}{100} + \frac{60}{100} \times \frac{10}{100}}$ $= \frac{3}{4}$	I II	$1\frac{1}{2}$ $\frac{1}{2}$
	OR		
38(iii)(b)	$P\left(\frac{E_2}{A'}\right) = \frac{\frac{60}{100} \times \frac{90}{100}}{\frac{40}{100} \times \frac{95}{100} + \frac{60}{100} \times \frac{90}{100}}$ $= \frac{27}{46}$	I II	$1\frac{1}{2}$ $\frac{1}{2}$