Class X Session 2026-27
Subject - Mathematics (Standard)
Sample Question Paper - 9

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment carrying 04 marks each.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

Section A

1. Raju bought a fish from a shop for his aquarium. The shop keeper takes out one fish from a tank containing 15 male fish and 18 female fish. The probability that the fish taken out is a male fish is **[1]**

   a) $\frac{6}{11}$ b) $\frac{5}{12}$

   c) $\frac{7}{11}$ d) $\frac{5}{11}$

2. A quadratic equation whose roots are $(3 - \sqrt{2})$ and $(3 + \sqrt{2})$ is: **[1]**

   a) $x^2 - 7 = 0$ b) $x^2 - 6x + 7 = 0$

   c) $9x^2 - 2 = 0$ d) $x^2 + 6x + 7 = 0$

3. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1cm and the height of the cone is equal to its radius. The volume of the solid is **[1]**

   a) $\pi\ cm^3$ b) $4\pi\ cm^3$

   c) $3\pi\ cm^3$ d) $2\pi\ cm^3$

4. Priya can row her boat at a speed of 5 km/hr in still water. If it takes her 1 hour more to row the boat 5.25 km upstream than to return downstream, find the speed of the stream. **[1]**

   a) 5 km/hr b) 4 km/hr

c) 3 km/hr d) 2 km/hr

5. In an AP, if a = 1, a_n = 20 and S_n = 399, then n is equal to **[1]**

a) 38 b) 42

c) 19 d) 21

6. If the point (x, 4) lies on a circle whose centre is at the origin and radius is 5 then x = **[1]**

a) ± 4 b) ± 3

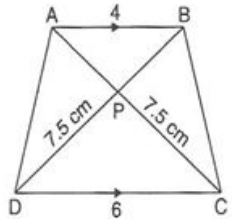
c) 0 d) ± 5

7. The zeroes of the quadratic polynomial $x^2 + 99x + 127$ are **[1]**

a) both positive b) both equal

c) one positive and one negative d) both negative

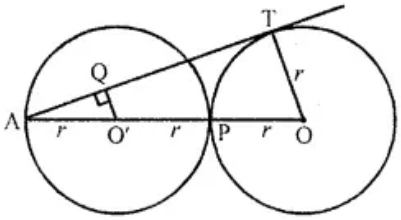
8. In the given figure, AP is equal to **[1]**



a) 7 cm b) 5.5 cm

c) 6 cm d) 5 cm

9. Two circles of same radii r and centres O and O' touch each other at P as shown in Figure. If O O' is produced to meet the circle C (O', r) at A and AT is a tangent to the circle C(O, r) such that O'Q $\perp$ AT. Then AO : AO' = **[1]**



a) 1:4 b) 3:1

c) 2:3 d) 3:2

10. A circle of radius 5.2 cm has two tangents AB and CD parallel to each other. What is the distance between the two tangents? **[1]**

a) 20.8 cm b) can't find

c) 5.2 cm d) 10.4 cm

11. If tan A = n tan B and sin A = m sin B, then $\cos^2 A$ = **[1]**

a) $\frac{m^2+1}{n^2+1}$ b) $\frac{m^2+1}{n^2-1}$

c) $\frac{m^2-1}{n^2-1}$ d) $\frac{m^2-1}{n^2+1}$

12. If p is prime, then H.C.F. and L.C.M. of p and p + 1 would be **[1]**

a) H.C.F. = p(p + 1), L.C.M. = 1 b) H.C.F. = 1, L.C.M. = p(p + 1)

c) H.C.F. = p, L.C.M. = p + 1 d) H.C.F. = p, L.C.M. = p(p + 1)

13. It is found that on walking x meters towards a chimney in a horizontal line through its base, the elevation of its top changes from 30° to 60°. The height of the chimney is **[1]**

a) $3\sqrt{2}x$ b) $2\sqrt{3}x$

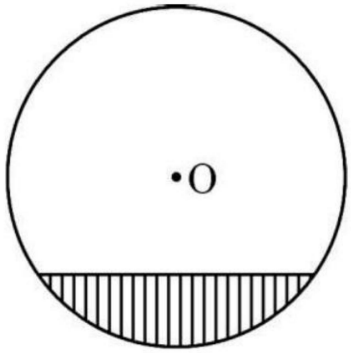
c) $\frac{2}{\sqrt{3}}x$ d) $\frac{\sqrt{3}}{2}x$

14. A man goes to a walking track twice a day in the shape of a sector with an angle of 123^0 and a radius of 138 m. Find the area covered by the man of the walking track in a day. **[1]**

a) 81765.6 m^2 b) 20882.8 m^2

c) 20441.4 m^2 d) 40882.8 m^2

15. In the given figure, the shaded region represents: **[1]**



a) minor segment b) major segment

c) minor sector d) major sector

16. Aarushi sold 100 lottery tickets in which 5 tickets carry prizes. If Priya purchased a ticket, what is the probability of Priya winning a prize? **[1]**

a) $\frac{17}{20}$ b) $\frac{1}{25}$

c) $\frac{19}{20}$ d) $\frac{1}{20}$

17. A number is selected at random from the numbers 3, 5, 5, 7, 7, 7, 9, 9, 9, 9. The probability that the selected number is their average is **[1]**

a) $\frac{7}{10}$ b) $\frac{1}{10}$

c) $\frac{9}{10}$ d) $\frac{3}{10}$

18. The most frequent value in the data is known as **[1]**

a) all the three b) median

c) mode d) mean

19. **Assrtion (A):** A toy is in the form of a cone mounted on a hemisphere with the same radius. The radius of the conical portion is 4 cm and its height is 3 cm. the surface area of the toy is 163.28 cm^2 . [Take $\pi = 3.14$] **[1]**
Reason (R): Volume of hemisphere is $\frac{2}{3}\pi r^2$

a) Both A and R are true and R is the b) Both A and R are true but R is not

correct explanation of A.

the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Common difference of the AP -5, -1, 3, 7, ... is 4. **[1]**

Reason (R): Common difference of the AP a, a + d, a + 2d, ... is given by d = 2nd term - 1st term.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Prove that $\sqrt{2}+\sqrt{3}$ is an irrational number. **[2]**

22. In triangle ABC and DEF, $\angle A = \angle E = 50°$, $\frac{AB}{ED} = \frac{AC}{EF}$ and $\angle F = 65°$. Show that AB = AC. **[2]**

23. The length of a chord of a bigger circle concentric with a circle of radius 3 cm is 8 cm. If the chord is tangent to the smaller circle, then find the radius of the bigger circle. **[2]**

24. If A = 60° and B = 30°, verify that: cos (A + B) = cos A cos B - sin A sin B **[2]**

OR

If $\sin\theta = \frac{a}{\sqrt{a^2+b^2}}, 0 < \theta < 90°$, find the values of $\cos\theta$ and $\tan\theta$

25. In a circle of radius 10 cm, an arc subtends an angle of 108° at the centre. What is the area of the sector in terms of π? **[2]**

OR

The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Section C

26. A wine seller had three types of wine. 403 liters of 1st kind, 434 liters of 2nd kind and 465 liters of 3rd kind. Find the least possible number of casks of equal size in which different types of wine can be filled without mixing. **[3]**

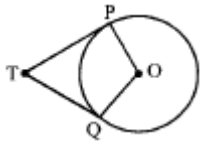
27. The line segment joining the points P (3,3) and Q (6, - 6) is trisected at the points A and B such that A is nearer to P. If A also lies on the line given by 2x + y + k = 0, find the value of k. **[3]**

28. If α, β are roots of the equation $2x^2$ - 6x + a = 0 and $2\alpha + 5\beta = 12$, find the value of a. **[3]**

OR

Solve the equation $\frac{4}{x} - 3 = \frac{5}{2x+3}$; $x \neq 0, -\frac{3}{2}$, for x.

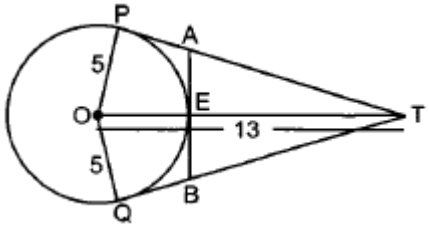
29. In the adjoining figure, TP and TQ are tangents to the circle with centre O such that $\angle POQ = 110°$. Then find $\angle PTQ$. **[3]**



OR

In figure, O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of

AB. where TP and TQ are two tangents to the circle.



30. Prove the trigonometric identity: **[3]**

$\sec^6\theta = \tan^6\theta + 3\tan^2\theta \sec^2\theta + 1$

31. From a window (60 metres high above the ground) of a house in street the angles of elevation and depression of the top and the foot of another house on opposite side of street are 60° and 45° respectively. Show that the height of the opposite house is 60 $(1+\sqrt{3})$ metres. **[3]**

Section D

32. The sum of a 2-digit number and the number obtained by reversing the order of its digits, is 121. The two digits differ by 3. **[5]**

i. Represent the above information in the form of pair of linear equations.

ii. Show that the equations have unique solution.

iii. Solve the equations and find the number.

OR

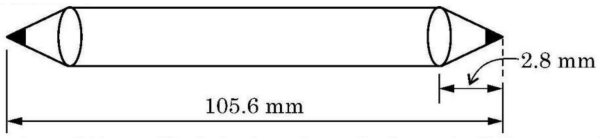
Susan invested certain amount of money in two schemes A and B, which offer interest at the rate of 8% per annum and 9% per annum, respectively. She received ₹ 1860 as annual interest. However, If she had interchanged the amount of investment in the two schemes, she would have received ₹ 20 more as annual interest. How much money did she invest in each scheme?

33. Find the coordinates of the points which divide the line segment joining A(-2, 2) and B(2, 8) into four equal parts. **[5]**

34. From a solid wooden cylinder of height 10 cm and radius 14 cm, a cylinder of radius 7 cm and height 5 cm is scooped out to form a cavity inside the solid cylinder. Find the total surface area of the remaining solid. **[5]**

OR

On the day of her examination, Riya sharpened her pencil from both ends as shown below:



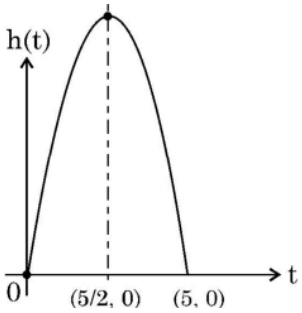
The diameter of the cylindrical and conical part of the pencil is 4.2 mm. If the height of each conical part is 2.8 mm and length of entire pencil is 105.6 mm, find the total surface area of the pencil.

35. In an A.P. of 40 terms, the sum of first 9 terms is 153 and the sum of last 6 terms is 687. Determine the first term and common difference of A.P. Also, find the sum of all the terms of the A.P. **[5]**

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

A ball is thrown in the air so that t seconds after it is thrown, its height h metre above its starting point is given by the polynomial $h = 25t - 5t^2$.



Observe the graph of the polynomial and answer the following questions:

i. Write zeroes of the given polynomial. **(1)**

ii. Find the maximum height achieved by ball. **(1)**

iii. a. After throwing upward, how much time did the ball take to reach to the height of 30 m? **(2)**

OR

b. Find the two different values of t when the height of the ball was 20 m. **(2)**

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Under the physical and health education a medical check up program was conducted in a Vidyalaya to improve the health and fitness conditions of the students. Reading of the heights of 50 students was obtained as given in the table below:



Height (in cm)	Number of students
135-140	2
140-145	8
145-150	10
150-155	15
155-160	6
160-165	5
165-170	4

i. Find the lower class limit of the modal class. (1)

ii. Find the median class. (1)

iii. Find the assumed mean of the data. (2)

OR

Find the median of the given data. (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Ashok wanted to determine the height of a tree on the corner of his block. He knew that a certain fence by the tree was 4 feet tall. At 3 PM, he measured the shadow of the fence to be 2.5 feet tall. Then he measured the tree's shadow to be 11.3 feet.



i. What is the height of the tree? (1)

ii. What will be length of shadow of tree at 12:00 pm? (1)

iii. Write the name triangle formed for this situation. (2)

OR

What will be the length of wall at 12:00 pm? (2)

Solution

Section A

1.

(d) $\frac{5}{11}$

Explanation:

Total number of fish = 15 + 18 = 33

Male fish = 15

Number of possible outcomes = 15

Number of total outcomes = 15 + 18 = 33

Required Probability = $\frac{15}{33} = \frac{5}{11}$

2.

(b) x^2 - 6x + 7 = 0

Explanation:

Let $\alpha = 3 - \sqrt{2}$ and $\beta = 3 + \sqrt{2}$

$\alpha + \beta = (3 - \sqrt{2}) + (3 + \sqrt{2}) = 6$

$\alpha\beta = (3 - \sqrt{2})(3 + \sqrt{2})$

= 9 - 2

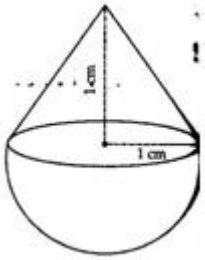
= 7

$\therefore$ Quadratic equation is = $x^2 - (\alpha + \beta)x + \alpha\beta$

Quadratic = x^2 - 6x + 7

3. **(a)** $\pi\ cm^3$

Explanation:



Radii of cone = r = 1 cm

Radius of hemisphere = r = 1 cm (h) = 1cm

Height of cone $(h) = 1$ h =1 cm

Volume of solid = Volume of cone + Volume of a hemisphere

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2 (h + 2r)$

$= \frac{1}{3} \times \pi \times (1)^2 (1 + 2 \times 1)$

$= \frac{1}{3} \times \pi \times 3 = \pi$ cm^3

4.

(d) 2 km/hr

Explanation:

Let the speed of the stream be x km/hr

Speed of the boat upstream = (5 - x)km/hr

Speed of the boat downstream = (5 + x)km/hr

Time taken for going 5.25 km upstream = $\frac{5.25}{5-x}$,hours

Time taken for going 5.25 km downstream = $\frac{5.25}{5+x}$ hours

Obviously, time taken for going 5.25 km upstream is more than the time taken for going 5.25 km, downstream

It is given that the time taken for going 5.25 km, upstream is 1 hour more than the time taken for going 5.25 downstream

$\therefore \frac{5.25}{5-x} - \frac{5.25}{5+x} = 1$

$\Rightarrow 5.25\left\{\frac{1}{5-x} - \frac{1}{5+x}\right\} = 1 \Rightarrow \frac{21}{4}\left(\frac{5+x-5+x}{(5-x)(5+x)}\right) = 1$

$\Rightarrow \frac{21}{4} \times \frac{2x}{25-x^2} = 1 \Rightarrow \frac{21}{2} \times \frac{x}{25-x^2} = 1$

$\Rightarrow 2x^2 + 21x - 50 = 0$

$\Rightarrow x(2x + 25) - 2(2x + 25) = 0 \Rightarrow (2x + 25)(x - 2) = 0$

$\Rightarrow x - 2 = 0, 2x + 25 = 0$

$\Rightarrow x = 2\left[\because x = \frac{-25}{2}\right]$

$\therefore$ speed of the stream = 2km/hr

5. **(a)** 38

Explanation:

Given

First term, a = 1,

Nth term, a_n = 20,

Sum of n terms, S_n = 399

Using the formula

$Sn = \frac{n}{2}(a + l)$

Where S_n = Sum of first n terms

n = no of terms

l = a_n = last term

$2S_n = n(a + a_n)$

2(399) = n(1 + 20)

n = 798/21

n = 38

6. **(b)** ± 3

Explanation:

Point A(x, 4) is on a circle with centre O(0, 0) and radius = 5

$\therefore OA = \sqrt{(x-0)^2 + (4-0)} = \sqrt{x^2 + 16}$

$\therefore \sqrt{x^2 + 16} = 5 \Rightarrow x^2 + 16 = 25$

Squaring on both sides, we get

$\Rightarrow x^2 = 25 - 16 = 9 = (\pm 3)^2$

$\therefore \ x = \pm 3$

7. **(d)** both negative

Explanation:

As the Discriminant of the given quadratic polynomial x^2 + 99x + 127 is more than Zero.

$\therefore$ Both the zeros are negative.

8.

(d) 5 cm

Explanation:

In triangles APB and CPD,

$\angle APB = \angle CPD$ [Vertically opposite]

$\angle$BAP $= \angle$ACD [Alternate angles as AB | | CD]

Then ΔAPB $\sim \Delta$CPD

Therefore, $\frac{\text{AB}}{\text{CD}} = \frac{\text{CP}}{\text{AP}}$

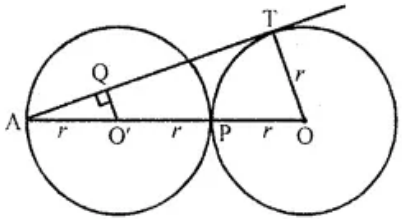
$\Rightarrow \frac{4}{6} = \frac{AP}{7.5}$ therefore AP = 5cm

9.

(b) 3:1

Explanation:

Two circles of equal radii touch each other externally at P. OO' produced meets at A.



From A, AT is the tangent to the circle (O, r)

O'Q $\perp$ AT

Now AO : AO' = 3r : r

= 3 : 1 = 3

10.

(d) 10.4 cm

Explanation:

We know that in a circle Tangents drawn at the ends of Diameter are Parallel.

So the distance between the Parallel tangent is = 2r

$= 2 \times 5.2$

= 10.4 cm

11.

(c) $\frac{m^2-1}{n^2-1}$

Explanation:

Given: tanA = n tanB

$\Rightarrow \frac{1}{\tan \text{B}} = \frac{n}{\tan \text{A}}$

$\Rightarrow \cot \text{B} = \frac{n}{\tan \text{A}}$

And sin A = m sin B

$\Rightarrow \frac{1}{\sin \text{B}} = \frac{m}{\sin \text{A}}$

$\Rightarrow$ cosec B = $\frac{n}{\sin \text{A}}$

Now, $\text{cosec}^2\text{B} - \cot^2\text{B} = 1$

$\Rightarrow \frac{m^2}{\sin^2\text{A}} - \frac{n^2}{\tan^2\text{A}} = 1$

$\Rightarrow \frac{m^2}{\sin^2 A} - \frac{n^2\cos^2 A}{\sin^2 A} = 1$

$\Rightarrow m^2 - n^2\cos^2 A = \sin^2 A$

$\Rightarrow m^2 - n^2\cos^2 A = 1 - \cos^2 A$

$\Rightarrow m^2 - 1 = (n^2 - 1)\cos^2 A$

$\Rightarrow \cos^2 A = \frac{m^2-1}{n^2-1}$

12.

(b) H.C.F. = 1, L.C.M. = p(p + 1)

Explanation:

Since, p is prime

$\therefore$ p and p + 1 has no common factor other than 1.

$\therefore$ H.C.F of p and p + 1 = 1

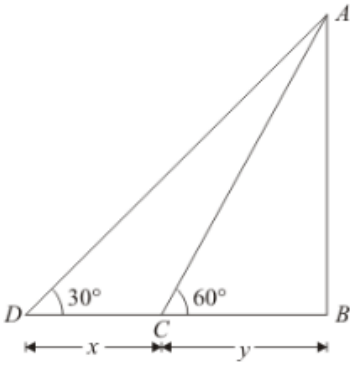
& L.C.M of p and p + 1 = p $\times$ (p + 1) = p(p + 1)

13.

(d) $\frac{\sqrt{3}}{2}x$

Explanation:

Let h be the height of chimney AB



Given that: angle of elevation changes from angle $\angle$D = 30° to $\angle$C = 60°.

Then Distance becomes CD = x and we assume BC = y

Here, we have to find the height of chimney.

So we use trigonometric ratios.

In a triangle ABC,

$\Rightarrow \tan C = \frac{AB}{BC}$

$\Rightarrow \tan 60^\circ = \frac{AB}{BC}$

$\Rightarrow \sqrt{3} = \frac{h}{y}$

$\Rightarrow y = \frac{h}{\sqrt{3}}$

Again in a triangle ABD,

$\Rightarrow \tan D = \frac{AB}{BC+CD}$

$\Rightarrow \tan 30^\circ = \frac{n}{y+x}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{y+x}$

$\Rightarrow \sqrt{3}$ h = y + x

$\Rightarrow \sqrt{3}h = \frac{h}{\sqrt{3}} + x \left[\text{ Put } y = \frac{h}{\sqrt{3}}\right]$

$\Rightarrow h\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = x$

$\Rightarrow h = \frac{x}{\sqrt{3}-\frac{1}{\sqrt{3}}}$

$\Rightarrow h = \frac{\sqrt{3}x}{2}$

14.

(d) 40882.8 m^2

Explanation:

The area of the sector $= \frac{x^\circ}{360^\circ} \times \pi r^2$

$= \frac{123^\circ}{360^\circ} \times \frac{22}{7} \times 138^2$

= 20441.4 m^2

Area covered by the man of the walking track in a day = 20441.4 + 20441.4

= 40882.8 m^2

15. **(a)** minor segment

Explanation:

minor segment

16.

(d) $\frac{1}{20}$

Explanation:

No. of lottery tickets = 100

No. of tickets carrying prizes = 5

∴ Probability of ticket buying a prized one

$\frac{m}{n} = \frac{5}{100} = \frac{1}{20}$

17.

(d) $\frac{3}{10}$

Explanation:

Total numbers are $\Sigma x_i = 10$

x	f
3	1
5	2
7	3
9	4

Average = $\frac{3\times1+5\times2+7\times3+9\times4}{10}$

= $\frac{3+10+21+36}{10} = \frac{70}{10} = 7$

∴ m = 3

∴ Probability of average number = $\frac{3}{10}$

18.

(c) mode

Explanation:

The most frequent value in the data is known as the Mode. e.g let us consider the following data set: 3,5,7,5,9,5,8,4 the mode is 5 since it occurs most often in data set.

19.

(c) A is true but R is false.

Explanation:

A is true but R is false.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Common difference, d = -1 - 1(-5) = 4

So, both A and R are true and R is the correct explanation of A.

Section B

21. Let us assume that $\sqrt{2}+\sqrt{3}$ is a rational number

Let $\sqrt{2}+\sqrt{3}=\frac{a}{b}$ Where a and b are co-prime positive integers

On squaring both sides, we get

$(\sqrt{2}+\sqrt{3})^2=\frac{a^2}{b^2}$

$2+3+2\sqrt{6}=\frac{a^2}{b^2}$

$5+2\sqrt{6}=\frac{a^2}{b^2}$

$2\sqrt{6}=\frac{a^2}{b^2}-5$

$2\sqrt{6}=\frac{a^2-5b^2}{b^2}$

$\sqrt{6}=\frac{a^2-5b^2}{2b^2}$

Now $\frac{a^2-5b^2}{2b^2}$ is a rational number.

This shows that $\sqrt{6}$ is a rational number.

But this contradicts the fact that $\sqrt{6}$ is an irrational number.

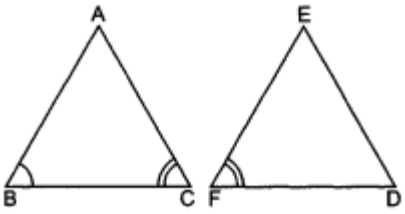
This contradiction has raised because we assume that $\left(\sqrt{2}+\sqrt{3}\right)$ is a rational number.

Hence, our assumption is wrong and $\left(\sqrt{2}+\sqrt{3}\right)$ is an irrational number.

22. In ABC and EDF

$\angle A = \angle E = 50°$ (Given)

$\frac{AB}{ED}=\frac{AC}{EF}$



$\therefore \quad \triangle ABC \sim \triangle EDF$

$\Rightarrow \quad \angle C = \angle F = 65^\circ$

$\therefore \quad \angle B = 180^\circ - 50^\circ - 65^\circ = 65^\circ$

$\Rightarrow \quad \angle \mathbf{B} = \angle \mathbf{C}$

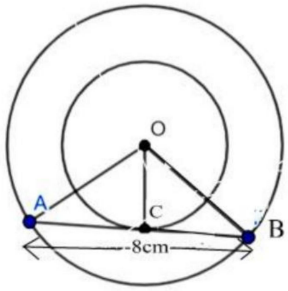
$\Rightarrow$ AB = AC (sides opp. to equal angles)

23. Clearly $AC = BC = 4\ cm$ (Perpendicular from centre of a circle to the chord bisects the chord)

$OB^2 = OC^2 + BC^2$(By Pythagoras theorem in ΔOBC)

$= 3^3 + 4^2 = 25$

$OB = 5$



$\therefore$ radius of the bigger circle is 5 cm

24. To verify: cos (A + B) = cos A cos B - sin A sin B
If A = 60° and B = 30°, then
To verify: cos (90°) = cos 60° cos 30° - sin60° sin 30°
Consider R.H.S. = cos 60° cos 30° - sin 60° sin 30°
= (1/2) $\times$ ($\sqrt{3}$/2) - ($\sqrt{3}$/2)(1/2)
= ($\sqrt{3}$/4) - ($\sqrt{3}$/4)
= 0 = cos 90° = L.H.S.
$\therefore$ L.H.S. = R.H.S.
Hence, verified.

OR

We have, $\sin\theta = \frac{a}{\sqrt{a^2+b^2}}$

$\therefore \quad \cos\theta = \sqrt{1-\sin^2\theta}$

$\Rightarrow \cos\theta = \sqrt{1-\frac{a^2}{a^2+b^2}}$

$= \sqrt{\frac{a^2+b^2-a^2}{a^2+b^2}} = \sqrt{\frac{b^2}{a^2+b^2}} = \frac{\sqrt{b^2}}{\sqrt{a^2+b^2}} = \frac{b}{\sqrt{a^2+b^2}}$

and, we know that

$\tan\theta = \frac{\sin\theta}{\cos\theta} \Rightarrow \tan\theta = \frac{a/\sqrt{a^2+b^2}}{b/\sqrt{a^2+b^2}} = \frac{a}{b}$

$\therefore \ \tan\theta = \frac{a}{b}$

25. We have given the radius of the circle and angle subtended at the center of the circle.
r = 10 cm
$\theta = 108^\circ$
Now we will find the area of the sector,
Area of the sector $= \frac{\theta}{360} \times \pi r^2$
Substituting the values we get,
$= \frac{108}{360} \times \pi \times 10^2$ ...(1)
Now we will simplify the equation (1) as below,
$= \frac{3}{10} \times \pi \times 100$
$= 3 \times \pi \times 10$
= 30π
Therefore, the area of the sector is 30π cm^2

OR

Here, r = 14 cm and $\theta = \frac{90^\circ}{3} = 30^\circ$
$\therefore$Area swept = $\frac{\theta}{360^\circ} \times \pi r^2$
=$\frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14$
= $\frac{154}{3}$ cm^2

Section C

26. For the least possible number of casks of equal size, the size of each cask must be of the greatest volume.
To get the greatest volume of each cask, we have to find the largest number which exactly divides 403, 434 and 465. That is nothing but the H.C.F of (403, 434, 465)
The H.C.F of (403, 434, 465) = 31 liters
Each cask must be of the volume 31 liters.
Req. No. of casks is
$= (\frac{403}{31}) + (\frac{434}{31}) + (\frac{465}{31})$
$= 13 + 14 + 15$
$= 42$
Hence, the least possible number of casks of equal size required is 42.

27. 

PQ is the line segment, A and B are the points of trisection of PQ.
We know that PA : QA = 1:2
So, the coordinates of A are
$\left(\frac{6\times1+3\times2}{2+1}, \frac{-6\times1+3\times2}{2+1}\right)$
$= \frac{12}{3}, 0$
$= (4, 0)$
Since, A lies on the line
2x + y + k = 0
$\Rightarrow 2 \times 4 + 0 + k = 0$
$\Rightarrow 8 + k = 0$
$\Rightarrow k = -8$

28. According to the question,
quadratic equation is $2x^2 - 6x + a = 0$
$\therefore \alpha, \beta$ are roots of the equation
$\therefore \alpha + \beta = \frac{-b}{a}$
$\Rightarrow \alpha + \beta = -\frac{-6}{2}$
$\Rightarrow \alpha + \beta = 3$
$\Rightarrow \alpha = 3 - \beta$ ...(i)
Also $2\alpha + 5\beta = 12$
$\Rightarrow 2(3 - \beta) + 5\beta = 12$ [using equation (i)]
$\Rightarrow 6 - 2\beta + 5\beta = 12$
$\Rightarrow 3\beta = 6$
$\Rightarrow \beta = 2$
where $\beta = 2$, eq. (i) becomes $\alpha = 3 - 2 = 1$
Now, product of roots = $\frac{c}{a}$
$\Rightarrow \alpha \cdot \beta = \frac{a}{2} \Rightarrow 1 \times 2 = \frac{a}{2} \Rightarrow a = 4.$

OR

The given equation is:
$\frac{4}{x} - 3 = \frac{5}{2x+3}$, $x \neq 0, -\frac{3}{2}$
$\Rightarrow \frac{4-3x}{x} = \frac{5}{2x+3}$
$\Rightarrow (4 - 3x)(2x + 3) = 5x$
$\Rightarrow 8x + 12 - 6x^2 - 9x = 5x$
$\Rightarrow -6x^2 - 6x + 12 = 0$

$\Rightarrow$ - 6 (x^2 + x - 2) = 0

$\Rightarrow$ x^2 + x - 2 = 0

Factorize the above quadratic equation we get

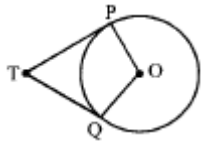
x^2 + 2x - x - 2 = 0

$\Rightarrow$x(x + 2) - 1(x + 2) = 0

$\Rightarrow$(x + 2)(x - 1) = 0

Therefore, Either x + 2 = 0 or x - 1 = 0

$\Rightarrow$x = -2, 1.

29. 

Since, the tangent at any point of a circle is perpendicular to the radius through the point of contact.

PT and QT are tangents to the circle with centre O.

$\therefore$ OP $\perp$ PT and OQ $\perp$ QT

i.e., $\angle$POQ = 110°, $\angle$OPT = 90° $\angle$OQT = 90°

In quad OPTQ we have,

$\angle$POQ + $\angle$OPT + $\angle$OQT + $\angle$PTQ = 360°

$\angle$PTQ = 360° – (110° + 90° + 90°)

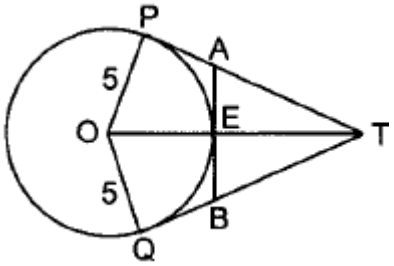
$\angle$PTQ = 360° – 290°

$\angle$PTQ = 70°

OR

According to the question,

O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects circle at E.



$\because$ OP $\perp$ TP [Radius from point of contact of the tangent]

$\therefore$ $\angle$OPT = 90°

In right $\triangle$OPT *

$OT^2 = OP^2 + PT^2$

$\Rightarrow (13)^2 = (5)^2 + PT^2 \Rightarrow$ PT = 12 cm

Let AP = x cm AE = AP $\Rightarrow$ AE = x cm

and AT = (12 - x)cm

TE = OT - OE = 13 - 5 = 8 cm

$\because$OE $\perp$ AB [Radius from the point of contact]

$\therefore$ $\angle$AEO = 90° $\Rightarrow$ $\angle$AET = 90°

In right $\triangle$AET,

$AT^2 = AE^2 + ET^2$

$(12 - x)^2 = x^2 + 8^2$

$\Rightarrow 144 + x^2 - 24x = x^2 + 64$

$\Rightarrow \quad 24x = 80 \Rightarrow x = \frac{80}{24} = \frac{10}{3}$cm

Also BE = AE = $\frac{10}{3}$ cm

$\Rightarrow \quad AB = \frac{10}{3} + \frac{10}{3} = \frac{20}{3}$ cm

30. We have to prove that :-

$\Rightarrow \sec^6\theta = \tan^6\theta + 3\tan^2\theta\sec^2\theta + 1$

$\Rightarrow \sec^6\theta - \tan^6\theta = 3\tan^2\theta\sec^2\theta + 1$

Now, LHS

$= \sec^6\theta - \tan^6\theta$

$= \left(\sec^2\theta\right)^3 - (\tan^2\theta)^3$

$= \left(\sec^2\theta - \tan^2\theta\right)\left[\left(\sec^2\theta\right)^2 + \sec^2\theta\tan^2\theta + \left(\tan^2\theta\right)^2\right]$ {Since, $a^3 - b^3 = (a - b)(a^2 - ab + b^2)$}

$= 1\left[\sec^4\theta + \sec^2\theta\tan^2\theta + \tan^4\theta\right] \quad \left[\because \sec^2\theta - \tan^2\theta = 1\right]$

$= \sec^4\theta + \tan^4\theta + \sec^2\theta\tan^2\theta$

$= (\sec^2\theta)^2 + (\tan^2\theta)^2 + \sec^2\theta\tan^2\theta$

Adding and subtracting $2\sec^2\theta\tan^2\theta$

$= (\sec^2\theta)^2 + (\tan^2\theta)^2 - 2\sec^2\theta\tan^2\theta + 2\sec^2\theta\tan^2\theta + \sec^2\theta\tan^2\theta$

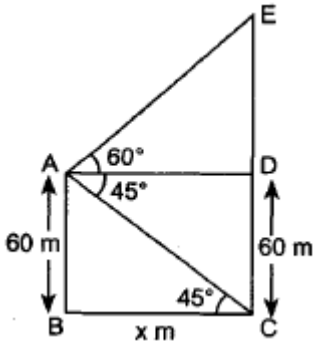
$= (\sec^2\theta - \tan^2\theta)^2 + 3\sec^2\theta\tan^2\theta \quad \left[\because (a-b)^2 = a^2 + b^2 - 2ab\right]$

$= 1 + 3\sec^2\theta\tan^2\theta \quad \left[\because \sec^2\theta - \tan^2\theta = 1\right]$

= RHS

Hence proved

31.



Let A be the window and CE be the opposite house

CD = AB = 60 m.....(i)

In rt. $\triangle$ABC, $\tan 45^\circ = \frac{60}{\text{BC}}$

$\Rightarrow \quad 1 = \frac{60}{\text{BC}}$

$\Rightarrow \quad \text{BC} = 60\text{m}$ ..... (ii)

AD = BC

$\therefore \quad AD = 60\text{m}$ [From(ii)] ....(iii)

In rt. $\triangle$ADE, $\tan 60^\circ = \frac{\text{DE}}{\text{AD}}$

$\Rightarrow \quad \sqrt{3} = \frac{DE}{60}$ [From (iii)]

$\Rightarrow \quad DE = 60\sqrt{3}\text{m}$

$\therefore$ Height of the opposite house

$CE = CD + DE = 60 + 60\sqrt{3}$

$= 60(1 + \sqrt{3})\text{m}$

Section D

32. Let the unit digit be y and tens digit be $x(x > y)$

The two-digit number will be $10x + y$

A.T.Q. $(10x + y) + (10y + x) = 121$

i. $\Rightarrow x + y = 11$ ...(i)

and $x - y = 3$ ...(ii)

ii. $\frac{1}{1} \neq \frac{1}{-1}$ therefore equations have unique solution

iii. Solving equations (1) and (2), we get $x = 7, y = 4$

$\therefore$ Number is 74

47 may be considered as the correct answer if $y > x$.

OR

Suppose she inverted x in Scheme A and y in Scheme B. Then,

$\frac{8x}{100} + \frac{9y}{100} = 1860$ and $\frac{8y}{100} + \frac{9x}{100} = 1880$

$\Rightarrow$ 8x + 9Y = 186000 and 9x + 8y = 188000

Adding and subtracting these two equations, we obtain

17(x + y) = 374000 and -x + y = 2000

$\Rightarrow$ x + y = 22000 .....(i)

and -x + y = 2000 .....(ii)

On multiplying Eq. (i) by 9 and Eq. (ii) by 8 and then subtracting them, we get

17y = 1000 [(9 $\times$ 186) - (8 $\times$ 188)]

= 1000 (1674 - 1504) = 1000 $\times$ 170

17y = 170000 $\Rightarrow$ y = 10000

On putting the value of y in Eq. (i), we get

8x + 9 $\times$ 10000 = 186000

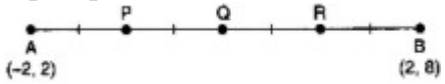
8x = 186000 - 90000

8x = 96000

x = 12000

So, the money invested in scheme A = ₹ 12,000 and in scheme B = ₹ 10,000

33. Let P (x_1, y_1) Q(x_2, y_2) and R(x_3, y_3) be the points which divide the line segment AB into four equal parts.



Then, P divides AB in the ratio 1 : 3 internally.

$x = \frac{mx_2 + nx_1}{m+n}$

$\therefore x_1 = \frac{(1)(2)+(3)(-2)}{1+3}$

$= \frac{2-6}{4} = -\frac{4}{4} = -1$

$y = \frac{my_2 + ny_1}{m+n}$

$y_1 = \frac{(1)(8)+(3)(2)}{1+3}$

$= \frac{8+6}{4} = \frac{14}{4} = \frac{7}{2}$

So, $\mathrm{P} \rightarrow \left(-1, \frac{7}{2}\right)$

Also, Q divides AB in the ratio 1 : 1 i.e.

Q is the mid point of AB

$x_2 = \frac{-2+2}{2} = 0$

$y_2 = \frac{2+8}{2} = \frac{10}{2} = 5$

So, $Q \rightarrow (0, 5)$

and, R divides AB in the ratio 3 : 1

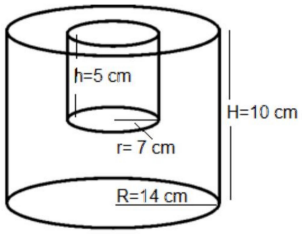
$\therefore x_2 = \frac{(3)(2)+(1)(-2)}{3+1}$

$= \frac{6-2}{4} = \frac{4}{4} = 1$

$y_3 = \frac{(3)(8)+(1)(2)}{3+1}$

$= \frac{24+2}{4} = \frac{26}{4} = \frac{13}{2}$

So, R $\rightarrow \left(1, \frac{13}{2}\right)$

34. 

Total surface area of the remaining solid

$=$ CSA of outer cylinder $+$ CSA of inner cylinder $+$ area of bases $+$ Area of ring

$= 2\pi RH + 2\pi r\ h + \left(\pi R^2 + \pi r^2\right) + \left(\pi R^2 - \pi r^2\right)$

$= 2\pi RH + 2\pi r\ h + 2\pi R^2$

$= 2 \times \frac{22}{7} \times 14 \times 10 + 2 \times \frac{22}{7} \times 7 \times 5 + 2 \times \frac{22}{7} \times 14 \times 14$

$= 880 + 220 + 1232$

$= 2332$ sq. cm

OR

$d = 4.2\ mm$

$r = 2.1\ mm$

Height of cylindrical part (h) $= 100\ mm$

Height of conical part $= 2.8\ mm$

Slant height $(l) = \sqrt{(2.8)^2 + (2.1)^2} = 3.5\ mm$

Curved Surface Area of Cylinder $= 2 \times \frac{22}{7} \times 2.1 \times 100 = 1320 mm^2$

Curved Surface Area of Cone $= \frac{22}{7} \times 2.1 \times 3.5 = 23.1 mm^2$

Total Surface Area of pencil = Curved Surface Area of cylinder + Curved Surface Area of two cones

$= 1366.2 mm^2$

35. Here n = 40,

$S_9 = \frac{9}{2}[2a + 8d]$ = 153 $\Rightarrow$ a + 4d = 17 ...(i)

and $S_{40} - S_{34}$ 687 or $a_{35} + a_{36} + a_{37} + a_{38} + a_{39} + a_{40} = 687$

$\Rightarrow$ 6a + 219d = 687 or 2a + 73d = 229 ...(ii)

solving (i) and (ii) to get a = 5, d = 3

Also, $S_{40} = \frac{40}{2}(10 + 39 \times 3)$ = 2540

Section E

36. i. Zeroes of the polynomial are 0 and 5

ii. Maximum height achieved by ball

$= 25 \times \frac{5}{2} - 5 \times \left(\frac{5}{2}\right)^2$

$= \frac{125}{4}$ or 31.25 m

iii. a. $-5t^2 + 25t = 30$

$\Rightarrow t^2 - 5t + 6 = 0$

$\Rightarrow$ (t - 2)(t - 3) = 0

$t \neq 3, t = 2$

OR

b. $-5t^2 + 25t = 20$

$\Rightarrow t^2 - 5t + 4 = 0$

$\Rightarrow$ (t - 4)(t - 1) = 0

$\Rightarrow$ t = 4, 1

37. i. The maximum class frequency is 15 belonging to class interval 150-155

$\therefore$ 150 - 155 is the modal class

lower limit (l) of modal class =150

ii.

Height (in cm)	frequency	C.F
135-140	2	2
140-145	8	10
145-150	10	20
150-155	15	35
155-160	6	41
160-165	5	46
165-170	4	50
	$\sum fi = 50$	

$\sum fi$ = 2 + 8 + 10 + 15 + 6 + 5 + 4 =50 =N

$\frac{N}{2} = \frac{50}{2}$ = 25

c.f just greater that $\frac{N}{2}$ i.e, 25 is 35

$\therefore$ Median class 150-155

iii.

Height (in cm)	frequency (f_i)	x_i
135-140	2	137.5
140-145	8	142.5
145-150	10	147.5
150-155	15	**152.5**
155-160	6	157.5
160-165	5	162.5
165-170	4	167.5

$x_i = \frac{\text{lower limit+upper limit}}{2}$

middle term of x_i,is the assumean mean

Hence, Assumed Mean = 152.5

OR

Median = $l\left(\frac{\frac{n}{2} - c.f}{f}\right) \times h$

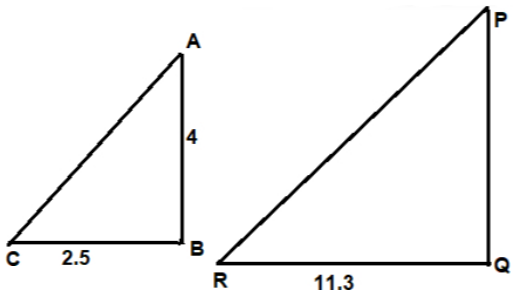
= 150 + $\left(\frac{25-20}{15}\right) \times 5$

= 150 + $\frac{5}{15} \times 5$

= 150 + $\frac{5}{3}$

= 150 + 1.67

= 151.67

38. i. 

Let AB be a wall and PQ is a tree

BC and QR are their shadow respectively at 3 p.m.

$\therefore \triangle ABC \sim \triangle PQR$

$\therefore \frac{AB}{PQ} = \frac{BC}{QR}$

$\frac{4}{PQ} = \frac{2.5}{11.3}$

$2.5 \times PQ = 4 \times 11.3$

$PQ = 18.08$

$\therefore$ height of tree = 18.08 feet

ii. 0

iii. Right triangle

OR

Zero