Class X Session 2026-27
Subject - Mathematics (Standard)
Sample Question Paper - 4

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

Read the following instructions carefully and follow them:

1. This question paper contains 38 questions.
2. This Question Paper is divided into 5 Sections A, B, C, D and E.
3. In Section A, Questions no. 1-18 are multiple choice questions (MCQs) and questions no. 19 and 20 are Assertion- Reason based questions of 1 mark each.
4. In Section B, Questions no. 21-25 are very short answer (VSA) type questions, carrying 02 marks each.
5. In Section C, Questions no. 26-31 are short answer (SA) type questions, carrying 03 marks each.
6. In Section D, Questions no. 32-35 are long answer (LA) type questions, carrying 05 marks each.
7. In Section E, Questions no. 36-38 are case study-based questions carrying 4 marks each with sub-parts of the values of 1,1 and 2 marks each respectively.
8. All Questions are compulsory. However, an internal choice in 2 Questions of Section B, 2 Questions of Section C and 2 Questions of Section D has been provided. An internal choice has been provided in all the 2 marks questions of Section E.
9. Draw neat and clean figures wherever required.
10. Take $\pi = 22/7$ wherever required if not stated.
11. Use of calculators is not allowed.

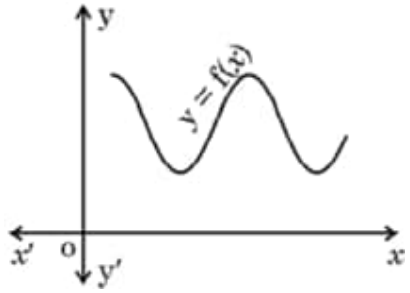
Section A

1. The least positive integer divisible by 20 and 24 is **[1]**

   a) 240 b) 360

   c) 120 d) 480

2. The graph of y = f(x) is shown in the figure for some polynomial f(x). **[1]**

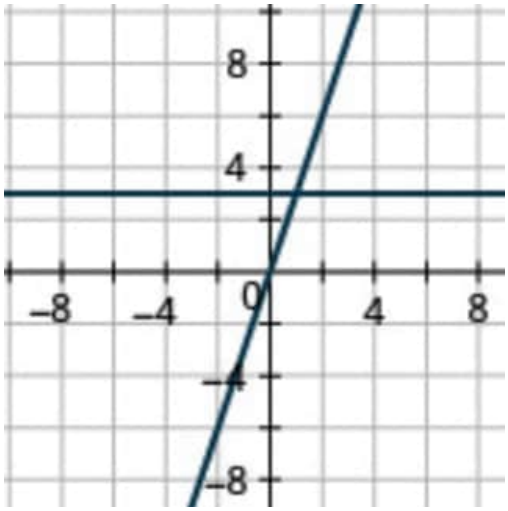


The number of zeroes of f(x) is

a) 2 b) 0

c) 3 d) 4

3. The number of solutions of two linear equations representing intersecting lines is/are **[1]**



a) 2 b) 0

c) 1 d) ∞

4. A quadratic equation whose roots are $(2 + \sqrt{3})$ and $(2 - \sqrt{3})$ is: **[1]**

a) $4x^2 - 3 = 0$ b) $x^2 + 4x + 1 = 0$

c) $x^2 - 4x + 1 = 0$ d) $x^2 - 1 = 0$

5. Which term of the AP 25, 20, 15,... is the first negative term? **[1]**

a) 9th b) 7th

c) 10th d) 8th

6. The equation of a line parallel to y-axis and at a distance of 5 units to the right of y-axis is: **[1]**

a) $y = -5$ b) $x = -5$

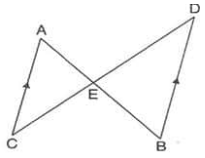
c) $y = 5$ d) $x = 5$

7. If (a, 0), (0, b) and (x, y) are collinear, then **[1]**

a) ay - bx = 1 b) ax + by = 1

c) ay + bx = ab d) ax - by = ab

8. In the adjoining figure $AC||BD$. If , EB = 4 cm, ED = 8 cm, AC = 6 cm, AE = 3 cm then CE and BD are respectively **[1]**

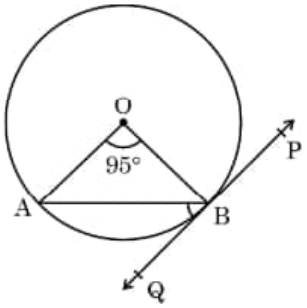


a) 4 cm, 6 cm. b) 7.5 cm, 9.5 cm.

c) 6 cm, 8 cm. d) 5 cm, 7 cm.

9. In the given figure, PQ is tangent to the circle centred at O. If $\angle AOB = 95^o$, then the **[1]**

measure of ∠ABQ will be



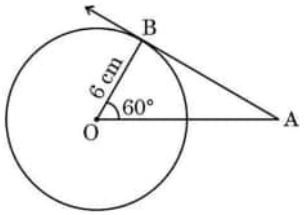
a) 85°

b) 95°

c) 47.5°

d) 42.5°

10. In the given figure, if AB is a tangent to the circle with centre O such that $OB = 6\ cm$ and $\angle AOB = 60^\circ$, then the length of OA is: **[1]**



a) 3 cm

b) $4\sqrt{3}\ cm$

c) 12 cm

d) $3\sqrt{3}\ cm$

11. $(\cos^4 A - \sin^4 A)$ on simplification, gives **[1]**

a) $2\cos^2 A + 1$

b) $2\sin^2 A + 1$

c) $2\cos^2 A - 1$

d) $2\sin^2 A - 1$

12. If $\sec\theta + \tan\theta = p$, then the value of $\sin\theta$ is **[1]**

a) $\frac{1+p^2}{p^2-1}$

b) $\frac{p^2-1}{p^2+1}$

c) $\frac{p^2+1}{p^2-1}$

d) $\frac{1-p^2}{p^2+1}$

13. A tower and a building standing on the ground are 20 m apart. The height of the tower is 2 times the height of the building. If at the mid-point of the line joining their feet, the angular elevation of their tops are complementary, then the height of the building is **[1]**

a) $\frac{2}{\sqrt{2}}$ m

b) $5\sqrt{2}$ m

c) $\sqrt{2}$ m

d) $\sqrt{3}$ m

14. The area of the sector of a circle with radius 6 cm which subtends an angle of 60° at the centre of the circle is: **[1]**

a) $\frac{142}{7}\ cm^2$

b) $\frac{132}{7}\ cm^2$

c) $\frac{152}{7}\ cm^2$

d) $\frac{122}{7}\ cm^2$

15. The angle of the sector of a circle whose area is one-eighth of the area of the circle is: **[1]**

a) 60°

b) 45°

c) 90° d) $22\frac{1}{2}^\circ$

16. A bag contains 3 red, 4 white and 7 green balls. A ball is drawn at random. The probability that the ball drawn is not of red colour is: **[1]**

a) $\frac{3}{11}$ b) $\frac{1}{11}$

c) $\frac{3}{14}$ d) $\frac{11}{14}$

17. Find the probability of getting 53 Fridays in a leap year. **[1]**

a) $\frac{4}{7}$ b) $\frac{3}{7}$

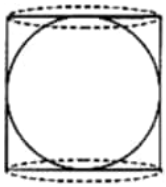
c) $\frac{2}{7}$ d) $\frac{5}{7}$

18. If the mean and median of a data are 10 and 11 respectively, then mode of the data is: **[1]**

a) 8 b) 20

c) 12 d) 13

19. **Assertion (A):** In the given figure, a sphere is inscribed in a cylinder. The surface area of the sphere is not equal to the curved surface area of the cylinder. **[1]**



Reason (R): Surface area of sphere is $4\pi r^2$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Arithmetic mean between 8 and 12 is 10. **[1]**
Reason (R): Arithmetic mean between two numbers a and b is given as $\frac{a+b}{2}$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

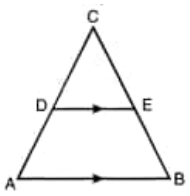
Section B

21. Given that HCF of two numbers is 23 and their LCM is 1449. If one of the numbers is 161, find the other. **[2]**

OR

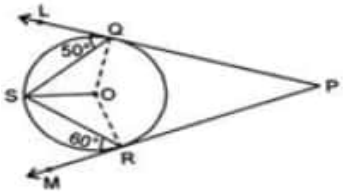
Prove that $\frac{2}{\sqrt{7}}$ is irrational.

22. In the given figure, $\angle A = \angle B$ and AD = BE. Show that DE $\|$AB. **[2]**



23. In figure, PQL and PRM are tangents to the circle with centre O at the points Q and R respectively and S is a point on the circle such that $\angle SQL = 50^o$ and **[2]**

$\angle SRM = 60^o$. Then, find $\angle QSR$.

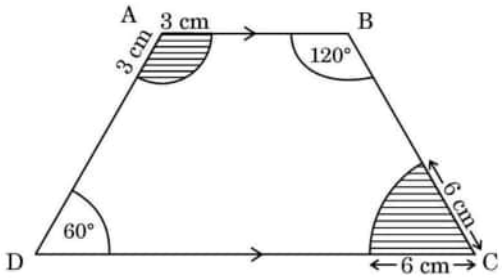


24. In a ΔABC right angled at B, if AB = 4 and BC = 3, find all the six trigonometric ratios of $\angle A$ **[2]**

OR

If sin (A + B) = 1 and cos (A - B) = 1, $0° \leq (A + B) \leq 90°$, $A \geq B$ find A and B.

25. In the given figure, ABCD is a trapezium with $AB \| DC$. Find the area of the shaded region. (Keep the answer in terms of π). **[2]**



Section C

26. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time and go in the same direction. After how many minutes will they meet again at the starting point? **[3]**

27. Read the following statement carefully and deduce about the sign of the constants p, q, and r. **[3]**

"The zeroes of a quadratic polynomial $px^2 + qx + r$ are both negatives."

28. An aircraft has 120 passenger seats. The number of seats occupied during 100 flights is given in the following table, calculate the mean number of seats: **[3]**

Number of seats	100-104	104-108	108-112	112-116	116-120
Frequency	15	20	32	18	15

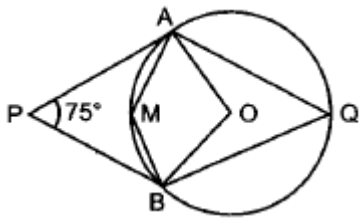
29. Check graphically whether the pair of equations x + 3y = 6 and 2x – 3y = 12 is consistent. If so, solve them graphically. **[3]**

OR

A jeweller has bars of 18-carat gold and 12-carat gold. How much of each must be melted together to obtain a bar of 16-carat gold, weighing 120 g? (Given: Pure gold is 24-carat).

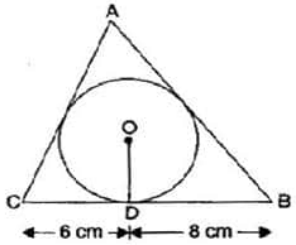
30. If,$\cot B = \frac{12}{5}$ prove that $\tan^2 B - \sin^2 B = \sin^4 B \cdot \sec^2 B$ **[3]**

31. In the given figure, O is the centre of the circle. Determine $\angle$AQB and $\angle$AMB, if PA and PB are tangents **[3]**



OR

A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see figure). Find the sides AB and AC.



Section D

32. During a medical checkup, height of 35 students of a class were recorded as follows: **[5]**

Height (in cm)	$90-100$	$100-110$	$110-120$	$120-130$	$130-140$	$140-150$
Number of Students	3	2	4	5	14	7

Find the difference between the mean height and median height.

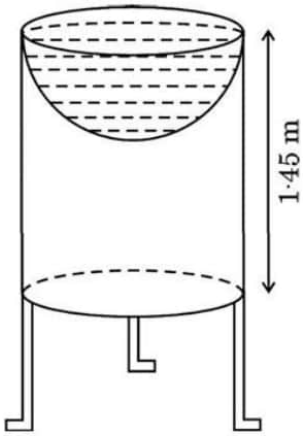
33. Prove that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then other two sides are divided in the same ratio. By using this theorem, prove that in $\triangle$ ABC if DE $\|$ BC then $\frac{AD}{BD} = \frac{AE}{AC}$. **[5]**

34. The length of the hypotenuse of a right-angled triangle exceeds the length of the base by 2 cm and exceeds twice the length of the altitude by 1 cm. Find the length of each side of the triangle. **[5]**

OR

A two-digit number is such that the product of its digits is 12. When 36 is added to this number, the digits interchange their places. Find the number.

35. Sukriti made a bird-bath for her garden in the shape of a hollow cylinder with a hemispherical depression at one end as shown in the figure. The height of the cylinder is 1.45 m and its radius is 30 cm. Find the total surface area of the bird-bath. **[5]**



OR

A toy is in the form of a cone mounted on a hemisphere. The diameter of the base of the cone is 7 cm and its height is 15.5 cm. Find the volume of the toy. (Use $\pi = 3.14$).

Section E

36. In a charming village, there is a street called 'Maplewood Avenue', where the houses are numbered from 1 to 49. A community group has decided to undertake a green renovation project, aiming to install solar panels on the houses. **[4]**

They have a few questions before they begin the project.



i. How many houses are there on 'Maplewood Avenue'? **(1)**

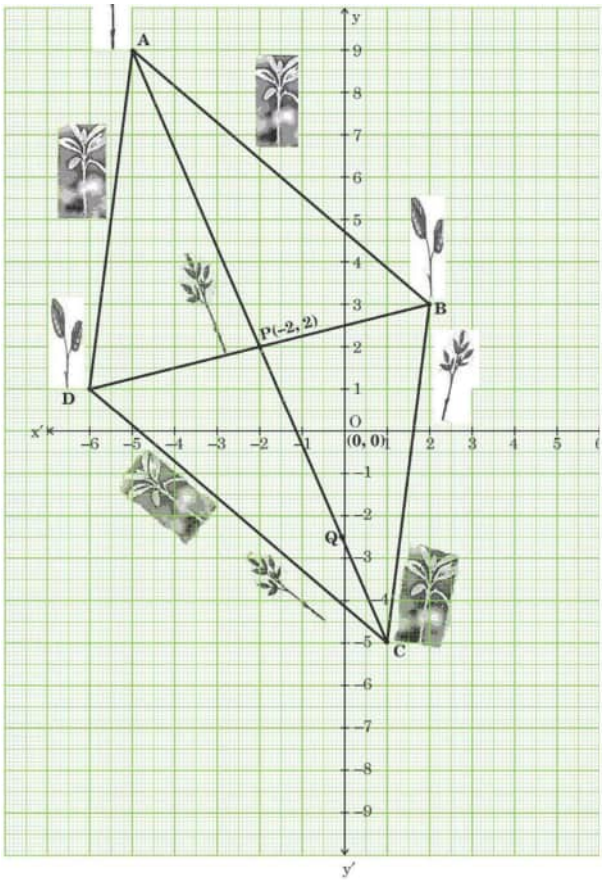
ii. The house numbers follow an Arithmetic Progression. What is the first term of the A.P. and the common difference? **(1)**

iii. a. If the group wanted to calculate the sum of all house numbers from 1 to 49, how much would that be? **(2)**

OR

b. What is the sum of the house numbers between 15 and 30? **(2)**

37. Trees act the natural filters. By planting trees in and around school premises, we create cleaner and healthier air for students and local residents, reducing respiratory problems. A school in Noida has proposed and organised a community drive on tree plantation under the title "Save Earth, Plant Trees". Students of that school have planted saplings in the field such that it formed a quadrilateral as shown in the figure $ABCD$. **[4]**



Based on the information given above, answer the following questions:

i. Find the distance between the two saplings at A and D. **(1)**

ii. a. One student plants one sapling at the mid-point of AD. Then he moves along a straight line parallel to DB and sows another sapling on AB. What are the coordinates of the positions of these two new saplings? **(2)**

OR

b. A new sapling is kept at a point M on DB such that $DM : MB = 3 : 1$. Find the coordinates of M. **(2)**

iii. The line segments AC and BD bisect each other at $P(-2, 2)$. Find the coordinates of C. **(1)**

38. An injured bird was found on the roof of a building. The building is 15 m high. A fireman was called to rescue the bird. The fireman used an adjustable ladder to reach the roof. He placed the ladder in such a way that the ladder makes an angle of 60° with the ground in order to reach the roof. **[4]**





Based on the above information, answer the following questions:

i. Find the length of the ladder used by the fireman to reach the roof. **(1)**

ii. Find the distance of the point on the ground at which the ladder was fixed from the bottom of the building. **(1)**

iii. In order to avoid skidding, the fireman placed the ladder in such a way that the bottom of the ladder touches the base of the wall which is opposite to the building, making an angle of 30° with the ground.

a. Draw a neat diagram to represent the above situation and hence find the width of the road between the building and the wall. **(2)**

OR

b. Find the length of the ladder used by the fireman in this case. **(2)**

Solution

Section A

1.

(c) 120

Explanation:

Least positive integer divisible by 20 and 24 is
LCM of (20, 24).

$20 = 2^2 \times 5$

$24 = 2^3 \times 3$

$\therefore$ LCM (20, 24) = $2^3 \times 3 \times 5 = 120$

Thus 120 is divisible by 20 and 24.

2.

(b) 0

Explanation:

Here y = f(x) is not intersecting or touching the X-axis.

$\therefore$ Number of zeroes of f(x) = 0

3.

(c) 1

Explanation:

The number of solutions of two linear equations representing intersecting lines is 1 because two linear equations representing intersecting lines has a unique solution.

4.

(c) $x^2 - 4x + 1 = 0$

Explanation:

let $\alpha = 2+\sqrt{3}$ and $\beta = 2-\sqrt{3}$ be the roots of quadratic equation.

$\alpha + \beta = 4$

$\alpha\beta = 4 - 3 = 1$

$\therefore$ Quadratic equation = $x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - 4x + 1 = 0$

5.

(b) 7th

Explanation:

Let $T_n < 0$. Then, $25 + (n - 1) \times (-5) < 0$.

$30 < 5n \Rightarrow 5n > 30 \Rightarrow n > 6$.

$\Rightarrow$ So, the required term is 7th.

6.

(d) $x = 5$

Explanation:

$x = 5$

7.

(c) ay + bx = ab

Explanation:

If given points are collinear, then the area of the triangle formed by these three points is 0.

$\therefore$ Area = $\frac{1}{2}$ | a(b - y) + 0(y - 0) + x(0 - b) | = 0

$\Rightarrow \frac{1}{2}$ | ab - ay - bx | = 0

$\Rightarrow$ ab - ay - bx = 0

$\Rightarrow$ ay + bx = ab

8.

(c) 6 cm, 8 cm.

Explanation:

Given: $AC||BD$. and AC = 6 cm, AE = 3 cm, EB = 4 cm, ED = 8 cm, In triangles ACE and DEB, $\angle$AEC = $\angle$DEB [Vertically opposite angles] $\angle$ECA = $\angle$EDB [Alternate angles as $AC||BD$]

$\therefore \triangle$ACE $\sim \triangle$DEB [AA similarity]

$\therefore \frac{EB}{AE} = \frac{ED}{EC}$

$\Rightarrow \frac{4}{3} = \frac{8}{EC}$

$\Rightarrow$ EC = $\frac{8\times3}{4}$ = 6 cm

Also $\frac{EB}{AE} = \frac{BD}{AC}$

$\Rightarrow \frac{4}{3} = \frac{BD}{6}$

$\Rightarrow$ BD = $\frac{4\times6}{3}$ = 8 cm

9.

(c) 47.5^o

Explanation:

47.5^o

10.

(c) 12 cm

Explanation:

12 cm

11.

(c) $2\cos^2A - 1$

Explanation:

The given expression is $\cos^4A - \sin^4A$.

Factorising the given expression, we have

$\cos^4A - \sin^4A = [(\cos^2A)^2 - (\sin^2A)^2]$

$= (\cos^2A + \sin^2A) \times (\cos^2A - \sin^2A)$ ...[$\because (a^2 - b^2) = (a + b)(a - b)$]

$= \cos^2A - \sin^2A$ ...[$\because \sin^2A + \cos^2A = 1$]

$= \cos^2A - (1 - \sin^2A)$

$= \cos^2A - 1 + \cos^2A$

$= 2\cos^2A - 1$

12.

(b) $\frac{p^2-1}{p^2+1}$

Explanation:

Given: $\sec\theta + \tan\theta = p$

$\Rightarrow \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = p$

$\Rightarrow \frac{1+\sin\theta}{\cos\theta} = p$

Squaring both sides, we get

$\Rightarrow \frac{(1+\sin\theta)^2}{\cos^2\theta} = p^2$

$\Rightarrow \frac{(1+\sin\theta)^2}{1-\sin^2\theta} = p^2$

$\Rightarrow \frac{(1+\sin\theta)^2}{(1+\sin\theta)(1-\sin\theta)} = p^2$

$\Rightarrow \frac{1+\sin\theta}{1-\sin\theta} = p^2$

$\Rightarrow 1 + \sin\theta = p^2 (1 - \sin\theta)$

$\Rightarrow 1 + \sin\theta = p^2 - p^2 \sin\theta$

$\Rightarrow \sin\theta + p^2 \sin\theta = p^2 - 1$

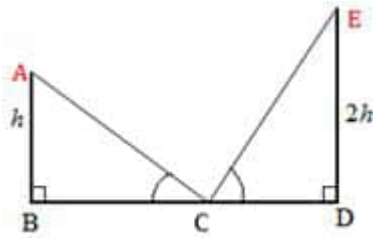
$\Rightarrow \sin\theta(1 + p^2) = p^2 - 1$

$\Rightarrow \sin\theta = \frac{p^2-1}{p^2+1}$

13.

(b) $5\sqrt{2}$ m

Explanation:



Let the height of the building = AB = h meters, then

Height of the tower = ED = 2h meters

According to question, $\angle ACB = \theta$ then $\angle EDC = 90^\circ - \theta$ And BC = CD = 10 m

Now, in triangle ABC, $\tan\theta = \frac{AB}{BC} \Rightarrow \tan\theta = \frac{h}{10}$ .....(i)

Now, in triangle EDC, $\tan(90^\circ - \theta) = \frac{ED}{CD}$

$\Rightarrow \cot\theta = \frac{2h}{10} = \frac{h}{5}$ .....(ii)

$h2h$ Multiplying eq. (i) and (ii), we get

$\tan\theta . \cot\theta = \frac{h}{10} \times \frac{h}{5} \Rightarrow 1 = \frac{h^2}{50}$

$\Rightarrow h^2 = 50 \Rightarrow h = 5\sqrt{2}$ m

14.

(b) $\frac{132}{7}$ cm^2

Explanation:

Angle of the sector is 60°

Area of sector = $(\frac{\theta}{360^o}) \times \pi r^2$

$\therefore$ Area of the sector with angle $60^o = (\frac{60^o}{360^o}) \times \pi r^2$ cm^2

$= (\frac{36}{6})\pi$ cm^2

$= 6 \times (\frac{22}{7})$ cm^2

$= \frac{132}{7}$ cm^2

15.

(b) 45°

Explanation:

45°

16.

(d) $\frac{11}{14}$

Explanation:

$\frac{11}{14}$

17.

(c) $\frac{2}{7}$

Explanation:

Leap year contains 366 days.

i.e, 52 weeks + 2 days

52 weeks contain 52 Fridays.

We will get 53 Fridays, if one of the remaining two days is a Friday. Total possibilities for two days are:

(Sunday, Monday), (Monday, Tuesday), (Tuesday, Wednesday), (Wednesday, Thursday), (Thursday, Friday), (Friday, Saturday), (Saturday, Sunday)

There are 7 possibilities and out of which 2 are favourable cases.

$\therefore$ Required probability = $\frac{2}{7}$

18.

(d) 13

Explanation:

We know that

3 Median= Mode + 2 Mean

3×11 = Mode + 2×10

33 - 20 = Mode

Mode = 13

19.

(d) A is false but R is true.

Explanation:

A is false but R is true.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

Section B

21. $HCF \times LCM = a \times b$

$\Rightarrow \quad 23 \times 1449 = 161 \times b$

$\Rightarrow \quad b = \frac{23 \times 1449}{161} = 207$

$\therefore$ Other number is 207

OR

Let us assume that $\frac{2}{\sqrt{7}}$ is rational. Then, there exist positive co-primes a and b such that

$\frac{2}{\sqrt{7}} = \frac{a}{b}$

$\sqrt{7} = \frac{2b}{a}$

As 2b and a are rational numbers.

Then $\frac{2b}{a}$ is rational number.

But $\sqrt{7}$ is not a rational number.

Since a rational number cannot be equal to an irrational number . Our assumption that $\frac{2}{\sqrt{7}}$ is rational number is wrong .

Hence $\frac{2}{\sqrt{7}}$ is an irrational number

22. In $\triangle$CAB, $\angle$A = $\angle$B(Given)

$\therefore$ AC = CB (By isosceles triangle property)

But, AD = BE (Given).....(i)

$\Rightarrow$ AC- CD = CB - BE

$\therefore$ CD = CE .....(ii)

Dividing equation (ii) by (i),

$\frac{CD}{AD} = \frac{CE}{BE}$

By converse of BPT,

DE $\parallel$ AB.

$\therefore$ If $\angle$A = $\angle$B and AD = BE then, DE $\parallel$ AB.

23. Here $\angle$OQS = $\angle$OQL - $\angle$SQL = 90° - 50° = 40°

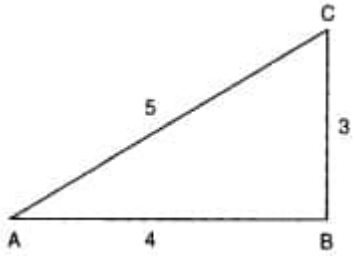
And $\angle$ORS = $\angle$ORM - $\angle$SQM 90° - 60° = 30° Since OS = OQ [Radii]

$\Rightarrow \angle$OSQ = $\angle$OQS = 40° [Angles opposite to equal sides] Again, since OS = OR [Radii]

$\Rightarrow \angle$OSR = $\angle$ORS = 30° [Angles opposite to equal sides]

$\therefore \angle$QSR = $\angle$OSQ+$\angle$OSR = 40° + 30° = 70°

24.



We have, $AB = 4 \; and \; BC = 3.$

Using Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$\Rightarrow \quad AC = \sqrt{AB^2 + BC^2}$

$\Rightarrow \quad AC = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$

When we consider the t-ratios of $\angle A$, we have

$Base = AB = 4, perpendicular = BC = 3$ and, $Hypotenuse = AC = 5.$

$\therefore \quad \sin A = \frac{BC}{AC} = \frac{3}{5}, \cos A = \frac{AB}{AC} = \frac{4}{5}, \tan A = \frac{BC}{AB} = \frac{3}{4}$

$cosecA = \frac{AC}{BC} = \frac{5}{3}, \sec A = \frac{AC}{AB} = \frac{5}{4}$ and , $\cot A = \frac{AB}{BC} = \frac{4}{3}$

OR

We have

sin (A + B) = 1

$\Rightarrow$ sin (A + B) = sin 90° ($\sin 90^\circ = 1$)

$\therefore$ A + B = 90° ..... (i)

and cos (A - B) = 1

$\Rightarrow$cos (A - B)= $\cos 0^\circ$ ($cos0^\circ = 1$)

$\therefore$ A - B = 0° ..... (ii)

Adding equation (i) and (ii), we get

A + B + A - B = 90° + 0°

2A = 90°

$A = \frac{90^o}{2} = 45^\circ$

Putting value of A in equation (i)

45° + B = 90°

$\Rightarrow$ B = 90° - 45° = 45°

25. ABCD is a trapezium.

$\therefore \angle A = 120^\circ$ and $\angle C = 60^\circ$

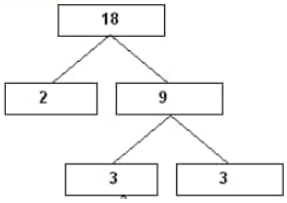
Area of shaded region $= \frac{120}{360} \times \pi \times (3)^2 + \frac{60}{360} \times \pi \times (6)^2$

$= 9\pi\ cm^2$

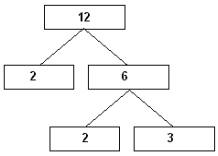
Section C

26. By taking LCM of time taken (in minutes) by Sonia and Ravi, We can get the actual number of minutes after which they meet again at the starting point after both start at the same point and at the same time, and go in the same direction.

$18 = 2 \times 3 \times 3 = 2 \times 3^2$



$12 = 2 \times 2 \times 3 = 2^2 \times 3$



LCM $(18, 12) = 2^2 \times 3^2 = 36$

Therefore, both Sonia and Ravi will meet again at the starting point after 36 minutes.

27. Sum of zeroes = $\frac{-q}{p}$ <0 [as zeroes are negative means sum of zeroes is negative]

So that $\frac{q}{p}$ > 0

$\Rightarrow\ q > 0,\ p > 0\ or\ q < 0,\ \ p <\ 0$..... (i)

Product of zeros = $\frac{r}{p}$ > 0 [as zeroes are negative means product of zeroes is positive]

$\Rightarrow\ r > 0,\ p > 0\ or\ r < 0,\ p\ < 0$ ..... (ii)

$\therefore$ From (i) and (ii), p, q and r will have same signs i.e.

Either $p > 0,\ q > 0,\ \ r > 0$

Or $p\ < 0,\ q < 0,\ r\ < 0.$

28. We first, find the classmark x_i of each class and then proceed as follows.

Class interval	Classmarks (x_i)	Frequency (f_i)	Deviation $d_i = x_i = a$	$f_i d_i$
100-104	102	15	-8	-120

104-108	106	20	-4	-80
108-112	A = 110	32	0	0
112-116	114	18	4	72
116-120	118	15	8	120
		$N = \sum f_i = 100$		$\Sigma f_i d_i = -8$

∴ Assumed mean, a = 110

Class width, h = 4

and total observations, N = 100

By step deviation method, Mean $(\bar{x}) = a + \frac{\sum f_i d}{\Sigma f_i}$

$= 110 + \left(\frac{-8}{100}\right)$

= 100 - 0.08

= 109.92

But seats cannot be in decimal, so the number of seats is 109.

29. The solution of pair of linear equations:

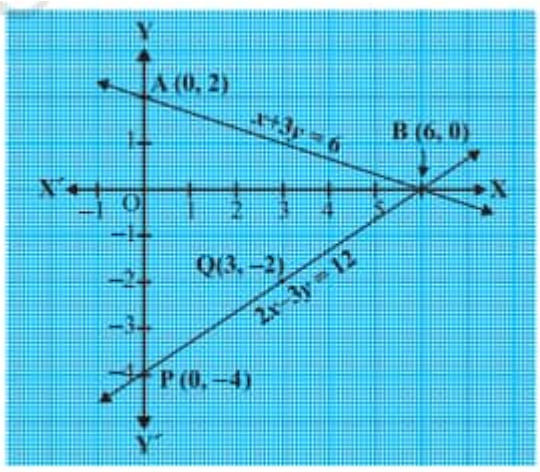
x + 3y = 6 and 2x – 3y = 12

x	0	6
$y = \frac{6-x}{3}$	2	0

and

x	0	3
$y = \frac{2x-12}{3}$	-4	-2

Plot the points A(0, 2), B(6, 0), P(0, – 4) and Q(3, – 2) on graph paper, and join the points to form the lines AB and PQ



We observe that there is a point B (6, 0) common to both the lines AB and PQ. So, the solution of the pair of linear equations is x = 6 and y = 0, i.e., the given pair of equations is consistent.

OR

Suppose the amount of 18-carat gold and 12-carat gold to be melted be x g and y g respectively.

According to the question,

$\frac{18}{24}x + \frac{12}{24}y = \frac{16}{24}(120)$

$\Rightarrow 18x + 12y = 16(120)$

$\Rightarrow 3x + 2y = 320$ .....(i)

The amount of each add up to 120 g,

$\Rightarrow x + y = 120$ .....(ii)

Multiplying (ii) by 2,

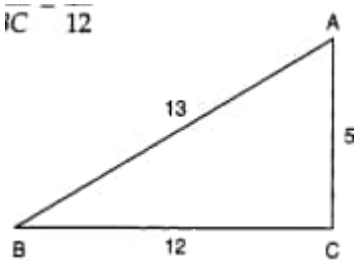
$\Rightarrow 2x + 2y = 240$ .....(iii)

Subtract (iii) from (i),

$\Rightarrow x = 80$

Substituting x = 80 in (ii), we get $y = 40$

$\therefore$ the amount of 18-carat gold is $80\ g$ and the amount of 12-carat gold is $40\ g$.

30. 

We have

$\cot B = \frac{\text{Base}}{\text{Perpendicular}} = \frac{12}{5}$

Let us draw a right triangle ABC, in which $\angle C = 90^\circ$ such that

Base = BC = 12 units and, Perpendicular = AC = 5 units.

Applying Pythagoras Theorem in,ΔBCA we get

$AB^2 = BC^2 + AC^2$

$\Rightarrow \quad AB^2 = 12^2 + 5^2 = 169$

$\Rightarrow \quad AB = \sqrt{169} = 13$

$\therefore \quad \sin B = \frac{AC}{AB} = \frac{5}{13}, \tan B = \frac{AC}{BC} = \frac{5}{12}$ and, $\sec B = \frac{AB}{BC} = \frac{13}{12}$

Now, L.H.S= $\tan^2 B - \sin^2 B$

$\Rightarrow \quad \text{L. H. S} = (\tan B)^2 - (\sin B)^2$

$\Rightarrow \quad \text{L. H. S} = \left(\frac{5}{12}\right)^2 - \left(\frac{5}{13}\right)^2 = \frac{25}{144} - \frac{25}{169}$

$\Rightarrow \quad \text{L. H. S} = 25\left(\frac{1}{144} - \frac{1}{169}\right) = 25\left(\frac{169-144}{144\times169}\right)$

$\Rightarrow \quad \text{L. H. S} = 25 \times \frac{25}{144\times169} = \frac{25\times25}{144\times169} = \frac{5^2\times5^2}{12^2\times13^2}$ .....(i)

and, $\text{R. H. S} = \sin^4 B \cdot \sec^2 B$

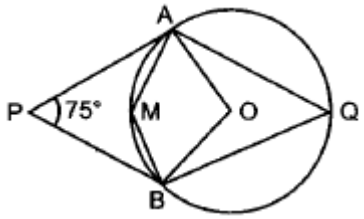
$\Rightarrow \quad \text{R. H. S} = (\sin B)^4(\sec B)^2 = \left(\frac{5}{13}\right)^4 \times \left(\frac{13}{12}\right)^2 = \frac{5^4\times13^2}{13^4\times12^2} = \frac{5^4}{13^2\times12^2} = \frac{5^2\times5^2}{13^2\times12^2}$ .....(ii)

From (i) and (ii), we have

$\tan^2 B - \sin^2 B = \sin^4 B \cdot \sec^2 B$

Hence proved.

31. Given,



$OA \perp PA$ and

$OB \perp PB$

$\therefore \angle \text{OAP} = 90^\circ$ and $\angle \text{OBP} = 90^\circ$

$\Rightarrow \angle \text{OAP} + \angle \text{OBP} = 180^\circ$

PBOA is cyclic quadrilateral

$\therefore \angle APB + \angle AOB = 180^\circ$

$\Rightarrow \angle AOB = 180^\circ - 75^\circ = 105^\circ$

Now $\angle AOB = 2\angle AQB$

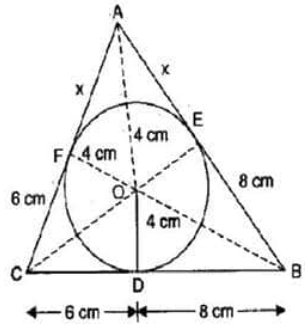
$\therefore \angle \text{AQB} = \frac{1}{2}(105^\circ) = 52.5^\circ$

Now $\angle AMB + \angle AQB = 180^\circ$

$\Rightarrow \angle AMB = 180^\circ - 52.5^\circ = 127.5^\circ$

OR

Join OE and OF. Also join OA, OB and OC.



Since BD = 8 cm

$\therefore$ BE = 8 cm

[Tangents from an external point to a circle are equal]

Since CD = 6 cm

$\therefore$ CF = 6 cm

[Tangents from an external point to a circle are equal]

Let AE = AF = x

Since OD = OE = OF = 4 cm [Radii of a circle are equal]

$\therefore$ Semi-perimeter of $\triangle$ ABC = $\frac{(x+6)(x+8) + (6+8)}{2} = \frac{(2x+28)}{2}$ = (x + 14)cm

$\therefore$ Area of $\triangle$ABC = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{(x+14)(x+14-14)(x+14-\overline{x+8})(x+14-\overline{x+6})}$

= $\sqrt{(x+14)(x)(8)(6)}$ cm^2

Now, Area of ΔABC = Area of $\triangle$OBC + Area of $\triangle$OCA + Area of $\triangle$OAB

$\Rightarrow \sqrt{(x+14)(x)(8)(6)} = \frac{(6+8)4}{2} + \frac{(x+6)4}{2} + \frac{(x+8)4}{2}$

$\Rightarrow \sqrt{(x+14)(x)(8)(6)}$ = 28 + 2x + 12 + 2x + 16

$\Rightarrow \sqrt{(x+14)(x)(8)(6)}$ = 4x + 56

$\Rightarrow \sqrt{(x+14)(x)(8)(6)}$ =4(x + 14)

Squaring both sides,

(x + 14) (x) (6) (8) = 16(x + 14)^2

$\Rightarrow$ 3x = x + 14

$\Rightarrow$ 2x = 14

$\Rightarrow$ x = 7

$\therefore$ AB = x + 8 = 7 + 8 = 15 cm

And AC = x + 6= 7 + 6 = 13 cm

Section D

32.

Height (in cm)	**Number of Students (f_i)**	x_i	$u_i = \frac{x_i - 115}{10}$	$f_i u_i$	cf
90-100	3	95	-2	-6	3
100-110	2	105	-1	-2	5
110-120	4	$115 = a$	0	0	9
120-130	5	125	1	5	14
130-140	14	135	2	28	28
140-150	7	145	3	21	35

Total	35			46	

$\text{Mean } = 115 + \frac{46}{35} \times 10$

$= \frac{897}{7}$ or 128.14 approx.

$\therefore$ Mean height is $\frac{897}{7}\ cm$ or 128.14 cm approx.

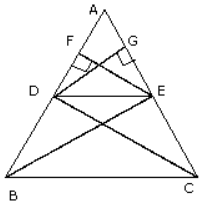
Median Class is $130 - 140$

$\text{Median } = 130 + \frac{\frac{35}{2} - 14}{14} \times 10$

$= 132.5$

$\therefore$ Median height is 132.5 cm

Difference of mean height and median height $= 132.5 - 128.14 = 4.36\ cm$

33. 

Given: In $\triangle$ ABC if DE $\|$ BC intersect AB at D and AC at E.

To Prove: $\frac{AD}{BD} = \frac{AE}{AC}$

Construction: Draw EF $\perp$ AB and DG $\perp$ AC and join DC and BE.

Proof: $ar\triangle ADE = \frac{1}{2} AD \times EF$

$ar\triangle DBE = \frac{1}{2} DB \times EF$

$\therefore \frac{ar\triangle ADE}{ar\triangle DBE} = \frac{\frac{1}{2} AD \times EF}{\frac{1}{2} DB \times EF} = \frac{AD}{DB}$ ....(i)

Similarly, $\therefore \frac{ar\triangle ADE}{ar\triangle DEC} = \frac{\frac{1}{2} AE \times DG}{\frac{1}{2} EC \times DG} = \frac{AE}{EC}$

Since $\triangle$DBE and $\triangle$DEC are on the same base and between the same parallels

$\therefore$ ar($\triangle$DBE) = ar ($\triangle$DEC)

$\Rightarrow \frac{1}{ar(\triangle DBE)} = \frac{1}{ar(\triangle DEC)}$

$\therefore \frac{ar\triangle ADE}{ar\triangle DBF} = \frac{ar\triangle ADE}{ar\triangle DFC}$

$\Rightarrow \frac{AD}{DB} = \frac{AB}{EC}$

$\because$ DE $\|$ BC

$\frac{AD}{DB} = \frac{AE}{EC}$

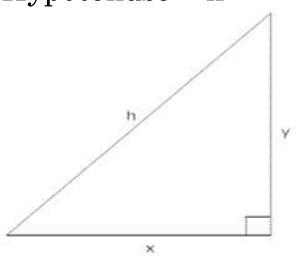
$\Rightarrow \frac{AD}{AD+DB} = \frac{AE}{AE+EC} \left[\because \frac{p}{q} = \frac{r}{s} \Rightarrow \frac{p}{p+q} = \frac{r}{r+s}\right]$

$\Rightarrow \frac{AD}{AB} = \frac{AE}{AC}$

34. Let base = x

Altitude = y

Hypotenuse = h



According to question,

h = x + 2

h = 2y + 1

$\Rightarrow$ x + 2 = 2y + 1

$\Rightarrow$ x + 2 - 1 = 2y

$\Rightarrow$ x - 1 = 2y

$\Rightarrow \frac{x-1}{2} = y$

And $x^2 + y^2 = h^2$

$\Rightarrow x^2 + \left(\frac{x-1}{2}\right)^2 = (x+2)^2$

$\Rightarrow x^2 - 15x + x - 15 = 0$

$\Rightarrow x^2 - 15x + x - 15 = 0$

$\Rightarrow$ (x - 15)(x + 1) = 0

$\Rightarrow$ x = 15 or x = -1

Base = 15 cm

Altitude = $\frac{x+1}{2} = 8\text{cm}$

Height, $h = 2 \times 8 + 1 = 17\text{cm}$

OR

Let unit digit be y

and ten's digit $= x$

hence, the two digit number $= 10x + y$,

ATQ

$xy = 12$ ...(i)

$10x + y + 36 = 10y + x$

$x - y + 4 = 0$ ...(ii)

From (i) and (ii)

$x^2 + 4x - 12 = 0$

$(x+6)(x-2) = 0$

Hence, $x = 2$

and $y = 6$

$\therefore$ Number $= 26$

35. Total surface area of bird-bath

= curved surface area of cylinder + curved surface area of hemisphere

$= 2\pi rh + 2\pi r^2 = 2\pi r(\ h + r)$

$= \frac{22}{7} \times 30(145 + 30)$

$= 2 \times \frac{22}{7} \times 30 \times 175 = 33000\ cm^2$

OR

According to question it is given that

Diameter of the base of the cone is = 7cm

Therefore radius = $\frac{7}{2}$ = 3.5cm

Total height of the toy = 14.5 cm

Height of the cone = 15.5 - 3.5 = 12 cm

Height of the hemisphere = 3.5 cm

According to question it is also given that

Volume of the toy = Volume of cone + Volume of hemisphere

$=\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^2$

={tex}\frac{1}{3}πr^2(2r+h){/tex}

$=\frac{1}{3} \times \frac{22}{7} \times (3.5)^2[2 \times 3.5 + 12]$

$=\frac{1}{3} \times 22 \times 1.75 \times 19$

$= 243.83 \text{ cm}^3$

Section E

36. i. 49 houses

ii. First term 'a' $= 1$, common difference $= 1$

iii. a. Sum $(S_{49}) = \frac{n}{2}[a + l]$

$= \frac{49}{2}[1 + 49] = 1225$

OR

b. Sum of house numbers between 15 & 30

$16 + 17 + 18 + \ldots . + 29$

$= \frac{n}{2}[2a + (n-1)d] = \frac{14}{2}[2 \times 16 + 13]$

$= 315$

37. i. Observing the graph, coordinators of points are $A(-5, 9)$ and $D(-6, 1)$

$AD = \sqrt{(-6+5)^2 + (1-9)^2} = \sqrt{65}$

ii. a. Mid point of $AD = \left(\frac{-6-5}{2}, \frac{1+9}{2}\right)$ i.e. $\left(\frac{-11}{2}, 5\right)$

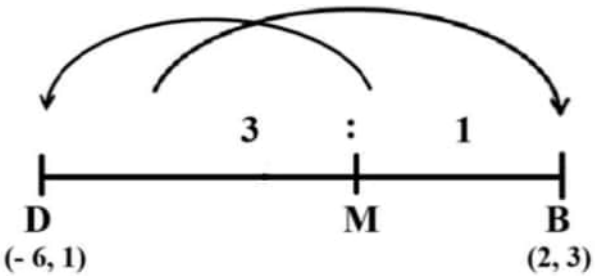
Student will sow another sapling at mid point of AB.

Point B is $(2, 3)$

Mid point of $AB = \left(\frac{2-5}{2}, \frac{3+9}{2}\right)$ i.e. $\left(\frac{-3}{2}, 6\right)$

OR

b.



Coordinates of $M = \left(\frac{6-6}{4}, \frac{9+1}{4}\right)$

i.e. $\left(0, \frac{5}{2}\right)$

iii. Coordinates of C are $(1, -5)$

38. i. Let the length of the ladder be 'a'

$\frac{15}{a} = \sin 60^\circ$

$a = \frac{30}{\sqrt{3}}$ or $10\sqrt{3}$

Thus the length of the ladder is $\frac{30}{\sqrt{3}}\ m$ or $10\sqrt{3}\ m$

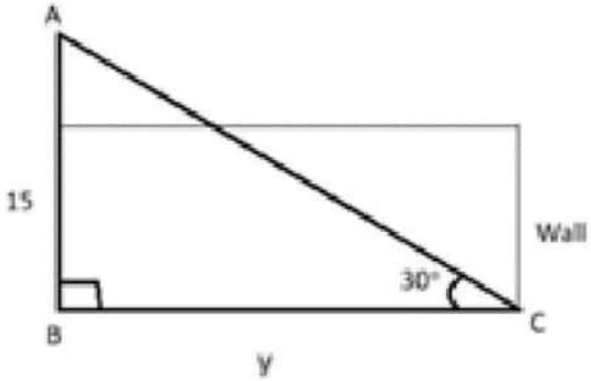
ii. Let the distance of the point on the ground be 'x'

$\frac{15}{x} = \tan 60^\circ$

$x = \frac{15}{\sqrt{3}}$ or $5\sqrt{3}$

Thus, the distance of the point on the ground is $\frac{15}{\sqrt{3}}\ m$ or $5\sqrt{3}\ m$

iii. a. Let the width of the road be y.



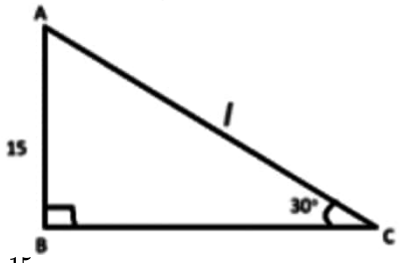
$\frac{15}{y} = \tan 30^\circ$

$y = 15\sqrt{3}$

Thus, the width of the road is $15\sqrt{3}\ m$.

OR

b. Let the length of the ladder be l.



$\frac{15}{l} = \sin 30^\circ$

$l = 30$

Thus, the length of the ladder is 30 m.