

**SET-3****Series : RPSQ4**प्रश्न-पत्र कोड
Q.P. Code**65/4/3**

रोल नं.

Roll No.

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परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।

Candidates must write the Q.P. Code on the title page of the answer-book.

**गणित****MATHEMATICS**

निर्धारित समय : 3 घण्टे

Time allowed : 3 hours

अधिकतम अंक : 80

Maximum Marks : 80

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं।
- कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में यथा स्थान पर प्रश्न का क्रमांक अवश्य लिखें।
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।
- Please check that this question paper contains 23 printed pages.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 38 questions.
- Please write down the Serial Number of the question in the answer-book at the given place before attempting it.
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.

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सामान्य निर्देश :

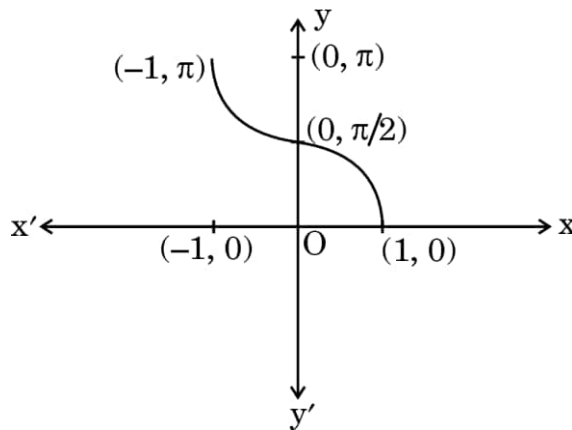
निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख्ती से पालन कीजिए :

- इस प्रश्न-पत्र में 38 प्रश्न हैं। सभी प्रश्न अनिवार्य हैं।
- यह प्रश्न-पत्र पाँच खण्डों में विभाजित है – क, ख, ग, घ एवं ङ।
- खण्ड क में प्रश्न संख्या 1 से 18 तक बहुविकल्पीय (MCQ) तथा प्रश्न संख्या 19 एवं 20 अभिकथन एवं तर्क आधारित 1 अंक के प्रश्न हैं।
- खण्ड ख में प्रश्न संख्या 21 से 25 तक अति लघु-उत्तरीय (VSA) प्रकार के 2 अंकों के प्रश्न हैं।
- खण्ड ग में प्रश्न संख्या 26 से 31 तक लघु-उत्तरीय (SA) प्रकार के 3 अंकों के प्रश्न हैं।
- खण्ड घ में प्रश्न संख्या 32 से 35 तक दीर्घ-उत्तरीय (LA) प्रकार के 5 अंकों के प्रश्न हैं।
- खण्ड ङ में प्रश्न संख्या 36 से 38 तक प्रकरण अध्ययन आधारित 4 अंकों के प्रश्न हैं।
- प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड ख के 2 प्रश्नों में, खण्ड ग के 3 प्रश्नों में, खण्ड घ के 2 प्रश्नों में तथा खण्ड ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है।
- कैल्कुलेटर का उपयोग वर्जित है।

खण्ड क

इस खण्ड में बहुविकल्पीय प्रश्न (MCQ) हैं, जिनमें प्रत्येक प्रश्न 1 अंक का है।

1. निम्न ग्राफ निरूपित करता है :



(A) $y = \cos^{-1} x$

(B) $y = \sec^{-1} x$

(C) $y = \sin^{-1} x$

(D) $y = \tan^{-1} x$



General Instructions :

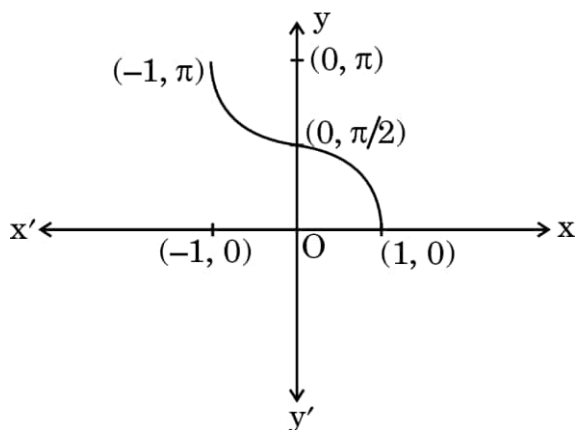
Read the following instructions very carefully and strictly follow them :

- (i) This question paper contains **38** questions. **All** questions are **compulsory**.
- (ii) This question paper is divided into **five** Sections – **A, B, C, D** and **E**.
- (iii) In **Section A**, Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.
- (iv) In **Section B**, Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.
- (v) In **Section C**, Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.
- (vi) In **Section D**, Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.
- (vii) In **Section E**, Questions no. **36** to **38** are case study based questions carrying **4** marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.
- (ix) Use of calculator is **not** allowed.

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. The following graph represents :



(A) $y = \cos^{-1} x$

(B) $y = \sec^{-1} x$

(C) $y = \sin^{-1} x$

(D) $y = \tan^{-1} x$



2. माना $A = [a_{ij}]$ कोटि 2×2 का आव्यूह है जिसके अवयव $a_{ij} = \frac{(i + 2j)^2}{3}$ द्वारा प्रदत्त हैं। तो A' है :

(A) $\begin{bmatrix} 12 & \frac{25}{3} \\ \frac{16}{3} & 3 \end{bmatrix}$

(B) $\begin{bmatrix} 12 & \frac{16}{3} \\ \frac{25}{3} & 3 \end{bmatrix}$

(C) $\begin{bmatrix} 3 & \frac{25}{3} \\ \frac{16}{3} & 12 \end{bmatrix}$

(D) $\begin{bmatrix} 3 & \frac{16}{3} \\ \frac{25}{3} & 12 \end{bmatrix}$

3. महत्तम पूर्णांक फलन $f(x) = [x]$, $0 < x < 3$, कितने बिंदुओं पर अवकलनीय नहीं है ?

- (A) केवल एक बिंदु पर (B) केवल दो बिंदुओं पर
(C) किसी भी बिंदु पर नहीं (D) तीन बिंदुओं पर

4. वक्र $y = x$ तथा x -अक्ष द्वारा $x = 0$ तथा $x = 2$ के बीच घिरे क्षेत्र का क्षेत्रफल है :

- (A) 2 वर्ग इकाई (B) $\frac{1}{2}$ वर्ग इकाई
(C) 1 वर्ग इकाई (D) 4 वर्ग इकाई

5. यदि $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x \neq 0 \\ k, & x = 0 \end{cases}$

$x = 0$ पर संतत है, तो k का मान है :

- (A) 0 (B) -2
(C) -1 (D) 2

6. एक बेलनाकार टैंक में $314 \text{ m}^3/\text{h}$ की दर से रेत भरा जा रहा है। यदि टैंक की त्रिज्या 10 m है, तो टैंक में रेत की ऊँचाई बढ़ने की दर है :

- (A) 1.1 m/h (B) 1 m/h
(C) $\pi \text{ m/h}$ (D) $\frac{\pi}{2} \text{ m/h}$



2. Let $A = [a_{ij}]$ be a 2×2 matrix whose elements are given by $a_{ij} = \frac{(i + 2j)^2}{3}$.

Then A' is given by :

- (A) $\begin{bmatrix} 12 & \frac{25}{3} \\ \frac{16}{3} & 3 \end{bmatrix}$ (B) $\begin{bmatrix} 12 & \frac{16}{3} \\ \frac{25}{3} & 3 \end{bmatrix}$
- (C) $\begin{bmatrix} 3 & \frac{25}{3} \\ \frac{16}{3} & 12 \end{bmatrix}$ (D) $\begin{bmatrix} 3 & \frac{16}{3} \\ \frac{25}{3} & 12 \end{bmatrix}$

3. The greatest integer function, $f(x) = [x]$, $0 < x < 3$ is **not** differentiable at how many points ?

- (A) At only one point (B) At only two points
(C) At no point (D) At three points

4. The area of the region bounded by the curve $y = x$ and x -axis, between $x = 0$ and $x = 2$ is :

- (A) 2 sq. units (B) $\frac{1}{2}$ sq. unit
(C) 1 sq. unit (D) 4 sq. units

5. If $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k

is :

- (A) 0 (B) -2
(C) -1 (D) 2

6. A cylindrical tank is being filled with sand at a rate of $314 \text{ m}^3/\text{h}$. If the radius of the tank is 10 m, then the height of sand in the tank increases at the rate of :

- (A) 1.1 m/h (B) 1 m/h
(C) π m/h (D) $\frac{\pi}{2}$ m/h



7. $\sec^{-1}(\sqrt{2}) + 2 \operatorname{cosec}^{-1}(-\sqrt{2})$ का मुख्य मान है :

(A) $-\frac{\pi}{2}$

(B) $-\frac{\pi}{4}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{2}$

8. यदि बिंदु $(2, 3)$, $(0, 4)$ तथा $(p, 2)$ संरेख हैं, तो p का मान है :

(A) $\frac{4}{7}$

(B) $-\frac{3}{7}$

(C) 4

(D) -4

9. e^{e^x} का x के सापेक्ष अवकलज है :

(A) $\log x$

(B) e^{e^x}

(C) $e^x e^{e^x}$

(D) $(e^x)^2$

10. $\int \frac{dx}{\sqrt{25 - 16x^2}}$ बराबर है :

(A) $\frac{1}{5} \sin^{-1} 4x + C$

(B) $\frac{1}{25} \sin^{-1} 16x + C$

(C) $\frac{1}{4} \sin^{-1} \frac{4x}{5} + C$

(D) $\frac{1}{16} \sin^{-1} \frac{4x}{5} + C$

11. यदि $\int_0^1 \frac{dx}{e^x + e^{-x}} = \tan^{-1} e + k$ है, तो k का मान है :

(A) e

(B) $\frac{\pi}{4}$

(C) 0

(D) $-\frac{\pi}{4}$

12. अवकल समीकरण $y = x \frac{dy}{dx} + 3 \left[2 - \left(\frac{dy}{dx} \right)^2 \right]$ की कोटि तथा घात का योगफल है :

(A) परिभाषित नहीं है

(B) 3

(C) 1

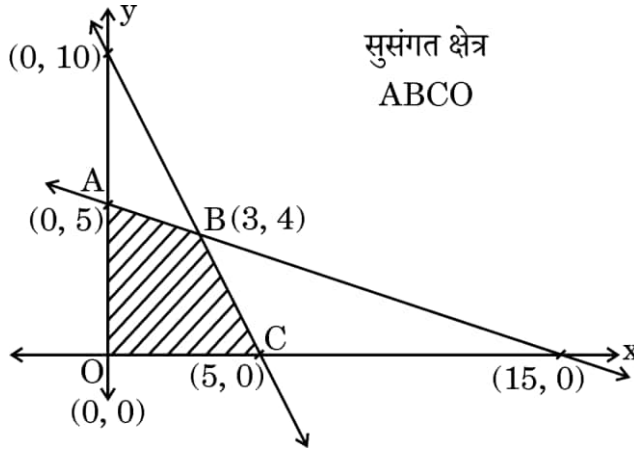
(D) 2



7. The principal value of $\sec^{-1}(\sqrt{2}) + 2 \operatorname{cosec}^{-1}(-\sqrt{2})$ is :
- (A) $-\frac{\pi}{2}$ (B) $-\frac{\pi}{4}$
(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$
8. If points (2, 3), (0, 4) and (p, 2) are collinear, then the value of p is :
- (A) $\frac{4}{7}$ (B) $-\frac{3}{7}$
(C) 4 (D) -4
9. Differential of e^{e^x} with respect to x is :
- (A) $\log x$ (B) e^{e^x}
(C) $e^x e^{e^x}$ (D) $(e^x)^2$
10. $\int \frac{dx}{\sqrt{25 - 16x^2}}$ is equal to :
- (A) $\frac{1}{5} \sin^{-1} 4x + C$ (B) $\frac{1}{25} \sin^{-1} 16x + C$
(C) $\frac{1}{4} \sin^{-1} \frac{4x}{5} + C$ (D) $\frac{1}{16} \sin^{-1} \frac{4x}{5} + C$
11. If $\int_0^1 \frac{dx}{e^x + e^{-x}} = \tan^{-1} e + k$, then the value of k is :
- (A) e (B) $\frac{\pi}{4}$
(C) 0 (D) $-\frac{\pi}{4}$
12. The sum of the order and the degree of the differential equation $y = x \frac{dy}{dx} + 3 \left[2 - \left(\frac{dy}{dx} \right)^2 \right]$ is :
- (A) not defined (B) 3
(C) 1 (D) 2



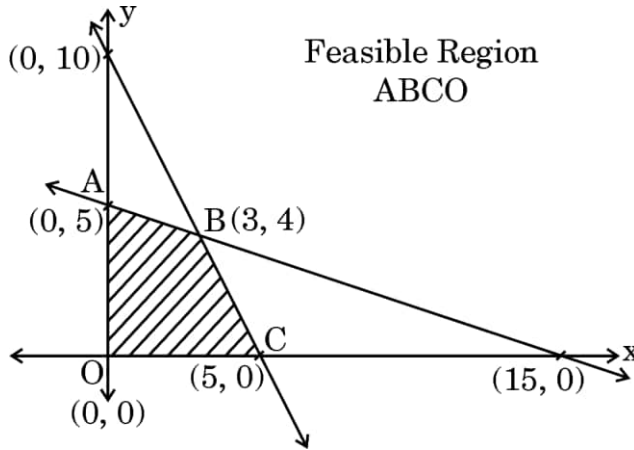
13. दिए गए ग्राफ में, एक रैखिक प्रोग्रामन समस्या (LPP) के उद्देश्य फलन $Z = px + qy$, $p, q > 0$ का अधिकतमीकरण करने के लिए सुसंगत क्षेत्र छायांकित किया गया है। यदि रेखाखण्ड AB के सभी बिंदुओं पर अधिकतम (Z) आता है, तो निम्नलिखित में से कौन-सा सही है ?



- (A) $p = 2q$ (B) $p = 3q$
 (C) $q = 3p$ (D) $q = 2p$
14. तीन बिंदु $A(0, 1, 1)$, $B(2, 0, -1)$ तथा $C(1, 0, 3)$ एक त्रिभुज ABC बनाते हैं। ΔABC का क्षेत्रफल है :
- (A) $\frac{\sqrt{53}}{2}$ वर्ग इकाई (B) $\sqrt{53}$ वर्ग इकाई
 (C) $\frac{\sqrt{11}}{2}$ वर्ग इकाई (D) $\sqrt{11}$ वर्ग इकाई
15. यदि $(3\hat{i} - 2\hat{j} + 5\hat{k}) \times (4\hat{i} + p\hat{j} + q\hat{k}) = \vec{0}$ है, तो p तथा q के मान हैं :
- (A) $p = -\frac{2}{3}, q = \frac{5}{3}$ (B) $p = -\frac{8}{3}, q = \frac{20}{3}$
 (C) $p = \frac{20}{3}, q = -\frac{8}{3}$ (D) $p = 0, q = 0$
16. निम्नलिखित में से कौन-सा एक रैखिक अवकल समीकरण **नहीं** है ?
- (A) $(1 + x^2) dy + 2xy dx = \cot x dx$
 (B) $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$
 (C) $x(1 + y^2) dx - y(1 + x^2) dy = 0$
 (D) $y dx - (x + 3y^2) dy = 0$



13. In the graph, the feasible region representing the Linear Programming Problem for maximising objective function $Z = px + qy$, $p, q > 0$ is shaded. If all points on segment AB give max (Z), then which of the following is true ?



- (A) $p = 2q$ (B) $p = 3q$
(C) $q = 3p$ (D) $q = 2p$
14. Three points $A(0, 1, 1)$, $B(2, 0, -1)$ and $C(1, 0, 3)$ form ΔABC . The ar (ΔABC) is :
- (A) $\frac{\sqrt{53}}{2}$ sq. units (B) $\sqrt{53}$ sq. units
(C) $\frac{\sqrt{11}}{2}$ sq. units (D) $\sqrt{11}$ sq. units
15. If $(3\hat{i} - 2\hat{j} + 5\hat{k}) \times (4\hat{i} + p\hat{j} + q\hat{k}) = \vec{0}$, then the values of p and q are :
- (A) $p = -\frac{2}{3}, q = \frac{5}{3}$ (B) $p = -\frac{8}{3}, q = \frac{20}{3}$
(C) $p = \frac{20}{3}, q = -\frac{8}{3}$ (D) $p = 0, q = 0$
16. Which of the following is **not** a Linear Differential Equation ?
- (A) $(1 + x^2) dy + 2xy dx = \cot x dx$
(B) $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$
(C) $x(1 + y^2) dx - y(1 + x^2) dy = 0$
(D) $y dx - (x + 3y^2) dy = 0$



17. असमिकाओं के निकाय $3x + y \geq 3$, $2x - y \geq -5$, $x, y \geq 0$ द्वारा निरूपित क्षेत्र है :

- (A) पहले चतुर्थांश में अपरिबद्ध
- (B) पहले चतुर्थांश में परिबद्ध
- (C) दूसरे चतुर्थांश में अपरिबद्ध
- (D) दूसरे चतुर्थांश में परिबद्ध

18. एक विशेष वस्तु की तीन दुकानों में उपलब्धता की प्रायिकता क्रमशः $\frac{4}{5}$, $\frac{3}{4}$ तथा $\frac{2}{3}$ है। यदि एक व्यक्ति इस वस्तु को खरीदने के लिए इन तीन दुकानों पर जाता है, तो इसके कम-से-कम एक दुकान पर उपलब्ध होने की प्रायिकता क्या है ?

- (A) $\frac{59}{60}$
- (B) 1
- (C) $\frac{1}{60}$
- (D) $\frac{9}{60}$

प्रश्न संख्या 19 और 20 अभिकथन एवं तर्क आधारित प्रश्न हैं। दो कथन दिए गए हैं जिनमें एक को अभिकथन (A) तथा दूसरे को तर्क (R) द्वारा अंकित किया गया है। इन प्रश्नों के सही उत्तर नीचे दिए गए कोडों (A), (B), (C) और (D) में से चुनकर दीजिए।

- (A) अभिकथन (A) और तर्क (R) दोनों सही हैं और तर्क (R), अभिकथन (A) की सही व्याख्या करता है।
- (B) अभिकथन (A) और तर्क (R) दोनों सही हैं, परन्तु तर्क (R), अभिकथन (A) की सही व्याख्या नहीं करता है।
- (C) अभिकथन (A) सही है, परन्तु तर्क (R) ग़लत है।
- (D) अभिकथन (A) ग़लत है, परन्तु तर्क (R) सही है।

19. अभिकथन (A) : यदि A तथा B दो ऐसे वर्ग आव्यूह हैं कि AB तथा BA दोनों परिभाषित हैं, तो यह आवश्यक नहीं है कि $AB = BA$.

तर्क (R) : समान कोटि के दो विकर्ण आव्यूहों का गुणनफल क्रमविनिमेय होता है।

20. अभिकथन (A) : फलन $f : N \rightarrow N$, जो $f(x) = x^3 + 2$, $\forall x \in N$ द्वारा परिभाषित है, एकैकी है परन्तु आच्छादी नहीं है।

तर्क (R) : क्योंकि सभी $y \in N$ (सह-प्रान्त) के लिए N (प्रान्त) में $x = (y - 2)^{1/3}$ का अस्तित्व नहीं है, जिससे $f(x) = x^3 + 2 = y$.



17. The region represented by the system of inequations $3x + y \geq 3$, $2x - y \geq -5$, $x, y \geq 0$ is :
- (A) unbounded in 1st quadrant
(B) bounded in 1st quadrant
(C) unbounded in 2nd quadrant
(D) bounded in 2nd quadrant
18. The probability that a particular item is available in three shops A, B and C is $\frac{4}{5}$, $\frac{3}{4}$ and $\frac{2}{3}$ respectively. If a person visits all the three shops to buy the item, then what is the probability that it will be available in at least one of the shops ?
- (A) $\frac{59}{60}$ (B) 1
(C) $\frac{1}{60}$ (D) $\frac{9}{60}$

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.
19. Assertion (A) : If A and B are two square matrices such that AB and BA are defined, then it is not necessary that $AB = BA$.
Reason (R) : Product of two diagonal matrices of same order is commutative.
20. Assertion (A) : A function $f : N \rightarrow N$ given by $f(x) = x^3 + 2$, $\forall x \in N$ is one-one but not onto.
Reason (R) : Since $\forall y \in N$ (Codomain), there does not exist $x = (y - 2)^{1/3}$ in N (Domain) such that $f(x) = x^3 + 2 = y$.



खण्ड ख

इस खण्ड में अति लघु-उत्तरीय (VSA) प्रकार के 5 प्रश्न हैं, जिनमें प्रत्येक के 2 अंक हैं।

21. (क) दर्शाइए कि फलन $f(x) = \begin{cases} \frac{\cos x}{-x + \frac{\pi}{2}}, & x \neq \frac{\pi}{2} \\ 1, & x = \frac{\pi}{2} \end{cases}$ $x = \frac{\pi}{2}$ पर संतत है।

अथवा

(ख) ज्ञात कीजिए कि $x = 2$ पर फलन $f(x) = \begin{cases} x - 1, & x < 2 \\ 2x - 3, & x \geq 2 \end{cases}$ अवकलनीय है या नहीं।

22. (क) माना भूमि पर पड़ी दो छड़ें सदिशों $4\hat{i} - \hat{j} + 3\hat{k}$ तथा $-2\hat{i} + \hat{j} - 2\hat{k}$ द्वारा निरूपित की गई हैं। इन दोनों छड़ों के लंबवत 5 m ऊँचाई के एक ध्वज-दण्ड को निरूपित किए जा सकने वाला सदिश ज्ञात कीजिए।

अथवा

(ख) यदि एक इकाई सदिश $\vec{a}$, x-अक्ष से $\frac{\pi}{4}$, y-अक्ष से $\frac{\pi}{3}$ तथा z-अक्ष से न्यून कोण θ बना रहा है, तो θ ज्ञात कीजिए तथा $\vec{a}$ के घटक ज्ञात कीजिए।

23. वह अंतराल ज्ञात कीजिए जिसमें/जिनमें फलन $f(x) = \frac{x}{3} + \frac{3}{x}$, $x \neq 0$ वर्धमान है।

24. यदि एक समांतर चतुर्भुज PQRS में, $\vec{PQ} = 2\hat{i} + 3\hat{j} + \hat{k}$ तथा $\vec{QR} = \hat{i} - 2\hat{j} + \hat{k}$ हैं, तो इसके विकर्णों $\vec{PR}$ तथा $\vec{SQ}$ के समानांतर इकाई सदिश ज्ञात कीजिए।

25. मान ज्ञात कीजिए :

$$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right) + \tan^{-1}\left(\tan\frac{2\pi}{3}\right)$$



SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. (a) Show that the function $f(x) = \begin{cases} \frac{\cos x}{-x + \frac{\pi}{2}}, & x \neq \frac{\pi}{2} \\ 1, & x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$.

OR

- (b) Find whether the function $f(x) = \begin{cases} x - 1, & x < 2 \\ 2x - 3, & x \geq 2 \end{cases}$ at $x = 2$ is differentiable or not.

22. (a) Let two rods placed on the ground be represented by vectors $4\hat{i} - \hat{j} + 3\hat{k}$ and $-2\hat{i} + \hat{j} - 2\hat{k}$. Find a vector representing a flag-post of height 5 m that has to be erected perpendicular to both the rods.

OR

- (b) A unit vector $\vec{a}$ is such that it makes an angle $\frac{\pi}{4}$ with x-axis, $\frac{\pi}{3}$ with y-axis and an acute angle θ with z-axis. Find θ and the components of $\vec{a}$.

23. Find the interval(s) for which the function $f(x) = \frac{x}{3} + \frac{3}{x}$, $x \neq 0$ is increasing.

24. If in a parallelogram PQRS, $\vec{PQ} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{QR} = \hat{i} - 2\hat{j} + \hat{k}$, then find the unit vectors parallel to the diagonals $\vec{PR}$ and $\vec{SQ}$.

25. Evaluate :

$$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right) + \tan^{-1}\left(\tan\frac{2\pi}{3}\right)$$



खण्ड ग

इस खण्ड में लघु-उत्तरीय (SA) प्रकार के 6 प्रश्न हैं, जिनमें प्रत्येक के 3 अंक हैं।

26. (क) रेखा $\frac{x-2}{3} = \frac{1-y}{2} = \frac{z-3}{2}$ पर एक ऐसा बिंदु ज्ञात कीजिए जो बिंदु (1, 2, 3) से $\sqrt{2}$ इकाई की दूरी पर है।

अथवा

- (ख) रेखाओं $\vec{r} = (4 + \lambda)\hat{i} + (2\lambda - 1)\hat{j} - 3\lambda\hat{k}$ तथा $\vec{r} = (1 + 2\mu)\hat{i} + (2 - 5\mu)\hat{k} + (4\mu - 1)\hat{j}$ के बीच की न्यूनतम दूरी ज्ञात कीजिए।

27. यदि $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$ है, तो $A^2 - 5A + 4I$ परिकलित कीजिए।

28. एक स्कूल में, वाद-विवाद प्रतियोगिता करवाने की प्रायिकता $\frac{1}{3}$ है जबकि प्रश्नोत्तरी प्रतियोगिता रखने की प्रायिकता $\frac{2}{3}$ है। भाग लेने वाली टीमों में टीम A में 4 लड़कियाँ तथा 6 लड़के हैं, जबकि टीम B में 7 लड़कियाँ तथा 3 लड़के हैं। यदि वाद-विवाद प्रतियोगिता होती है, तो टीम A में से विद्यार्थी चुने जाने हैं, जबकि प्रश्नोत्तरी प्रतियोगिता के लिए टीम B में से विद्यार्थियों का चयन होना है। यदि टीमों में से केवल दो विद्यार्थी चुनने हैं, तो एक लड़का तथा एक लड़की के चुने जाने की प्रायिकता ज्ञात कीजिए।

29. निम्नलिखित रैखिक प्रोग्रामन समस्या को ग्राफ द्वारा हल कीजिए :
व्यवरोधों

$$x + 2y \leq 28$$

$$3x + y \leq 24$$

$$x \geq 2$$

$$x, y \geq 0$$

के अंतर्गत $Z = 20x + 10y$ का अधिकतमीकरण कीजिए।



SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26. (a) Find a point on the line $\frac{x-2}{3} = \frac{1-y}{2} = \frac{z-3}{2}$ at a distance of $\sqrt{2}$ units from the point (1, 2, 3).

OR

- (b) Find the shortest distance between the lines

$$\vec{r} = (4 + \lambda)\hat{i} + (2\lambda - 1)\hat{j} - 3\lambda\hat{k}$$

$$\vec{r} = (1 + 2\mu)\hat{i} + (2 - 5\mu)\hat{k} + (4\mu - 1)\hat{j}.$$

27. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then compute $A^2 - 5A + 4I$.

28. In a school, the probability of holding a debate competition is $\frac{1}{3}$ and that of a quiz competition is $\frac{2}{3}$. In the two participating teams, A has 4 girls and 6 boys and B has 7 girls and 3 boys. If a debate competition is held, the students are selected from team A and for the quiz competition they are selected from team B. If only two students are to be chosen from the teams, then find the probability that one will be a girl and the other a boy.

29. Solve the following Linear Programming Problem graphically :

Maximize $Z = 20x + 10y$

subject to constraints

$$x + 2y \leq 28$$

$$3x + y \leq 24$$

$$x \geq 2$$

$$x, y \geq 0$$



30. (क) माना $A = \mathbb{R} - \{3\}$ तथा $B = \mathbb{R} - \{1\}$ है। एक फलन $f : A \rightarrow B$ इस प्रकार परिभाषित है कि $f(x) = \left(\frac{x-2}{x-3} \right)$ है। जाँच कीजिए कि क्या f एकैकी व आच्छादी है।

अथवा

- (ख) माना n एक निश्चित धन पूर्णांक है तथा संबंध R समुच्चय Z में इस प्रकार परिभाषित है कि $R = \{(x, y) : (x - y), n \text{ से भाज्य है}, x, y \in Z\}$ । ज्ञात कीजिए कि क्या R एक तुल्यता संबंध है।

31. (क) यदि $xy = e^{x-y}$ है, तो $\frac{dy}{dx}$ ज्ञात कीजिए।

अथवा

- (ख) $\tan^{-1} \left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right)$ का $\cos^{-1} x^2$ के सापेक्ष अवकलज ज्ञात कीजिए।

खण्ड घ

इस खण्ड में 4 दीर्घ-उत्तरीय (LA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 5 अंक हैं।

32. (क) ज्ञात कीजिए :

$$\int \frac{x}{(x-1)(x^2+4)} dx$$

अथवा

- (ख) मान ज्ञात कीजिए :

$$\int_0^1 \frac{x \tan^{-1} x}{(1+x^2)^{3/2}} dx$$

33. समाकलन के प्रयोग से $y = 5x + 4$, $y = 0$, $x = -1$ तथा $x = 1$ के द्वारा परिबद्ध क्षेत्र का क्षेत्रफल ज्ञात कीजिए।



30. (a) Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. A function $f : A \rightarrow B$ is defined by $f(x) = \left(\frac{x-2}{x-3} \right)$. Find whether f is one-one and onto.

OR

- (b) Let n be a fixed positive integer. A relation R is defined in set Z such that $R = \{(x, y) : (x - y) \text{ is divisible by } n, x, y \in Z\}$. Determine if R is an equivalence relation.

31. (a) If $xy = e^{x-y}$, then find $\frac{dy}{dx}$.

OR

- (b) Differentiate $\tan^{-1} \left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right)$ with respect to $\cos^{-1} x^2$.

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32. (a) Find :

$$\int \frac{x}{(x-1)(x^2+4)} dx$$

OR

- (b) Evaluate :

$$\int_0^1 \frac{x \tan^{-1} x}{(1+x^2)^{3/2}} dx$$

33. Using integration, find the area of the region bounded by $y = 5x + 4$, $y = 0$, $x = -1$ and $x = 1$.



34. (क) अवकल समीकरण $y e^y dx = (y^3 + 2x e^y) dy$, जब $y(0) = 1$ को हल कीजिए।

अथवा

(ख) अवकल समीकरण $(x^3 - 3xy^2) dx = (y^3 - 3x^2y) dy$ का व्यापक हल ज्ञात कीजिए।

35. बिंदु $P(1, 2, 3)$ से रेखा $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$ पर डाले गए लंब की लंबाई ज्ञात कीजिए।

बिंदु P को लंब पाद से मिलाने वाली लंबवत रेखा का समीकरण भी ज्ञात कीजिए।

खण्ड ड

इस खण्ड में 3 प्रकरण-अध्ययन आधारित प्रश्न हैं, जिनमें प्रत्येक के 4 अंक हैं।

प्रकरण अध्ययन – 1

36. एक कारपेंटर एक ऐसे घनाभाकार लकड़ी के बक्से को बनाना चाहता है कि इसकी लंबाई व चौड़ाई का योगफल इसकी ऊँचाई से 3 cm अधिक हो। इसकी लंबाई के दुगुने तथा चौड़ाई के तिगुने को ऊँचाई में जोड़ने पर 10 cm आता है। इसकी चौड़ाई को इसकी ऊँचाई के 7 गुने में जोड़ने पर इसकी लंबाई के तिगुने से 1 cm कम आता है।

उपर्युक्त सूचना के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

(i) विभिन्न विमाओं को दर्शाते हुए समीकरणों को लिखिए तथा उन्हें आव्यूह समीकरण $AX = B$ के रूप में व्यक्त कीजिए। 1

(ii) ज्ञात कीजिए कि क्या A^{-1} का अस्तित्व है। अपने उत्तर की पुष्टि कीजिए। 1

(iii) (क) A^{-1} ज्ञात कीजिए। 2

अथवा

(iii) (ख) $A^2 + 7 I$ ज्ञात कीजिए। 2



34. (a) Solve the differential equation $y e^y dx = (y^3 + 2x e^y) dy$, when $y(0) = 1$.

OR

- (b) Find the general solution of the differential equation
 $(x^3 - 3xy^2) dx = (y^3 - 3x^2y) dy$.

35. Find the length of the perpendicular drawn from the point $P(1, 2, 3)$ to the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$. Also, find the equation of the perpendicular line joining P and the foot of the perpendicular.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. A carpenter needs to design a wooden box in the shape of a cuboid such that the sum of its length and breadth is 3 cm more than its height. Twice of its length, thrice of its breadth and its height add up to 10 cm. Its breadth added to 7 times its height is 1 cm less than 3 times its length.

On the basis of the above information, answer the following questions :

- (i) Write the equations representing the various dimensions and express them as the matrix equation $AX = B$. 1
- (ii) Find if A^{-1} exists. Justify your answer. 1
- (iii) (a) Find A^{-1} . 2

OR

- (iii) (b) Find $A^2 + 7I$. 2



प्रकरण अध्ययन – 2

37. एक NGO ने एक चैरिटी इवेंट आयोजित किया जिसमें उन्होंने बच्चों को सर्दी से बचाने के लिए ऊनी टोपियाँ बाँटने का फैसला किया। बाँटने वाली टोपियाँ तीन अलग-अलग बक्सों में हैं, बॉक्स I में 30 लाल रंग की टोपियाँ हैं, बॉक्स II में 20 लाल तथा 10 हरी टोपियाँ हैं, तथा बॉक्स III में 30 हरे रंग की टोपियाँ हैं। बॉक्स i चुनकर उसमें से एक टोपी लेने की प्रायिकता $\frac{i}{6}$ है, जहाँ $i = 1, 2, 3$ है।

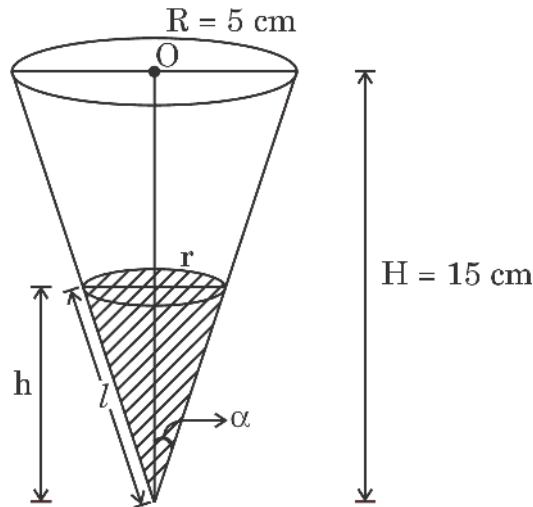
उपर्युक्त सूचना पर आधारित, निम्नलिखित प्रश्नों के उत्तर दीजिए :

एक व्यक्ति एक टोपी चुनता है।

- उसके एक लाल रंग की टोपी चुनने की प्रायिकता क्या है? 2
- यदि वह एक हरी टोपी चुनता है, तो इस टोपी के बॉक्स II में से चुने जाने की प्रायिकता क्या है? 2

प्रकरण अध्ययन – 3

38. एक बर्थडे पार्टी पर, बच्चों को शंक्वाकार कपों में संतरे का जूस दिया जा रहा था, जैसा कि आकृति में दर्शाया गया है।



प्रत्येक कप की गहराई 15 cm है तथा त्रिज्या 5 cm है। इस कप में जूस $0.1 \text{ cm}^3/\text{s}$ की दर से डाला जा रहा है।



Case Study – 2

37. An NGO organises a charity event in which they decide to distribute woollen caps to protect children from winter. The caps to be distributed are in three separate boxes, Box I has 30 red caps, Box II has 20 red and 10 green caps, and Box III has 30 green caps. The probability that a Box i is selected and a cap picked out is $\frac{i}{6}$, where $i = 1, 2, 3$.

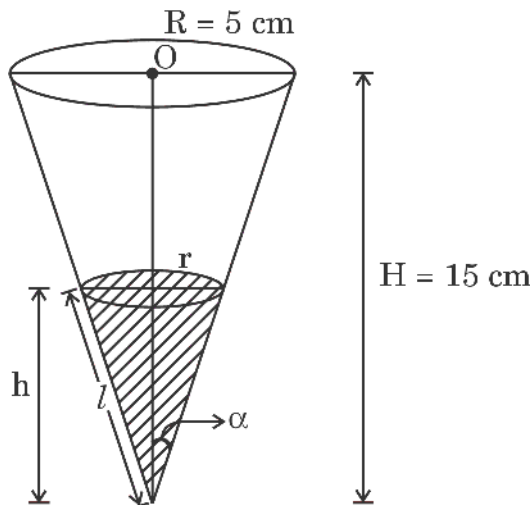
Based on the above information, answer the following questions :

A person selects a cap.

- (i) What is the probability that he selects a red cap ? 2
- (ii) If he selects a green cap, what is the probability that the cap has come from Box II ? 2

Case Study – 3

38. At a birthday party, children are being served orange juice in conical cups, as shown in the figure.



Each cup is 15 cm deep and has a radius 5 cm. The juice is being poured into this cup at a rate of $0.1 \text{ cm}^3/\text{s}$.



उपर्युक्त सूचना के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

- (i) कप में जूस की ऊँचाई h तथा कप में जूस के पृष्ठ की त्रिज्या r में संबंध स्थापित कीजिए जबकि शंकु का अर्ध-शीर्ष कोण α है। 1
- (ii) कप में जूस का तल किस दर से ऊपर चढ़ रहा है, जब कप में जूस की गहराई 6 cm है? 1
- (iii) (क) जिस समय कप में जूस की गहराई 6 cm है, उस समय जूस के ऊपरी पृष्ठीय क्षेत्रफल के बढ़ने की दर ज्ञात कीजिए। 2

अथवा

- (iii) (ख) जब कप में जूस की गहराई 6 cm है, उस समय कप के गीले पृष्ठीय क्षेत्रफल (wetted surface area) के बढ़ने की दर ज्ञात कीजिए। 2



On the basis of the above information, answer the following questions :

- (i) Establish a relation between the height h of the juice in the cup and radius r of the surface of the juice in the cup, if the semi-vertical angle of the cone is α . 1
- (ii) At what rate is the juice level in the cup rising when the juice is 6 cm deep ? 1
- (iii) (a) When the juice is 6 cm deep, then find at what rate is the upper surface area of juice increasing ? 2

OR

- (iii) (b) When the juice is 6 cm deep, then find the rate at which the wetted surface area of the cup is increasing. 2

65/4/3

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Senior Secondary School Certificate Examination, 2026

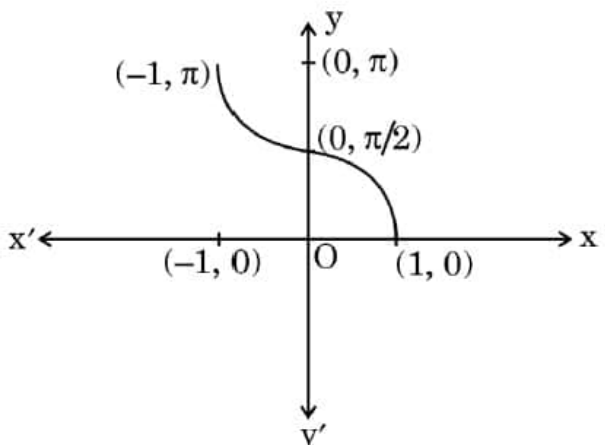
MATHEMATICS (041) (PAPER CODE 65/4/3)

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1.	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2.	“Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its’ leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC.”
3.	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one’s own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-X, while evaluating two competency-based questions, please try to understand given answer and even if reply is not from marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.
4.	The Marking scheme carries only suggested value points for the answers These are in the nature of Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5.	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6.	Evaluators will mark(√) wherever answer is correct. For wrong answer CROSS ‘X’ be marked. Evaluators will not put right (✓)while evaluating which gives an impression that answer is correct and no marks are awarded. This is most common mistake which evaluators are committing.
7.	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8.	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9.	If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note “Extra Question” .
10.	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.

11.	A full scale of marks 0 to 80 has to be used. Please do not hesitate to award full marks if the answer deserves it.
12.	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
13.	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
14.	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
15.	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
16.	The Examiners should acquaint themselves with the guidelines given in the “ Guidelines for spot Evaluation ” before starting the actual evaluation.
17.	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
18.	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

MARKING SCHEME
MATHEMATICS CLASS XII (Subject Code-041)
(PAPER CODE: 65/4/3)

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Steps	Marks
	SECTION A		
	Q. Number 1 to 20 are multiple choice questions of 1 mark each.		
1.	The following graph represents :  (A) $y = \cos^{-1} x$ (B) $y = \sec^{-1} x$ (C) $y = \sin^{-1} x$ (D) $y = \tan^{-1} x$		
Ans.	(A) $y = \cos^{-1} x$		1
2.	Let $A = [a_{ij}]$ be a 2×2 matrix whose elements are given by $a_{ij} = \frac{(i+2j)^2}{3}$. Then A' is given by : (A) $\begin{bmatrix} 12 & \frac{25}{3} \\ \frac{16}{3} & 3 \end{bmatrix}$ (B) $\begin{bmatrix} 12 & \frac{16}{3} \\ \frac{25}{3} & 3 \end{bmatrix}$ (C) $\begin{bmatrix} 3 & \frac{25}{3} \\ \frac{16}{3} & 12 \end{bmatrix}$ (D) $\begin{bmatrix} 3 & \frac{16}{3} \\ \frac{25}{3} & 12 \end{bmatrix}$		
Ans.	(D) $\begin{bmatrix} 3 & \frac{16}{3} \\ \frac{25}{3} & 12 \end{bmatrix}$		1

3.	The greatest integer function, $f(x) = [x]$, $0 < x < 3$ is not differentiable at how many points ? (A) At only one point (B) At only two points (C) At no point (D) At three points		
Ans.	(B) At only two points		1
4.	The area of the region bounded by the curve $y = x$ and x-axis, between $x = 0$ and $x = 2$ is : (A) 2 sq. units (B) $\frac{1}{2}$ sq. unit (C) 1 sq. unit (D) 4 sq. units		
Ans.	(A) 2 sq. units		1
5.	If $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is: (A) 0 (B) -2 (C) -1 (D) 2		
Ans.	(D) 2		1
6.	A cylindrical tank is being filled with sand at a rate of $314 \text{ m}^3/\text{h}$. If the radius of the tank is 10 m, then the height of sand in the tank increases at the rate of : (A) 1.1 m/h (B) 1 m/h (C) π m/h (D) $\frac{\pi}{2}$ m/h		
Ans.	(B) 1 m/h		1
7.	The principal value of $\sec^{-1}(\sqrt{2}) + 2 \operatorname{cosec}^{-1}(-\sqrt{2})$ is (A) $-\frac{\pi}{2}$ (B) $-\frac{\pi}{4}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$		
Ans.	(B) $-\frac{\pi}{4}$		1

8.	If points (2, 3), (0, 4) and (p, 2) are collinear, then the value of p is : (A) $\frac{4}{7}$ (B) $-\frac{3}{7}$ (C) 4 (D) - 4		
Ans.	(C) 4		1
9.	Differential of e^{e^x} with respect to x is : (A) $\log x$ (B) e^{e^x} (C) $e^x \cdot e^{e^x}$ (D) $(e^x)^2$		
Ans.	(C) $e^x \cdot e^{e^x}$		1
10.	$\int \frac{dx}{\sqrt{25-16x^2}}$ is equal to : (A) $\frac{1}{5} \sin^{-1} 4x + c$ (B) $\frac{1}{25} \sin^{-1} 16x + c$ (C) $\frac{1}{4} \sin^{-1} \frac{4x}{5} + c$ (D) $\frac{1}{16} \sin^{-1} \frac{4x}{5} + c$		
Ans.	(C) $\frac{1}{4} \sin^{-1} \frac{4x}{5} + c$		1
11.	If $\int_0^1 \frac{dx}{e^x + e^{-x}} = \tan^{-1} e + k$, then the value of k is : (A) e (B) $\frac{\pi}{4}$ (C) 0 (D) $-\frac{\pi}{4}$		
Ans.	(D) $-\frac{\pi}{4}$		1
12.	The sum of the order and the degree of the differential equation $y = x \frac{dy}{dx} + 3 \left[2 - \left(\frac{dy}{dx} \right)^2 \right]$ is : (A) not defined (B) 3 (C) 1 (D) 2		
Ans.	(B) 3		1

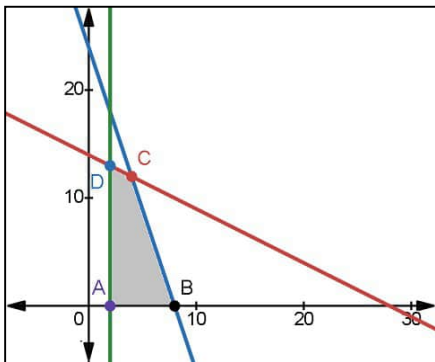
16.	Which of the following is not a Linear Differential Equation? $(A) (1 + x^2)dy + 2xydx = \cot x dx$ $(B) y + \frac{d}{dx}(xy) = x(\sin x + \log x)$ $(C) x(1 + y^2)dx - y(1 + x^2)dy = 0$ $(D) ydx - (x + 3y^2)dy = 0$		
Ans.	$(C) x(1 + y^2)dx - y(1 + x^2)dy = 0$		1
17.	The region represented by the system of inequations $3x + y \geq 3$, $2x - y \geq -5$, $x, y \geq 0$ is : (A) unbounded in 1 st quadrant (B) bounded in 1 st quadrant (C) unbounded in 2 nd quadrant (D) bounded in 2 nd quadrant		
Ans.	(A) unbounded in 1 st Quadrant		1
18.	The probability that a particular item is available in three shops A, B and C is $\frac{4}{5}, \frac{3}{4}$ and $\frac{2}{3}$ respectively. If a person visits all the three shops to buy the item, then what is the probability that it will be available in at least one of the shops ? $(A) \frac{59}{60}$ $(B) 1$ $(C) \frac{1}{60}$ $(D) \frac{9}{60}$		
Ans.	$(A) \frac{59}{60}$		1
19.	Assertion (A): If A and B are two square matrices such that AB and BA are defined, then it is not necessary that $AB = BA$. Reason (R): Product of two diagonal matrices of same order is commutative.		
Ans.	(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).		1

20.	Assertion (A): A function $f : \mathbb{N} \rightarrow \mathbb{N}$ given by $f(x) = x^3 + 2, \forall x \in \mathbb{N}$ is one-one but not onto. Reason (R): Since $\forall y \in \mathbb{N}$ (codomain), there does not exist $x = (y - 2)^{\frac{1}{3}}$ in $\mathbb{N}$ (Domain) such that $f(x) = x^3 + 2 = y$.		
Ans.	(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).		1
SECTION B Q. Numbers 21 to 25 are very short answer questions of 2 marks each.			
21. (a)	Show that the function $f(x) = \begin{cases} \frac{\cos x}{-x + \frac{\pi}{2}}, & x \neq \frac{\pi}{2} \\ 1, & x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$.		
Sol.	For showing: $\lim_{x \rightarrow \frac{\pi}{2}} f(x) = 1$ For showing: $f\left(\frac{\pi}{2}\right) = 1$ $\therefore f(x)$ is continuous at $x = \frac{\pi}{2}$.	I II III	1 $\frac{1}{2}$ $\frac{1}{2}$
OR			
21. (b)	Find whether the function $f(x) = \begin{cases} x - 1, & x < 2 \\ 2x - 3, & x \geq 2 \end{cases}$ at $x = 2$ is differentiable or not.		
Sol.	Getting $LHD = 1$ Getting $RHD = 2$ $\therefore LHD \neq RHD$, so $f(x)$ is not differentiable at $x = 2$.	I II III	1 $\frac{1}{2}$ $\frac{1}{2}$

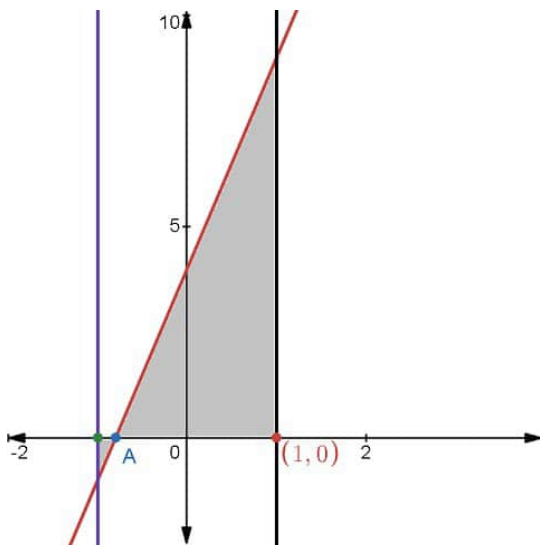
22. (a)	Let two rods placed on the ground be represented by vectors $4\hat{i} - \hat{j} + 3\hat{k}$ and $-2\hat{i} + \hat{j} - 2\hat{k}$. Find a vector representing a flag-post of height 5 m that has to be erected perpendicular to both the rods.		
Sol.	Let $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$ $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 3 \\ -2 & 1 & -2 \end{vmatrix} = -\hat{i} + 2\hat{j} + 2\hat{k}$ Now, Required vector $= 5 \left(\frac{\vec{a} \times \vec{b}}{ \vec{a} \times \vec{b} } \right) = \begin{cases} \frac{-5}{3}\hat{i} + \frac{10}{3}\hat{j} + \frac{10}{3}\hat{k} \\ or \\ \frac{5}{3}\hat{i} - \frac{10}{3}\hat{j} - \frac{10}{3}\hat{k} \end{cases}$ Alternative approach: Let $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$ Let required vector be $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$ so, $25 = x^2 + y^2 + z^2, 4x - y + 3z = 0, -2x + y - 2z = 0$ on solving we get, $z = -2x, y = z \Rightarrow \frac{100}{9} = z^2 \Rightarrow z = \pm \frac{10}{3}$ $\therefore$ Required vector, $\vec{p} = \begin{cases} \frac{-5}{3}\hat{i} + \frac{10}{3}\hat{j} + \frac{10}{3}\hat{k} \\ or \\ \frac{5}{3}\hat{i} - \frac{10}{3}\hat{j} - \frac{10}{3}\hat{k} \end{cases}$	I II	1 1 (For any one result)
OR			
22. (b)	A unit vector $\vec{a}$ is such that it makes an angle $\frac{\pi}{4}$ with x-axis, $\frac{\pi}{3}$ with y-axis and an acute angle θ with z-axis. Find θ and the components of $\vec{a}$.		

	$\cos^2\left(\frac{\pi}{4}\right) + \cos^2\left(\frac{\pi}{3}\right) + \cos^2\theta = 1 \Rightarrow \cos\theta = \frac{1}{2} \text{ (As, } \theta \text{ is acute)}$ $\Rightarrow \theta = \frac{\pi}{3} \text{ \& Components of } \vec{a} = \frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2}$	I	1
		II	1
23.	Find the interval(s) in which the function $f(x) = \frac{x}{3} + \frac{3}{x}, x \neq 0$ is increasing.		
Sol.	$f'(x) = \frac{1}{3} - \frac{3}{x^2} = \frac{x^2 - 9}{3x^2}$ For increasing function, $f'(x) = 0 \Rightarrow x^2 - 9 = 0$ $\Rightarrow x = -3, 3$ $\therefore f(x)$ is increasing on $x \in (-\infty, -3) \cup (3, \infty)$.	I	1
		II	$\frac{1}{2}$
		III	$\frac{1}{2}$
24.	If in a parallelogram $PQRS, \overrightarrow{PQ} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\overrightarrow{QR} = \hat{i} - 2\hat{j} + \hat{k}$, then find the unit vectors parallel to the diagonals $\overrightarrow{PR}$ and $\overrightarrow{SQ}$.		
Sol.	$\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR} = 3\hat{i} + \hat{j} + 2\hat{k},$ So, unit vector parallel to $\overrightarrow{PR} = \frac{3}{\sqrt{14}}\hat{i} + \frac{1}{\sqrt{14}}\hat{j} + \frac{2}{\sqrt{14}}\hat{k}$ Now, $\overrightarrow{SQ} = \overrightarrow{PQ} + \overrightarrow{SP} = \overrightarrow{PQ} - \overrightarrow{PS} = \overrightarrow{PQ} - \overrightarrow{QR} = \hat{i} + 5\hat{j}$ Thus, unit vector parallel to $\overrightarrow{SQ} = \frac{1}{\sqrt{26}}\hat{i} + \frac{5}{\sqrt{26}}\hat{j}$	I	1
		II	1
25.	Evaluate: $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin\left(\frac{-\pi}{2}\right)\right) + \tan^{-1}\left(\tan\frac{2\pi}{3}\right)$		2
Sol.	$\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin\left(\frac{-\pi}{2}\right)\right) + \tan^{-1}\left(\tan\frac{2\pi}{3}\right)$ $= -\frac{\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4} - \frac{\pi}{3}$ $= \frac{-5\pi}{12}$	I	$1\frac{1}{2}$ (For any 3 correct result)
		II	$\frac{1}{2}$

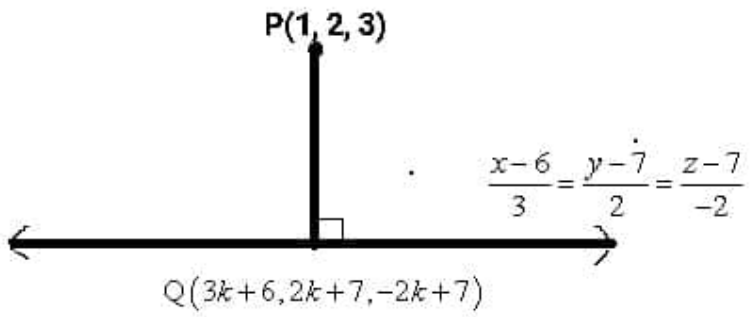
27.	If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then compute $A^2 - 5A + 4I$.		
Sol.	$A^2 = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$ $A^2 - 5A + 4I = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ $A^2 - 5A + 4I = \begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$	I II III	$1\frac{1}{2}$ ($\frac{1}{2}$ for any three correct elements) 1 $\frac{1}{2}$
28.	In a school, the probability of holding a debate competition is $\frac{1}{3}$ and that of a quiz competition is $\frac{2}{3}$. In the two participating teams, A has 4 girls and 6 boys and B has 7 girls and 3 boys. If a debate competition is held, the students are selected from team A and for the quiz competition they are selected from team B. If only two students are to be chosen from the teams, then find the probability that one will be a girl and the other a boy.		
Sol :	Let D be the event a debate competition is held, Q be the event a quiz competition is held and E be the event that one girl & one boy are chosen. $P(E D) = 2 \times \frac{4}{10} \times \frac{6}{9} = \frac{8}{15}, P(E Q) = 2 \times \frac{7}{10} \times \frac{3}{9} = \frac{7}{15}$ $\text{Now, } P(E) = \frac{1}{3} \times \frac{8}{15} + \frac{2}{3} \times \frac{7}{15} = \frac{22}{45}$	I II	1+1 1

29.	Solve the following Linear programming problem graphically: Maximize $Z = 20x + 10y$ subject to the constraints $x + 2y \leq 28$ $3x + y \leq 24$ $x \geq 2$ $x \geq 0, y \geq 0$												
Sol.	 <table border="1" data-bbox="709 571 1261 770"><thead><tr><th>Corner point</th><th>Value of $Z=20x+10y$</th></tr></thead><tbody><tr><td>A (2, 0)</td><td>40</td></tr><tr><td>B (8,0)</td><td>160</td></tr><tr><td>C (4, 12)</td><td>200</td></tr><tr><td>D (2, 13)</td><td>170</td></tr></tbody></table> Thus, maximum value of Z is 200 at $x = 4$ and $y = 12$	Corner point	Value of $Z=20x+10y$	A (2, 0)	40	B (8,0)	160	C (4, 12)	200	D (2, 13)	170	I	1 ½
Corner point	Value of $Z=20x+10y$												
A (2, 0)	40												
B (8,0)	160												
C (4, 12)	200												
D (2, 13)	170												
		II	1										
		III	½										
30. (a)	Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. A function $f : A \rightarrow B$ is defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Find whether f is one-one and onto.												
Sol.	Let $f(a) = f(b)$ for some $a, b \in \mathbb{R} - \{3\}$ so, $\left(\frac{a-2}{a-3}\right) = \left(\frac{b-2}{b-3}\right) \Rightarrow ab - 2b - 3a + 6 = ab - 3b - 2a + 6$ $\Rightarrow a = b$, Thus f is one-one function. Now, let us assume for some $y \in \mathbb{R} - \{1\}$, $y = f(x) = \frac{x-2}{x-3}$ For getting, $x = \frac{3y-2}{y-1}$ Range = $\mathbb{R} - \{1\}$ = co-domain, Thus f is onto function.	I	1										
		II	½										
		III	1										
		IV	½										
30. (b)	Let n be a fixed positive integer. A relation R is defined in set $\mathbb{Z}$ such that $R = \{(x, y) : (x - y) \text{ is divisible by } n, x, y \in \mathbb{Z}\}$. Determine if R is an equivalence relation.												

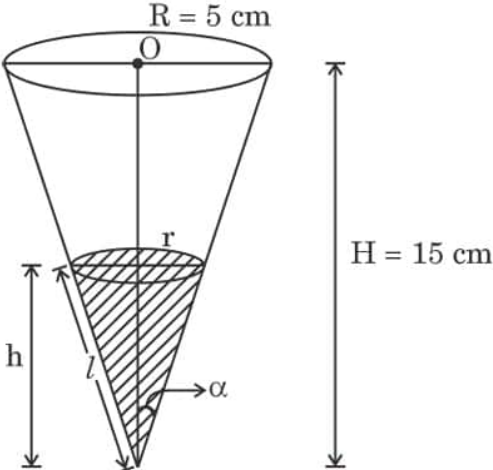
31. (b)	Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$ with respect to $\cos^{-1} x^2$.		
Sol.	Let $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$ & put $x^2 = \cos \theta$ $y = \tan^{-1}\left(\frac{\sqrt{1+\cos \theta}-\sqrt{1-\cos \theta}}{\sqrt{1+\cos \theta}+\sqrt{1-\cos \theta}}\right)$ $y = \tan^{-1}\left(\frac{\cos\left(\frac{\theta}{2}\right)-\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)+\sin\left(\frac{\theta}{2}\right)}\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\right)$ Thus, $y = \left(\frac{\pi}{4}-\frac{\theta}{2}\right) \Rightarrow \frac{dy}{d\theta} = \frac{-1}{2}$	I II III	$\frac{1}{2}$ $1\frac{1}{2}$ 1
SECTION D			
Q. Numbers 32 to 35 are long answer questions of 5 marks each.			
32. (a)	Find : $\int \frac{x}{(x-1)(x^2+4)} dx$		
	Let $\frac{x}{(x-1)(x^2+4)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+4}$ on solving we get, $A = \frac{1}{5}, B = \frac{-1}{5}, C = \frac{4}{5}$ so, $\int \frac{x}{(x-1)(x^2+4)} dx = \frac{1}{5} \int \frac{dx}{x-1} - \frac{1}{10} \int \frac{2x dx}{x^2+4} + \frac{4}{5} \int \frac{dx}{x^2+4}$ $\int \frac{x}{(x-1)(x^2+4)} dx = \frac{1}{5} \log x-1 - \frac{1}{10} \log(x^2+4) + \frac{2}{5} \tan^{-1}\left(\frac{x}{2}\right) + c$	I II III	1 $1\frac{1}{2}$ $2\frac{1}{2}$
OR			
32. (b)	Evaluate : $\int_0^1 \frac{x \tan^{-1} x}{(1+x^2)^{\frac{3}{2}}} dx$.		

	Let $x = \tan t \Rightarrow dx = \sec^2 t dt$ $I(\text{Let}) = \int_0^1 \frac{x \tan^{-1} x}{(1+x^2)^{\frac{3}{2}}} dx = \int_0^{\frac{\pi}{4}} \frac{t \tan t}{\sec^3 t} \sec^2 t dt = \int_0^{\frac{\pi}{4}} t \sin t dt$ $I = (-t \cos t + \sin t) \Big _0^{\frac{\pi}{4}}$ $I = \frac{-\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} \text{ or } \frac{-\pi + 4}{4\sqrt{2}}$	I	1
		II	2
		III	1
		IV	1
33.	Using integration, find the area of the region bounded by $y = 5x + 4$, $y = 0$, $x = -1$ and $x = 1$.		
Sol.	 Required Area = $\left \int_{-1}^{-\frac{4}{5}} y dx \right + \int_{-\frac{4}{5}}^1 y dx$ Area = $-\int_{-1}^{-\frac{4}{5}} (5x + 4) dx + \int_{-\frac{4}{5}}^1 (5x + 4) dx$ Area = $-\left(\frac{(5x+4)^2}{10} \right)_{-1}^{-\frac{4}{5}} + \left(\frac{(5x+4)^2}{10} \right)_{-\frac{4}{5}}^1 = \frac{1}{10} + \frac{81}{10} = \frac{41}{5} \text{ or } \frac{41}{5} \text{ sq. units}$	I	1 (For correct graph)
		II	1
		III	1
		IV	2

34. (a)	Solve the differential equation: $ye^y dx = (y^3 + 2xe^y)dy$, when $y(0) = 1$.		
Sol.	Given differential eq. can be written as $\frac{dx}{dy} - \frac{2}{y}x = y^2e^{-y}$	I	1
	Integrating factor $= e^{\int \frac{-2}{y} dy} = \frac{1}{y^2}$	II	1
	General solution: $x\left(\frac{1}{y^2}\right) = \int \frac{1}{y^2} \cdot y^2 e^{-y} dy$	III	1
	$x\left(\frac{1}{y^2}\right) = -e^{-y} + c$ or $x = y^2(-e^{-y} + c)$	IV	1
	Now, when $x = 0, y = 1: 0 = -e^{-1} + c \Rightarrow c = e^{-1}$	V	$\frac{1}{2}$
	Required particular solution is, $x = y^2(e^{-1} - e^{-y})$	VI	$\frac{1}{2}$
34. (b)	Find the general solution of the differential equation: $(x^3 - 3xy^2)dx = (y^3 - 3x^2y) dy$.		
Sol.	$\frac{dy}{dx} = \frac{x^3 - 3xy^2}{y^3 - 3x^2y}$	I	1
	Put $y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$, so	II	1
	$v + x \frac{dv}{dx} = \frac{1 - 3v^2}{v^3 - 3v} \Rightarrow x \frac{dv}{dx} = \frac{1 - 3v^2}{v^3 - 3v} - v = \frac{1 - v^4}{v^3 - 3v}$		
	$\Rightarrow \int \frac{3v - v^3}{v^4 - 1} dv = \int \frac{1}{x} dx$	III	1
	$\Rightarrow \frac{3}{4} \log \left \frac{v^2 - 1}{v^2 + 1} \right - \frac{1}{4} \log v^4 - 1 = \log x + c$	IV	$\frac{1}{2}$
	$\therefore \frac{3}{4} \log \left \frac{y^2 - x^2}{y^2 + x^2} \right - \frac{1}{4} \log \left \frac{y^4 - x^4}{x^4} \right = \log x + c$	V	$\frac{1}{2}$

35.	Find the length of the perpendicular drawn from the point P(1, 2, 3) to the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$. Also, find the equation of the perpendicular line joining P and the foot of the perpendicular.		
Sol :	Let $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2} = k$ So any general point on the given line is Q(3k + 6, 2k + 7, -2k + 7).  Direction ratio's of PQ are $\langle 3k + 5, 2k + 5, -2k + 4 \rangle$ Direction ratio's of line are $\langle 3, 2, -2 \rangle$ $\Rightarrow 3(3k + 5) + 2(2k + 5) - 2(-2k + 4) = 0$ $\Rightarrow k = -1$ So, Foot of the perpendicular Q(3, 5, 9) Required distance (PQ) = $\sqrt{4 + 9 + 36} = 7$ units Required equation of line PQ: $\left. \begin{aligned} \vec{r} &= (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k}) \\ \text{or} \\ \frac{x-1}{2} &= \frac{y-2}{3} = \frac{z-3}{6} \end{aligned} \right\}$	I	1
		II	1
		III	1
		IV	1
		V	1
SECTION E			
This section (Q. 36 to 38) has 3 case study based questions of 4 marks each.			
36.	A carpenter needs to design a wooden box in the shape of a cuboid such that the sum of its length and breadth is 3 cm more than its height. Twice of its length, thrice of its breadth and its height add up to 10 cm. Its breadth added to 7 times its height is 1 cm less than 3 times its length.		

On the basis of the above information, answer the following questions: (i) Write the equations representing the various dimensions & express them as the matrix equation $AX = B$. (ii) Find if A^{-1} exists. Justify your answer. (iii) (a) Find A^{-1}. OR (iii) (b) Find $A^2 + 7I$.		
(i) $\begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ -3 & 1 & 7 \end{pmatrix} \begin{pmatrix} l \\ b \\ h \end{pmatrix} = \begin{pmatrix} 3 \\ 10 \\ -1 \end{pmatrix}$ $A \quad X = B$ (ii) $A = -8 \neq 0$, so A^{-1} exists. (iii) (a) $A^{-1} = \frac{1}{ A }(\text{adj}A) = \frac{1}{-8} \begin{pmatrix} 20 & -8 & 4 \\ -17 & 4 & -3 \\ 11 & -4 & 1 \end{pmatrix}$ OR (iii) (b) $A^2 = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ -3 & 1 & 7 \end{pmatrix} \begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ -3 & 1 & 7 \end{pmatrix} = \begin{pmatrix} 6 & 3 & -7 \\ 5 & 12 & 8 \\ -22 & 7 & 53 \end{pmatrix}$ Thus, $A^2 + 7I = \begin{pmatrix} 13 & 3 & -7 \\ 5 & 19 & 8 \\ -22 & 7 & 60 \end{pmatrix}$ Alternative : If $A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{pmatrix}$, then $A = 8$ & $A^{-1} = \frac{1}{8} \begin{pmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{pmatrix}$ $A^2 = \begin{pmatrix} 0 & 5 & 7 \\ 11 & 10 & -6 \\ -20 & 7 & 45 \end{pmatrix}$, $A^2 + 7I = \begin{pmatrix} 7 & 5 & 7 \\ 11 & 17 & -6 \\ -20 & 7 & 52 \end{pmatrix}$ Note: In this question, students may take different A by rearranging of terms so check the correctness of the solution accordingly	I I I I I II I II	1 1 2 (1 Mark for any 4 correct entries) 1½ ½ 1+2 1 1

37.	An NGO organises a charity event in which they decide to distribute woollen caps to protect children from winter. The caps to be distributed are in three separate boxes, Box I has 30 red caps, Box II has 20 red and 10 green caps, & Box III has 30 green caps. The probability that a Box i is selected and a cap picked out is $\frac{i}{6}$, where $i = 1, 2, 3$. Based on the above information, answer the following questions : A person selects a cap. (i) What is the probability that he selects a red cap ? (ii) If he selects a green cap, what is the probability that the cap has come from Box II ?		
Sol.	(i) Let B_1, B_2 & B_3 be the event that box I, box II and box III selected respectively. $P(B_1) = \frac{1}{6}, P(B_2) = \frac{2}{6}, P(B_3) = \frac{3}{6}$ Now, $P(\text{selecting a red cap}) = \frac{1}{6} \times 1 + \frac{2}{6} \times \frac{2}{3} + \frac{3}{6} \times 0 = \frac{7}{18}$ (ii) $P(\text{selecting a green cap}) = 1 - \frac{7}{18} = \frac{11}{18}$ Now, $P\left(\frac{B_2}{\text{Green cap}}\right) = \frac{\frac{2}{6} \times \frac{1}{3}}{\frac{11}{18}} = \frac{2}{11}$	I II I II	1 1 1 1
38.	At a birthday party, children are being served orange juice in conical cups, as shown in the figure. 		

	Each cup is 15 cm deep and has a radius 5 cm. The juice is being poured into this cup at a rate of $0.1 \text{ cm}^3/\text{s}$. On the basis of the above information, answer the following questions: (i) Establish a relation between the height h of the juice in the cup and radius r of the surface of the juice in the cup, if the semi-vertical angle of the cone is α. (ii) At what rate is the juice level in the cup rising when the juice is 6 cm deep ? (iii) (a) When the juice is 6 cm deep, then find at what rate is the upper surface area of juice increasing ? OR (iii) (b) When the juice is 6 cm deep, then find the rate at which the wetted surface area of the cup is increasing.		
Sol.	(i) $\tan \alpha = \frac{r}{h} = \frac{5}{15} \Rightarrow r = \frac{h}{3}$ (ii) Volume, $V = \frac{1}{27} \pi h^3 \Rightarrow \frac{dV}{dt} = \frac{1}{9} \pi h^2 \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{40\pi} \text{ cm/sec}$ (iii) (a) Upper surface area of juice, $A = \pi r^2 = \frac{\pi h^2}{9}$ $\frac{dA}{dt} = \frac{2\pi h}{9} \frac{dh}{dt} = \frac{1}{30} \text{ cm}^2/\text{sec}$ OR (iii) (b) Wetted surface area, $S = \pi r l = \frac{\sqrt{10} \pi h^2}{9}$ $\frac{dS}{dt} = \frac{2\sqrt{10} \pi h}{9} \frac{dh}{dt} = \frac{\sqrt{10}}{30} \text{ cm}^2/\text{sec}$	I I I II I II	1 1 1 1 1 1