SUBJECT: MATHEMATICS (041)
SAMPLE QUESTION PAPER
CLASS- XII (2026 - 27)

Maximum Marks:80 **Time Allowed: 3 Hours**

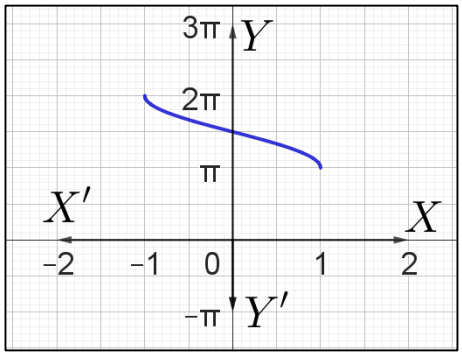
General Instructions:

Read the following instructions very carefully and strictly follow them:

1. This question paper contains 38 questions. All questions are compulsory.
2. This question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Question no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Question no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Question no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Question no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Question no. 36 to 38 are case study-based questions carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one sub-part each in 2 questions of Section E.
9. Draw neat and clean figures wherever required. Take $\pi = \frac{22}{7}$ wherever required, if not stated.
10. Use of calculators is not allowed.

SECTION – A **(20 x 1 = 20)**

This section comprises of 18 multiple choice questions and two assertion and reason type questions of 1 mark each.

Q.No.	Questions	Marks
Q1.	Identify the function whose graph is given below:  (A) $\cos^{-1}\left(\frac{x}{2}\right)$ (B) $\cos^{-1}(2x)$ (C) $\pi - \cos^{-1} x$ (D) $\pi + \cos^{-1} x$	**1**

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	For Visually Impaired: Find the range of $\frac{\pi}{2} + \sec^{-1}\left(\frac{x}{2}\right)$ (A) $[0, \pi]$ (B) $[0, \pi] - \left\{\frac{\pi}{2}\right\}$ (C) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ (D) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right] - \{\pi\}$	
Q2.	Domain of the function: $\sec^{-1}(2x)$ is (A) $(-\infty, \infty)$ (B) $\left(-\frac{1}{2}, \frac{1}{2}\right)$ (C) $\left(-\infty, -\frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$ (D) $\left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$	**1**
Q3.	Value of $\cot^{-1}\left(2\cos\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right)$ (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$	**1**
Q4.	If $A = \begin{bmatrix} 2 & 1 & -2 \\ 3a & 0 & 4 \\ b & 2c & 1 \end{bmatrix}$ is a symmetric matrix, then value of $\sqrt{6a + 2b + 3c}$ is: (A) -2 (B) 2 (C) ± 2 (D) 4	**1**
Q5.	If P and Q are matrices of same order such that $P = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$, $\lvert Q \rvert = 3$, then find $\lvert adj(PQ) \rvert$ (A) -3 (B) 1 (C) 6 (D) 36	**1**
Q6.	If $\begin{bmatrix} 1 & x \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \end{bmatrix} = O$, then value of x is (A) -2 (B) -1 (C) 1 (D) 2	**1**
Q7.	The derivative of $\cos^{-1}(2x^2 - 1)$ with respect to $\cos^{-1}(x)$ is (A) 2 (B) ± 1 (C) $\frac{1}{2}$ (D) -2	**1**
Q8.	If a, b, c are arbitrary constants such that the function defined by $f(x) = \begin{cases} ax^2 + b\,,\ b \neq 0\ ,\ x \leq 1 \\ bx^2 + ax + c\,, x > 1 \end{cases}$ is differentiable at $x = 1$, then which of the following could be a triplet for (a, b, c)? (A) (3,2,0) (B) (0,0,3) (C) $\left(\frac{3}{2}, 2, 0\right)$ (D) $\left(3, \frac{3}{2}, 0\right)$	**1**
Q9.	If $y = a\left\{x + 2a\ \sin\left(\frac{x}{a}\right)\right\}, a > 0, x \in [0, \pi a]$; is a decreasing function, then the interval in which x lies is (A) $\left(\frac{5\pi}{6}, \pi\right]$ (B) $\left(\frac{2\pi}{3}, \pi\right]$ (C) $\left(\frac{5\pi a}{6}, \pi a\right]$ (D) $\left(\frac{2\pi a}{3}, \pi a\right]$	**1**

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Q10.	The values of x for which the derivative of the function $y = x^4$ is greater than the value of the function are (A) $0 < x < 4$ (B) $0 < x \leq 4$ (C) $x < 0 \; or \; x > 4$ (D) $x < 0 \; or \; x \geq 4$	**1**
Q11.	The degree and order of the differential equation $y - x\left(\frac{dy}{dx}\right)^3 = a\left(y + \frac{d^2y}{dx^2}\right)^2$ are respectively (A) $3, 1$ (B) $2, 2$ (C) $3, 2$ (D) $4, 2$	**1**
Q12.	If $\int \frac{1}{1+e^{2x}} dx = x - P \; log(1 + e^{2x}) + C$, then '$P$' is equal to (A) $-\frac{1}{2}$ (B) $-\frac{1}{3}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$	**1**
Q13.	If vector $2\hat{\imath} - u\hat{\jmath} + 3\hat{k}$ has magnitude 4, then value of 'u' is (A) 1 (B) $\sqrt{2}$ (C) $\pm\sqrt{3}$ (D) ± 2	**1**
Q14.	Angle made by the vector $\hat{\imath} + \hat{\jmath} + \sqrt{2}\,\hat{k}$ with positive direction of z-axis is (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$	**1**
Q15.	A vector perpendicular to the vectors $\hat{\imath} + \hat{\jmath}$ and $\hat{\imath} - \hat{\jmath}$ is (A) $-2\,\hat{\jmath}$ (B) $3\hat{k}$ (C) $2\hat{\imath}$ (D) $2\hat{\imath} + \hat{k}$	**1**
Q16.	The probability that it rains on a particular day is $\frac{1}{5}$. Assuming weather conditions of any two days are independent, find the probability that it rains on three consecutive days. (A) $\frac{4}{5}$ (B) $\frac{1}{5}$ (C) $\frac{1}{25}$ (D) $\frac{1}{125}$	**1**
Q17.	A die is thrown twice. What is the probability that at least one number is 4, given the sum of the numbers appearing is a multiple of 5? (A) $\frac{3}{5}$ (B) $\frac{4}{5}$ (C) $\frac{3}{7}$ (D) $\frac{4}{7}$	**1**
Q18.	A and B are two independent events such that $P(A) = \frac{1}{6}, P(B) = k$. If the probability of occurrence of exactly one of them is $\frac{1}{3}$, then find the value of k. (A) $\frac{1}{4}$ (B) $\frac{2}{3}$ (C) $\frac{3}{4}$ (D) $\frac{5}{6}$	**1**

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	Assertion-Reason Based Questions **Directions: Question numbers 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R).** **Select the correct answer from the options (A), (B), (C) and (D) as given below:** (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A). (B) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A). (C) Assertion (A) is true but Reason (R) is false. (D) Assertion (A) is false but Reason (R) is true.	
Q19.	Assertion: The value of the integral $\int_{-1}^{1}\|x-2\|\,dx$ is 4 Reason: If $f(-x) = f(x)$ then $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$	**1**
Q20.	Assertion: The general solution of a differential equation represents a family of curves. Reason: The derivative of an arbitrary constant is zero.	**1**
SECTION – B (5 x 2 = 10) **This section comprises of 5 very short answer (VSA) type questions of 2 marks each**		
Q21 (A).	Evaluate $\int \frac{1}{x\sqrt{x^6-1}}dx$	**2**
	OR	
Q21 (B).	Evaluate $\int \frac{\cos x-\sin x}{\sqrt{\sin(2x)}}\,dx$	
Q22.	If $y = \sin(x+y)$ and $\frac{dy}{dx} = 0$, then prove that $y = \pm 1$.	**2**
Q23 (A).	A line passing through the points $(2,6,5), and\ (3,p,7)$ is parallel to the line joining the points $(-4,3,4)$ and $(-1,0,q)$. Find the value of $\frac{q}{p}$.	**2**
	OR	
Q23 (B).	A rectangle ABCD is such that $\overrightarrow{AC} = \vec{u}, and\ \overrightarrow{BD} = \vec{v}$, then find the vectors representing four sides of the rectangle (in a cyclic order starting from $\overrightarrow{AB}$).	
Q24.	For two non-zero vectors $\vec{a}$ and $\vec{b}$, $\|\vec{a}\| = \sqrt{6}$, $\vec{a}\cdot\vec{b} = 1$ and $\vec{b}\times\vec{a} = 3\hat{\imath} + 4\hat{\jmath} - 2\hat{k}$. Find $\|\vec{b}\|$.	**2**

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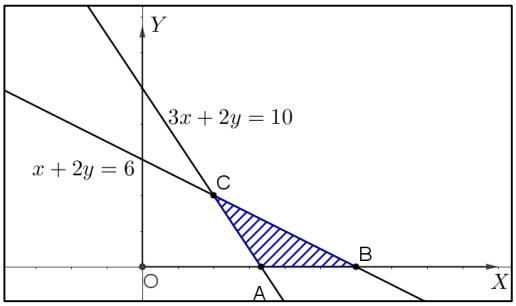
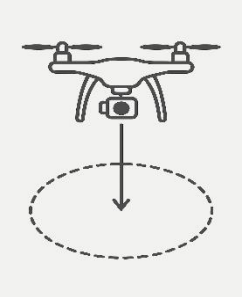
Q25.	The feasible region of a linear programming problem is shown in the given figure:  Write all the constraints on the decision variables involved **For Visually Impaired:** For a linear programming problem, the maximum value of an objective function $Z = 3x + 4y$ exists at the points (p, q) and $\left(2p, \frac{q}{2}\right)$. Find the relation between p and q.	**2**
	SECTION – C (6 x 3 = 18) **This section comprises of 6 short answer (SA) type questions of 3 marks each.**	
Q26.	For two matrices, $P = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ and $Q = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 0 \end{bmatrix}$, find PQ and QP, whichever is defined. Also, find the inverse of the product matrix, if it exists.	**3**
Q27.	The circular area of vision on the ground by a surveillance drone fitted with camera increases at the rate of $44\ m^2$ per metre as it gains vertical height. The radius of the circle on the ground increases at the rate of 6 m/s. Find the rate at which the height of the drone increases with time, at the instant when the radius of the circular area on the ground is 14 m. $\left(Use\ \pi = \frac{22}{7}\right)$ 	**3**
Q28 (A).	If $x = sin^3 t, y = cos^3 t$, $Evaluate\ \frac{d^2y}{dx^2}\ at\ \left(\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}\right)$.	**3**
	OR	
Q28 (B).	If $y = x^{logx} + (logx)^x$; $x > 1$, then evaluate $\frac{dy}{dx}$	
Q29 (A).	Find the particular solution of the differential equation $xdy + (y - x^2e^x)dx = 0$, given that y (1) =0	**3**
	OR	
Q29 (B).	Find the particular solution of the differential equation $(x^2 + y^2)dy = xydx$, given that $y(0) = 1$	

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Q30 (A).	Find the foot of the perpendicular drawn from the point (2,1,4) on the line $x-2=\frac{2y+3}{4}=2-z$. Also find the equation of the line passing through this point and the point (2,1,4).	**3**
	OR	
Q30 (B).	Find the angle between the lines: $\frac{x-1}{2}=\frac{y-1}{2}=\frac{z-2}{1}$ and $\frac{x-5}{2}=y-\frac{1}{2}=\frac{3-z}{2}$. Also find the cartesian equation of a line passing through the point (1,1,1) and perpendicular to both lines.	
Q31.	Solve the linear programming problem graphically: Maximize $Z=40x+30y$ Subject to constraints: $x+y\leq 6, 2x\geq y, y\geq 2$ **For Visually Impaired:** For a linear programming problem, vertices of the feasible region are $(20,0),(40,0),(30,10),(0,20)$ and $(0,10)$. Minimize and maximize the objective function $Z=50x-20y$	**3**
	SECTION – D (4 x 5 = 20) **This section comprises of 4 long answer (LA) type questions of 5 marks each**.	
Q32 (A).	Let $A=N\times N$, where N is the set of natural numbers and a relation R is defined on A such that $(a,b)R(c,d)\ if\ and\ only\ if\ a^2+d^2=b^2+c^2$. Check whether R is an equivalence relation or not?	**5**
	OR	
Q32 (B).	A relation $R:(-\infty,\infty)\to[0,\infty)$ is defined as $R=\{(x,y):y=x^2+5\}$ Draw its graph and check whether it is a function or not. If the given relation is a function, then examine whether the function is injective, and whether it is surjective or not. **For Visually Impaired:** A relation $R:(-\infty,\infty)\to[0,\infty)$ is defined as $R=\{(x,y):y=x^2+7\}$. Check whether it is a function or not. If the given relation is a function, then examine whether the function is injective, surjective, or both, and conclude whether it is bijective or not.	
Q33.	Evaluate $\int_{-a}^{a}\sqrt{\frac{a-x}{a+x}}\,dx$	**5**
Q34 (A).	Using Integration, find the area of the region $\{(x,y):0\leq y\leq\sqrt{4-x^2}, x\geq 2-y\}$.	**5**
	OR	
Q34 (B).	Using Integration, find the area of the region bounded by the curve $y^2=2x$ and the line $x-2y=0$.	

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	For Visually Impaired:	
Q34 (A).	Using Integration, find the area of the region enclosed within $25x^2 + 16y^2 = 400$	**5**
	OR	
Q34 (B).	Using Integration find the area of the region in the first quadrant bounded by the curve $y^2 = 8x$, and the lines $x = 2, x = 5$ and x-axis.	
Q35.	A fitness app syncs with four types of smartwatches: Alpha, Beta, Gamma and Delta. 45% of the users use Alpha, 30% use Beta,15% use Gamma and 10% use Delta. Due to device errors, the probabilities of getting a false heart-rate alert from the Alpha, Beta, Gamma and Delta smartwatches are respectively 0.01, 0.02, 0.05, and 0.01. Based on the given information, answer the following questions: (i) Find the probability that a randomly selected user gets a false alert. (ii) If a false alert occurs, what is the probability that it came from a Beta smartwatch?	**5**

SECTION – E (3 x 4 = 12)

This section comprises of 3 case-study/passage-based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub parts of 2 marks each.

Q36. Two security sensors in a multi-storey mall emit laser beams for movement detection. Three of these beams travel along the following lines:

$l_1: \vec{r} = (\lambda + 1)\hat{\imath} + (3\lambda + 2)\hat{\jmath} + (3 - \lambda)\hat{k}$ and

$l_2: \vec{r} = 2(\mu + 1)\hat{\imath} + (3\mu - 1)\hat{\jmath} + (\mu + 1)\,\hat{k}$

$l_3: \vec{r} = (\gamma + 3)\hat{\imath} + (3\gamma - 1)\hat{\jmath} + (1 - \gamma)\,\hat{k}$



Based on the given information, answer the following questions:

(i) Show that the lines l_1 and l_2 are not parallel to each other. **1**

(ii) Show that the lines l_1 and l_3 are parallel to each other. **1**

(iii) (a) Find the shortest distance between the lines l_1 and l_2.

OR

(b) Find the shortest distance between the lines l_1 and l_3. **2**

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Q37.	Water is essential for life, and rivers are vital water sources across the world. The Yamuna Action Plan (YAP), one of India's largest river restoration projects, was launched to clean the Yamuna, but its effectiveness (WQI) the Water Quality Index must be measured through cost-benefit, economic and environmental analysis. Therefore, comprehensive pollutant treatment, prevention of further contamination, and community awareness are crucial for sustainable river management. Implementing the same concept to the river Yamuna, if an initial base line WQI is 35, a quadratic expression is formed to measure the impact of investment of ₹x crore on the WQI i.e., $W(x)$ as $W(x) = 35 + 1.6x - 0.02x^2$, where x is in Rupees in crores  Use the given $W(x)$ to answer the following questions: (i) Find the value of x at the stationary point for the function $W(x)$. (ii) Find the maximum value of the WQI using the second derivative test? (iii) (a) If the investment is ₹ 35 crores, find by how much the WQI is short of its maximum achievable value. **Or** (b) Evaluate the WQI when the investment is ₹60 crore. Determine whether this indicates an enhancement or a diminishing return in our objective when compared with the optimum WQI.	**1** **1** **2**
Q38.	To celebrate their child's birthday, a family decided to distribute fruits among children in three orphanages. The fruit vendor packed three different types of baskets, according to the quantity required at each orphanage, details are given as follows: Basket A contains 5 kg Apples, 10 kg Oranges and 5 kg Pomegranate Basket B contains 10 kg Apples, 10 kg Oranges and 5kg Pomegranate Basket C contains 8 kg Apples, 15 kg Oranges and 7kg Pomegranate Cost of basket A, B and C are respectively ₹2750, ₹3500, and ₹4100.  Based on the given information, answer the following questions: (i) Write the matrix form of the equations involved in this situation and find the adjoint of the coefficient matrix involved. (ii) Using inverse of the matrix, calculate the price of each fruit per kg.	**2** **2**

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MATHEMATICS (041)
MARKING SCHEME
CLASS XII (2026-27)

SECTION: A (Solution of MCQs of 1 Mark each, no part marks)

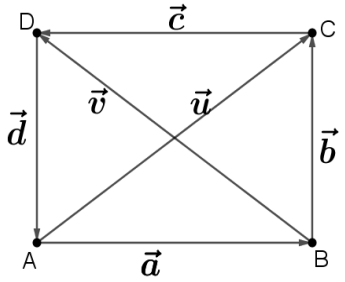
Q. No.	ANS	HINTS/SOLUTION	Marks
1.	**(D)**	$\pi + \cos^{-1} x$	**1**
	(D)	**For Visually Impaired:** $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right] - \{\pi\}$	**1**
2.	**(D)**	$\left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$	**1**
3.	**(A)**	$\cot^{-1}\left(2\cos\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right) = \cot^{-1}\left(2\cos\left(\frac{1}{2}\times\frac{\pi}{3}\right)\right)$ $= \cot^{-1}\left(2\cos\left(\frac{\pi}{6}\right)\right) = \cot^{-1}\left(2\times\frac{\sqrt{3}}{2}\right) = \cot^{-1}(\sqrt{3}) = \frac{\pi}{6}$	**1**
4.	**(B)**	$3a = 1, b = -2, 2c = 4 \Rightarrow \sqrt{6a + 2b + 3c} = \sqrt{2 - 4 + 6} = \sqrt{4} = 2$	**1**
5.	**(C)**	$\lvert adj(PQ)\rvert = \lvert PQ\rvert = \lvert P\rvert\lvert Q\rvert = 2 \times 3 = 6$ ($\because$ order of P and $Q = 2 \times 2$)	**1**
6.	**(D)**	$\begin{bmatrix}1 & x\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}\begin{bmatrix}4\\ -1\end{bmatrix} = O \Rightarrow \begin{bmatrix}1 & 2x\end{bmatrix}\begin{bmatrix}4\\ -1\end{bmatrix} = O \Rightarrow 4 - 2x = 0 \Rightarrow x = 2$	**1**
7.	**(A)**	Let $y = \cos^{-1}(2x^2 - 1)$ and $u = \cos^{-1}(x)$ $\Rightarrow y = \cos^{-1}(2\cos^2 u - 1)$ as $\cos u = x$ $\Rightarrow y = \cos^{-1}(\cos(2u)) = 2u$ $\Rightarrow \frac{dy}{du} = 2$	**1**
8.	**(D)**	$f(x)$ is differentiable $\Rightarrow f(x)$ is continuous $\Rightarrow c = 0$ and $a = 2b$ $\Rightarrow (a, b, c)$ becomes $(2b, b, 0)$ which is satisfied by option (D) $(3, \frac{3}{2}, 0)$ only	**1**
9.	**(D)**	y is decreasing $\Rightarrow \frac{dy}{dx} < 0 \Rightarrow a\{1 + 2\cos\left(\frac{x}{a}\right)\} < 0 \Rightarrow \cos\left(\frac{x}{a}\right) < -\frac{1}{2}$ $\Rightarrow \frac{x}{a} \in \left(\frac{2\pi}{3}, \pi\right] \Rightarrow x \in \left(\frac{2\pi a}{3}, \pi a\right]$	**1**
10.	**(A)**	$\frac{dy}{dx} > y \Rightarrow 4x^3 > x^4 \Rightarrow x^3(4 - x) > 0 \Rightarrow x(x - 4) < 0 \Rightarrow 0 < x < 4$	**1**
11.	**(B)**	The order is 2 and the term containing $\frac{d^2y}{dx^2}$ has the maximum power 2 in the equation, so the degree is 2	**1**
12.	**(D)**	$\int \frac{1}{1+e^{2x}}dx = \int\left(\frac{1+e^{2x}}{1+e^{2x}} - \frac{e^{2x}}{1+e^{2x}}\right)dx = x - \frac{1}{2}log\lvert 1 + e^{2x}\rvert + C$ So $P = \frac{1}{2}$	**1**
13.	**(C)**	Magnitude of $2\hat{\imath} - u\hat{\jmath} + 3\hat{k}$ = 4 $\Rightarrow \sqrt{4 + u^2 + 9} = 4$ $\Rightarrow u^2 = 16 - 13 = 3 \Rightarrow u = \pm\sqrt{3}$	**1**
14.	**(B)**	Direction cosines of the vector $\hat{\imath} + \hat{\jmath} + \sqrt{2}\,\hat{k}$ are $\frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}$ Let angle = γ. Then $\cos\gamma = \frac{1}{\sqrt{2}} \Rightarrow \gamma = \frac{\pi}{4}$	**1**
15.	**(B)**	Both the vectors $\hat{\imath} + \hat{\jmath}$ and $\hat{\imath} - \hat{\jmath}$ are in XY-plane So a vector perpendicular to both is $3\hat{k}$	**1**

16.	**(D)**	The probability that it rains on a particular day is $\frac{1}{5}$. Probability that it rains on three consecutive days = $\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}=\frac{1}{125}$	**1**
17.	**(D)**	Let the events be A: sum of the numbers appeared is a multiple of 5 B: at least one number is 4 $A=\{(1,4),(2,3),(3,2),(4,1),(4,6),(5,5),(6,4)\}$ $A\cap B=\{(1,4),(4,1),(4,6),(6,4)\}$ So required probability = $P(B\|A)=\frac{4}{7}$	**1**
18.	**(A)**	Probability of occurrence of any one of them is $\frac{1}{3}$ $\Rightarrow P(A'\cap B)+P(A\cap B')=\frac{1}{3}\Rightarrow P(A')P(B)+P(A)P(B')=\frac{1}{3}$ $\Rightarrow\left(1-\frac{1}{6}\right)k+\frac{1}{6}(1-k)=\frac{1}{3}\Rightarrow k\left(\frac{5}{6}-\frac{1}{6}\right)=\frac{1}{3}-\frac{1}{6}\Rightarrow k=\frac{1}{6}\times\frac{6}{4}=\frac{1}{4}$	**1**
19.	**(B)**	Both assertion and reason are true, but reason is not the correct explanation of the assertion $\int_{-1}^{1}\|x-2\|\,dx=\int_{-1}^{1}(-x+2)\,dx=4$	**1**
20.	**(A)**	Both assertion and reason are true, and reason is the correct explanation of the assertion.	**1**

Section –B

[This section comprises of solution of very short answer type questions (VSA) of 2 marks each]

21(A).	$I=\int\frac{1}{x\sqrt{x^6-1}}dx=\frac{1}{6}\int\frac{6x^5}{x^6\sqrt{x^6-1}}dx$ Let $x^6-1=t^2 \quad\Rightarrow 6x^5dx=2tdt$ $I=\frac{1}{3}\int\frac{1}{(t^2+1)}dt=\frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)+C$	½ 1+½
	OR	
21(B).	Let $I=\int\frac{\cos x-\sin x}{\sqrt{\sin(2x)}}\,dx=\int\frac{\cos x-\sin x}{\sqrt{1+\sin(2x)-1}}\,dx=\int\frac{\cos x-\sin x}{\sqrt{(sinx+cosx)^2-1}}\,dx$ Let $sinx+cosx=t \quad\Rightarrow(\cos x-\sin x)dx=dt$ $I=\int\frac{1}{\sqrt{t^2-1}}\,dt=log\left\|t+\sqrt{t^2-1}\right\|+C$ $=log\left\|sinx+cosx+\sqrt{(sinx+cosx)^2-1}\right\|+C$ $=log\left\|sinx+cosx+\sqrt{\sin(2x)}\right\|+C$	½ ½ ½ ½
22.	$y=\sin(x+y)\Rightarrow\sin^{-1}y=x+y$ $\Rightarrow\frac{1}{\sqrt{1-y^2}}=\frac{dx}{dy}+1\ \Rightarrow\ \frac{dx}{dy}=\frac{1-\sqrt{1-y^2}}{\sqrt{1-y^2}}$ $\Rightarrow\frac{dy}{dx}=\frac{\sqrt{1-y^2}}{1-\sqrt{1-y^2}}$ So, $\frac{dy}{dx}=0\ \Rightarrow y=\pm1$	**1** **1**

	Alternative way: $y = \sin(x+y) \Rightarrow \frac{dy}{dx} = \cos(x+y)\left(1+\frac{dy}{dx}\right)$ $\Rightarrow \frac{dy}{dx}(1-\cos(x+y)) = \cos(x+y)$ $\Rightarrow \cos(x+y) = 0$ as $\frac{dy}{dx} = 0$ $\Rightarrow \sin(x+y) = \pm 1 \Rightarrow y = \pm 1$	**1** **1**
23(A).	Given points are $A(2,6,5), B(3,p,7), C(-4,3,4)$ and $D(-1,0,q)$ Direction ratios of the line $\overleftrightarrow{AB}$ are $\langle 1,\ p-6,\ 2\rangle$ and Direction ratios of the line $\overleftrightarrow{CD}$ are $\langle 3,-3,q-4\rangle$ As the lines are parallel, so we have $\frac{1}{3} = \frac{p-6}{-3} = \frac{2}{q-4}$ $\Rightarrow -1 = p-6 \ and\ q-4 = 3\times 2$ $\Rightarrow p = -1+6 = 5,\ q = 6+4 = 10$ Thus, $\frac{q}{p} = \frac{10}{5} = 2$	**1** **½** **½**
	OR	
23 (B).	 Let vectors representing the four sides of the rectangle are as follows $\overrightarrow{AB} = \vec{a}, \overrightarrow{BC} = \vec{b}, \overrightarrow{CD} = \vec{c}, and\ \overrightarrow{DA} = \vec{d}$ Then $\overrightarrow{AC} = \vec{u} = \vec{a}+\vec{b}$ …(i)and $\overrightarrow{BD} = \vec{v} = \vec{b}+\vec{c} = \vec{b}-\vec{a}$ …(ii) Solving (i) and (ii), $\vec{a} = \frac{1}{2}(\vec{u}-\vec{v})\ and\ \vec{b} = \frac{1}{2}(\vec{u}+\vec{v})$ Also, $\vec{c} = -\vec{a} = \frac{1}{2}(\vec{v}-\vec{u})$ and $\vec{d} = -\vec{b} = -\frac{1}{2}(\vec{u}+\vec{v})$	**½** **1** **½**
24.	Given $\lvert\vec{a}\rvert = \sqrt{6}$, $\vec{a}\cdot\vec{b} = 1$ and $\vec{b}\times\vec{a} = 3\hat{\imath}+4\hat{\jmath}-2\hat{k}$ So $\lvert\vec{a}\times\vec{b}\rvert = \lvert\vec{b}\times\vec{a}\rvert = \sqrt{29}$ We know, $\lvert\vec{a}\times\vec{b}\rvert^2 + \lvert\vec{a}\cdot\vec{b}\rvert^2 = \lvert\vec{a}\rvert^2\lvert\vec{b}\rvert^2$ $\Rightarrow 29+1 = 6\lvert\vec{b}\rvert^2 \Rightarrow \lvert\vec{b}\rvert^2 = \frac{30}{6} = 5 \Rightarrow \lvert\vec{b}\rvert = \sqrt{5}$	**½** **½** **1**
25.	Constraints are $3x+2y \geq 10, x+2y \leq 6,$ $y \geq 0$	**1½** **½**

	For Visually Impaired: As maximum value exists at (p, q) and $\left(2p, \frac{q}{2}\right)$ So $Z\ at\ (p,q) = Z\ at\left(2p, \frac{q}{2}\right)$ $\Rightarrow 3p + 4q = 3 \times 2p + 4 \times \frac{q}{2}$ $\Rightarrow 3p + 4q = 6p + 2q$ $\Rightarrow 6p - 3p = 4q - 2q$ $\Rightarrow 3p = 2q$	**1** **1**

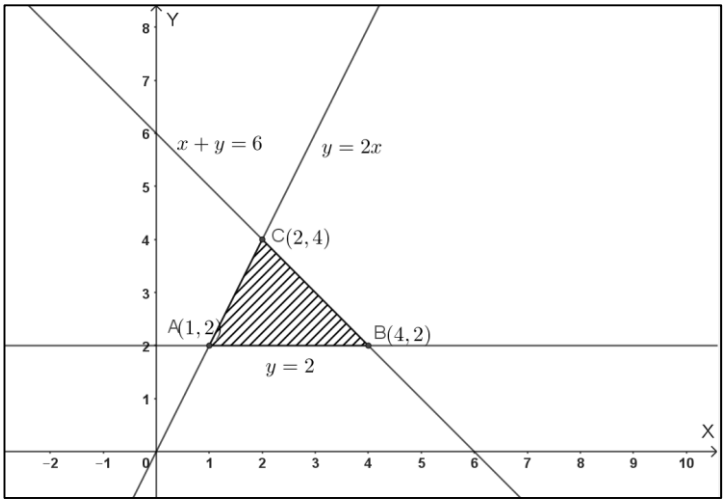
Section –C

[This section comprises of solution of short answer type questions (SA) of 3 marks each]

26(A)	Let $A = PQ$ and $B = QP$. Then $A = PQ = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix}$ $B = QP = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 4 \\ 2 & 0 & 0 \end{bmatrix}$ Here $\lvert A\rvert = 4 \neq 0$ So, A is a non-singular matrix, hence its inverse exists. Now $A^{-1} = \frac{1}{\lvert A\rvert} adj\ A = \frac{1}{4}\begin{bmatrix} 2 & 0 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$ Here $\lvert B\rvert = 0$ So, B is a singular matrix, hence its inverse does not exist.	**1** **1** **½** **½**
27.	Let A be the area of the circular region covered by the drone and let r be its radius. Let h be the height of the drone at any time t. Given $\frac{dA}{dh} = 44\ m^2\ per\ metre$ and $\frac{dr}{dt} = 6\ m/s$ To find $\frac{dh}{dt}$ when r = 14 m. $A = \pi r^2 \quad \Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt} \quad \Rightarrow \frac{dA}{dh}.\frac{dh}{dt} = 2\pi r \frac{dr}{dt}$ $44\ \frac{dh}{dt} = 2(\frac{22}{7})6r \quad \Rightarrow \left.\frac{dh}{dt}\right]_{r=14m} = 12$ m/s The height of the drone increases at the rate of 12 m/s at that moment.	**½** **1 ½** **1**
28(A).	$\frac{dx}{dt} = 3sin^2t\ cos\ t$ and $\frac{dy}{dt} = -3\ cos^2t\ sin\ t$ $\frac{dy}{dx} = \left.\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right. = -cot\ t$ $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right).\frac{dt}{dx} = cosec^2t\left(\frac{1}{3sin^2t\cos t}\right) = \frac{1}{3}(sec\ t.cosec^4t)$ At the point $\left(\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}\right)$, $sin^3t = \frac{1}{2\sqrt{2}} \Rightarrow t = \frac{\pi}{4}$ $\left.\frac{d^2y}{dx^2}\right]_{t=\frac{\pi}{4}} = \frac{4\sqrt{2}}{3}$	**1** **1** **1**

	OR	
28(B).	Let $y = x^{logx} + (logx)^x = u + v$ where $u = x^{logx}$ and $v = (logx)^x$ $log\ u = (log\ x)^2$ and $log\ v = x.log(log\ x)$ $\frac{1}{u}\frac{du}{dx} = 2(\ log\ x).\frac{1}{x}$ and $\frac{1}{v}\frac{dv}{dx} = log(log\ x) + \frac{1}{log\ x}$ $\frac{du}{dx} = x^{logx}\left(2(\ log\ x).\frac{1}{x}\right)$ and $\frac{dv}{dx} = (logx)^x\left(log(log\ x) + \frac{1}{log\ x}\right)$ $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ $\Rightarrow \frac{dy}{dx} = \frac{2}{x}\ x^{logx}\ log\ x + \ (logx)^x\left(log(log\ x) + \frac{1}{log\ x}\right)$ **Alternative Solution:** Since $x = e^{logx}$ $y = x^{logx} + (logx)^x = e^{\log(x^{logx})} + e^{\log((logx)^x)}$ $= e^{(\log x)^2} + e^{x\log(\log x)}$ $\frac{dy}{dx} = e^{(\log x)^2}\left[\frac{2\log x}{x}\right] + e^{x\log(\log x)}[\log(\log x) + x\left(\frac{1}{\log x}\right)\left(\frac{1}{x}\right)]$ $OR\ \ x^{logx}\left[\frac{2\log x}{x}\right] + (logx)^x[\log(\log x) + \left(\frac{1}{\log x}\right)]$	**1** **1** **1** **1** **1+1**
29(A).	$xdy + (y - x^2e^x)dx = 0$ is a linear differential equation given by $\frac{dy}{dx} + \frac{1}{x}\ y = xe^x$ I.F = $e^{\int\frac{1}{x}dx} = \ e^{log\ x} = x$ Solution is $y.x = \int x^2e^x\ dx$ using by parts $yx = x^2e^x - \int 2xe^x\ dx$ using by parts again $yx = x^2e^x - 2\{xe^x - e^x\} + C$ y (1) = 0 $\Rightarrow\ \ C = -e$ Particular solution is $yx = x^2e^x - 2\{xe^x - e^x\} - e = e^x\{x^2 - 2x + 2\} - e$	**½** **½** **½** **½** **½** **½**
	OR	
29(B).	$\frac{dy}{dx} = \frac{xy}{x^2+y^2}$ is a homogeneous differential equation. Let $y = v\ x \qquad \Rightarrow \frac{dy}{dx} = v + x\ \frac{dv}{dx}$ So, equation reduces to	

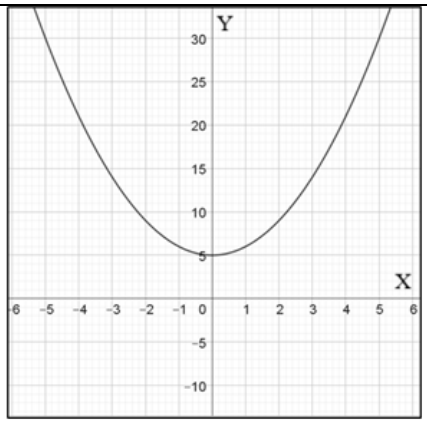
	$v + x\,\frac{dv}{dx} = \frac{v}{1+v^2} \quad \Rightarrow x\,\frac{dv}{dx} = \frac{v-v-v^3}{1+v^2}$	**1**
	$\int \frac{1+v^2}{v^3}dv = -\int \frac{dx}{x}$	
	$\int(\frac{1}{v^3}+\frac{1}{v})\ dv = -log\ \lvert x\rvert + \log C \Rightarrow \frac{-1}{2v^2} + \log\lvert v\rvert = -\log\lvert x\rvert + \log C$	
	$\log\left\lvert\frac{y}{C}\right\rvert = \frac{x^2}{2y^2}$ or $y = C\,e^{\frac{x^2}{2y^2}}$	**1**
	$C = 1$ when $x = 0$ and $y = 1$.	
	The particular solution is $y = e^{\frac{x^2}{2y^2}}$	**1**
30(A).	A point on the line $x - 2 = \frac{2y+3}{4} = 2 - z = k$ *is* $P\left(k+2, \frac{4k-3}{2}, 2-k\right)$	**½**
	And direction ratios of the given line are $\langle 1, 2, -1\rangle$.	
	Direction Ratios of the line joining the points $A(2,1,4)$ and this point P are	
	$\langle k+2-2, \frac{4k-3}{2} - 1, 2-k-4\rangle = \langle k, \frac{4k-5}{2}, -k-2\rangle$	**½**
	As $\overrightarrow{AP}$ is perpendicular to the given line, So	
	$1\times k + 2\times\frac{4k-5}{2} + (-1)\times(-k-2) = 0$	
	$\Rightarrow k + 4k - 5 + k + 2 = 0 \Rightarrow 6k - 3 = 0 \Rightarrow k = \frac{1}{2}$	**½**
	So, foot of the perpendicular is	
	$P\left(k+2, \frac{4k-3}{2}, 2-k\right) = \left(\frac{1}{2}+2, \frac{2-3}{2}, 2-\frac{1}{2}\right) = \left(\frac{5}{2}, \frac{-1}{2}, \frac{3}{2}\right)$	**½**
	Direction ratios of the line $\overleftarrow{AP}$ are $\langle k, \frac{4k-5}{2}, -k-2\rangle = \langle\frac{1}{2}, -\frac{3}{2}, -\frac{5}{2}\rangle$ or $\langle 1, -3, -5\rangle$	**½**
	And equation of $\overleftarrow{AP}$ is	
	$\frac{x-2}{1} = \frac{y-1}{-3} = \frac{z-4}{-5}$ OR $x - 2 = \frac{1-y}{3} = \frac{4-z}{5}$	**½**
	OR	
30(B).	Given lines are $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$ and	
	$\frac{x-5}{2} = y - \frac{1}{2} = \frac{3-z}{2} \Rightarrow \frac{x-5}{2} = \frac{y-\frac{1}{2}}{1} = \frac{z-3}{-2}$	**½**
	Let the angle between the lines be θ, then	
	$\cos\theta = \frac{2\times2+2\times1+1\times(-2)}{\sqrt{9}\times\sqrt{9}} = \frac{4}{9} \Rightarrow \theta = \cos^{-1}\left(\frac{4}{9}\right)$	**1**
	Now required line will be in the direction of the vector	
	$\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 2 & 2 & 1\\ 2 & 1 & -2\end{vmatrix} = -5\hat{\imath} + 6\hat{\jmath} - 2\hat{k}$	**1**
	So required line is $\frac{x-1}{-5} = \frac{y-1}{6} = \frac{z-1}{-2}$ OR $\frac{x-1}{5} = \frac{y-1}{-6} = \frac{z-1}{2}$	**½**

31.	 Feasible region is bounded with three vertices $A(1,2), B(4,2), C(2,4)$ Now $Z_A = 100, Z_B = 220, Z_C = 200$ So, Maximum Value of Z is 220 at the point $B(4,2)$.	**1 ½** **1** **½**
	For Visually Impaired: The vertices of the feasible region are $A(20,0), B(40,0), C(30,10),\ D(0,20)$ and $E(0,10)$. Now $Z_A = 50 \times 20 - 20 \times 0 = 1000$ $Z_B = 50 \times 40 - 20 \times 0 = 2000$ $Z_C = 50 \times 30 - 20 \times 10 = 1300$ $Z_D = 50 \times 0 - 20 \times 20 = -400$ $Z_E = 50 \times 0 - 20 \times 10 = -200$ Minimum value of Z is -400 at $D(0,20)$ Maximum Value of Z is 2000 at $B(40,0)$	**2** **1**

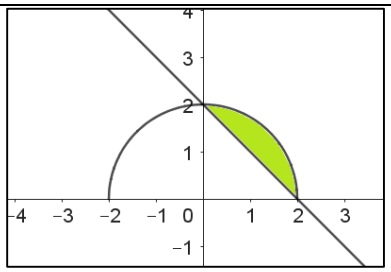
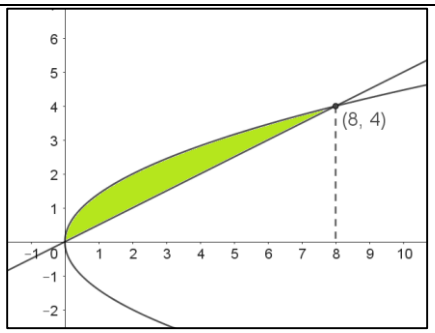
Section –D

[This section comprises of solution of long answer type questions (LA) of 5 marks each]

32(A).	Given relation R is defined on $A = N \times N$ such that $(a,b)R(c,d)\ if\ and\ only\ if\ a^2 + d^2 = b^2 + c^2$ Reflexive: Let $(p,q) \in A = N \times N$ Then as $p, q \in N$, so $p^2 + q^2 = q^2 + p^2$ $\Rightarrow (p,q)R(p,q)\ \forall (p,q) \in N \times N$ $\Rightarrow R$ is a reflexive relation. Symmetric: Let $(p,q)\ and\ (r,s) \in A = N \times N$ such that $(p,q)\ R\ (r,s)$ $\Rightarrow p^2 + s^2 = q^2 + r^2$ $\Rightarrow q^2 + r^2 = p^2 + s^2$ $\Rightarrow r^2 + q^2 = s^2 + p^2$ $\Rightarrow (r,s)\ R\ (p,q)$ Thus, R is a symmetric relation. Transitive:	**1 ½** **1 ½**

	Let $(p,q),(r,s),(u,v) \in A$ such that $(p,q)\ R\ (r,s)\ and\ (r,s)R(u,v)$ $\Rightarrow p^2+s^2=q^2+r^2$ ...(i) and $r^2+v^2=s^2+u^2$ ...(ii) Adding (i) and (ii), $p^2+s^2+r^2+v^2=q^2+r^2+s^2+u^2$ $\Rightarrow p^2+v^2=q^2+u^2$ $\Rightarrow (p,q)\ R\ (u,v)$ Thus, R is a transitive relation So, R is a reflexive, symmetric and transitive relation. Hence R is an equivalence relation	**1 ½** **½**
	OR	
32(B).	 Yes, R is a function as the graph shows every element of the set $(-\infty,\infty)$ has a unique image in the set $[0,\infty)$. Now, let us denote this function by $f:(-\infty,\infty)\rightarrow[0,\infty)$ as $f(x)=x^2+5$ Injective Function: Here $f(-1)=(-1)^2+5=6$ and $f(1)=1^2+5=6$ So $f(-1)=f(1)=6$ Thus f is a many-one function, so not an injective function. Surjective Function: Range of $f(x)=\ [5,\infty) \neq$ co-domain$=[0,\infty)$ So, it is not surjective. **Alternate Method:** Correct graph as above Yes, R is a function as the graph shows every element of the set $(-\infty,\infty)$ has a unique image in the set $[0,\infty)$. Now, let us denote this function by $f:(-\infty,\infty)\rightarrow[0,\infty)$ as $f(x)=x^2+5$ Injective Function: Let $a,b \in (-\infty,\infty)$, such that $f(a)=f(b)$ $\Rightarrow a^2+5=b^2+5 \Rightarrow a^2=b^2 \Rightarrow a=\pm b$ So f is not injective. Surjective Function: For $y=1$ there is no pre-image as if $1=x^2+5 \Rightarrow x^2=-4$ not possible. So, it is not surjective. **For Visually Impaired:** For every element a of the set $(-\infty,\infty)$, $a^2 \in [0,\infty)$ and therefore $a^2+7 \in [7,\infty)$ Hence every element of the set $(-\infty,\infty)$ has a unique image in the set $[0,\infty)$.	**1 for correct graph** **1** **1 ½** **1 ½** **1** **1** **1 ½** **1 ½**

	So, $f:(-\infty,\infty)\to[0,\infty)$, $f(x)=x^2+7$ is a function. Now, let us denote this function by $f:(-\infty,\infty)\to[0,\infty)$ as $f(x)=x^2+7$ Injective Function: Here $f(-1)=(-1)^2+7=8$ and $f(1)=1^2+7=8$ So $f(-1)=f(1)=8$ Thus f is a many-one function, so not an injective function. Surjective Function: Range of $f(x)=\ [7,\infty)\neq$ co-domain$=[0,\infty)$ So, it is not surjective. Thus $f(x)$ is neither injective nor surjective and hence not bijective	**1** **1 ½** **1 ½** **1**
33.	Let $I=\int_{-a}^{a}\sqrt{\frac{a-x}{a+x}}\,dx$ $I=\int_{-a}^{a}\sqrt{\frac{a-x}{a+x}\times\frac{a-x}{a-x}}\,dx$ $=\int_{-a}^{a}\frac{a-x}{\sqrt{a^2-x^2}}dx=\int_{-a}^{a}\frac{a}{\sqrt{a^2-x^2}}dx-\int_{-a}^{a}\frac{x}{\sqrt{a^2-x^2}}dx=I_1-I_2$ $I_1=a\int_{-a}^{a}\frac{1}{\sqrt{a^2-x^2}}dx$ $=2a\int_{0}^{a}\frac{1}{\sqrt{a^2-x^2}}dx$ using $\int_{-a}^{a}f(x)\,dx=2\int_{0}^{a}f(x)\,dx$ if $f(-x)=f(x)$ $I_1=2a\ \left[\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a=2a\left(\frac{\pi}{2}\right)=\pi a$ and $I_2=\int_{-a}^{a}\frac{x}{\sqrt{a^2-x^2}}dx=0$ using $\int_{-a}^{a}f(x)\,dx=0$ if $f(-x)=-f(x)$ $I=\ I_1-I_2=\pi a$ **Alternate Method:** Let $x=a\cos 2\theta\ \Rightarrow dx=-2\,a\sin(2\,\theta)\,d\theta$ When $x=-a,\theta=\frac{\pi}{2}$ and $x=a,\theta=0$ $I=\int_{-a}^{a}\sqrt{\frac{a-x}{a+x}}\,dx\ =-2\,a\int_{\frac{\pi}{2}}^{0}\tan\theta\sin(2\theta)\,d\theta$ $I=\ 2\,a\int_{0}^{\frac{\pi}{2}}\tan\theta\sin(2\theta)\,d\theta$ .....(i) using $\int_a^b f(x)dx=-\int_b^a f(x)dx$ $I=\ 2a\int_{0}^{\frac{\pi}{2}}\tan\left(\frac{\pi}{2}-\theta\right)\sin(\pi-2\,\theta)d\theta$ using $\int_0^a f(x)dx=\int_0^a f(a-x)dx$ $I=\ 2a\int_{0}^{\frac{\pi}{2}}\cot(\theta)\sin(2\theta)\,d\theta$ .....(ii) Adding equations (i) and (ii) $2I=2a\int_{0}^{\frac{\pi}{2}}(\tan\theta+\cot\theta)\sin(2\theta)\,d\theta$ $\Rightarrow I=\ 2a\int_{0}^{\frac{\pi}{2}}(sin^2\theta+cos^2\theta)\,d\theta$ $\Rightarrow I=2a\int_{0}^{\frac{\pi}{2}}1\,d\theta=2a\left(\frac{\pi}{2}\right)=\pi a$	 **½** **1** **1** **1** **½** **½** **½** **1** **1** **1** **1**

34(A).	 1 mark for correct figure $y = \sqrt{4 - x^2}$ is a semi-circle with radius 2 units and centre (0,0) $x = 2 - y$ which is x + y = 2 is a line with intercepts 2 on each axis. Curves meet at (0,2) and (2,0) Area = $\int_0^2 \left(\sqrt{4 - x^2} - (2 - x)\right) dx$ $= \left[\frac{x}{2}\sqrt{4 - x^2} + 2 \sin^{-1}\left(\frac{x}{2}\right) - 2x + \frac{x^2}{2}\right]_0^2$ $= \pi - 4 + 2 = (\pi - 2)$ sq. units	**1** **1** **1** **1** **1**
	OR	
34(B).	 1 mark for correct figure. $y^2 = 2x$ represents a parabola, whose points of intersection with the given line $x = 2y$ are $(0,0)$, $(8,4)$ Area = $\int_0^8 \left(\sqrt{2x} - \frac{x}{2}\right) dx$ $= \left[\sqrt{2}\frac{2}{3}x^{\frac{3}{2}} - \frac{x^2}{4}\right]_0^8$ $= \frac{64}{3} - 16 = \frac{16}{3}$ sq. units	**1** **1** **1** **1** **1**
34(A).	**For Visually Impaired:** $25x^2 + 16y^2 = 400$ is an Ellipse with the standard equation as $\frac{x^2}{16} + \frac{y^2}{25} = 1$ $\Rightarrow y = \frac{5}{4}\sqrt{4^2 - x^2}$ Area = $4\int_0^4 \left(\frac{5}{4}\sqrt{4^2 - x^2}\right) dx$ $= 5\int_0^4 \left(\sqrt{4^2 - x^2}\right) dx$ $= 5\left[\frac{x}{2}\sqrt{16 - x^2} + 8 \sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$ $= 5[4(\pi)] = 20\pi$ sq. units	**½** **1** **1** **1½** **1**
	OR	

34(B).	$y^2 = 8x$ represents a parabola, and the area of the region above the x-axis, below the parabola, between the lines $x = 2$ and $x = 5$ is given by Area = $\int_2^5 (\sqrt{8x})\ dx$ $= \left[2\sqrt{2}\frac{2}{3}x^{\frac{3}{2}}\right]_2^5$ $= \sqrt{2}\left(\frac{4}{3}\right)[5\sqrt{5} - 2\sqrt{2}] = \frac{4}{3}[5\sqrt{10} - 4]$ sq. units	½ **2** **1 ½** **1**
35.	Let the events be E_1 : User uses Alpha smartwatch E_2 : User uses Beta smartwatch E_3 : User uses Gamma smartwatch E_4 : User uses Delta smartwatch A : User gets a false heart-rate alert Clearly $\{E_1, E_2, E_3, E_4\}$ is a partition of the sample space. Then $P(E_1)$=0.45, $P(E_2) = 0.3, P(E_3) = 0.15, P(E_4) = 0.1$ Now $P(A\|E_1) = 0.01,\ P(A\|E_2) = 0.02, P(A\|E_3) = 0.05, P(A\|E_4) = 0.01$ (i) By Theorem of Total Probability, The total probability that a randomly selected user gets a false alert $= P(A)$ $= P(E_1)P(A\|E_1) + P(E_2)P(A\|E_2) + P(E_3)P(A\|E_3) + P(E_4)P(A\|E_4)$ $= 0.45 \times 0.01 + 0.3 \times 0.02 + 0.15 \times 0.05 + 0.1 \times 0.01$ $= 0.0045 + 0.006 + 0.0075 + 0.001 = 0.019$ (ii) By Bayes Theorem, The probability that false alert came from a Beta smartwatch $= P(E_2\|A)$ $= \frac{P(E_2)P(A\|E_2)}{P(E_1)P(A\|E_1)+P(E_2)P(A\|E_2)+P(E_3)P(A\|E_3)+P(E_4)P(A\|E_4)}$ $= \frac{0.3\times0.02}{0.019} = \frac{0.006}{0.019} = \frac{6}{19}$	**1** **1** **1 ½** **1 ½**

Section –E

[This section comprises solution of 3 case- study/passage-based questions of 4 marks]

36.	(i) Given lines are $l_1: \vec{r} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} + \lambda(\hat{\imath} + 3\hat{\jmath} - \hat{k}) \equiv \vec{a} + \lambda\,\vec{b}$ and $l_2: \vec{r} = 2\hat{\imath} - \hat{\jmath} + \hat{k} + \mu(2\hat{\imath} + 3\hat{\jmath} + \hat{k}) \equiv \vec{c} + \mu\,\vec{d}$ Here $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k},\ \vec{b} = \hat{\imath} + 3\hat{\jmath} - \hat{k},\ \vec{c} = 2\hat{\imath} - \hat{\jmath} + \hat{k},\ \vec{d} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ For direction vectors $\vec{b}$ and $\vec{d}$, $\vec{b} \neq \lambda\,\vec{d}$, for any $\lambda \in R$ $\Rightarrow$ $\vec{b}$ is not parallel to $\vec{d}$ So, lines are not parallel. (ii) Given lines are $l_1: \vec{r} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} + \lambda(\hat{\imath} + 3\hat{\jmath} - \hat{k}) \equiv \vec{a} + \lambda\,\vec{b}$ and $l_3: \vec{r} = 3\hat{\imath} - \hat{\jmath} + \hat{k} + \gamma(\hat{\imath} + 3\hat{\jmath} - \hat{k}) \equiv \vec{e} + \gamma\,\vec{f}$	**1** **1**

	Since $\vec{b} = \vec{f}$, the lines l_1 and l_3 are parallel.	
	(iii) (a) $\vec{c} - \vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k} - (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) = \hat{\imath} - 3\hat{\jmath} - 2\hat{k}$ $\vec{b} \times \vec{d} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 3 & -1 \\ 2 & 3 & 1 \end{vmatrix} = 6\hat{\imath} - 3\hat{\jmath} - 3\hat{k} = 3(2\hat{\imath} - \hat{\jmath} - \hat{k})$ $(\vec{c} - \vec{a}).(\vec{b} \times \vec{d}) = (\hat{\imath} - 3\hat{\jmath} - 2\hat{k}).3(2\hat{\imath} - \hat{\jmath} - \hat{k}) = 3(2 + 3 + 2) = 21 \neq 0$ So, the lines are skew lines. $\lvert\vec{b} \times \vec{d}\rvert = 3\sqrt{2^2 + (-1)^2 + (-1)^2} = 3\sqrt{6}$ Thus, the shortest distance between the given lines is $D = \left\lvert \frac{(\vec{c} - \vec{a}).(\vec{b} \times \vec{d})}{\lvert\vec{b} \times \vec{d}\rvert} \right\rvert = \left\lvert \frac{21}{3\sqrt{6}} \right\rvert = \frac{7}{\sqrt{6}} = \frac{7\sqrt{6}}{6}\ units$	**1** **1**
	OR (b) $l_1 : \vec{r} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} + \lambda(\hat{\imath} + 3\hat{\jmath} - \hat{k}) \equiv \vec{a} + \lambda\,\vec{b}$ and $l_3 : \vec{r} = 3\hat{\imath} - \hat{\jmath} + \hat{k} + \gamma(\hat{\imath} + 3\hat{\jmath} - \hat{k}) \equiv \vec{e} + \gamma\,\vec{b}$ Since the lines are parallel, the shortest distance = $\frac{\lvert\vec{b} \times (\vec{e} - \vec{a})\rvert}{\lvert\vec{b}\rvert}$ $= \frac{1}{\sqrt{11}} \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 3 & -1 \\ 2 & -3 & -2 \end{vmatrix} = \frac{\lvert -9\hat{\imath} - 9\hat{k}\rvert}{\sqrt{11}} = \frac{9\sqrt{22}}{11} units$	**1** **1**
37.	(i) $W(x) = 35 + 1.6x - 0.02x^2$ $W'(x) = 1.6 - 0.04x = 0 \Rightarrow x = 40$ Therefore, the value of x is 40 crores at the stationary point of $W(x)$.	**1**
	(ii) $W''(x) = -0.04 < 0 \Rightarrow W(x)$ is maximum at $x = 40$ by the second derivative test. Thus, the WQI is maximum when ₹ 40 crores are spent on the revival. Max. WQI = $W(40) = 35 + 64 - 32 = 67$	**½** **½**
	(iii) (a) If the investment is ₹35 crores, then the WQI = $W(35)$ $W(35) = 35 + (1.6)(35) - (0.02)(35)^2$ $= (2.6)(35) - (0.02)(1225)$ $= 91 - 24.5 = 66.5$ Also, 67-66.5 = 0.5 So, the WQI is short by 0.5 units of its maximum level. **OR** (b) $W(60) = 35 + 96 - 72 = 59$ 59 is less than the maximum value 67 of WQI, even when the investment has been increased from 40 crores to 60 crores. This shows that the returns are diminishing.	**1** **1** **1** **1**

38.	(i) Taking price of each kg of apples, oranges and pomegranate as ₹x, ₹y, ₹z respectively. Matrix form is $AX = B$, where $A = \begin{pmatrix} 5 & 10 & 5 \\ 10 & 10 & 5 \\ 8 & 15 & 7 \end{pmatrix}, X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, B = \begin{pmatrix} 2750 \\ 3500 \\ 4100 \end{pmatrix}$ Here $\lvert A \rvert = 25 \neq 0$ So, the system is consistent and has unique solution Adjoint of the matrix A = adjA = $\begin{pmatrix} -5 & 5 & 0 \\ -30 & -5 & 25 \\ 70 & 5 & -50 \end{pmatrix}$ (ii) Unique solution of the system is given by $X = A^{-1}B = \frac{1}{25}\begin{pmatrix} -5 & 5 & 0 \\ -30 & -5 & 25 \\ 70 & 5 & -50 \end{pmatrix}\begin{pmatrix} 2750 \\ 3500 \\ 4100 \end{pmatrix}$ $\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 150 \\ 100 \\ 200 \end{pmatrix} \Rightarrow x = 150, y = 100, z = 200$ So the price of Apples, Oranges and Pomegranate are respectively ₹150 per kg, ₹100 per kg, ₹200 per kg.	**1** **1** **1** **1**