

**SET-1****Series : PQSR5**प्रश्न-पत्र कोड
Q.P. Code**65/5/1**

रोल नं.

Roll No.

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परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।

Candidates must write the Q.P. Code on the title page of the answer-book.

**गणित****MATHEMATICS**

निर्धारित समय : 3 घण्टे

Time allowed : 3 hours

अधिकतम अंक : 80

Maximum Marks : 80

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं।
- कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में यथा स्थान पर प्रश्न का क्रमांक अवश्य लिखें।
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।
- Please check that this question paper contains 23 printed pages.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 38 questions.
- Please write down the Serial Number of the question in the answer-book at the given place before attempting it.
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.

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सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख्ती से पालन कीजिए :

- (i) इस प्रश्न-पत्र में 38 प्रश्न हैं। सभी प्रश्न अनिवार्य हैं।
- (ii) यह प्रश्न-पत्र पाँच खण्डों में विभाजित है – क, ख, ग, घ एवं ङ।
- (iii) खण्ड क में प्रश्न संख्या 1 से 18 तक बहुविकल्पीय (MCQ) तथा प्रश्न संख्या 19 एवं 20 अभिकथन एवं तर्क आधारित 1 अंक के प्रश्न हैं।
- (iv) खण्ड ख में प्रश्न संख्या 21 से 25 तक अति लघु-उत्तरीय (VSA) प्रकार के 2 अंकों के प्रश्न हैं।
- (v) खण्ड ग में प्रश्न संख्या 26 से 31 तक लघु-उत्तरीय (SA) प्रकार के 3 अंकों के प्रश्न हैं।
- (vi) खण्ड घ में प्रश्न संख्या 32 से 35 तक दीर्घ-उत्तरीय (LA) प्रकार के 5 अंकों के प्रश्न हैं।
- (vii) खण्ड ङ में प्रश्न संख्या 36 से 38 तक प्रकरण अध्ययन आधारित 4 अंकों के प्रश्न हैं।
- (viii) प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड ख के 2 प्रश्नों में, खण्ड ग के 3 प्रश्नों में, खण्ड घ के 2 प्रश्नों में तथा खण्ड ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है।
- (ix) कैल्कुलेटर का उपयोग वर्जित है।

खण्ड क

इस खण्ड में बहुविकल्पीय प्रश्न (MCQ) हैं, जिनमें प्रत्येक प्रश्न 1 अंक का है।

1. यदि $A = \begin{bmatrix} -p & q \\ r & p \end{bmatrix}$ ऐसा आव्यूह है कि $A^2 = I$ है, तो :

(A) $1 + p^2 + qr = 0$

(B) $1 - p^2 - qr = 0$

(C) $1 - p^2 + qr = 0$

(D) $1 + p^2 - qr = 0$

2. यदि A एक ऐसा वर्ग आव्यूह है कि $A^2 = A$ है, तो $(A - I)^3 - A$ बराबर है :

(A) I

(B) -I

(C) A

(D) A^2



General Instructions :

Read the following instructions very carefully and strictly follow them :

- (i) This question paper contains **38** questions. **All** questions are **compulsory**.
- (ii) This question paper is divided into **five** Sections – **A, B, C, D** and **E**.
- (iii) In **Section A**, Questions no. **1** to **18** are Multiple Choice Questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.
- (iv) In **Section B**, Questions no. **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.
- (v) In **Section C**, Questions no. **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.
- (vi) In **Section D**, Questions no. **32** to **35** are Long Answer (LA) type questions carrying **5** marks each.
- (vii) In **Section E**, Questions no. **36** to **38** are case study based questions carrying **4** marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.
- (ix) Use of calculator is **not** allowed.

SECTION A

This section comprises Multiple Choice Questions (MCQs) of 1 mark each.

1. If matrix $A = \begin{bmatrix} -p & q \\ r & p \end{bmatrix}$ is such that $A^2 = I$, then :
- (A) $1 + p^2 + qr = 0$ (B) $1 - p^2 - qr = 0$
(C) $1 - p^2 + qr = 0$ (D) $1 + p^2 - qr = 0$
2. If A is a square matrix such that $A^2 = A$, then $(A - I)^3 - A$ is equal to :
- (A) I (B) -I
(C) A (D) A^2



3. प्रतिलोम त्रिकोणमितीय फलनों के लिए, निम्नलिखित में से कौन-सी मुख्य मानीय शाखा सही परिभाषित **नहीं** है ?

- (A) $\tan^{-1} : \mathbb{R} \rightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
 (B) $\sec^{-1} : \mathbb{R} - (-1, 1) \rightarrow [0, \pi] - \left\{\frac{\pi}{2}\right\}$
 (C) $\cot^{-1} : \mathbb{R} \rightarrow (0, \pi)$
 (D) $\operatorname{cosec}^{-1} : \mathbb{R} - (-1, 1) \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

4. माना $A = \begin{bmatrix} 0 & -3 & 4 \\ 1 & 0 & 2 \end{bmatrix}$ तथा $B = \begin{bmatrix} -3 & 0 & 1 \\ 2 & 4 & 0 \end{bmatrix}$ है। यदि $A + B + C = O$ है, तो आव्यूह C है :

- (A) $\begin{bmatrix} -3 & -3 & 5 \\ 3 & 4 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 3 & 3 & 5 \\ -3 & -4 & -2 \end{bmatrix}$
 (C) $\begin{bmatrix} 3 & 3 & -5 \\ -3 & -4 & -2 \end{bmatrix}$ (D) $\begin{bmatrix} -3 & -3 & -5 \\ 3 & 4 & 2 \end{bmatrix}$

5. यदि A एक व्युत्क्रमणीय आव्यूह है, तो निम्नलिखित में से कौन-सा सत्य **नहीं** है ?

- (A) $\operatorname{adj} A$ अव्युत्क्रमणीय है (B) $(\operatorname{adj} A)^{-1} = (\operatorname{adj} A^{-1})$
 (C) $|A| \neq 0$ (D) A^{-1} का अस्तित्व है

6. यदि $f(x) = \begin{cases} \frac{x^2 - 4x - 5}{x + 1}, & x \neq -1 \\ k, & x = -1 \end{cases}$

$x = -1$ पर एक संतत फलन है, तो k का मान है :

- (A) कोई वास्तविक मान (B) 6
 (C) -1 (D) -6



3. For the inverse trigonometric functions, which of the following Principal Value Branch is **not** correctly defined ?

- (A) $\tan^{-1} : \mathbb{R} \rightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
- (B) $\sec^{-1} : \mathbb{R} - (-1, 1) \rightarrow [0, \pi] - \left\{\frac{\pi}{2}\right\}$
- (C) $\cot^{-1} : \mathbb{R} \rightarrow (0, \pi)$
- (D) $\operatorname{cosec}^{-1} : \mathbb{R} - (-1, 1) \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

4. Let $A = \begin{bmatrix} 0 & -3 & 4 \\ 1 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 0 & 1 \\ 2 & 4 & 0 \end{bmatrix}$. If $A + B + C = O$, then matrix C is :

- (A) $\begin{bmatrix} -3 & -3 & 5 \\ 3 & 4 & 2 \end{bmatrix}$
- (B) $\begin{bmatrix} 3 & 3 & 5 \\ -3 & -4 & -2 \end{bmatrix}$
- (C) $\begin{bmatrix} 3 & 3 & -5 \\ -3 & -4 & -2 \end{bmatrix}$
- (D) $\begin{bmatrix} -3 & -3 & -5 \\ 3 & 4 & 2 \end{bmatrix}$

5. If A is a non-singular matrix, then which of the following is **not** true ?

- (A) $\operatorname{adj} A$ is singular
- (B) $(\operatorname{adj} A)^{-1} = (\operatorname{adj} A^{-1})$
- (C) $|A| \neq 0$
- (D) A^{-1} exists

6. If $f(x) = \begin{cases} \frac{x^2 - 4x - 5}{x + 1}, & x \neq -1 \\ k, & x = -1 \end{cases}$

is continuous at $x = -1$, then the value of k is :

- (A) Any real value
- (B) 6
- (C) -1
- (D) -6



7. यदि ΔABC जिसके शीर्ष $A(3, 1)$, $B(-2, 1)$ तथा $C(0, k)$ हैं, का क्षेत्रफल 5 वर्ग इकाई है, तो k के मान हैं :
- (A) 3, 1 (B) -1, 3
(C) -1, 2 (D) 0, 2
8. $\cos^{-1} \left(\frac{\sin x + \cos x}{\sqrt{2}} \right)$, $-\frac{\pi}{4} < x < \frac{\pi}{4}$ का x के सापेक्ष अवकलज है :
- (A) -1 (B) 1
(C) $\frac{\pi}{4}$ (D) $-\frac{\pi}{4}$
9. फलन $f(x) = (x - 2)^2 + 5$ का अंतराल $[-3, 2]$ में निरपेक्ष निम्नतम मान है :
- (A) -3 (B) 2
(C) 5 (D) 30
10. $\int \frac{1}{\sqrt{1 + \cos 2x}} dx$ बराबर है :
- (A) $\log \cos x + C$ (B) $\frac{1}{\sqrt{2}} \log |\sec x + \tan x| + C$
(C) $\frac{1}{\sqrt{2}} \log |\sec x - \tan x| + C$ (D) $\log \sin 2x + C$
11. $\int_{-5}^{-1} \frac{1}{x} dx$ का मान है :
- (A) $-\log 5$ (B) x^6
(C) $\log(-5)$ (D) x^{-6}
12. एक चींटी को कागज़ की एक शीट पर एक सीधी रेखा में रेंगते हुए देखा गया है जिसका समीकरण $y = 2x - 4$ द्वारा प्रदत्त है। चींटी द्वारा तय किए गए पृष्ठ का क्षेत्रफल जो y -अक्ष, x -अक्ष तथा $x = 1$ द्वारा परिबद्ध है, है :
- (A) 1 वर्ग इकाई (B) 3 वर्ग इकाई
(C) 2 वर्ग इकाई (D) 4 वर्ग इकाई



7. If the area of ΔABC with vertices $A(3, 1)$, $B(-2, 1)$ and $C(0, k)$ is 5 sq. units, then values of k are :
- (A) 3, 1 (B) -1, 3
(C) -1, 2 (D) 0, 2
8. Derivative of $\cos^{-1} \left(\frac{\sin x + \cos x}{\sqrt{2}} \right)$, $-\frac{\pi}{4} < x < \frac{\pi}{4}$ with respect to x is :
- (A) -1 (B) 1
(C) $\frac{\pi}{4}$ (D) $-\frac{\pi}{4}$
9. Absolute minimum value of $f(x) = (x - 2)^2 + 5$ in the interval $[-3, 2]$ is :
- (A) -3 (B) 2
(C) 5 (D) 30
10. $\int \frac{1}{\sqrt{1 + \cos 2x}} dx$ is equal to :
- (A) $\log \cos x + C$ (B) $\frac{1}{\sqrt{2}} \log |\sec x + \tan x| + C$
(C) $\frac{1}{\sqrt{2}} \log |\sec x - \tan x| + C$ (D) $\log \sin 2x + C$
11. The value of $\int_{-5}^{-1} \frac{1}{x} dx$ is equal to :
- (A) $-\log 5$ (B) x^6
(C) $\log(-5)$ (D) x^{-6}
12. An ant is observed crawling on a sheet of paper along a straight line given by equation $y = 2x - 4$. Area of the surface covered by the ant bounded by y -axis, x -axis and $x = 1$ is :
- (A) 1 sq. unit (B) 3 sq. units
(C) 2 sq. units (D) 4 sq. units



13. अवकल समीकरण $1 + \left(\frac{d^3y}{dx^3}\right)^3 = \lambda \frac{d^2y}{dx^2}$ की कोटि तथा घात है :
- (A) कोटि = 3, घात = 3 (B) कोटि = 2, घात = 2
(C) कोटि = 3, घात = 1 (D) कोटि = 2, घात = 1
14. अवकल समीकरण $\frac{dy}{dx} = e^{3x-y}$ का व्यापक हल है :
- (A) $3e^y = e^{3x} + C$ (B) $\log(3x - y) = C$
(C) $e^{3x-y} = C$ (D) $-e^y + 3e^{3x} = C$
15. रैखिक व्यवरोधों के एक निकाय द्वारा बने सुसंगत क्षेत्र के कोनीय बिंदु (0, 0), (0, 40), (20, 40), (60, 20) तथा (60, 0) हैं। यदि LPP का उद्देश्य फलन $Z = 4x + 3y$ है, तो इसका अधिकतम मान है :
- (A) 200 (B) 300
(C) 240 (D) 120
16. यदि बिंदु (24, n) का स्थिति सदिश $\vec{p}$ इस प्रकार है कि $|\vec{p}| = 25$ है, तो n के मान हैं :
- (A) ± 49 (B) ± 5
(C) ± 1 (D) ± 7
17. यदि सदिश $\vec{a} = 3\hat{i} + 2\hat{j} + \lambda\hat{k}$ तथा $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$, एक डॉक्टर के क्लिनिक के बाहर लगे हुए रेड क्रॉस साइन की दो स्ट्रिप्स का निरूपण करते हैं, तो λ का मान है :
- (A) 1 (B) $\frac{5}{2}$
(C) $\frac{2}{5}$ (D) 0



13. The order and degree of the differential equation

$$1 + \left(\frac{d^3 y}{dx^3} \right)^3 = \lambda \frac{d^2 y}{dx^2} \text{ is :}$$

- (A) Order = 3, Degree = 3 (B) Order = 2, Degree = 2
(C) Order = 3, Degree = 1 (D) Order = 2, Degree = 1

14. The general solution for the differential equation $\frac{dy}{dx} = e^{3x-y}$ is :

- (A) $3e^y = e^{3x} + C$ (B) $\log(3x - y) = C$
(C) $e^{3x-y} = C$ (D) $-e^y + 3e^{3x} = C$

15. The corner points of the feasible region determined by the system of linear constraints are (0, 0), (0, 40), (20, 40) (60, 20) and (60, 0). If the objective function of an LPP is $Z = 4x + 3y$, then the maximum value is :

- (A) 200 (B) 300
(C) 240 (D) 120

16. If position vector $\vec{p}$ of a point (24, n) is such that $|\vec{p}| = 25$, then the value of n is :

- (A) ± 49 (B) ± 5
(C) ± 1 (D) ± 7

17. If vectors $\vec{a} = 3\hat{i} + 2\hat{j} + \lambda\hat{k}$ and $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$, represent the two strips of the Red Cross sign placed outside a doctor's clinic, then the value of λ is :

- (A) 1 (B) $\frac{5}{2}$
(C) $\frac{2}{5}$ (D) 0



18. यदि $3P(A) = P(B) = \frac{3}{5}$ तथा $P(A|B) = \frac{2}{3}$ है, तो $P(A \cup B)$ है :

- (A) $\frac{3}{5}$ (B) $\frac{1}{5}$
(C) $\frac{2}{15}$ (D) $\frac{2}{5}$

प्रश्न संख्या 19 और 20 अभिकथन एवं तर्क आधारित प्रश्न हैं। दो कथन दिए गए हैं जिनमें एक को अभिकथन (A) तथा दूसरे को तर्क (R) द्वारा अंकित किया गया है। इन प्रश्नों के सही उत्तर नीचे दिए गए कोडों (A), (B), (C) और (D) में से चुनकर दीजिए।

- (A) अभिकथन (A) और तर्क (R) दोनों सही हैं और तर्क (R), अभिकथन (A) की सही व्याख्या करता है।
(B) अभिकथन (A) और तर्क (R) दोनों सही हैं, परन्तु तर्क (R), अभिकथन (A) की सही व्याख्या नहीं करता है।
(C) अभिकथन (A) सही है, परन्तु तर्क (R) गलत है।
(D) अभिकथन (A) गलत है, परन्तु तर्क (R) सही है।

19. अभिकथन (A) : समुच्चय $\{1, 2, 3\}$ पर परिभाषित संबंध

$R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$ एक तुल्यता संबंध है।

तर्क (R) : एक संबंध जो स्वतुल्य, सममित तथा संक्रामक है, वह एक तुल्यता संबंध होता है।

20. अभिकथन (A) : LPP : $Z = x + 2y$ का न्यूनतमीकरण करना, व्यवरोधों $2x + y \geq 3$, $x + 2y \geq 6$, $x, y \geq 0$ के अंतर्गत, पर विचार कीजिए। न्यूनतम Z अपरिमित रूप से अनेक बिंदुओं पर आता है। सुसंगत क्षेत्र के कोनीय बिंदु (0, 3) तथा (6, 0) हैं।

तर्क (R) : यदि दो कोनीय बिंदुओं पर उद्देश्य फलन का समान न्यूनतम मान आता है, तो इन बिंदुओं को मिलाने वाले रेखा-खंड के प्रत्येक बिंदु पर समान न्यूनतम मान आता है।



18. If $3P(A) = P(B) = \frac{3}{5}$ and $P(A|B) = \frac{2}{3}$, then $P(A \cup B)$ is :

(A) $\frac{3}{5}$

(B) $\frac{1}{5}$

(C) $\frac{2}{15}$

(D) $\frac{2}{5}$

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Assertion (A) is false, but Reason (R) is true.

19. Assertion (A) : A relation R on the set $\{1, 2, 3\}$ defined as $R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$ is an equivalence relation.

Reason (R) : A relation that is reflexive, symmetric and transitive is an equivalence relation.

20. Assertion (A) : Consider a Linear Programming Problem with minimise $Z = x + 2y$ subject to constraints $2x + y \geq 3$, $x + 2y \geq 6$, $x, y \geq 0$ which gives minimum Z at infinitely many points. The corner points of feasible region are (0, 3) and (6, 0).

Reason (R) : If two corner points produce the same minimum value of the objective function, then every point on the line segment joining the points will give the same minimum value.



खण्ड ख

इस खण्ड में अति लघु-उत्तरीय (VSA) प्रकार के 5 प्रश्न हैं, जिनमें प्रत्येक के 2 अंक हैं।

21. $\sin \left[\tan^{-1} \tan \left(\frac{3\pi}{4} \right) \right]$ का मान ज्ञात कीजिए।

22. (क) $x \log x$ के सापेक्ष x^x का अवकलन कीजिए।

अथवा

(ख) यदि $y = P \cos ux + Q \sin ux$ है, तो दर्शाइए कि $\frac{d^2y}{dx^2} + u^2y = 0$.

23. x के वह मान ज्ञात कीजिए जिनके लिए $f(x) = \frac{x-3}{x+1}$, $x \neq -1$, एक वर्धमान फलन है।

24. (क) तीन मधुमक्खियाँ क्रमशः सदिशों $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = 4\hat{j} - 2\hat{k}$ तथा $\vec{c} = 3\hat{i} + 2\hat{k}$ की दिशा में उड़ती पाई गईं। λ का वह मान ज्ञात कीजिए ताकि $\vec{a} + \lambda \vec{b}$ का पथ $\vec{c}$ के लंबवत हो।

अथवा

(ख) यदि A, B तथा C तीन असरेखीय बिंदु इस प्रकार हैं कि $\vec{AB} = \hat{i} + 2\hat{j} - \hat{k}$ तथा $\vec{AC} = 2\hat{i} - 3\hat{j}$ है, तो त्रिभुज ABC का क्षेत्रफल ज्ञात कीजिए।

25. निम्नलिखित रेखाओं के युग्म के बीच का कोण ज्ञात कीजिए :

$$\frac{x-2}{3} = \frac{y+5}{2} = \frac{1-z}{-6} \text{ तथा}$$

$$\frac{x-7}{1} = \frac{y}{2} = \frac{6-z}{-2}$$



SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. Evaluate $\sin \left[\tan^{-1} \tan \left(\frac{3\pi}{4} \right) \right]$.

22. (a) Differentiate x^x with respect to $x \log x$.

OR

(b) If $y = P \cos ux + Q \sin ux$, show that $\frac{d^2y}{dx^2} + u^2y = 0$.

23. Determine the values of x for which $f(x) = \frac{x-3}{x+1}$, $x \neq -1$ is an increasing function.

24. (a) Three honey bees were found flying along the vectors $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = 4\hat{j} - 2\hat{k}$ and $\vec{c} = 3\hat{i} + 2\hat{k}$ respectively. Find the value of λ such that the path for $\vec{a} + \lambda \vec{b}$ is perpendicular to $\vec{c}$.

OR

(b) If A , B and C be three non-collinear points such that $\vec{AB} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{AC} = 2\hat{i} - 3\hat{j}$, then find the area of ΔABC .

25. Find the angle between the following pair of lines :

$$\frac{x-2}{3} = \frac{y+5}{2} = \frac{1-z}{-6} \text{ and}$$

$$\frac{x-7}{1} = \frac{y}{2} = \frac{6-z}{-2}$$



खण्ड ग

इस खण्ड में लघु-उत्तरीय (SA) प्रकार के 6 प्रश्न हैं, जिनमें प्रत्येक के 3 अंक हैं।

26. हवा निकलने के कारण एक गोलाकार गुब्बारे का आयतन इस प्रकार कम हो रहा है कि आयतन में कमी, किसी समय पर उसके पृष्ठीय क्षेत्रफल के समानुपाती है। दर्शाइए कि इसकी त्रिज्या एक निश्चित दर से कम हो रही है।

27. (क) ज्ञात कीजिए :

$$\int \frac{x - \sin x}{1 - \cos x} dx$$

अथवा

(ख) मान ज्ञात कीजिए :

$$\int_0^2 \frac{1}{\sqrt{x^2 + 2x + 3}} dx$$

28. अवकल समीकरण $(x + 2y^3) dy = y dx$ का हल ज्ञात कीजिए।

29. ग्राफ द्वारा निम्नलिखित रेखिक प्रोग्रामन समस्या का हल ज्ञात कीजिए :
व्यवरोधों

$$2x + y \leq 1000$$

$$x + y \leq 800$$

$$x, y \geq 0$$

के अंतर्गत $Z = \frac{2x}{5} + \frac{3y}{10}$ का अधिकतमीकरण कीजिए।

30. (क) माना तीन खिलौने A, B तथा C एक सीधी रेखा में रखे गए हैं। यदि A, B तथा C के स्थिति सदिश क्रमशः $55\hat{i} - 2\hat{j}$, $5\hat{i} + 8\hat{j}$ तथा $a\hat{i} - 52\hat{j}$ हैं, तो 'a' का मान ज्ञात कीजिए।

अथवा

(ख) यदि $\vec{a}$, $\vec{b}$ तथा $\vec{c}$ मात्रक सदिश हैं, तो सिद्ध कीजिए कि

$$|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 \leq 9.$$



SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26. A spherical balloon loses its volume due to escape of air from it in such a way that decrease of volume at any instant is proportional to its surface area. Show that the radius is decreasing at a constant rate.

27. (a) Find :

$$\int \frac{x - \sin x}{1 - \cos x} dx$$

OR

- (b) Evaluate :

$$\int_0^2 \frac{1}{\sqrt{x^2 + 2x + 3}} dx$$

28. Solve the differential equation $(x + 2y^3) dy = y dx$.

29. Solve the following Linear Programming Problem graphically :

$$\text{Maximize } Z = \frac{2x}{5} + \frac{3y}{10}$$

subject to constraints

$$2x + y \leq 1000$$

$$x + y \leq 800$$

$$x, y \geq 0.$$

30. (a) Let three toys A, B and C be placed in the same straight line. If the position vectors of A, B and C are $55\hat{i} - 2\hat{j}$, $5\hat{i} + 8\hat{j}$ and $a\hat{i} - 52\hat{j}$ respectively, find the value of 'a'.

OR

- (b) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors, then prove that

$$|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 \leq 9.$$



31. (क) एक पासे को उछाला गया। निम्न घटनाओं पर विचार कीजिए :

$$A = \{1, 2, 5\}, B = \{3, 5\}, C = \{2, 3, 4, 5\}$$

अतः निम्न ज्ञात कीजिए :

(i) $P(A|C)$ तथा $P(C|A)$

(ii) $P(A \cap B|C)$ तथा $P(A \cup B|C)$

अथवा

(ख) एक डिब्बे में 1 से 6 तक संख्यांकित 6 कार्ड हैं। एक विद्यार्थी को डिब्बे में से एक-एक करके प्रतिस्थापन सहित दो कार्ड निकालने तथा उन पर अंकित संख्याओं को नोट करने के लिए कहा गया। माना दोनों कार्डों पर आई संख्याओं का योगफल 10 आना घटना A है तथा पहले कार्ड पर 4 को छोड़ कर अन्य संख्या आना घटना B है।

$P(A)$ और $P(B)$ ज्ञात कीजिए। अतः ज्ञात कीजिए कि क्या घटनाएँ A तथा B स्वतंत्र घटनाएँ हैं या नहीं।

खण्ड घ

इस खण्ड में 4 दीर्घ-उत्तरीय (LA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 5 अंक हैं।

32. एक व्यक्ति बाज़ार से फल खरीदने जाता है, तो दुकानदार उसे बताता है कि 4 सेब, 3 संतरे तथा 2 केलों का मूल्य ₹ 60 है; 2 सेब, 4 संतरे तथा 6 केलों का मूल्य ₹ 90 है; जबकि 6 सेब, 2 संतरे तथा 3 केलों का मूल्य ₹ 70 है। आव्यूह विधि के प्रयोग से प्रत्येक प्रकार के एक-एक फल का मूल्य ज्ञात कीजिए।

33. (क) यदि $y\sqrt{x^2+1} = \log \sqrt{x^2+1} - x$ है, तो दर्शाइए कि

$$(x^2 + 1) \frac{dy}{dx} + xy + 1 = 0.$$

अथवा

(ख) $x^{\cot x} + \frac{2x^2 - 3}{2x^2 - x + 2}$ का x के सापेक्ष अवकलज ज्ञात कीजिए।



31. (a) A die is rolled. Consider events :
- $A = \{1, 2, 5\}$, $B = \{3, 5\}$, $C = \{2, 3, 4, 5\}$
- and hence find :
- (i) $P(A | C)$ and $P(C | A)$
- (ii) $P(A \cap B | C)$ and $P(A \cup B | C)$

OR

- (b) A box contains 6 cards numbered 1 to 6. A student is asked to pick up two cards, one by one after replacement and note down the numbers on the cards. Let A be the event of getting sum of the numbers on two cards as 10, and B, the event of a number other than 4 on the first card selected.
- Find $P(A \text{ and } B)$ and find whether the events A and B are independent events or not.

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32. A man goes to buy fruits from the market. The shopkeeper informs him that 4 apples, 3 oranges and 2 bananas cost ₹ 60; 2 apples, 4 oranges and 6 bananas cost ₹ 90; whereas 6 apples, 2 oranges and 3 bananas cost ₹ 70. Using matrix method, find the cost of one fruit of each kind.

33. (a) If $y\sqrt{x^2+1} = \log \sqrt{x^2+1} - x$, show that

$$(x^2 + 1) \frac{dy}{dx} + xy + 1 = 0.$$

OR

- (b) Find the differential of $x^{\cot x} + \frac{2x^2 - 3}{2x^2 - x + 2}$ with respect to x.



34. वक्र $\{(x, y) : 100x^2 + 25y^2 = 2500\}$ का स्केच बनाइए तथा वक्र द्वारा परिबद्ध क्षेत्र का क्षेत्रफल, समाकलन विधि से ज्ञात कीजिए।
35. (क) बिंदु $(0, 2, 3)$ से रेखा $\frac{-x-3}{-5} = \frac{1-y}{-2} = \frac{3z+12}{9}$ पर खींचे गए लंब का पाद ज्ञात कीजिए। अतः लंब की लंबाई ज्ञात कीजिए।

अथवा

- (ख) यदि दो रेखाओं $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ तथा $\vec{r} = (p\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$ के बीच की न्यूनतम दूरी $\frac{3}{\sqrt{2}}$ इकाई है, तो p का मान ज्ञात कीजिए।

खण्ड ड

इस खण्ड में 3 प्रकरण-अध्ययन आधारित प्रश्न हैं, जिनमें प्रत्येक के 4 अंक हैं।

प्रकरण अध्ययन – 1

36. एक स्कूल चाहता है कि कक्षा XII के विद्यार्थी दुनिया के पर्यावरण को ध्यान में रखते हुए 'सस्टेनेबिलिटी' पर एक प्रोजेक्ट करें। उन्होंने इसमें भाग लेने वाले विद्यार्थियों को एक निबंध लेखन प्रतियोगिता के आधार पर चुना।

इस प्रोजेक्ट के लिए 80 में से 7 विद्यार्थियों को चुना गया तथा उन्हें दो समुच्चयों में बाँटा गया :

समुच्चय $A = \{G_1, G_2, G_3, G_4\}$ लड़कियों का समुच्चय है, तथा

समुच्चय $B = \{B_1, B_2, B_3\}$ लड़कों का समुच्चय है।

उपर्युक्त सूचना के आधार पर निम्नलिखित प्रश्नों के उत्तर दीजिए :

- (i) समुच्चय $A \rightarrow$ समुच्चय B तक कितने संबंध संभव हैं ?

1



34. Sketch the curve $\{(x, y) : 100x^2 + 25y^2 = 2500\}$ and find the area of the region enclosed by it, using integration.
35. (a) Find the foot of the perpendicular from the point $(0, 2, 3)$ on the line $\frac{-x-3}{-5} = \frac{1-y}{-2} = \frac{3z+12}{9}$ and hence find the length of the perpendicular.

OR

- (b) Find the value of p if the shortest distance between the lines

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda (\hat{i} - \hat{j} + \hat{k}) \text{ and}$$

$$\vec{r} = (p\hat{i} - \hat{j} - \hat{k}) + \mu (2\hat{i} + \hat{j} + 2\hat{k})$$

$$\text{is } \frac{3}{\sqrt{2}} \text{ units.}$$

SECTION E

This section comprises 3 Case Study based questions of 4 marks each.

Case Study – 1

36. A school wants the students of class XII to do a project on ‘Sustainability’ keeping the world environment in mind. They select the student participants on the basis of an essay writing competition.

7 students out of 80 are selected for the project and are categorized into two sets such that :

Girl students belong to Set A = $\{G_1, G_2, G_3, G_4\}$,

Boy students belong to Set B = $\{B_1, B_2, B_3\}$.

Based on the above information, answer the following questions :

- (i) How many relations are possible from Set A $\rightarrow$ Set B ?

1



(ii) माना संबंध $R : A \rightarrow B$ इस प्रकार है कि

$$R = \{(G_1, B_1), (G_2, B_2), (G_3, B_2), (G_4, B_3), (G_1, B_2)\} \text{ है।}$$

क्या R एक एकैकी फलन है ? अपने उत्तर की पुष्टि कीजिए।

1

(iii) (क) माना संबंध $R : A \rightarrow A$ इस प्रकार परिभाषित है कि

$$R = \{(x, y), x, y \in A, x \text{ तथा } y \text{ शहर की एक ही कॉलोनी के विद्यार्थी हैं}\}$$

जाँच कीजिए कि क्या R एक तुल्यता संबंध है।

2

अथवा

(iii) (ख) जाँच कीजिए कि क्या कोई फलन $f : B \rightarrow A$ एकैकी आच्छादी है। अपने उत्तर के समर्थन में कारण दीजिए।

2

प्रकरण अध्ययन – 2

37. देश के लोगों को एक विशेष प्रकार के इंफेक्शन से बचाने के लिए मार्केट में तीन तरह की वैक्सीन A_1 , A_2 तथा A_3 उपलब्ध हैं। एक सर्वेक्षण के अनुसार यह पाया गया कि 25% आबादी को वैक्सीन A_1 , 35% आबादी को वैक्सीन A_2 तथा 40% आबादी को वैक्सीन A_3 दी गई तथा यह भी पाया गया कि वैक्सीन A_1 , A_2 तथा A_3 की इंफेक्शन से बचाव करने की प्रायिकताएँ क्रमशः 60%, 55% तथा 50% थीं।

उपर्युक्त सूचना के आधार पर निम्नलिखित प्रश्नों के उत्तर दीजिए :

प्रायिकता ज्ञात कीजिए कि

(i) एक A_2 वैक्सीन लेने वाले व्यक्ति को इंफेक्शन हो जाए।

1

(ii) एक यादृच्छया चुने गए व्यक्ति का इंफेक्शन से बचाव हो जाए।

1



(ii) Let R be a relation from $A \rightarrow B$ such that

$$R = \{(G_1, B_1), (G_2, B_2), (G_3, B_2), (G_4, B_3), (G_1, B_2)\}.$$

Is R an injective function ? Justify your answer.

1

(iii) (a) Let the relation R from $A \rightarrow A$ be such that

$$R = \{(x, y), x, y \in A, x \text{ and } y \text{ are students from the same colony in the city}\}$$

Verify if R is an equivalence relation.

2

OR

(iii) (b) Verify if any function $f : B \rightarrow A$ is bijective. Give reason to support your answer.

2

Case Study – 2

37. There are three types of vaccines A_1, A_2, A_3 , available in the market to protect the population of the country from spread of certain infection. According to a survey conducted, it was found that 25% of the population was given Vaccine A_1 , 35% of the population was given Vaccine A_2 and 40% of the population was given Vaccine A_3 . The survey also stated that the probabilities that Vaccines A_1, A_2 and A_3 would protect against the infection were 60%, 55% and 50% respectively.

Based on the above information, answer the following questions :

Find the probability that :

(i) The person taking vaccine A_2 will get infected.

1

(ii) If a person is chosen randomly, he/she will be protected from the infection.

1



- (iii) (क) एक यादृच्छया चुना गया व्यक्ति इंफेक्शन से ग्रसित पाया गया, तो उसने वैक्सीन A_1 ली थी।

2

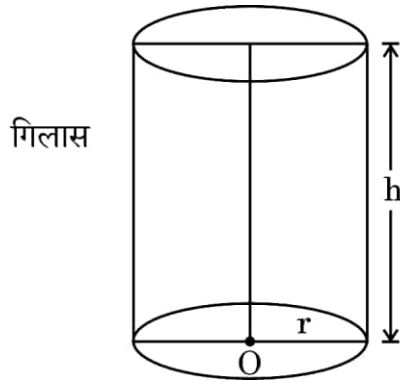
अथवा

- (iii) (ख) एक यादृच्छया चुना गया व्यक्ति इंफेक्शन से मुक्त पाया गया, तो उसे वैक्सीन A_3 दी गई थी।

2

प्रकरण अध्ययन – 3

38. एक कंपनी ऊपर से खुले बेलनाकार गिलास बनाती है। उत्पाद में एकरूपता बनाए रखने के लिए उन्होंने बनाए गए गिलासों का पृष्ठीय क्षेत्रफल निश्चित कर दिया।



उपर्युक्त सूचना के आधार पर निम्नलिखित प्रश्नों के उत्तर दीजिए :

यदि एक गिलास का आयतन V , ऊँचाई h तथा वृत्तीय आधार की त्रिज्या r है, तो :

- (i) इसके आयतन का आधार की त्रिज्या के सापेक्ष अवकलज ज्ञात कीजिए, जहाँ पृष्ठीय क्षेत्रफल नियत है।
- (ii) यदि यह कंपनी प्रत्येक गिलास के आयतन का अधिकतमीकरण करना चाहती है, तो ऊँचाई और आधार की त्रिज्या में संबंध स्थापित कीजिए।

2

2



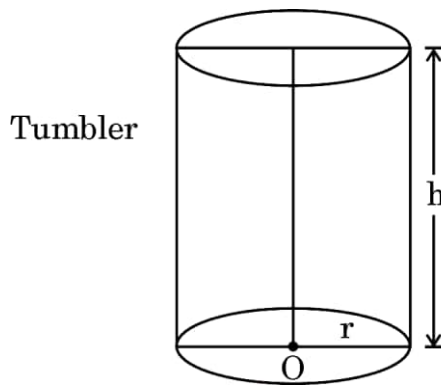
- (iii) (a) The person was given Vaccine A_1 , given that the randomly chosen person is infected. 2

OR

- (iii) (b) The person was given Vaccine A_3 , given that the randomly chosen person is not infected. 2

Case Study – 3

38. A company produces cylindrical tumblers, open from the top. Since they want uniformity in the product, they fix the surface area of the tumblers produced.



Based on the above information, answer the following questions :

If for a tumbler, V is its volume, h the height and r the radius of the circular base, then :

- (i) Differentiate its volume with respect to radius of the base, where the surface area is constant. 2
- (ii) If the company wants to maximize the volume of each tumbler, then establish a relation between its height and the radius of the base. 2

Marking Scheme Strictly Confidential (For Internal and Restricted use only) Senior Secondary Certificate Examination, 2026 MATHEMATICS (041) (PAPER CODE 65/5/1)	
General Instructions: -	
1.	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2.	“Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its leakage to the public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in Newspaper/Website, etc. may invite action under various rules of the Board and BNS.”
3.	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one’s own interpretation or any other consideration. The Marking Scheme should be strictly adhered to and religiously followed. However, while evaluating, Answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-XII, while evaluating the competency-based questions, please try to understand the given Answer and even if reply is not from a marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.
4.	The Marking Scheme carries only suggested value points for the Answers. These are Guidelines only and do not constitute the complete Answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5.	The Head-Examiner must go through the first five Answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining Answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6.	Evaluators will mark (✓) wherever Answer is correct. For wrong Answer CROSS ‘X’ be marked. Evaluators will not put right (✓) while evaluating which gives the impression that the Answer is correct, and no marks are awarded. This is the most common mistake which evaluators are committing.
7.	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left- hand margin and encircled. This may be followed strictly.
8.	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9.	If a student has attempted an extra question, Answer to the question deserving more marks should be retained and the other Answer scored out with a note “Extra Question” .
10.	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
11.	A full scale of marks__ (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the Answer deserves it.

12.	Every examiner must necessarily do evaluation work for full working hours, i.e., 8 hours every day and evaluate 20 Answer books per day in main subjects and 25 Answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
13.	Ensure that you do not make the following common types of errors committed by the Examiner in the past: - • Leaving Answer or part thereof unassessed in an Answer book. • Giving more marks for an Answer than assigned to it. • Wrong totaling of marks awarded on an Answer. • Wrong transfer of marks from the inside pages of the Answer book to the title page. • Wrong question wise totaling on the title page. • Wrong totaling of marks of the two columns on the title page. • Wrong grand total. • Marks in words and figures not tallying/not same. • Wrong transfer of marks from the Answer book to online award list. • Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect Answer.) • Half or a part of the Answer marked correct and the rest as wrong, but no marks awarded.
14.	While evaluating the Answer books if the Answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
15.	Any unassessed portion, non-carrying over marks to the title page, or total error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
16.	The Examiners should acquaint themselves with the guidelines given in the “ Guidelines for Spot Evaluation ” before starting the actual evaluation.
17.	Every Examiner shall also ensure that all the Answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
18.	The candidates are entitled to obtain a photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each Answer as given in the Marking Scheme.

MARKING SCHEME
MATHEMATICS (Subject Code-041)
(PAPER CODE: 65/5/1)

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Steps	Marks
	SECTION A Q. Number 1 to 20 are multiple choice questions of 1 mark each.		
1.	If matrix $A = \begin{bmatrix} -p & q \\ r & p \end{bmatrix}$ is such that $A^2 = I$, then : (A) $1 + p^2 + qr = 0$ (B) $1 - p^2 - qr = 0$ (C) $1 - p^2 + qr = 0$ (D) $1 + p^2 - qr = 0$		
Sol.	(B) $1 - p^2 - qr = 0$		1
2.	If A is a square matrix such that $A^2 = A$, then $(A - I)^3 - A$ is equal to : (A) I (B) -I (C) A (D) A^2		
Sol.	(B) -I		1
3.	For the inverse trigonometric functions, which of the following Principal Value Branch is not correctly defined ? (A) $\tan^{-1} : \mathbb{R} \rightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (B) $\sec^{-1} : \mathbb{R} - (-1, 1) \rightarrow [0, \pi] - \left\{\frac{\pi}{2}\right\}$ (C) $\cot^{-1} : \mathbb{R} \rightarrow (0, \pi)$ (D) $\operatorname{cosec}^{-1} : \mathbb{R} - (-1, 1) \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$		
Sol.	(D) $\operatorname{cosec}^{-1} : \mathbb{R} - (-1, 1) \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$		1
4.	Let $A = \begin{bmatrix} 0 & -3 & 4 \\ 1 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 0 & 1 \\ 2 & 4 & 0 \end{bmatrix}$. If $A + B + C = O$, then matrix C is : (A) $\begin{bmatrix} -3 & -3 & 5 \\ 3 & 4 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 3 & 3 & 5 \\ -3 & -4 & -2 \end{bmatrix}$ (C) $\begin{bmatrix} 3 & 3 & -5 \\ -3 & -4 & -2 \end{bmatrix}$ (D) $\begin{bmatrix} -3 & -3 & -5 \\ 3 & 4 & 2 \end{bmatrix}$		

Sol.	(C) $\begin{bmatrix} 3 & 3 & -5 \\ -3 & -4 & -2 \end{bmatrix}$		1
5.	If A is a non-singular matrix, then which of the following is not true ? (A) adj A is singular (B) $(\text{adj } A)^{-1} = (\text{adj } A^{-1})$ (C) $A \neq 0$ (D) A^{-1} exists		
Sol.	(A) adj A is singular		1
6.	If $f(x) = \begin{cases} \frac{x^2 - 4x - 5}{x + 1}, & x \neq -1 \\ k, & x = -1 \end{cases}$ is continuous at $x = -1$, then the value of k is : (A) Any real value (B) 6 (C) -1 (D) -6		
Sol.	(D) -6		1
7.	If the area of ΔABC with vertices A(3, 1), B(-2, 1) and C(0, k) is 5 sq. units, then values of k are : (A) 3, 1 (B) -1, 3 (C) -1, 2 (D) 0, 2		
Sol.	(B) -1, 3		1
8.	Derivative of $\cos^{-1} \left(\frac{\sin x + \cos x}{\sqrt{2}} \right)$, $-\frac{\pi}{4} < x < \frac{\pi}{4}$ with respect to x is : (A) -1 (B) 1 (C) $\frac{\pi}{4}$ (D) $-\frac{\pi}{4}$		
Sol.	(A) -1		1
9.	Absolute minimum value of $f(x) = (x - 2)^2 + 5$ in the interval $[-3, 2]$ is : (A) -3 (B) 2 (C) 5 (D) 30		
Sol.	(C) 5		1

10.	$\int \frac{1}{\sqrt{1+\cos 2x}} dx$ is equal to : (A) $\log \cos x + C$ (B) $\frac{1}{\sqrt{2}} \log \sec x + \tan x + C$ (C) $\frac{1}{\sqrt{2}} \log \sec x - \tan x + C$ (D) $\log \sin 2x + C$		
Sol.	(B) $\frac{1}{\sqrt{2}} \log \sec x + \tan x + C$		1
11.	The value of $\int_{-5}^{-1} \frac{1}{x} dx$ is equal to : (A) $-\log 5$ (B) x^6 (C) $\log (-5)$ (D) x^{-6}		
Sol.	(A) $-\log 5$		1
12.	An ant is observed crawling on a sheet of paper along a straight line given by equation $y = 2x - 4$. Area of the surface covered by the ant bounded by y-axis, x-axis and $x = 1$ is : (A) 1 sq. unit (B) 3 sq. units (C) 2 sq. units (D) 4 sq. units		
Sol.	(B) 3 sq. units		1
13.	The order and degree of the differential equation $1 + \left(\frac{d^3 y}{dx^3} \right)^3 = \lambda \frac{d^2 y}{dx^2}$ is : (A) Order $\equiv 3$, Degree = 3 (B) Order = 2, Degree = 2 (C) Order = 3, Degree = 1 (D) Order = 2, Degree = 1		
Sol.	(A) Order = 3, Degree = 3		1
14.	The general solution for the differential equation $\frac{dy}{dx} = e^{3x-y}$ is : (A) $3e^y = e^{3x} + C$ (B) $\log (3x - y) = C$ (C) $e^{3x-y} = C$ (D) $-e^y + 3e^{3x} = C$		
Sol.	(A) $3e^y = e^{3x} + C$		1

15.	The corner points of the feasible region determined by the system of linear constraints are (0, 0), (0, 40), (20, 40) (60, 20) and (60, 0). If the objective function of an LPP is $Z = 4x + 3y$, then the maximum value is : (A) 200 (B) 300 (C) 240 (D) 120		
Sol.	(B) 300		1
16.	If position vector $\vec{p}$ of a point (24, n) is such that $ \vec{p} = 25$, then the value of n is : (A) ± 49 (B) ± 5 (C) ± 1 (D) ± 7		
Sol.	(D) ± 7		1
17.	If vectors $\vec{a} = 3\hat{i} + 2\hat{j} + \lambda\hat{k}$ and $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$, represent the two strips of the Red Cross sign placed outside a doctor's clinic, then the value of λ is : (A) 1 (B) $\frac{5}{2}$ (C) $\frac{2}{5}$ (D) 0		
Sol.	(C) $\frac{2}{5}$		1
18.	If $3P(A) = P(B) = \frac{3}{5}$ and $P(A B) = \frac{2}{3}$, then $P(A \cup B)$ is : (A) $\frac{3}{5}$ (B) $\frac{1}{5}$ (C) $\frac{2}{15}$ (D) $\frac{2}{5}$		
Sol.	(D) $\frac{2}{5}$		1

	Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below. (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.		
19.	Assertion (A) : A relation R on the set {1, 2, 3} defined as $R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$ is an equivalence relation. Reason (R) : A relation that is reflexive, symmetric and transitive is an equivalence relation.		
Sol.	(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).		1
20.	Assertion (A) : Consider a Linear Programming Problem with minimise $Z = x + 2y$ subject to constraints $2x + y \geq 3$, $x + 2y \geq 6$, $x, y \geq 0$ which gives minimum Z at infinitely many points. The corner points of feasible region are (0, 3) and (6, 0). Reason (R) : If two corner points produce the same minimum value of the objective function, then every point on the line segment joining the points will give the same minimum value.		
Sol.	(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).		1
SECTION B			
Q. Numbers 21 to 25 are very short answer questions of 2 marks each.			
21.	Evaluate $\sin \left[\tan^{-1} \tan \left(\frac{3\pi}{4} \right) \right]$.		
Sol.	Given expression = $\sin[\tan^{-1}(-1)]$ or $\sin [\tan^{-1} \tan (\pi - \frac{\pi}{4})]$ $= \sin\left(-\frac{\pi}{4}\right)$ $= -\frac{1}{\sqrt{2}}$	I II III	1 $\frac{1}{2}$ $\frac{1}{2}$

22.	(a) Differentiate x^x with respect to $x \log x$. OR (b) If $y = P \cos ux + Q \sin ux$, show that $\frac{d^2y}{dx^2} + u^2y = 0$.		
Sol.	(a) Let $u = x^x$ and $v = x \log x$ $\frac{du}{dx} = x^x(1 + \log x)$ $\frac{dv}{dx} = 1 + \log x$ $\therefore \frac{du}{dv} = x^x$ OR (b) $\frac{dy}{dx} = -P u \sin ux + Q u \cos ux$ $\frac{d^2y}{dx^2} = -P u^2 \cos ux - Q u^2 \sin ux$ $\frac{d^2y}{dx^2} = -u^2(P \cos ux + Q \sin ux)$ $\therefore \frac{d^2y}{dx^2} + u^2y = 0$	I II III IV I II III IV	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
23.	Determine the values of x for which $f(x) = \frac{x-3}{x+1}$, $x \neq -1$ is an increasing function.		
Sol.	$f'(x) = \frac{(x+1).1 - (x-3).1}{(x+1)^2}$ $= \frac{4}{(x+1)^2} > 0$ $\therefore f(x)$ is an increasing function for all $x \in \mathbb{R} - \{-1\}$	I II III	1 $\frac{1}{2}$ $\frac{1}{2}$

24.	(a) Three honey bees were found flying along the vectors $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = 4\hat{j} - 2\hat{k}$ and $\vec{c} = 3\hat{i} + 2\hat{k}$ respectively. Find the value of λ such that the path for $\vec{a} + \lambda \vec{b}$ is perpendicular to $\vec{c}$. OR (b) If A, B and C be three non-collinear points such that $\vec{AB} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{AC} = 2\hat{i} - 3\hat{j}$, then find the area of ΔABC.		
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Sol.	As $\vec{a} + \lambda \vec{b}$ is perpendicular to $\vec{c}$ $\therefore (\vec{a} + \lambda \vec{b}) \cdot \vec{c} = 0$ i.e. $\vec{a} \cdot \vec{c} + \lambda \vec{b} \cdot \vec{c} = 0$ which gives $8 + \lambda(-4) = 0$ $\lambda = 2$ OR Area of $\Delta ABC = \frac{1}{2} \vec{AB} \times \vec{AC}$ $= \frac{1}{2} -3\hat{i} - 2\hat{j} - 7\hat{k}$ $= \frac{\sqrt{62}}{2}$	I II III I II	$\frac{1}{2}$ 1 $\frac{1}{2}$ 1 1
25.	Find the angle between the following pair of lines : $\frac{x-2}{3} = \frac{y+5}{2} = \frac{1-z}{-6} \text{ and }$ $\frac{x-7}{1} = \frac{y}{2} = \frac{6-z}{-2}$		
Sol.	Drs of given lines are $\langle 3, 2, 6 \rangle$ and $\langle 1, 2, 2 \rangle$ Let θ be angle between given lines $\therefore \cos \theta = \frac{3 \times 1 + 2 \times 2 + 6 \times 2}{7 \times 3}$ Thus, $\theta = \cos^{-1} \left(\frac{19}{21} \right)$	I II III	$\frac{1}{2}$ 1 $\frac{1}{2}$
SECTION C Q. Numbers 26 to 31 are short answer questions of 3 marks each.			

26.	A spherical balloon loses its volume due to escape of air from it in such a way that decrease of volume at any instant is proportional to its surface area. Show that the radius is decreasing at a constant rate.		
Sol.	Let r be the radius, $V = \frac{4}{3}\pi r^3$ be the volume $\frac{dV}{dt} = -k(4\pi r^2) \quad \text{where } k > 0$ $4\pi r^2 \frac{dr}{dt} = -4k\pi r^2$ $\therefore \frac{dr}{dt} = -k \text{ which is a constant}$ Thus, radius is decreasing at a constant rate	I II III	1 1 1
27.	(a) Find : $\int \frac{x - \sin x}{1 - \cos x} dx$ OR (b) Evaluate : $\int_0^2 \frac{1}{\sqrt{x^2 + 2x + 3}} dx$		
Sol.	(a) Let $I = \int \frac{x - \sin x}{1 - \cos x} dx$ $= \int \frac{x}{1 - \cos x} dx - \int \frac{\sin x}{1 - \cos x} dx$ $= \int \frac{x}{2 \sin^2 \frac{x}{2}} dx - \int \frac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \sin^2 \frac{x}{2}} dx$ $= -\frac{x}{2} \cdot 2 \cot \frac{x}{2} - \frac{1}{2} \int -2 \cot \frac{x}{2} dx - \int \cot \frac{x}{2} dx$ $= -x \cot \frac{x}{2} + C$ OR (b) $I = \int_0^2 \frac{1}{\sqrt{x^2 + 2x + 3}} dx$ $= \int_0^2 \frac{1}{\sqrt{(x+1)^2 + 2}} dx$ $= \left[\log (x + 1 + \sqrt{x^2 + 2x + 3}) \right]_0^2$ $= \log (3 + \sqrt{11}) - \log (1 + \sqrt{3}) \text{ or } \log \left(\frac{3 + \sqrt{11}}{1 + \sqrt{3}} \right)$	I II III IV I II III	$\frac{1}{2}$ 1 1 $\frac{1}{2}$ 1 1 1
28.	Solve the differential equation $(x + 2y^3) dy = y dx$.		
Sol.	Given differential equation can be written as		

	$\frac{dx}{dy} - \frac{x}{y} = 2y^2$ Integrating factor = $e^{\int -\frac{1}{y} dy} = e^{-\log y} = \frac{1}{y}$ Solution is $x \cdot \frac{1}{y} = \int 2y dy + C$ i.e., $x \cdot \frac{1}{y} = y^2 + C$ or $x = y^3 + Cy$	I	$\frac{1}{2}$								
		II	1								
		III	1								
		IV	$\frac{1}{2}$								
29.	Solve the following Linear Programming Problem graphically : Maximize $Z = \frac{2x}{5} + \frac{3y}{10}$ subject to constraints $2x + y \leq 1000$ $x + y \leq 800$ $x, y \geq 0.$										
Sol	Corner Points Value of Z <table><tr><td>(0, 0)</td><td>0</td></tr><tr><td>(500, 0)</td><td>200</td></tr><tr><td>(200, 600)</td><td>260</td></tr><tr><td>(0, 800)</td><td>240</td></tr></table> $Z_{\max} = 260$ at $x = 200, y = 600$	(0, 0)	0	(500, 0)	200	(200, 600)	260	(0, 800)	240	I	Correct Lines and Shading $1\frac{1}{2}$
(0, 0)	0										
(500, 0)	200										
(200, 600)	260										
(0, 800)	240										
		II	1								
		III	$\frac{1}{2}$								

30.	(a) Let three toys A, B and C be placed in the same straight line. If the position vectors of A, B and C are $55\hat{i} - 2\hat{j}$, $5\hat{i} + 8\hat{j}$ and $a\hat{i} - 52\hat{j}$ respectively, find the value of 'a'. OR (b) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors, then prove that $\vec{a} - \vec{b} ^2 + \vec{b} - \vec{c} ^2 + \vec{c} - \vec{a} ^2 \leq 9$.		
Sol.	(a) $\overrightarrow{AB} = -50\hat{i} + 10\hat{j}$ and $\overrightarrow{BC} = (a - 5)\hat{i} - 60\hat{j}$ As $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are collinear vectors $\therefore \frac{-50}{a-5} = \frac{10}{-60}$ $a = 305$ OR (b) $\vec{a} = \vec{b} = \vec{c} = 1$ Now, consider $\vec{a} - \vec{b} ^2 + \vec{b} - \vec{c} ^2 + \vec{c} - \vec{a} ^2$ $= 2 \vec{a} ^2 + 2 \vec{b} ^2 + 2 \vec{c} ^2 - 2\vec{a} \cdot \vec{b} - 2\vec{b} \cdot \vec{c} - 2\vec{c} \cdot \vec{a}$ $= 3 \vec{a} ^2 + 3 \vec{b} ^2 + 3 \vec{c} ^2 - (\vec{a} ^2 + \vec{b} ^2 + \vec{c} ^2 + 2\vec{a} \cdot \vec{b} + 2\vec{b} \cdot \vec{c} + 2\vec{c} \cdot \vec{a})$ $= 9 - \vec{a} + \vec{b} + \vec{c} ^2 \leq 9$ $\vec{a} - \vec{b} ^2 + \vec{b} - \vec{c} ^2 + \vec{c} - \vec{a} ^2 \leq 9$	I II III I II III IV	1 1 1 $\frac{1}{2}$ $1\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
31.	(a) A die is rolled. Consider events : $A = \{1, 2, 5\}$, $B = \{3, 5\}$, $C = \{2, 3, 4, 5\}$ and hence find : (i) $P(A C)$ and $P(C A)$ (ii) $P(A \cap B C)$ and $P(A \cup B C)$ OR (b) A box contains 6 cards numbered 1 to 6. A student is asked to pick up two cards, one by one after replacement and note down the numbers on the cards. Let A be the event of getting sum of the numbers on two cards as 10, and B, the event of a number other than 4 on the first card selected. Find $P(A \text{ and } B)$ and find whether the events A and B are independent events or not.		
Sol.	(a) (i) $P(A C) = \frac{n(A \cap C)}{n(C)} = \frac{2}{4}$ or $\frac{1}{2}$	I	1

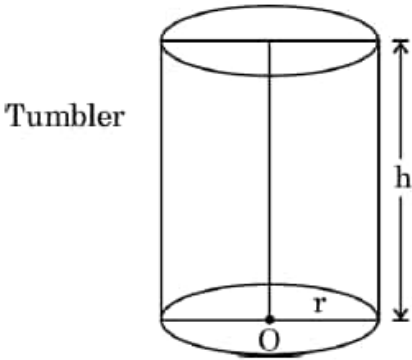
	$\text{and } P(C A) = \frac{n(C \cap A)}{n(A)} = \frac{2}{3}$ $(ii) \quad P(A \cap B C) = \frac{n(A \cap B \cap C)}{n(C)} = \frac{1}{4}$ $\text{and } P(A \cup B C) = \frac{n[(A \cup B) \cap C]}{n(C)} = \frac{3}{4}$ OR (b) $A = \{(4, 6), (6, 4), (5, 5)\}$ or $n(A) = 3$ $n(B) = 30$ $n(A \cap B) = 2$ $P(A \cap B) = \frac{2}{36}$ $P(A) = \frac{3}{36} = \frac{1}{12}$ $P(B) = \frac{30}{36} = \frac{5}{6}$ $P(A) \cdot P(B) = \frac{5}{72} \neq P(A \cap B)$ Hence A and B are not independent	II	$\frac{1}{2}$
		III	1
		IV	$\frac{1}{2}$
		I	$\frac{1}{2}$
		II	$\frac{1}{2}$
		III	$\frac{1}{2}$
		IV	$\frac{1}{2}$
		V	$\frac{1}{2}$
		VI	$\frac{1}{2}$
	SECTION D		
	Q. Numbers 32 to 35 are long answer questions of 5 marks each.		
32.	A man goes to buy fruits from the market. The shopkeeper informs him that 4 apples, 3 oranges and 2 bananas cost ₹ 60; 2 apples, 4 oranges and 6 bananas cost ₹ 90; whereas 6 apples, 2 oranges and 3 bananas cost ₹ 70. Using matrix method, find the cost of one fruit of each kind.		
Sol.	Let cost of one apple, one orange, one banana be ₹ x, ₹ y and ₹ z respectively According to the question, $\left. \begin{array}{l} 4x + 3y + 2z = 60 \\ 2x + 4y + 6z = 90 \\ 6x + 2y + 3z = 70 \end{array} \right\}$ Let $A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$ $A = 50 \neq 0 \Rightarrow A^{-1}$ exists System becomes $AX = B$. So, $X = A^{-1}B$ $adjA = \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$ $A^{-1} = \frac{1}{50} \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$ $X = \frac{1}{50} \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix} \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$ $X = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$	I	1
		II	1
		III	$1\frac{1}{2}$
		IV	$\frac{1}{2}$
		V	1

	$\therefore$ cost of one apple, one orange, one banana is ₹ 5, ₹ 8 and ₹ 8 respectively		
33.	(a) If $y\sqrt{x^2+1} = \log \sqrt{x^2+1} - x$, show that $(x^2 + 1) \frac{dy}{dx} + xy + 1 = 0.$ OR (b) Find the differential of $x^{\cot x} + \frac{2x^2-3}{2x^2-x+2}$ with respect to x.		
Sol.	(a) Differentiating both sides with respect to x, we get $\sqrt{x^2+1} \frac{dy}{dx} + y \cdot \frac{x}{\sqrt{x^2+1}} = \frac{1}{\sqrt{x^2+1}} \cdot \frac{x}{\sqrt{x^2+1}} - 1$ $(x^2 + 1) \frac{dy}{dx} + y \cdot x = \frac{x}{\sqrt{x^2+1}} - \sqrt{x^2+1}$ Note: In case student, considers $\sqrt{x^2+1} - x$ as part of log and writes following solution Differentiating both sides with respect to x, we get $\sqrt{x^2+1} \frac{dy}{dx} + y \cdot \frac{x}{\sqrt{x^2+1}} = \frac{1}{\sqrt{x^2+1}-x} \cdot \left(\frac{x}{\sqrt{x^2+1}} - 1 \right)$ $\sqrt{x^2+1} \frac{dy}{dx} + y \cdot \frac{x}{\sqrt{x^2+1}} = \frac{1}{\sqrt{x^2+1}-x} \cdot \left(\frac{x-\sqrt{x^2+1}}{\sqrt{x^2+1}} \right)$ $\sqrt{x^2+1} \frac{dy}{dx} + y \cdot \frac{x}{\sqrt{x^2+1}} = \frac{-1}{\sqrt{x^2+1}}$ gives $(x^2 + 1) \frac{dy}{dx} + y \cdot x = -1 \text{ or } (x^2 + 1) \frac{dy}{dx} + x \cdot y + 1 = 0$ OR (b) Let $u = x^{\cot x}$ and $v = \frac{2x^2-3}{2x^2-x+2}$ $\log u = \cot x \cdot \log x$ $\frac{1}{u} \frac{du}{dx} = \frac{\cot x}{x} + \log x (-\operatorname{cosec}^2 x)$ $\frac{du}{dx} = x^{\cot x} \left(\frac{\cot x}{x} - \operatorname{cosec}^2 x \cdot \log x \right)$ $\frac{dv}{dx} = \frac{(2x^2-x+2)(4x) - (2x^2-3)(4x-1)}{(2x^2-x+2)^2}$ $= \frac{-2x^2 + 20x - 3}{(2x^2-x+2)^2}$ Required derivative = $x^{\cot x} \left(\frac{\cot x}{x} - \operatorname{cosec}^2 x \cdot \log x \right) + \frac{-2x^2 + 20x - 3}{(2x^2-x+2)^2}$	I II I II III IV I II III IV V VI VII	2 + 2 1 2 1 1 1 $\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$ 1 1 $\frac{1}{2}$

Sol.	(a) Given line is $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3} = \lambda$ Any general point on line l is $(5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$ Drs of given line are $\langle 5, 2, 3 \rangle$ Drs of perpendicular line are $\langle 5\lambda - 3, 2\lambda - 1, 3\lambda - 7 \rangle$ As lines are perpendicular $\therefore 5(5\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 7) = 0$ gives, $\lambda = 1$ $\therefore$ coordinates of foot of perpendicular are $(2, 3, -1)$ length of perpendicular = $\sqrt{21}$ OR (b) Let $\vec{a}_1 = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{a}_2 = p\hat{i} - \hat{j} - \hat{k}$, and $\vec{b}_1 = \hat{i} - \hat{j} + \hat{k}$, $\vec{b}_2 = 2\hat{i} + \hat{j} + 2\hat{k}$ Here $(\vec{a}_2 - \vec{a}_1) = (p-1)\hat{i} - 3\hat{j} - 2\hat{k}$ $\vec{b}_1 \times \vec{b}_2 = -3\hat{i} + 3\hat{k}$ S.D. = $\left \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{ \vec{b}_1 \times \vec{b}_2 } \right$ $\frac{-3p-3}{3\sqrt{2}} = \pm \frac{3}{\sqrt{2}}$ gives $p = -4, 2$	I	1
		II	$\frac{1}{2}$
		III	$\frac{1}{2}$
		IV	1
		V	1
		VI	1
SECTION E			
This section (Q. 36 to 38) has 3 case study based questions of 4 marks each.			
36.	Case Study - 1 A school wants the students of class XII to do a project on 'Sustainability' keeping the world environment in mind. They select the student participants on the basis of an essay writing competition. 7 students out of 80 are selected for the project and are categorized into two sets such that : Girl students belong to Set A = $\{G_1, G_2, G_3, G_4\}$, Boy students belong to Set B = $\{B_1, B_2, B_3\}$. Based on the above information, answer the following questions : (i) How many relations are possible from Set A $\rightarrow$ Set B ?		1

	(ii) Let R be a relation from $A \rightarrow B$ such that $R = \{(G_1, B_1), (G_2, B_2), (G_3, B_2), (G_4, B_3), (G_1, B_2)\}.$ Is R an injective function ? Justify your answer. 1		
	(iii) (a) Let the relation R from $A \rightarrow A$ be such that $R = \{(x, y), x, y \in A, x \text{ and } y \text{ are students from the same colony in the city}\}$ Verify if R is an equivalence relation. 2 OR (iii) (b) Verify if any function $f : B \rightarrow A$ is bijective. Give reason to support your answer. 2		
Sol.	(i) Number of relations from A to B is 2^{12} or 4096 (ii) No R is not injective It is not a function as G_1 has two images B_1 and B_2 (iii) (a) R is reflexive as $(x, x) \in R \quad \forall x \in A$ because x and x are students from same colony Let $(x, y) \in R$ so x and y are students from same colony Hence y and x are students from same colony Thus, $(y, x) \in R$ $\therefore R$ is symmetric Let $(x, y) \in R$ and $(y, z) \in R$ so x and y are students from same colony, y and z are students from same colony Thus, x and z are students from same colony $\therefore (x, z) \in R$ Thus, R is transitive As R is reflexive, symmetric and transitive, hence R is an equivalence relation OR (iii) (b) Any function f from B to A will not be bijective As f from B to A cannot be surjective because $n(B) < n(A)$ or A and B are finite sets and $n(A) \neq n(B)$	I I II I II III IV I II	1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1 1

37.	Case Study – 2 There are three types of vaccines A_1, A_2, A_3, available in the market to protect the population of the country from spread of certain infection. According to a survey conducted, it was found that 25% of the population was given Vaccine A_1, 35% of the population was given Vaccine A_2 and 40% of the population was given Vaccine A_3. The survey also stated that the probabilities that Vaccines A_1, A_2 and A_3 would protect against the infection were 60%, 55% and 50% respectively. Based on the above information, answer the following questions : Find the probability that : (i) The person taking vaccine A_2 will get infected. 1 (ii) If a person is chosen randomly, he/she will be protected from the infection. 1 (iii) (a) The person was given Vaccine A_1, given that the randomly chosen person is infected. 2 OR (iii) (b) The person was given Vaccine A_3, given that the randomly chosen person is not infected. 2		
Sol.	(i) $P(\text{Person taking vaccine } A_2 \text{ will be infected}) = 45\% \text{ or } 0.45$ (ii) $P(\text{Person is protected from infection})$ $= \frac{25}{100} \cdot \frac{60}{100} + \frac{35}{100} \cdot \frac{55}{100} + \frac{40}{100} \cdot \frac{50}{100}$ $= \frac{5425}{10000} \text{ or } 0.5425$ (iii) (a) $P(A_1/\text{person is infected}) = \frac{\frac{25}{100} \cdot \frac{40}{100}}{\frac{25}{100} \cdot \frac{40}{100} + \frac{35}{100} \cdot \frac{45}{100} + \frac{40}{100} \cdot \frac{50}{100}}$ $= \frac{1000}{4575} \text{ or } \frac{40}{183}$ OR (iii) (b) $P(A_3/\text{person is not infected}) = \frac{\frac{40}{100} \cdot \frac{50}{100}}{\frac{25}{100} \cdot \frac{60}{100} + \frac{35}{100} \cdot \frac{55}{100} + \frac{40}{100} \cdot \frac{50}{100}}$ $= \frac{2000}{5425} \text{ or } \frac{80}{217}$	I I II I II I II	1 $\frac{1}{2}$ $\frac{1}{2}$ $1\frac{1}{2}$ $\frac{1}{2}$ $1\frac{1}{2}$ $\frac{1}{2}$

38.	Case Study - 3 A company produces cylindrical tumblers, open from the top. Since they want uniformity in the product, they fix the surface area of the tumblers produced.  Based on the above information, answer the following questions : If for a tumbler, V is its volume, h the height and r the radius of the circular base, then : (i) Differentiate its volume with respect to radius of the base, where the surface area is constant. 2 (ii) If the company wants to maximize the volume of each tumbler, then establish a relation between its height and the radius of the base. 2		
Sol.	(i) Surface area $S = 2\pi rh + \pi r^2$ $V = \pi r^2 h = \frac{\pi r^2 (S - \pi r^2)}{2\pi r} = \frac{(Sr - \pi r^3)}{2}$ $\frac{dV}{dr} = \frac{(S - 3\pi r^2)}{2}$ (ii) $\frac{dV}{dr} = 0$ gives $S = 3\pi r^2$ $\frac{d^2V}{dr^2} = -3\pi r < 0$ V is maximum when $S = 3\pi r^2$ Thus, $h = \frac{3\pi r^2 - \pi r^2}{2\pi r} = r$	I II III I II III	$\frac{1}{2}$ 1 $\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$